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The Settlers of "Catanbinatorics"

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Catan (formerly known as *The Settlers of Catan*) is a board game based on property development and resource trading. Like many other games, *Catan* contains opportunities for the application of game theory, probability, and statistics (see, e.g., [**1**]). However, some games also provide interesting contexts for exploring combinatorics (see, e.g., [**3**]). *Catan* is one such game due to its game board design which allows players to "construct" a new board every time they play by randomly arranging nineteen hexagonal tiles, eighteen number tokens, and nine port (harbor) markers according to a set of given parameters. To many, this leads to seemingly endless possible boards, but a mathematician will likely raise the "Catanbinatorics" question of exactly how many possible boards exist. In this paper we use basic combinatorial techniques to explore this question. We also address two related counting problems by focusing on parts of the game board design. The first reconsiders the way in which we count the arrangements of number tokens based on their role in the game. The second explores two methods of counting non-equivalent ways to arrange only the resource tiles. One might expect that no longer considering the number tokens and ports would simplify calculations, however, removing these components surprisingly makes the problem more complex (and interesting!) to solve.

A Brief History of *Catan*

Catan is an award-winning, internationally popular, easy-to-learn strategy board game which has been credited with revolutionizing the board game industry [**7**]. Since its introduction in Germany in 1995, Klaus Teuber's innovative game has received numerous awards including, but not limited to, Spiel des Jahres Game of the Year (1995), Meeples Choice Award (1995), Games Magazine Hall of Fame (2005), and GamesCon Vegas Game of the Century (2015) [**2**]. As of 2015, *Catan* has sold over 22 million copies and has been translated into over 30 different languages [**9**]. It has inspired several expansions and themed game variations, as well as several digital adaptations for platforms such as Microsoft, Nintendo, Xbox, iPod/iPhone, and Facebook [**2**]. The seasoned *Catan* player may even notice some subtle differences between editions of the base game. We will not explore these expansions and variations.

The complexities of *Catan* as a strategy game have received attention in both professional and recreational domains. Computer scientists have praised *Catan* as a scenario ripe with potential for artificial intelligence and programming analysis (see, e.g.,

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[**6**, **11**, **13**]). Mathematicians have highlighted how one might mathematize the choices made during initial settlement placement by using statistics and expected value to assign values to potential settlement locations based on players' individual strategies [**1**]. *Catan* has also received significant attention in various amateur circles via blog posts and other unreviewed works. Several of these focus on counting problems related to *Catan*, including how many distinct possible boards exist (see, e.g., [**8**, **10**, **12**]). Many present correct information, and furthermore some discuss approaches similar to what appears in this paper. However, the mathematics presented here was developed independently. Due to the general unreliability of unreviewed information, we assert the value of an authoritative and mathematically accurate exploration that is widely accessible while still substantive and interesting. We hope the reader will find that this paper satisfies these goals.

Board Construction

According to the *Catan* game rules, the board is assembled in three stages: the resource tiles, the number tokens, and the ports. The first step is positioning the nineteen hexagonal resource tiles in a larger roughly hexagonal configuration shown in Figure 1. These tiles designate which resources will be produced by each location on the board. There are four lumber tiles, four grain tiles, four wool tiles, three brick tiles, three ore tiles, and one desert tile which does not produce any resources.

Next the number tokens are arranged, one per hexagon resource tile with the exception of the desert. The number tokens are labeled "2" through "12" (excluding "7"), with one "2," one "12," and two of each for numbers "3" through "11" (except "7"). These tokens are placed in one-to-one correspondence with the resource tiles and dictate when each resource will be produced during the game; at the start of each turn, a player rolls a pair of dice and the resource tiles whose label matches the roll will produce resources for any player who has a settlement adjacent to the tile. The game rules dictate that tokens with red numbers (labeled "6" or "8") cannot be next to one another; however, for the purposes of this article, we are opting to ignore this restriction in favor of an entirely random set-up design.

