# IS YOUR RELIEF VALVE SIZING METHOD TRULY RIGOROUS? 

Ugur Guner, PhD, P.E.<br>Bryan Research \& Engineering, Inc.

BRYAN, TX, USA


#### Abstract

Pressure relief valves are devices that protect equipment from excessive overpressure. In case of emergency situations, they should ensure a sufficient discharge of mass to reduce the pressure below the recommended pressure limits. The purpose of relief sizing is to determine the required area that can hold the required mass discharge from the valve under different overpressure scenarios. The discharged mass can be vapor, liquid, supercritical fluids or two-phase fluids. This article focuses on relief valve sizing methods for vapor phase and supercritical fluids at choked flow. The American Petroleum Institute (API Standard 520 ${ }^{1,2}$ ) recommends basing vapor-phase sizing methods on an ideal gas flow assumption. ${ }^{1}$ This assumption has been addressed by several groups in the industry and may lead to high levels of deviation in cases of near critical and supercritical fluids ${ }^{3,4}$. In the most recent version of API 520, a rigorous approach for calculating mass flux through the valves is introduced in addition to the existing, ideal gas based models. ${ }^{2}$ Furthermore, API Standard 520 suggests using a real gas isentropic coefficient calculation method as an alternative to the ideal gas specific heat ratio for sizing relief valves. This paper will compare different vapor and supercritical vapor vent sizing approaches and compare their performance against a rigorous model. The rigorous model performs many isentropic flashes using appropriate thermodynamic packages offered in ProMax ${ }^{7}$. The performance of each method is assessed through both pure component and mixture examples. In ProMax, a rigorous sizing method can be conveniently applied along with other alternative methods for API 520 sizing calculations ${ }^{7}$.


## INTRODUCTION

Pressure relief valves (PRV) are the primary means of excessive overpressure protection. A pressure relief device is designed to open, relieve excessive pressure, reclose, and prevent further flow of fluid after normal conditions have been restored. PRVs consist of an inlet nozzle connected to the vessel, a movable disc that controls the flow through the nozzle, and a spring that controls the position of the disc. The operation of conventional spring loaded PRVs is based on a force balance. The spring-load is preset to apply a force that is opposite in amount to the pressure force exerted by the fluid on the other side when it is at the set pressure. When the pressure at the inlet of the valve is below the set pressure of the valve, the disc is seated on the nozzle to prevent flow through the nozzle.

The purpose of relief valve sizing is to determine the proper discharge area of the relief device and diameter of the associated inlet and outlet piping. Although an orifice is commonly used to describe the minimum flow area constricted in the valve, the geometry resembles a nozzle and area is determined by applying the equation for flow in an isentropic nozzle. ${ }^{5,6}$ The required orifice area for a relief valve is;

$$
\begin{equation*}
A=\frac{\dot{m}}{K_{D} * G_{n}} \tag{1}
\end{equation*}
$$

,where $A$ is the area of the valve, $\dot{m}$ is the mass flow rate through the valve, $G_{n}$ is the mass flux and $\mathrm{K}_{\mathrm{D}}$ is the discharge coefficient.

The value of mass flow rate, $\dot{m}$ is determined by energy and mass balances on the vessel under the conditions of a specific relief scenario: a run-away reaction, an external fire, loss of cooling, thermal expansion of a liquid, control valve failure, etc. Calculation of mass flow rate discharged through the relief system is not within the scope of this paper.

The mass flux, $G_{n}$ is calculated from either an appropriate theoretical model or numerically. The discharge coefficient $K_{D}$ accounts for the difference between the predicted ideal nozzle and the actual mass flux in the valve. It is available from the valve vendors.

It is important that the relief area be neither too large nor too small. An undersized valve would not provide the required overpressure protection, whereas an oversized valve will result in excessive flow. This can adversely affect the opening and closing characteristics of the relief valve, resulting in possible damage to the valve. Unexpected high flow due to oversizing also results in undersized discharge piping and effluent handling systems downstream of the valve. In addition, the cost of an oversized relief valve will be higher. Over predicted mass flux leads to an undersized valve, while under predicted mass flux results in an oversized valve. Hence, it is crucial to calculate mass flux correctly. In the next section, we will investigate theoretical models for mass flux through relief valves.

