

ISSN: 0975-766X CODEN: IJPTFI Research Article

# Available Online through <u>www.ijptonline.com</u> EFFECTS OF THERMAL RADIATION AND CHEMICAL REACTION ON AN UNSTEADY MHD FLOW OF A JEFFREY FLUID PAST A VERTICAL POROUS PLATE WITH SUCTION

M. Eswara Rao, <sup>\*</sup>S. Sreenadh and B. Sumalatha

\*Department of Mathematics, Sri Venkateswara University, Tirupati – 517502, A.P. <u>Email: profsreenadh@gmail.com</u>

Received on: 03-12-2017

Accepted on: 28-12-2017

### Abstract

An unsteady flow of an incompressible Jeffrey fluid past a semi infinite vertical plate with time dependent suction is examined. The dimensionless governing equations are solved by employing perturbation technique. The expressions for the velocity, temperature and concentration are obtained. Numerical results for velocity, temperature, concentration, Skin friction, Nusselt number, Sherwood number are shown in various graphs and discussed for embedded flow parameters. It is observed that the velocity increases due to increase in the Jeffrey parameter  $\lambda_1$  and Eckret number. Further increase in the Prandtl number leads to decrease in the temperature field.

Keywords: Jeffrey fluid, thermal radiation and chemical reaction.

# 1. Introduction

The study of boundary layer flow behavior and heat transfer characteristics of a Jeffrey fluid past a semi infinite vertical plate with time dependent suction under the influence of magnetic field and chemical reaction has extensive technological applications in the astrophysical, geophysical and engineering problems such as chemical catalytic reactors, thermal insulators, drying of porous solids, enhanced oil and gas recovery and underground energy transport. MHD free convection flows have significant applications in the field of stellar and planetary magnetosphere, aeronautical plasma flows, chemical engineering and electronics. Inspite of these enormous applications of boundary layer theory, Chambre and Young [1] have studied the flow of chemically reacting species at the boundary. The effect of heat generation or absorption on hydromagnetic three dimensional free convection flows over a vertical stretching sheet was discussed by Chamkha [2]. Rapits [3] has investigated the

two dimensional free convection flow through porous medium bounded by a vertical infinite porous plate in the presence of radiation. Youn J Kim [4] has studied the unsteady MHD convective heat transfer past a semi infinite vertical porous moving plate with variable suction. Md Abdus Samad and Mohammad Mansur Rahman [5] investigated the thermal radiation interaction on an absorbing emitting fluid permitted by a transversely applied magnetic field past a moving vertical porous plate embedded in a porous medium with time dependent suction and temperature. Dulal Pal et al. [6] studied the combined effect of MHD and ohmic heating in unsteady twodimensional boundary layer slip flow, heat and mass transfer of a viscous incompressible fluid past a vertical permeable plate with the diffusion of species in the presence of thermal radiation incorporating first-order chemical reaction. Singh and Rakesh Kumar [7] analyzed the effects of chemical reaction and heat generation absorption on unsteady MHD free convection heat and mass transfer flow of an electrically conducting, viscous, incompressible fluid past an infinite hot vertical porous plate through porous medium when the plate temperature is span wise cosinusoidally fluctuating with time. Dulal Pal et al. [8] presented a analytical study for the problem of unsteady hydromagnetic heat and mass transfer for a micropolar fluid bounded by semi-infinite vertical permeable plate in the presence of first-order chemical reaction, thermal radiation and heat absorption. A uniform magnetic field acts perpendicularly to the porous surface which absorbs the micropolar fluid with a time-dependent suction velocity. The basic partial differential equations are reduced to a system of nonlinear ordinary differential equations which are solved analytically using perturbation technique. Nazibuddin Ahmed and Kishor Kumar Das [9] examined the effect of thermal radiation and chemical reaction on magnetohydrodynamic convective mass transfer flow of an unsteady viscous incompressible eclectically conducting fluid past a semi-infinite vertical permeable plate embedded in a porous medium in slip flow regime. Perturbation technique is applied to convert the governing nonlinear partial differential equations in to a system of ordinary differential equations which are solved analytically. Kalidas Das [10] investigated the effect of chemical reaction and viscous dissipation on MHD mixed convective heat and mass transfer flow of a viscous, incompressible, electrically conducting second grade fluid past a semiinfinite stretching sheet in the presence of thermal diffusion and thermal radiation with Rosseland approximation. Eshetu Haile et al. [11] explore the effects of heat and mass transfer through a porous media of MHD flow of nanofluids with thermal radiation, viscous dissipation and chemical reaction. Mamta Goyal and Kiran Kumari [12]

investigated the effects of radiation absorption, heat absorption/generation and chemical reaction on unsteady heat and mass MHD oscillatory flow of a visco-elastic fluid between two inclined plates. The equations of continuity, linear momentum, energy and diffusion, which govern the flow field, are solved by using a regular perturbation method. Free convective flow of a Jeffrey fluid in a vertical deformable porous stratum is investigated by Sreenadh et al. [13] and noticed that the effect of increasing Jeffrey parameter is to increase the skin friction in the deformable porous stratum. Aivesimi et al. [14] Examined the magnetohydrodynamic (MHD) flow of unsteady convective third grade fluid in a cylindrical system and it is observed that velocity decreases and increases with increasing magnetic field and porosity, temperature increases as magnetic field increases. Srinivas et al. [15] studied the free convection effects on the flow and heat transfer of a Jeffrey fluid confined between two long, parallel, vertical plates moving with equal velocities but in opposite directions. Perturbation analysis of magnetohydrodynamics oscillatory flow on convective-radiative heat and mass transfer of micropolar fluid in a porous medium with chemical reaction was made by Dulal Pal and Sukanta Biswas [16]. Jhankal et al. [17] made an analysis to study the problem of boundary layer forced convective flow and heat transfer of an incompressible fluid past a plate embedded in a Darcy-Forchheimer porous medium in presence of transverse magnetic field and thermal radiation term is considered in the energy equation. They found that the physical parameters such as Porous medium parameter, the Inertial parameter, the Magnetic parameter, the Prandtl number, and the Radiation parameter have significant effects on the flow and heat transfer.