The final phase of board construction involves placing ports at different designated locations around the larger hexagonal configuration, also shown in Figure 1. The ports allow players to trade two of a specified resource type for one of any other or to trade three of any common type for one of any other. There is one port for each of the five resources (lumber, brick, wool, grain, and ore), and four ports which allow any resource to be traded at the reduced three-for-one rate.

How Many Boards?

One's first instinct when counting the number of boards may be to consider it as no more than a relatively straight-forward combinatorial problem for permutations with repeated elements, similar to counting the number of possible arrangements of the letters in the word MISSISSIPPI. We begin with the resource tiles, number tokens, and ports, and then account for equivalent boards under symmetries.

In laying out the resource tiles we begin with the 19 tile locations and choose four for the lumber, four for the grain, four for the wool, three for the ore, and three for the brick, with the remaining spot designated as the desert. So the number of ways to arrange just the resource tiles would be

$$\binom{19}{4,4,4,3,3,1} = \binom{19}{4}\binom{15}{4}\binom{11}{4}\binom{7}{3}\binom{4}{3} = 244{,}432{,}188{,}000. \tag{1}$$

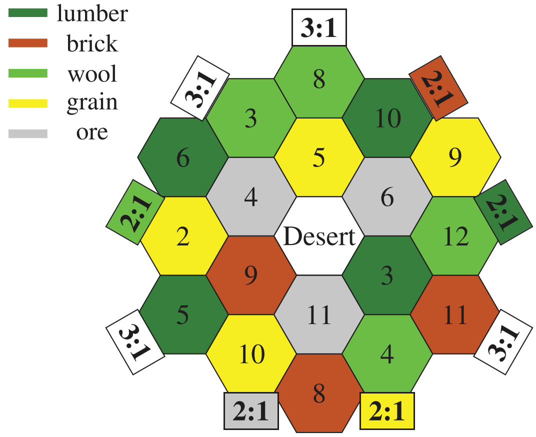


Figure 1 A sample board.

Then, adding in the number tokens would require selecting two of the eighteen non-desert spots for each number token from "3" to "11," excluding "7," and then choosing one of the remaining two spots for the "2" and the other by default for the "12." So the number of ways to place the number tokens would be

$$\binom{18}{2,2,2,2,2,2,2,2,1,1} = \binom{18}{2}\binom{16}{2}\binom{14}{2}\binom{12}{2}\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{1}$$
$$= 25{,}009{,}272{,}288{,}000.$$

Adding the ports is a much simpler process because we would simply choose four locations for the three-for-one ports and then distribute the five resource-specific ports among the remaining five positions, for a total of

$$\binom{9}{4} \cdot 5! = 15{,}120.$$

To obtain the total number of possible boards, we multiply the number of arrangements for each of these three components, i.e., the resource tiles, number tokens, and ports, to get more than 9.2429635×10^{28} possible configurations of the resources numbers and ports. While this may seem like the final number of boards, there is one more factor which must be taken into account. Because of the way *Catan* is played, the structure of the game board depends only on how various elements of the board are arranged relative to one another. The general arrangement of resource tiles, number tokens, and ports has $(120n)^\circ$ rotational symmetry for $n = 0, 1, 2$ and three lines of symmetry; see Figure 2. Any such rotation or reflection of a given board will create a new configuration while maintaining all salient adjacencies among board elements, and therefore can be thought of as equivalent to the original board. So for any given board there are five other equivalent boards as related by possible reflections, rotations, and combinations thereof. So we must divide our previous total number of configurations by six in order to account for the six equivalent versions of the same board. This leaves us with more than 1.5404939×10^{28} boards. But this is not the end; there are a few more im*port*ant "Catanbinatorial" questions to consider.