## METHODS

Mass flux in PRVs is modeled using an isentropic nozzle equation. The expression for the mass flux ( $G$ ) in an ideal (isentropic) nozzle is obtained directly from an energy balance in the nozzle ${ }^{2,5,6}$.

$$
\begin{equation*}
G=\rho_{n} u_{n}=\rho_{n}\left(-2 \int_{P_{1}}^{P_{n}} \frac{d P}{\rho}\right) \tag{2}
\end{equation*}
$$

where $P_{1}$ is the pressure at the valve entrance, $P$ is the fluid pressure, $P_{n}$ is the downstream pressure (pressure at the nozzle exit or nozzle throat), $\rho_{n}$ is the density at the nozzle exit (throat), and $u_{n}$ is the velocity at the exit.

When a compressible fluid moves from a high-pressure upstream condition to a low downstream pressure across a nozzle, orifice, or a pipe, it expands. As a result, its density decreases and velocity increases. For a given inlet condition and with decreasing downstream pressure, the mass flux in the nozzle increases due to the expansion and flow area reduction until a limiting velocity is reached in the nozzle. This is called choked or critical flow. The limiting velocity is the sonic velocity of the fluid at the throat condition. The mass flux that corresponds to the sonic velocity is known as the critical mass flux. The pressure at which critical mass flux occurs is called the critical flow pressure. When the downstream pressure is lower than the critical flow pressure, mass flux will remain constant at the maximum value.

In order to solve Eq. 2 analytically, a relationship between pressure and density (or specific volume) is needed. For vapors and gases with a constant isentropic expansion coefficient, the expression for the pressure and specific volume relationship along an isentropic path can be shown as ${ }^{2,8}$;

$$
\begin{equation*}
P v^{n}=P_{1} v_{1}^{n} \tag{3}
\end{equation*}
$$

where $P_{1}$ is the pressure at the inlet, $v_{1}$ is the specific volume at the inlet, and $n$ is the isentropic expansion coefficient. $P$ and $v$ are the pressure and specific volume within the isentropic path. The major assumptions in the derivation of Eq. 3 are that the gas follows an isentropic path and the isentropic coefficient is constant along this path.

Combining Eq. 2 and 3 and the definition of sonic velocity, the mass flux relation for choked flow can be obtained as;

$$
\begin{equation*}
G=\sqrt{n P_{1} \rho_{1}} \sqrt{\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \tag{4}
\end{equation*}
$$

In this equation, $P_{1}$ and $\rho_{1}$ are the inlet gas pressure and density, $n$ is the isentropic expansion coefficient. The expansion coefficient is assumed to be constant along the isentropic nozzle path.

For gases, the density at the valve inlet can be written as ${ }^{8}$;

$$
\begin{equation*}
\rho_{1}=\frac{Z P_{1} M_{w}}{R T_{1}} \tag{5}
\end{equation*}
$$

where $Z$ is the compressibility, $M_{w}$ is the molecular weight, $T_{1}$ is the temperature at the inlet, $P_{1}$ is the pressure at the inlet, and $R$ is the gas constant.

## Calculating Isentropic Expansion Coefficients

## Ideal gas Specific Heat Ratio

In the derivation of relief valve sizing equations, the common assumption is that the gas behaves ideally when following the isentropic path between upstream and downstream valve conditions. However, the inlet gas density is calculated using the real gas equation (Eq. 5). An ideal gas flow assumption leads to the following expression for the critical mass flux

$$
\begin{equation*}
G=\sqrt{\kappa P_{1} \rho_{1}} \sqrt{\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}} \tag{6}
\end{equation*}
$$

where $\kappa$ is the ideal gas isentropic coefficient and

$$
\begin{equation*}
\kappa=\frac{C_{p}^{o}}{C_{v}^{o}} \tag{7}
\end{equation*}
$$

It can be seen from Eq. 7 that the ideal gas isentropic coefficient is simply equal to the ideal gas specific heat ratio. Similar to Eq. 4, the derivation of mass flux in Eq. 6 is based on the assumption that the ideal gas specific heat ratio is constant along the isentropic path. Therefore, the ideal gas specific heat ratio at the valve inlet conditions can be used.