The present work investigates the effects of thermal radiation and chemical reaction on unsteady MHD flow of a Jeffrey fluid past a vertical porous plate with time dependent suction. Applying perturbation technique, the governing equations are solved analytically.

# 2. Mathematical Formulation

We consider the unsteady laminar boundary layer flow of an incompressible, radiating and electrically conducting Jeffrey fluid past a semi infinite vertical plate with time dependent suction with the effect of chemical reaction. The x'-axis is along the plate in upward direction and y'-axis normal to it. A constant magnetic field is applied in the transverse direction to the flow. In addition a homogeneous first order chemical reaction between fluid and the species concentration is considered, in which the rate of chemical reaction is directly proportional to the species

*S. Sreenadh\*et al. /International Journal of Pharmacy & Technology* concentration. All the properties of the fluid is assumed to be constant except that the influence of the density variation with temperature is considered only in the body force term. Under the Boussinesq approximation the flow field is governed by the following equations.

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \frac{\upsilon}{1 + \lambda_1} \frac{\partial^2 u'}{\partial {y'}^2} + \beta_T g \left( T' - T_\infty \right) + \beta_c g \left( C' - C_\infty \right) + \left( V_0 - u' \right) \frac{\sigma B_o^2}{\rho} + \left( V_0 - u' \right) \frac{\upsilon}{k' \left( 1 + \lambda_1 \right)}$$
(2)

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{K}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r'}{\partial y'} + \frac{\upsilon}{c_p \left(1 + \lambda_1\right)} \left(\frac{\partial u'}{\partial y'}\right)^2$$
(3)

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D_m \frac{\partial^2 C'}{\partial {y'}^2} - K_r \left( C' - C_{\infty} \right) \tag{4}$$

where u', v' are the velocity components in x', y' directions respectively. t' is time, g is the acceleration due to the gravity, v is the kinematic viscosity,  $\rho$  is the fluid density,  $\sigma$  is the electric conductivity of the fluid,  $B_0^2$  is the constant transverse magnetic field, K' is the permeability of the porous medium,  $\beta_T$ ,  $\beta_C$  are the thermal and concentration expansion coefficients, T' is the temperature of the fluid in the boundary layer,  $C_{\infty}$  is the species concentration in the fluid far away from the plate,  $C_p$  is the specific heat at constant pressure,  $q'_r$  radiative heat flux,  $D_m$  is the mass diffusivity,  $K_T$  is the thermal duffusion ratio,  $K_r$  is the chemical reaction rate on the species concentration.

The boundary conditions for the velocity, temperature and concentration are

At 
$$y' = 0$$
,  $u' = u_p$ ,  $T' = T'_w$ ,  $C' = C'_w$  (5)

$$y' \to \infty, \ u' = U'(t') = V_o(1 + \varepsilon e^{\omega t'}), \quad T' \to T_{\infty}, \quad C' \to C_{\infty}$$
 (6)

where  $T'_{w}$  and  $C'_{w}$  are the wall temperature and concentration of the plate. From Continuity equation (1), it is clear that v' is a constant or a function of time only. We assume that  $V'_{o} = -V_{o} \left(1 + \varepsilon e^{\omega t'}\right)$  (7)

Here  $v_0 > 1$  and  $\varepsilon << 1$  and the negative sign indicates that the suction velocity is towards the plate.

Assuming Rosseland approximation which leads to the radiative heat flux  $q_r$  is given by  $q'_r = -\frac{4\sigma_s}{3K_e}\frac{\partial T'^4}{\partial y'}$  (8)

in which  $\sigma_s$  is the Stefan-Boltzmann constant and  $K_e$  is the mean absorption coefficient. By the assumption that the temperature differences within the flow are sufficiently small, expanding  $T'^4$  in a Taylors series about  $T_{\infty}$  and neglecting higher order terms we get,

$$T'^{4} = 4T_{\infty}^{3}T' - 3T_{\infty}^{4}$$
<sup>(9)</sup>

We introduce the following quantities to make the governing equations and boundary conditions dimensionless.

$$y = \frac{V_{0}}{\upsilon} y', \quad \omega = \frac{\upsilon}{V_{0}^{2}} \omega', \quad u = \frac{1}{V_{0}} u', \quad v = \frac{1}{V_{0}} v'$$

$$t = \frac{V_{0}^{2}}{\upsilon} t', \quad u_{p} = \frac{u_{p}'}{V_{0}}, \quad \theta = \frac{T' - T_{\infty}}{T_{w}' - T_{\infty}}, \quad C = \frac{C' - C_{\infty}}{C_{w}' - C_{\infty}}, \quad Ec = \frac{V_{0}^{2}}{c_{p} \left(T_{w} - T_{\infty}\right)}$$

$$Gr = \frac{\upsilon g \beta_{T} \left(T_{w}' - T_{\infty}\right)}{V_{0}^{3}}, \quad Gc = \frac{\upsilon g \beta_{c} \left(C_{w}' - C_{\infty}\right)}{V_{0}^{3}}, \quad M = \frac{\sigma B_{0}^{2} \upsilon}{\rho V_{0}^{2}}$$

$$K = \frac{V_{0}^{2} k'}{\upsilon^{2}}, \quad \gamma = \frac{\upsilon k_{r}}{V_{0}^{2}}, \quad Sc = \frac{\gamma}{D_{m}}, \quad \Pr = \frac{\mu c_{p}}{K}, \quad N = \frac{4T_{\infty}^{3} \sigma_{s}}{kk_{e}}$$
(10)