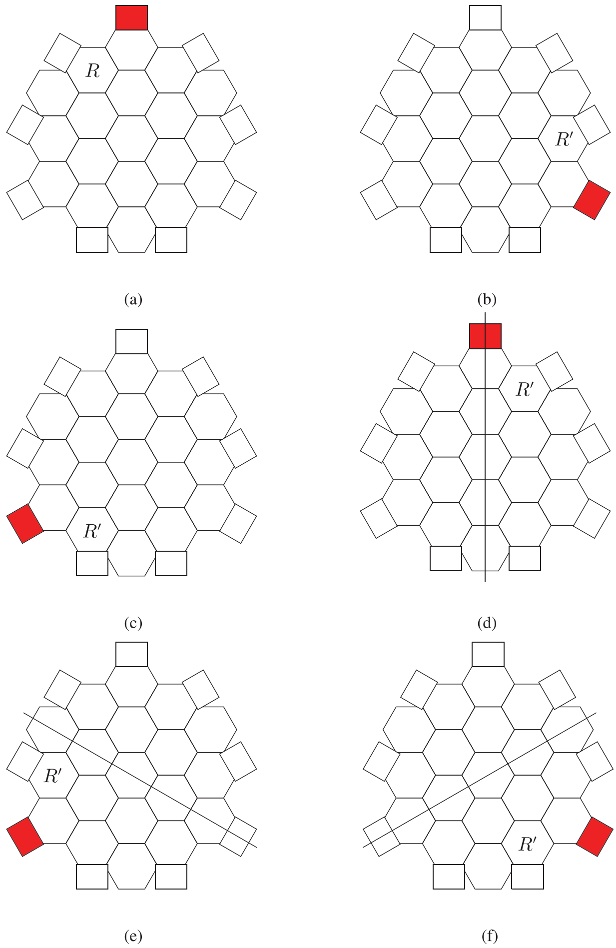


Figure 2 This figure contains images of six equivalent boards; they are simplified in that only one port and one resource tile are labeled to make the equivalences more visible. If (a) is considered the original board, (b) can be obtained by rotating (a) 120° clockwise, (c) can be obtained by rotating (a) 240° clockwise, and (d), (e), and (f) can each be obtained by reflecting (a) across the line shown on each respective board.

Equivalence among number token configurations

The strategy for counting the arrangements of the number tokens provided earlier can also be refined for equivalent configurations based on how the number tokens act during game play. The primary purpose of the numbers involves connecting the production of resources to the roll of a pair of standard six-sided dice. Each turn begins with a dice roll; resources are then produced by the resource tiles which have number tokens that match the number produced by the roll and are collected by any player(s) who have a settlement adjacent to the producing resource tiles. Because of this function, one may wish to think of the number tokens based on their probability of being rolled rather than the actual number printed on each. For example, a "6" and an "8" are essentially the same because they are equally likely to be rolled. Under this assumption there are two number tokens with probability $1/36$, and four each of tokens with probabilities $2/36$, $3/36$, $4/35$, and $5/36$. So instead of placing two "6" tokens and two "8" tokens, one can imagine distributing four tokens with probability 5/36.

This would reduce our calculation to

$$\binom{18}{4,4,4,4,2} = \binom{18}{4}\binom{14}{4}\binom{10}{4}\binom{6}{4} = 9{,}648{,}639{,}000.$$

This result is significantly smaller than our original estimate which contained nearly 25 trillion more possibilities. However, such groupings may seem a bit hasty to the seasoned *Catan* payer. Although in the long term a 6 and an 8 are equally likely to be rolled, the game has a much different feel depending on if your settlements are adjacent to duplicate number tokens such as two tokens labeled "6" or diversified number tokens such as a "6" and an "8." In order to take this into account and still consider duplicate boards we consider only the possibility of switching *pairs* of numbers, such as switching the two "6" tokens with the two "8" tokens. Because there are five pairs of numbers with the same probability of being rolled, we can simply divide the original calculation by 2^5, one 2 for each pair that could be switched. This brings our total to

$$\frac{\binom{18}{2,2,2,2,2,2,2,2,1,1}}{2^5} = \frac{\binom{18}{2}\binom{16}{2}\binom{14}{2}\binom{12}{2}\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{1}}{2^5} = 781{,}539{,}759{,}000.$$

Replacing the original calculation for number tokens with this equivalent one would again reduce the total number of boards to more than 4.8140434×10^{26} possible boards. This is still a lot of boards. If we counted one possible board every second of every day for 365 days a year, it would still take over 1.5×10^{19} years to go through them all!