## Real Gas Specific Heat Ratio

API 520 stresses that most simulators will provide the real gas specific heat ratio and that using the real gas specific heat ratio may lead to undersized valves ${ }^{2}$. ProMax reports both ideal gas and real gas specific heat ratios. To test the impact of using real gas specific heat ratio instead of ideal gas specific heat ratio on PRV sizing, the critical mass flux based on the real gas specific heat ratio can be written as;

$$
\begin{equation*}
G=\sqrt{n^{*} P_{1} \rho_{1}} \sqrt{\left(\frac{2}{n^{*}+1}\right)^{\frac{n^{*}+1}{n^{*}-1}}} \tag{8}
\end{equation*}
$$

Eq. 8 is simply obtained by replacing $\kappa$ in Eq. 6 with $n^{*}$. The term $n^{*}$ is the real gas specific heat ratio and is shown as

$$
n^{*}=\frac{C_{p}}{C_{v}}
$$

where $C_{p}$ and $C_{v}$ are the real gas specific heat at constant pressure and volume, respectively.

## Real Gas Isentropic Coefficient

Another approach for calculating mass flux is to calculate the isentropic expansion coefficient for real gases. The analytical derivation for the real gas isentropic coefficient requires a valid thermodynamic model that describes the pressure-volume relationship. The analytical derivation can get complicated and the coefficient may change along the isentropic path. In the event of a constant isentropic coefficient, an expression for the isentropic coefficient can be derived as ${ }^{2,4}$;

$$
\begin{equation*}
n=-\frac{v}{P}\left(\frac{\partial P}{\partial v}\right)_{T} \frac{C_{p}}{C_{v}} \tag{10}
\end{equation*}
$$

Since the isentropic coefficient is assumed constant along the isentropic path, expression in Eq. 10 can be calculated at the valve inlet conditions. However, it is still a complex task to get an analytical expression for the derivative term at the right hand side of Eq. 10. It can be calculated numerically as;

$$
\begin{equation*}
n \approx-\frac{v_{1}}{P_{1}}\left(\frac{\Delta P}{\Delta v}\right)_{T} \frac{C_{p}}{C_{v}} \tag{11}
\end{equation*}
$$

where $\Delta P$ is the change in pressure and $\Delta v$ is the change in specific volume.
The derivative $(\Delta P / \Delta v)_{T}$ is calculated at inlet conditions using ProMax ${ }^{7}$. The gas at the inlet pressure (relief pressure) $P_{1}$ is expanded to a pressure; $P_{1}-\Delta P$. The change in pressure $\Delta P$ is set to a small value (10 psi used in calculations). The temperature at the outlet of the expansion process is set equal to the inlet temperature (relief temperature), $T_{1}$. Upon choosing an appropriate thermodynamic package in ProMax, flashes are performed both at the inlet $\left(P_{1}, T_{1}\right)$ and outlet conditions $\left(\left(P_{1}-\Delta P\right), T_{1}\right)$ to obtain a specific volume change $\Delta v$. The real gas specific heat ratio, $C_{p} / C_{v}$, is calculated at the inlet flash. The calculated real gas isentropic coefficient can be plugged into Eq. 4 to obtain critical mass flux.

## Direct Integration Method

The energy balance for an isentropic nozzle path that is given in Eq. 2 is valid irrespective of non-ideality or compressibility for any homogenous fluid. Therefore, it can be taken as the reference model for calculating valve mass flux. This general expression can be used when the pressure-density relationship at constant entropy is available. This relationship is not available analytically for most thermodynamic models. However, the integral in Eq. 2 can be evaluated numerically by direct summation over small pressure intervals. Eq. 2 can be discretized as;

$$
\begin{equation*}
G \approx \rho_{n}\left(-2 \sum_{P_{1}}^{P_{n}} \frac{P^{(i+1)}-P^{(i)}}{\bar{\rho}_{i}}\right)^{1 / 2} \tag{12}
\end{equation*}
$$

where $P^{(i+1)}$ and $P^{(i)}$ are the consecutive pressures at a pressure increment, $\bar{\rho}_{i}$ is the average density in the isentropic path between pressures: $P^{(i)}$ and $P^{(i+1)}, \rho_{n}$ is the density at the downstream conditions, $P_{n}$ is the downstream pressure. The pressure domain between valve upstream pressure $P_{1}$ and valve downstream pressure $P_{2}$ is divided into $m$ pressure steps. The step size is

$$
\begin{equation*}
\Delta P^{(i)}=P^{(i)}-P^{(i+1)} \tag{13}
\end{equation*}
$$

for $i=0, . ., m$, where

$$
\begin{equation*}
P^{(0)}=P_{1} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
P^{(m)}=P_{n} \tag{15}
\end{equation*}
$$