The dimensionless governing equations and boundary conditions are

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon e^{\omega t}\right) \frac{\partial u}{\partial y} = A_1 \frac{\partial^2 u}{\partial y^2} + Gr\theta + GcC + R_1 \left(1 - u\right)$$
(11)

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon e^{\omega t}\right) \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} \left[1 + \frac{4N}{3}\right] + EcA_1 \left(\frac{\partial u}{\partial y}\right)^2$$
(12)

$$\frac{\partial C}{\partial t} - \left(1 + \varepsilon e^{\omega t}\right) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - \gamma C \tag{13}$$

The corresponding boundary conditions are

at 
$$y = 0$$
,  $u = u_p$ ,  $\theta = 1$ ,  $C = 1$  (14)

as 
$$y \to \infty$$
,  $u = (1 + \varepsilon e^{\omega t}), \quad \theta \to 0, \quad C \to 0$  (15)

#### 3. Solution of the Problem

S. Sreenadh\*et al. /International Journal of Pharmacy & Technology Since the equations (11), (12) and (13) are highly nonlinearly coupled partial differential equations, they cannot be solved in closed form. However, analytical solutions to the above equations could be possible. Since  $\varepsilon$  is small we can write perturbation expansions of the form

$$u(y,t) = u_0(y) + \varepsilon e^{\omega t} u_1(y)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{\omega t} \theta_1(y)$$

$$C(y,t) = C_0(y) + \varepsilon e^{\omega t} C_1(y)$$

$$(16)$$

Substituting the expansions (16) in equations (11) - (13) and equating the harmonic and non-harmonic terms, we get

$$A_{1}u_{0}'' + u_{0}' - u_{0}R_{1} = |-Gr\theta_{0} - GcC_{0} - R_{1}$$
(17)

$$A_{1}u_{1}'' + u_{1}' - u_{1}(R_{1} + \omega) = |-Gr\theta_{1} - GcC_{1} - u_{0}'$$
(18)

$$\left[1 + \frac{4N}{3}\right]\theta_0'' + \Pr \theta_0' = \left|-EcA_1 \operatorname{Pr}\left(u_0'\right)^2\right]$$
(19)

$$A_2 \theta_1'' + \Pr \theta_1' - \Pr \omega \theta_1 = \Pr \theta_0' - 2EcA_1 \Pr u_0' u_1'$$
<sup>(20)</sup>

$$C_0'' + ScC_0' - Sc\gamma C_0 = 0$$
<sup>(21)</sup>

$$C_1'' + ScC_1' - R_3 ScC_1 = -ScC_0'$$
<sup>(22)</sup>

The corresponding boundary conditions are

at 
$$y = 0$$
,  $u_0 = u_p$ ,  $u_1 = 0$ ,  $\theta_0 = 1$ ,  $\theta_1 = 0$ ,  $C_0 = 1$ ,  $C_1 = 0$  (23)

as 
$$y \to \infty$$
,  $u_0 \to 1$ ,  $u_1 \to 1$ ,  $\theta_0 \to 0$ ,  $\theta_1 \to 0$ ,  $C_0 \to 0$ ,  $C_1 \to 0$  (24)

In order to solve the non-linear coupled equations (17) - (22) subject to boundary conditions given in equations (23) - (24), we assume that the Eckert number Ec is small. So, it is used as the perturbation parameter. Then we write

$$u_{0}(y) = u_{01}(y) + Ecu_{02}(y)$$

$$u_{1}(y) = u_{11}(y) + Ecu_{12}(y)$$

$$\theta_{0}(y) = \theta_{01}(y) + Ec\theta_{02}(y)$$

$$\theta_{1}(y) = \theta_{11}(y) + Ec\theta_{12}(y)$$

$$C_{0}(y) = C_{01}(y) + EcC_{02}(y)$$

$$C_{1}(y) = C_{11}(y) + EcC_{12}(y)$$
(25)

# S. Sreenadh\*et al. /International Journal of Pharmacy & Technology Using expansions (25) in equations (17) – (22), we get the following equations

$$A_{1}u_{01}'' + u_{01}' - R_{1}u_{01} = |-Gr\theta_{01} - GcC_{01} - R_{1}$$
<sup>(26)</sup>

$$A_{1}u_{02}'' + u_{02}' - R_{1}u_{02} = -Gr\theta_{02} - GcC_{02}$$
<sup>(27)</sup>

$$A_{1}u_{11}'' + u_{11}' - R_{2}u_{11} = -Gr\theta_{11} - GcC_{11} - u_{01}$$
<sup>(28)</sup>

$$A_{1}u_{12}'' + u_{12}' - R_{2}u_{12} = -Gr\theta_{12} - GcC_{12} - u_{02}'$$
<sup>(29)</sup>