Counting Resource Configurations

In the following section, we explain two ways to count the total number of possible configurations of the 19 resource tiles alone, without considering the number tokens or ports. Why isn't the answer 244,432,188,000 as calculated earlier in equation (1)? If we are only placing the resource tiles, we actually have a significantly more complicated system of symmetries to explore. When considering complete boards, the presence of the number tokens and ports eliminates these symmetries. Thus, this work must be considered as its own problem and cannot inform the board counting argument. The reader may wish to pause while admiring this mathematical oddity: one might expect that removing the number tokens and ports from consideration would make calculations easier. However, this simplification surprisingly makes the problem more complex to solve.

Hence, our main objective in this section is to account for this more complicated system of symmetries of resource tile configurations. We do so in two ways. The first uses a simple and readily accessible direct approach without any heavy machinery. The second approach is more elegant and makes use of abstract algebra.

First, we note that no nontrivial rotation (less than 360°) of a configuration can ever produce itself. Indeed, there are no fixed configurations under 60° or 300° rotations because there are not six copies of any single resource (see Figure 3a). Similarly, there are no fixed configurations under 120° or 240° rotations because there are not six sets of three like resources (see Figure 3b). Finally, there are no fixed configurations under a 180° rotation because there are not nine pairs of like resources (see Figure 3c).

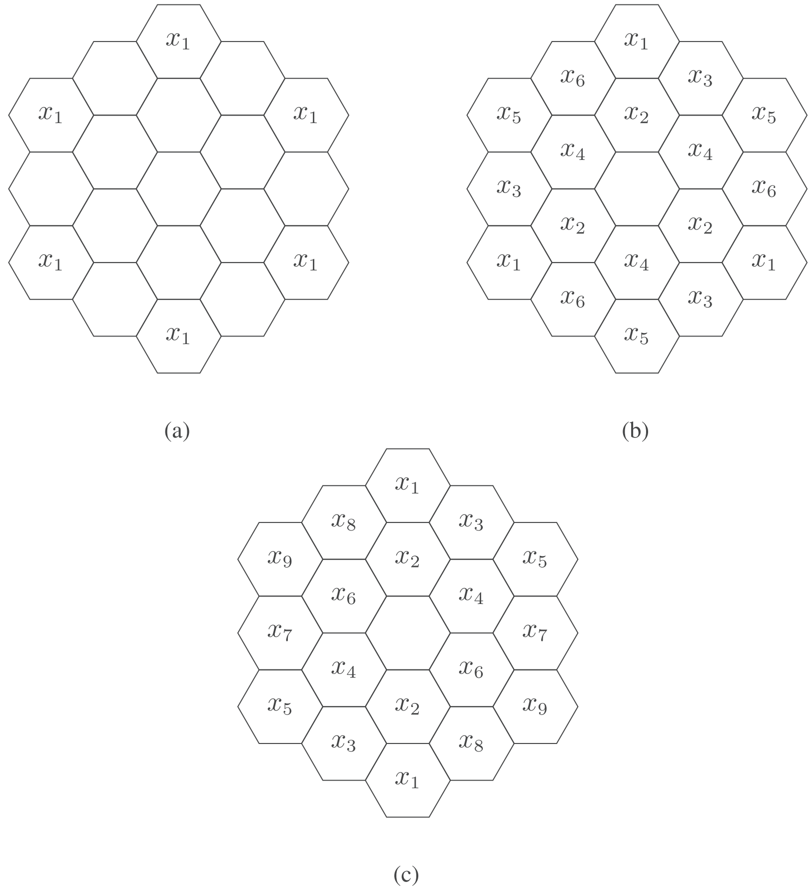


Figure 3 Nontrivial rotations of less than 360° do not fix any configuration.