The step size, $s$, in each increment is equal to a fraction of the pressure at that step.

$$
\begin{equation*}
\Delta P^{(i)}=s P^{(i)} \tag{16}
\end{equation*}
$$

In this equation, $s$ is set to 0.05 . Average density is taken as the arithmetic average of the two densities.

$$
\begin{equation*}
\bar{\rho}_{i}=\frac{\rho_{i}+\rho_{i+1}}{2} \tag{17}
\end{equation*}
$$

These calculations are performed in ProMax. First, a valid thermodynamic package for the system is chosen. Then, fluid is flashed at each pressure increment and inlet entropy using the chosen thermodynamic environment. Pressure, average density and outlet density resulting from the flash calculations at each increment are plugged into Eq. 12 and summed over all $m$ steps to evaluate integration. The only error associated with this technique is related to the numerical error that is introduced due to the discretization. Error can be reduced by choosing smaller step size (by decreasing the parameter, $s$ ) at the expense of increased computation load.

## COMPARISON

In the previous section, different theoretical models for evaluating mass flux for relief valves are presented. Direct integration is the most general, rigorous and accurate method, as it deals with real gasses and does not require an assumption of a constant isentropic coefficient. Hence, it can be used as a reference model for mass flux calculations. The other models are benchmarked against the direct integration method in different case studies to assess validity of assumptions in these models.

The methods are compared based on the percent deviation of the calculated mass flux from the rigorous direct integration method mass flux calculation for different systems at various inlet reduced pressures.

Inlet reduced pressure is the ratio of inlet pressure to the critical pressure of the gas. Critical pressure is the pressure at the thermodynamic critical point of the substances above which substances cannot be liquefied. Critical pressure should not be confused with the critical flow pressure that defines the downstream pressure at which flow becomes choked. Inlet reduced critical flow ratio is defined as;

$$
\begin{equation*}
r=\frac{P_{1}}{P_{c}} \tag{18}
\end{equation*}
$$

where $P_{c}$ is the critical pressure determined using ProMax.
Percent deviation of the mass flux is defined as

$$
\begin{equation*}
e=100 \times \frac{G-G^{*}}{G^{*}} \tag{19}
\end{equation*}
$$

where $e$ is the percent mass flux deviation of a method from the direct integration method. The term $G^{*}$ is the mass flux calculated using the direct integration method, and $G$ is the mass flux calculation in question.

The mass flux calculation methods have been compared with the direct integration method for the following three cases at different inlet reduced pressures. All thermophysical properties are calculated using the PengRobison equation of state in ProMax. ${ }^{7}$

Case Study \#1: Air 250K: This case involves air venting at 250 K at different inlet reduced pressures. Fig. 1 shows the mass flux deviations for all three methods. Mass flux that is calculated using the ideal gas isentropic coefficient (Eq. 6) under predicts the mass flux at all reduced pressures. This leads to an oversized valve (Eq. 1). The results are satisfactory for low pressures, however, the deviation increases to 15 percent at very high pressures. Using the real gas specific heat ratio (Eq. 8) over predicts the mass flux, hence, under sizes the valve. This is in accordance with API's comment on the possibility of under sizing valves when real gas specific heat ratio is used. At very high pressures, however, this method under predicts the flux and gives lower percent deviation than the ideal gas specific heat ratio method. Mass flux calculation using the real gas isentropic coefficient outperforms other methods and results in deviation within $2 \%$ for all inlet reduced pressures.


Figure 1. Percent mass flux deviation from the direct integration method for air at 250 K for different reduced pressures (Case 1)

Case Study \#2: Saturated n-Butane: This case evaluates the discharge of saturated $n$-butane at five different inlet reduced pressures from 0.2 to 0.9. The results are plotted in Figure 2. The ideal gas specific heat ratio method over predicts the flux, which results in an undersized valve. The degree of over prediction increases up to 20 percent as the thermodynamic critical point is approached. This is opposite to the trend observed in the first case study for this method.