$$A_2 \theta_{01}'' + \Pr \theta_{01}' = 0 \tag{30}$$

$$A_2 \theta_{02}'' + \Pr \theta_{02}' = -A_1 \Pr \left( u_{01}' \right)^2$$
(31)

$$A_2 \theta_{11}^{\prime\prime} + \Pr \theta_{11}^{\prime} - \Pr \omega \theta_{11} = -\Pr \theta_{01}^{\prime\prime}$$
(32)

$$A_{2}\theta_{12}'' + \Pr \theta_{12}' - \Pr \omega \theta_{12} = -\Pr \theta_{02}' - 2A_{1} \Pr u_{11}' u_{01}'$$
(33)

$$C_{01}'' + ScC_{01}' - Sc\gamma C_{01} = 0$$
(34)

$$C_{02}'' + ScC_{02}' - Sc\gamma C_{02} = 0 \tag{35}$$

$$C_{11}'' + ScC_{11}' - R_3 ScC_{11} = -ScC_{01}'$$
(36)

$$C_{12}'' + ScC_{12}' - R_3 ScC_{12} = -ScC_{02}'$$
(37)

The corresponding boundary conditions are

at 
$$y = 0$$
,  $u_{01} = u_p$ ,  $u_{02} = 0$ ,  $u_{11} = 0$ ,  $u_{12} = 0$ ,  $\theta_{01} = 1$ ,  $\theta_{02} = 0$   
 $\theta_{11} = 0$ ,  $\theta_{12} = 0$ ,  $C_{01} = 1$ ,  $C_{02} = 0$ ,  $C_{11} = 0$ ,  $C_{12} = 0$  } } (38)

as 
$$y \to \infty$$
,  $u_{01} = 1$ ,  $u_{02} = 0$ ,  $u_{11} = 1$ ,  $u_{12} = 0$ ,  $\theta_{01} = 0$ ,  $\theta_{02} = 0$   
 $\theta_{11} = 0$ ,  $\theta_{12} = 0$ ,  $C_{01} = 0$ ,  $C_{02} = 0$ ,  $C_{11} = 0$ ,  $C_{12} = 0$ } (39)

By solving equations (26)-(37) subject to the boundary conditions given in (38) - (39), we get the velocity, temperature and concentration distributions as follows,

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$$u_{01} = 1 + B_1 e^{-m_5 y} - t_3 e^{-m_3 y} - t_4 e^{-m_1 y}$$
(40)

$$S. Sreenadh*et al. /International Journal of Pharmacy & Technology \\ u_{02} = B_4 e^{-m_7 y} + t_{16} e^{-m_3 y} + t_{17} e^{-2m_5 y} + t_{18} e^{-2m_3 y} + t_{19} e^{-2m_1 y} + t_{20} e^{-(m_5 + m_3) y} + t_{21} e^{-(m_5 + m_1) y} + t_{22} e^{-(m_5 + m_1) y}$$
(41)

$$u_{11} = 1 + B_3 e^{-m_6 y} + t_{11} e^{-m_1 y} + t_{12} e^{-m_2 y} + t_{13} e^{-m_3 y} + t_{14} e^{-m_4 y} + t_{15} e^{-m_5 y}$$
(42)

$$u_{12} = B_{6}e^{-m_{9}y} + t_{42}e^{-m_{8}y} + t_{43}e^{-m_{3}y} + t_{44}e^{-2m_{5}y} + t_{45}e^{-2m_{3}y} + t_{46}e^{-2m_{1}y} + t_{47}e^{-(m_{5}+m_{3})y} + t_{48}e^{-(m_{3}+m_{1})y} + t_{49}e^{-(m_{5}+m_{1})y} + t_{50}e^{-(m_{5}+m_{6})y} + t_{51}e^{-(m_{5}+m_{2})y} + t_{52}e^{-(m_{5}+m_{4})y} + t_{53}e^{-2m_{5}y} + t_{54}e^{-(m_{3}+m_{6})y} + t_{55}e^{-(m_{3}+m_{2})y} + t_{56}e^{-2m_{3}y} + t_{57}e^{-(m_{3}+m_{4})y} + t_{58}e^{-(m_{1}+m_{6})y} + t_{59}e^{-2m_{1}y} + t_{60}e^{-(m_{1}+m_{2})y} + t_{61}e^{-(m_{1}+m_{4})y}$$
(43)

$$\theta_{01} = e^{-m_3 y} \tag{44}$$

$$\theta_{02} = B_2 e^{-m_3 y} + t_5 e^{-2m_5 y} + t_6 e^{-2m_3 y} + t_7 e^{-2m_1 y} + t_8 e^{-(m_5 + m_3) y} + t_9 e^{-(m_3 + m_1) y} + t_{10} e^{-(m_5 + m_1) y}$$
(45)

$$\theta_{11} = t_2 \left[ e^{-m_3 y} - e^{-m_4 y} \right] \tag{46}$$

$$\theta_{12} = B_5 e^{-m_8 y} + t_{23} e^{-m_3 y} + t_{24} e^{-2m_5 y} + t_{25} e^{-2m_3 y} + t_{26} e^{-2m_1 y} + t_{27} e^{-(m_3 + m_5)y} + t_{28} e^{-(m_3 + m_1)y} + t_{29} e^{-(m_1 + m_5)y} + t_{30} e^{-(m_5 + m_6)y} + t_{31} e^{-(m_5 + m_2)y} + t_{32} e^{-(m_5 + m_4)y} + t_{33} e^{-2m_5 y} + t_{34} e^{-(m_3 + m_6)y} + t_{35} e^{-(m_3 + m_2)y} + t_{36} e^{-2m_3 y}$$
(47)  
+  $t_{37} e^{-(m_3 + m_4)y} + t_{38} e^{-(m_1 + m_6)y} + t_{39} e^{-2m_1 y} + t_{40} e^{-(m_1 + m_2)y} + t_{41} e^{-(m_1 + m_4)y}$ 

$$C_{01} = e^{-m_1 y} (48)$$

$$C_{02} = 0$$
 (49)

$$C_{11} = t_1 \left[ e^{-m_1 y} - e^{-m_2 y} \right]$$
(50)

$$C_{12} = 0$$
 (51)