A Direct Approach We begin by placing the desert; there are four choices, up to rotational and reflectional symmetry. For further explanation on type a and type b lines of symmetry, see Figures 4 and 5, respectively.

- **Case A:** The desert lies in the outer ring on a type a line of symmetry.
- **Case B:** The desert lies in the outer ring on a type b line of symmetry.
- **Case C:** The desert lies in the inner ring.
- **Case D:** The desert is the middle tile.

We begin with **case A**; without loss of generality, suppose the desert is placed in the uppermost location as shown in Figure 4.

Let T denote the set of all such configurations. Now,

$$|T| = \frac{18!}{4!4!4!3!3!} = 12{,}864{,}852{,}000.$$

Observe that any configuration that is NOT symmetric across line a will actually be counted twice: once for each of the two equivalent configurations. Denote by S_a the set of configurations that have reflectional symmetry across line a. Note that in all such configurations, the pair of resources placed in locations labeled $x_1, x_2, \ldots, x_7$

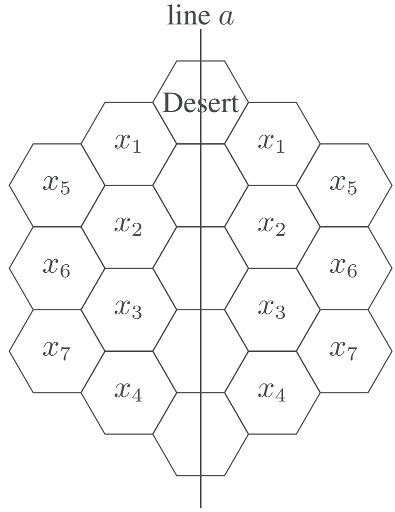


Figure 4 A case A configuration with symmetry across line *a*.

in Figure 4 must be the same. As there are eight available pairs (two pairs of wool tiles, two pairs of lumber, two pairs of grain, one pair of brick, and one pair of ore) in addition to a spare brick and ore, we proceed based on which of the eight pairs is not chosen.

If we leave out a pair of wool, lumber, or grain, then there are $\binom{3}{1} \cdot \frac{7!}{2!2!}$ ways to place the pairs because there are $\binom{3}{1}$ ways to choose which pair to exclude, and $\frac{7!}{2!2!}$ ways to place the 7 remaining pairs, dividing by 2!2! to account for the fact that there are two identical pairs which may each be interchanged without changing the configuration. We then must multiply by $\binom{4}{2} \cdot 2$, the number of ways to place the remaining tiles (the excluded pair of tiles, one brick, and one ore). Similarly, if we leave out a pair of brick or ore, then there are $\binom{2}{1} \cdot \frac{7!}{2!2!2!} \cdot \binom{4}{1}$ configurations. Hence, we have

$$|S_a| = \binom{3}{1} \cdot \frac{7!}{2!2!} \cdot \binom{4}{2} \cdot 2 + \binom{2}{1} \cdot \frac{7!}{2!2!2!} \cdot \binom{4}{1} = 50{,}400.$$

Therefore, since each configuration in $T \setminus S_a$ is double counted, the total number of distinct case A configurations up to symmetry is

$$\frac{|T| - |S_a|}{2} + |S_a| = \frac{|T| + |S_a|}{2} = 6{,}432{,}451{,}200.$$

We next consider **case B**; without loss of generality, suppose the desert is placed as shown in Figure 5.