The real gas specific heat ratio method also over predicts the mass flux and under sizes the valve. The deviation for this method increases to over $50 \%$ with increasing inlet reduced pressure. The real gas isentropic coefficient method, on the other hand, slightly under predicts the mass flux. The mass flux deviation remains within $-5 \%$ at all inlet reduced pressures.


Figure 2. Percent mass flux deviation from the direct integration method for saturated n-Butane (Case 2)

Case Study \#3: Hydrocarbon Mixture @ 200 F: This case study considers a gas mixture that consists of 90 mole $\% \mathrm{C} 1,5 \mathrm{~mole} \% \mathrm{C} 2,3 \mathrm{~mole} \% \mathrm{C} 3$ and 2 mole\% C4. Figure 3 displays the change in mass flux deviation for this mixture at 200 F with reduced pressure. For all inlet reduced pressures, the ideal gas specific heat ratio method slightly under predicts the flux (over sizing the valve) with the error increasing as reduced pressure increases. The real gas specific heat ratio method over predicts the mass flux by up to $3 \%$ from reduced pressures from 0.2-2.7. This trend is reversed at higher pressure. The ideal gas method under predicts the flux up to $18 \%$. The deviation for the real gas specific heat ratio method remains less compared to the ideal gas case at high pressures. The real gas isentropic coefficient method, on the other hand, slightly under predicts at low pressures (inlet reduced pressures from 0.2 to 1 ). It over predicts the flux at the supercritical region and errors remain within the $3 \%$ range.


Figure 3. Percent mass flux deviation from the direct integration method for vapor mixture at 200F (Case 3)

## CONCLUSIONS

This paper examines different mass flux calculation methods for relief valve sizing. These are the Ideal Gas Specific Heat Ratio, Real Gas Specific Heat Ratio, and Real Gas Isentropic coefficient method. All methods are benchmarked against the rigorous direct integration method using percent mass flux deviation criteria. The results show that using the ideal gas isentropic coefficient can lead to errors up to $20 \%$. This method under predicts the mass flux for most cases, but also can over predict the flux as in the saturated n-butane case. The real gas specific heat ratio method follows a similar trend but remains slightly over the ideal gas specific heat ratio curve. The Real Gas Isentropic Coefficient method compared closely and consistently, against the benchmark direct integration method in most cases with errors generally below $5 \%$.

The results reveal that of the assumptions, the most consistent as compared to direct integration was the use of real gas isentropic coefficients. Still, with modern simulators capable of rigorously calculating the integral for mass flux, the best choice may be to avoid these assumptions all together, and instead simply use the direct integration method for calculating mass flux.

## NOMENCLATURE


$\mathrm{m}^{\wedge} 2$
$\mathrm{~J} /$ mole-K
$\mathrm{J} /$ mole-K
$\mathrm{J} /$ mole-K
$\mathrm{J} /$ mole-K
$\%$
$\mathrm{kg} / \mathrm{m}^{\wedge} 2-\mathrm{s}$
Mass Flux at the valve throat or exit increment counter used for summation purposes

Discharge coefficient of the valve
Molecular weight of the gas
Mass flow rate through the valve
Real gas isentropic coefficient
Real gas specific heat ratio
Pressure at the valve inlet
Pressure at the valve exit
Pressure
Gas constant
Inlet reduced pressure
Fraction of pressure at each increment
Temperature of the gas at the valve inlet
Velocity at the valve throat or exit
Specific volume of the gas at an arbitrary point in the valve
Specific volume of the gas at the valve inlet
Compressibility of the gas
Ideal Gas isentropic coefficient or ideal gas specific heat ratio

| $\Delta P$ | Pa | Differential pressure used in real gas isentropic coefficient <br> calculation |
| :--- | :--- | :--- |
| $\Delta P^{(i)}$ | Pa | Differential pressure used in direct integration method |
| $\Delta v$ | $\mathrm{~m}^{\wedge} 3 / \mathrm{kg}$ | Differential specific volume |
| $\rho_{n}$ | $\mathrm{~kg} / \mathrm{m}^{\wedge} 3$ | Gas Density at the valve throat or exit |
| $\rho$ | $\mathrm{kg} / \mathrm{m}^{\wedge} 3$ | Gas Density within the valve |
| $\bar{\rho}_{i}$ | $\mathrm{~kg} / \mathrm{m}^{\wedge} 3$ | Average gas density between consecutive pressure increments |

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