The expressions for constants are given in the Appendix.

The Skin-friction Coefficient ( $C_f$ ), Nusselt number (Nu) and Sherwood number (Sh) at the plate in the dimensionless form are given by

$$C_{f} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_{0}}{\partial y} + \varepsilon e^{\omega t} \frac{\partial u_{1}}{\partial y}\right)_{y=0}$$
$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial \theta_{0}}{\partial y} + \varepsilon e^{\omega t} \frac{\partial \theta_{1}}{\partial y}\right)_{y=0}$$
$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left(\frac{\partial C_{0}}{\partial y} + \varepsilon e^{\omega t} \frac{\partial C_{1}}{\partial y}\right)_{y=0}$$

#### 4. Results and Discussions

The unsteady flow of an incompressible Jeffrey fluid past a semi infinite vertical plate with time dependent suction is investigated. Regular Perturbation technique is employed in solving non linearly coupled partial differential equations. The Numerical results show the effects of various pertaining parameter such as thermal Grashof number (Gr), solutal Grashof number (Gc), Magnetic parameter (M), radiation parameter (N), Jeffrey parameter ( $\lambda_1$ ), Prandtl number (Pr), Schmidt number (Sc), Chemical reaction parameter ( $\gamma$ ), Permeability parameter (K) and Eckert number (Ec) on velocity, temperature and concentration fields. For numerical analysis we used the following parametric values. M = 1, K=2, Gr = 4, Gc = 2, Sc = 0.2, t=1,  $\varepsilon = 0.2$ ,  $\omega = 0.1$ ,

P r = 0.71, N = 2,  $u_p = 0.5$  and  $\gamma = 2$ .

Figures 5-7 show the effect of above mentioned physical parameters on velocity field. Figure 1 shows the effect of thermal Grashof number on the velocity field. Since the thermal Grashof number represents the relative strength of the thermal buoyancy force to the viscous hydrodynamic force, the increasing values of thermal Grashof number cause to increase the velocity of the flow field throughout the boundary layer region i.e., the buoyancy forces are enhanced by the Grashof number near the plate but away from the plate the buoyancy forces and all other forces become weak and fluid on the free stream surface has zero velocity. Figure 2 depicts the effect of solutal Grashof number for mass transfer on velocity. It is observed that the velocity increases with increase in Gc due to buoyancy effect. Figure 3 illustrate the effect of Magnetic parameter on velocity field. It is observed that the velocity reduces by increasing parameter M. This is due to the Lorentz force which retards the fluid motion near the plate and away from the plate this force become weak so fluid comes to rest. The effect of radiation parameter on velocity is depicted in Figure 4. It is observed that the velocity increases as the radiation parameter increases. From Figure 5, it is noticed that velocity increases with increasing values of Jeffrey parameter. Figure 6 depicts the effect of the permeability parameter on the flow field. The permeability parameter increases the transient velocity of the flow field. The contribution of Eckert number on the velocity profiles is noticed in Figure 7. An increase in Eckert number contributes to the increase in the velocity field.

Temperature profiles with effects of radiation parameter and prandtl number are shown in Figures 8-9. Since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby from Figure 8 it is noticed that

the temperature profiles decreases as the radiation parameter increases. Figure 9 illustrates that the temperature profiles decrease with increase in the values of the Prandtl number. Temperature profiles drawn with Pr = 0.71represent profiles for air at  $20^{\circ}c$ . This figure also shows that temperature for water (Pr=1.0) reduces at a greater speed compared to air. Thus it is pointed out that the temperature of the flow field diminishes as the Prandtl number increases. Further, it is observed that higher the Prandtl number, sharper the reduction in the temperature. Figures 10-11 shows the effect of Schmidt number and chemical reaction parameter on concentration field. Figure 10 demonstrates that the concentration level of the fluid drops due to increasing Schmidt number indicating the fact that the mass diffusivity raises the concentration level steadily. From Figure 11, it is observed from this figure that the concentration distribution decreases at all points of the flow field with increase in the chemical reaction parameter  $\delta$ . This shows that the diffusive species with higher value of chemical reaction parameter have retarding effect on the concentration distribution in the solutal boundary layer. The effect of radiation parameter on Nusselt number which measures the rate of heat transfer is shown in Figure 12. It is noticed that the Nusselt number at y = 0 reduces with the increasing values of radiation parameter. The Sherwood number which measures the rate of mass transfer at the wall y = 0 is shown in Figures 13-14 for different values of Schimdt number and chemical reaction parameter respectively. It is found that Sherwood number enhances with the increasing values of Schmidt number and chemical reaction parameter. Figures 15-16 shows the effects of magnetic, Jeffrey and radiation parameters on skin friction coefficient. It is evident that the skin friction enhances with the increase in the values of Jeffrey and radiation parameters whereas it has opposite behavior in case of magnetic parameter.