As in the previous case, let T denote the set of all such configurations, double counting those that are not symmetric across line b; once again,

$$|T| = \frac{18!}{4!4!4!3!3!} = 12{,}864{,}852{,}000.$$

We let S_b denote the set of configurations that have reflectional symmetry across line b. This time, all eight pairs must be placed as illustrated in Figure 5, followed by the left-over brick and ore, and so

$$|S_b| = \frac{8!}{2!2!2!} \cdot 2 = 10{,}080.$$

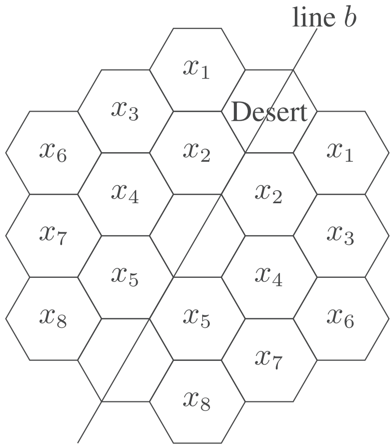


Figure 5 A case B configuration with symmetry across line b.

Hence, the total number of distinct case B configurations up to symmetry is

$$\frac{|T|-|S_b|}{2}+|S_b|=\frac{|T|+|S_b|}{2}=6{,}432{,}431{,}040.$$

As can be seen by comparing Figure 6 to Figure 4, the number of **case C** configurations is equal to the number of case A configurations, namely 6,432,451,200.

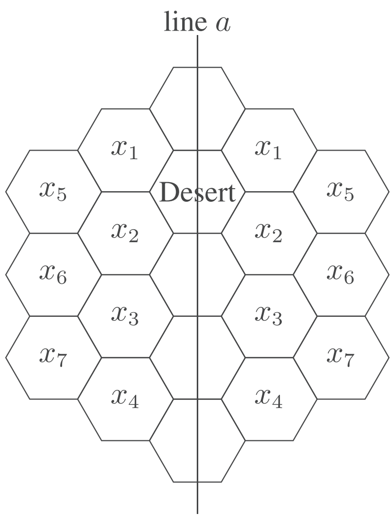
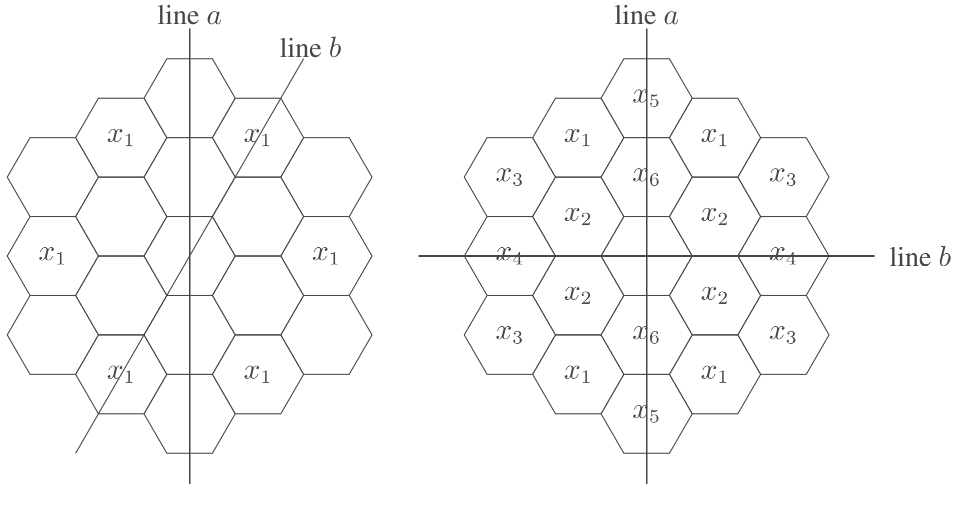


Figure 6 A case C configuration with symmetry across line a.

Finally, we consider **case D**. This case requires a bit more care due to the central location of the desert. It appears at first glance that we must consider rotational symmetry, but we already explained why this type of symmetry is impossible (see Figure 3 and the discussion immediately preceeding the "A Direct Approach" section). Therefore, we need only account for three potential lines of symmetry of type a and three potential lines of symmetry of type b. However, no configuration can be simultaneously fixed by a reflection of type a and a reflection of type b because this would either require six copies of a single resource (see Figure 7a) or three sets of four like resources and three additional pairs of like resources (see Figure 7b).



(a) (b)

Figure 7 No configuration can be reflected onto itself using both type a and type b reflections.

Hence, the total number of distinct configurations up to symmetry is

$$\frac{1}{6} \cdot \left(\frac{|T| - 3\left(|S_a| + |S_b|\right)}{2} + 3\left(|S_a| + |S_b|\right) \right) = \frac{|T| + 3\left(|S_a| + |S_b|\right)}{12}$$
$$= 1{,}072{,}086{,}120,$$

where we divided by six because each configuration will be counted six times, one for each of the six possible rotations of the board.

Combining all cases, the total number of configurations up to symmetry is

$$6{,}432{,}451{,}200 + 6{,}432{,}431{,}040 + 6{,}432{,}451{,}200 + 1{,}072{,}086{,}120 = 20{,}369{,}419{,}560.$$

"Ore" would you prefer a more elegant approach?

A More Elegant Approach In this alternative approach, we will use Burnside's lemma to simplify our counting problem. We will need the following concepts.

Definition. Let G be a group of permutations on a set S (in other words, each element of G is a bijection $\phi : S \to S$). For any ϕ in G, define

$$\text{fix}(\phi) = \{i \in S | \phi(i) = i\}.$$

In other words, fix(ϕ) is the set of all elements of S that are fixed by ϕ.

Burnside's lemma is a statement about orbits. Again, let G be a group of permutations on a set S. Then for any $s \in S$, the orbit of G on s is the set of all elements that s can be mapped to by an element of G; i.e., $orb_G(s) = \{\phi(s) | \phi \in G\}$. Then there exist $s_1, \ldots, s_n$ such that $orb_G(s_1), \ldots, orb_G(s_n)$ are disjoint and their union is S. The choice of $s_1, \ldots, s_n$ is usually not unique; however, the number of orbits of G on S, i.e., the value of n, is fixed for a given G and S. The purpose of Burnside's lemma is to calculate this number.

Theorem (Burnside's Lemma)**.** *Let G be a finite group of permutations on a set S. Then the number of orbits of G on S is*

$$\frac{1}{|G|}\sum_{\phi\in G}|\mathrm{fix}(\phi)|.$$

For more information about these concepts, consult an abstract algebra text such as [**4**] or [**5**].

In our problem, the group of permutations G is D_6, the dihedral group whose elements are the 12 symmetries of a regular hexagon (six rotations and six reflections). The set of all resource configurations, without removing symmetric configurations, will be the set S; recall from (1) that $|S| = 244{,}432{,}188{,}000$. For a given resource configuration s, the orbit of s is the set of all resource configurations that we can obtain by applying the symmetries in $G = D_6$ (rotations and reflections) to s.

Let's begin by calculating $\mathrm{fix}(\phi)$ for the six rotations ϕ. When ϕ is the rotation of $0°$ (i.e., the identity element of D_6), ϕ fixes every element of S. Hence,

$$|\mathrm{fix}(\phi)| = |S| = 244{,}432{,}188{,}000.$$

Furthermore, notice that if ϕ is a rotation of $(60n)°$, $n = 1, \ldots, 5$, then $|\mathrm{fix}(\phi)| = 0$; see Figure 3 and the discussion before the "A Direct Approach" section.

This leaves only the reflectional symmetries across lines of type a and b as described previously. Before proceeding, recall that there are eight available pairs of resource tiles (2 pairs of wool tiles, 2 pairs of lumber, 2 pairs of grain, 1 pair of brick, and 1 pair of ore), as well as 1 additional brick, ore, and desert.

We'll let F_a denote a reflectional symmetry (F for flip) across a line of type a. Then to fill the seven pairs of locations (marked by $x_1, \ldots, x_7$ in Figure 8a), we choose from the eight pairs of resources. We must choose the pair to exclude, place the seven pairs, and then place the remaining five resources along line a. Since there are two cases (depending on whether the excluded pair is a wool, lumber, or grain, or is a brick or ore), we have

$$\begin{aligned}|\mathrm{fix}(F_a)| &= \binom{3}{1}\cdot\binom{7}{1,1,2,1,2}\cdot\binom{5}{2}\cdot 3! + \binom{2}{1}\cdot\binom{7}{2,2,1,2}\cdot\binom{5}{3}\cdot 2!\\ &= 252{,}000.\end{aligned}$$

Similarly, consider reflections across a line of type b. Then there are eight pairs of locations to fill with the eight pairs of resources; see Figure 8b.