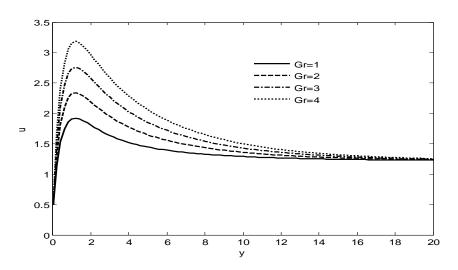


Figure 1. Velocity profiles for different thermal Grashof number.

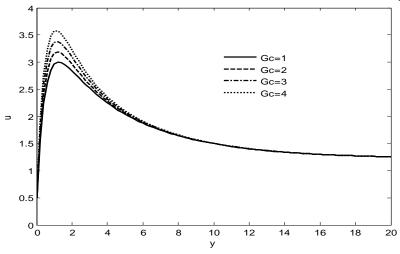


Figure 2. Velocity profiles for different solutal Grashof number.

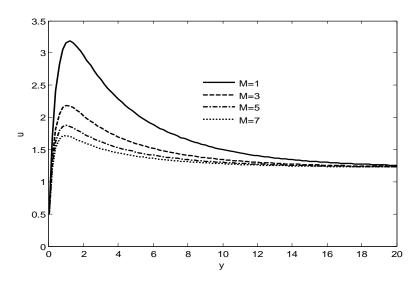


Figure 3. Velocity profiles for different magnetic parameter.

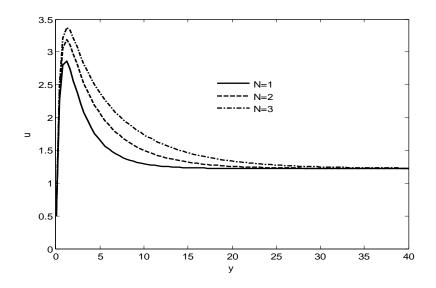


Figure 4. Velocity profiles for different radiation parameter.

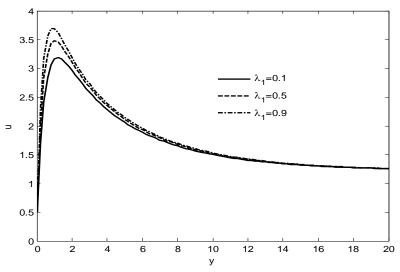


Figure 5. Velocity profiles for different Jeffrey parameter.

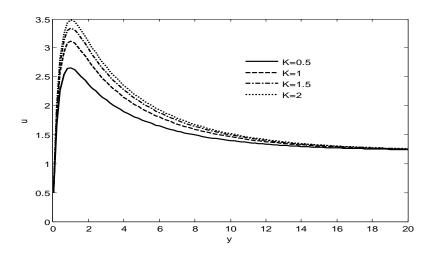


Figure 6. Velocity profiles for different permeability parameter.

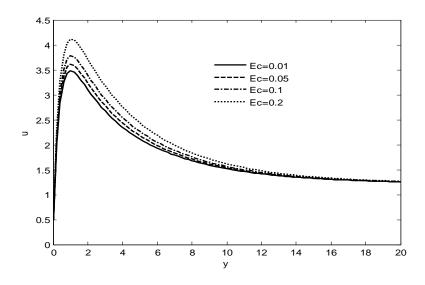


Figure 7. Velocity profiles for different Eckert number.

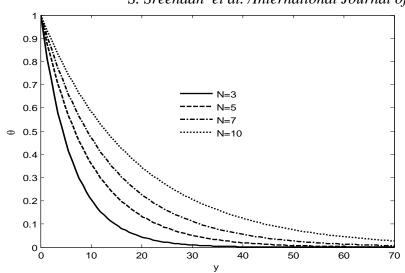


Figure 8. Temperature profiles for different radiation parameter.

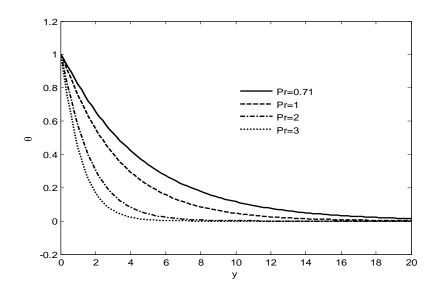


Figure 9. Temperature profiles for different Prandtl number.

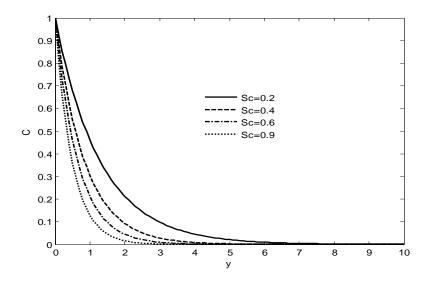


Figure 10. Concentration profiles for different Schmidt number.

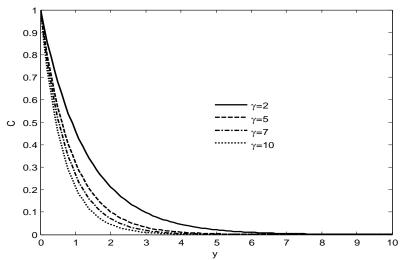


Figure 11. Concentration profiles for different chemical reaction parameter.