The remaining three locations are filled with the leftover brick, ore, and desert. Hence,

$$|\mathrm{fix}(F_b)| = \binom{8}{2,1,2,1,2}\cdot 3! = 30{,}240.$$

Finally, we are ready to apply Burnside's lemma. Since there are three reflectional symmetries of type a and three of type b, and Figure 7 illustrates why $F_a \cap F_b = \emptyset$, the number of distinct configurations up to symmetry is

$$\frac{1}{12}\Big(244{,}432{,}188{,}000 + 3(252{,}000) + 3(30{,}240)\Big) = 20{,}369{,}419{,}560.$$

Of course, this is the same number as we obtained using the more direct approach!

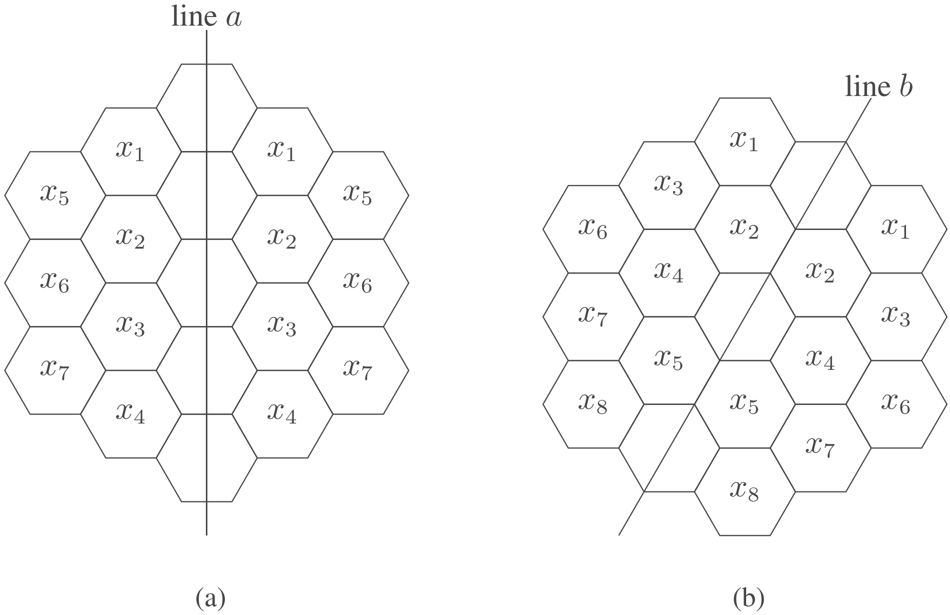


Figure 8 Configurations fixed under reflections across line a and line b, respectively.

Conclusion

The “Catanbinatorics” presented in this article provide a first insight into the combinatorial potential of this game board. The way the game itself is played provides motivation for considering additional restrictions on board configurations such as rules about resource or number adjacency, or limiting which number tokens might be placed on which resource tiles. Counting the boards within these restrictions could require still other rich combinatorial techniques. Additional counting problems may be considered for similar boards with different quantities or types of tiles. Although the number of possible boards is not actually endless, this game may provide countless opportunities for the exploration of interesting “Catanbinatorics.”

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Summary. *Catan* is a dynamic property-building and trading board game in which players build a new board every time they play by arranging tiles, number tokens, and port markers. In this paper, we count the number of possible boards, consider different ways of counting the number tokens based on probability, and count the number of non-equivalent tile arrangements in two ways: one using a direct approach, the other taking advantage of more elegant techniques from abstract algebra.

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