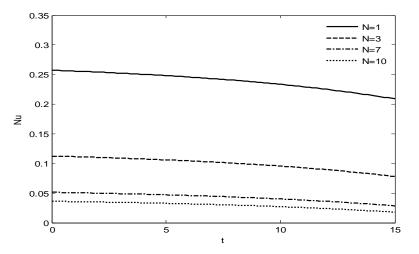


Figure 12. Nusselt number for different radiation parameter.

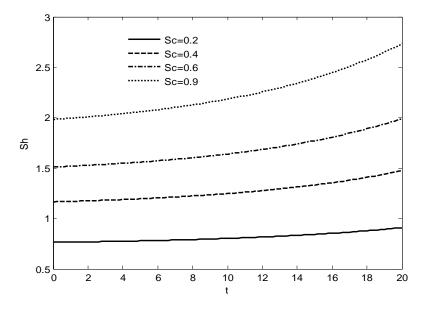


Figure 13.Sherwood number for different Schmidt number.

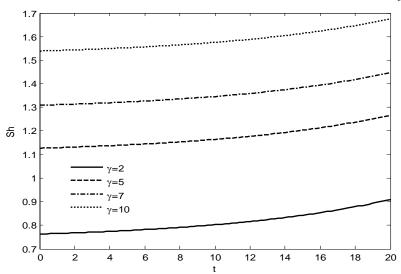


Figure 14.Sherwood number for different chemical reaction parameter.

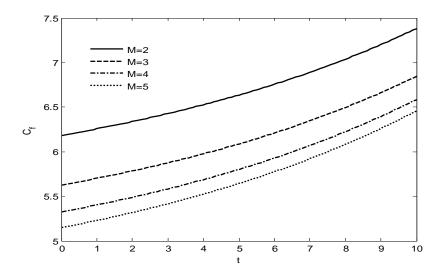


Figure 15.Skin friction for different magnetic parameter.

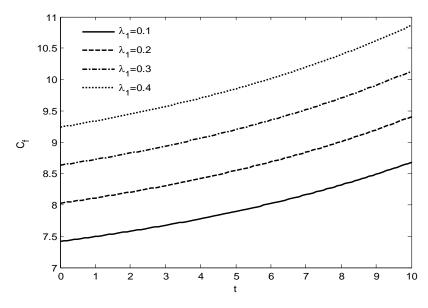


Figure 16. Skin friction for different Jeffrey parameter.

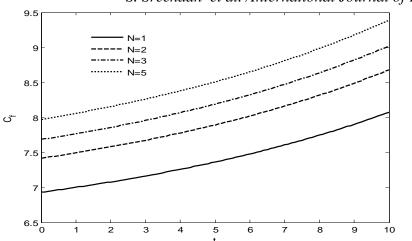


Figure 17.Skin friction for different radiation parameter.

#### **5.** Conclusions

An unsteady flow of an incompressible Jeffrey fluid past a semi infinite vertical plate with time dependent suction is examined analytically by using perturbation technique. Some graphical results are also plotted and discussed. Some noteworthy observations are as follows:

- 1. The effect of increasing values of thermal and solutal Grashof number, radiation parameter, Jeffrey parameter, permeability parameter and Eckret number on velocity profiles, results in increasing the velocity of the flow field where as velocity decreases with the increasing values of magnetic parameter.
- 2. The effect of Prandtl number decreases the temperature profiles whereas the temperature increase with the increasing values of radiation parameter.
- 3. Concentration profiles are decreasing with the increasing values of Schimidt number and chemical reaction parameter.
- 4. The effect of increasing values of radiation parameter shows opposite behavoiur on skin friction coefficient and Nusselt number.

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# **Corresponding Author:**

# S. Sreenadh\*,

Email: profsreenadh@gmail.com

<u>Appendix</u>

$$A_{1} = \frac{1}{1 + \lambda_{1}}, R_{1} = M + \frac{A_{1}}{K}, R_{2} = R_{1} + \omega, A_{2} = 1 + \frac{4N}{3}, R_{3} = R_{1} + \omega$$

$$m_{1} = \frac{1}{2} \Big[ Sc + \sqrt{Sc^{2} + 4Sc\gamma} \Big], m_{2} = \frac{1}{2} \Big[ Sc + \sqrt{Sc^{2} + 4ScR_{3}} \Big], m_{3} = \frac{Pr}{A_{2}}, m_{4} = \frac{1}{2A_{2}} \Big[ Pr + \sqrt{Pr^{2} + 4PrA_{2}\omega} \Big],$$

$$m_{5} = \frac{1}{2A_{1}} \Big[ 1 + \sqrt{1 + 4A_{1}R_{1}} \Big], m_{6} = \frac{1}{2A_{1}} \Big[ 1 + \sqrt{1 + 4A_{1}R_{2}} \Big]$$

$$m_{7} = \frac{1}{2A_{1}} \Big[ 1 + \sqrt{1 + 4A_{1}R_{1}} \Big], m_{8} = \frac{1}{2A_{2}} \Big[ Pr + \sqrt{Pr^{2} + 4PrA_{2}\omega} \Big], m_{9} = \frac{1}{2A_{1}} \Big[ 1 + \sqrt{1 + 4A_{1}R_{2}} \Big]$$

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$$B_1 = u_p - 1 + t_3 + t_4$$
,  $B_2 = -(t_5 + t_6 + t_7 + t_8 + t_9 + t_{10})$ ,  $B_3 = -(1 + t_{11} + t_{12} + t_{13} + t_{14} + t_{15})$ 

$$B_4 = -(t_{16} + t_{17} + t_{18} + t_{19} + t_{20} + t_{21} + t_{22}),$$

$$B_{5} = -(t_{23} + t_{24} + t_{25} + t_{26} + t_{27} + t_{28} + t_{29} + t_{30} + t_{31} + t_{32} + t_{33} + t_{34} + t_{35} + t_{36} + t_{37} + t_{38} + t_{39} + t_{40} + t_{41})$$

$$B_{6} = -(t_{42} + t_{43} + t_{44} + t_{45} + t_{46} + t_{47} + t_{48} + t_{49} + t_{50} + t_{51} + t_{52} + t_{53} + t_{54} + t_{55} + t_{56} + t_{57} + t_{58} + t_{59} + t_{60} + t_{61})$$

$$t_{1} = \frac{Scm_{1}}{m_{1}^{2} - Scm_{1} - R_{3}Sc}, t_{2} = \frac{\Pr m_{3}}{A_{2}m_{3}^{2} - \Pr m_{3} - \Pr \omega}, t_{3} = \frac{Gr}{A_{1}m_{3}^{2} - m_{3} - R_{1}}, t_{4} = \frac{Gc}{A_{1}m_{1}^{2} - m_{1} - R_{1}}$$

$$t_{5} = \frac{-A_{1} \operatorname{Pr} m_{5}^{2} B_{1}^{2}}{4A_{2} m_{5}^{2} - 2 \operatorname{Pr} m_{5}}, t_{6} = \frac{-A_{1} \operatorname{Pr} m_{3}^{2} t_{3}^{2}}{4A_{2} m_{3}^{2} - 2 \operatorname{Pr} m_{3}}, t_{7} = \frac{-A_{1} \operatorname{Pr} m_{1}^{2} t_{4}^{2}}{4A_{2} m_{1}^{2} - 2 \operatorname{Pr} m_{1}}$$

$$t_8 = \frac{2A_1 \operatorname{Pr} m_5 B_1 t_3 m_3}{A_2 (m_5 + m_3)^2 - \operatorname{Pr} (m_5 + m_3)}, t_9 = \frac{-2A_1 \operatorname{Pr} m_1 t_4 t_3 m_3}{A_2 (m_1 + m_3)^2 - \operatorname{Pr} (m_1 + m_3)}$$

$$t_{10} = \frac{2A_{1} \operatorname{Pr} m_{1}t_{4}B_{1}m_{5}}{A_{2}(m_{1}+m_{5})^{2} - \operatorname{Pr}(m_{1}+m_{5})}, \quad t_{11} = \frac{-Gct_{1}-t_{4}m_{1}}{A_{1}m_{1}^{2}-m_{1}-R_{2}}, \quad t_{12} = \frac{Gct_{1}}{A_{1}m_{2}^{2}-m_{2}-R_{2}} \quad t_{13} = \frac{-Grt_{2}-t_{3}m_{3}}{A_{1}m_{3}^{2}-m_{3}-R_{2}},$$

$$t_{14} = \frac{Grt_2}{A_1m_4^2 - m_4 - R_2}, \quad t_{15} = \frac{B_1m_5}{A_1m_5^2 - m_5 - R_2}, \quad t_{16} = \frac{-GrB_2}{A_1m_3^2 - m_3 - R_1}$$

$$t_{17} = \frac{-Grt_5}{4A_1m_5^2 - 2m_5 - R_1}, t_{18} = \frac{-Grt_6}{4A_1m_3^2 - 2m_3 - R_1}, t_{19} = \frac{-Grt_7}{4A_1m_1^2 - 2m_1 - R_1}$$

$$t_{20} = \frac{-Grt_8}{A_1(m_5 + m_3)^2 - (m_5 + m_3) - R_1}, t_{21} = \frac{-Grt_9}{A_1(m_1 + m_3)^2 - (m_1 + m_3) - R_1}$$

$$t_{22} = \frac{-Grt_{10}}{A_1(m_5 + m_1)^2 - (m_5 + m_1) - R_1}, t_{23} = \frac{\Pr B_2 m_3}{A_2 m_3^2 - \Pr m_3 - \Pr \omega}, t_{24} = \frac{2\Pr t_5 m_5}{4A_2 m_5^2 - 2\Pr m_5 - \Pr \omega}$$

$$t_{25} = \frac{2 \operatorname{Pr} t_6 m_3}{4 A_2 m_3^2 - 2 \operatorname{Pr} m_3 - \operatorname{Pr} \omega}, \quad t_{26} = \frac{2 \operatorname{Pr} t_7 m_1}{4 A_2 m_1^2 - 2 \operatorname{Pr} m_1 - \operatorname{Pr} \omega}$$

$$t_{27} = \frac{\Pr t_8(m_5 + m_3) - 2\Pr A_1 B_1 m_5 t_{13} m_3 + 2\Pr A_1 t_3 m_3 t_{15} m_5}{A_2(m_5 + m_3)^2 - \Pr(m_5 + m_3) - \Pr \omega}, t_{28} = \frac{\Pr t_9(m_1 + m_3) + 2\Pr A_1 t_3 m_1 t_{11} m_3 + 2\Pr A_1 t_4 m_3 t_{13} m_1}{A_2(m_3 + m_3)^2 - \Pr(m_1 + m_3) - \Pr \omega}, t_{28} = \frac{\Pr t_9(m_1 + m_3) + 2\Pr A_1 t_3 m_1 t_{11} m_3 + 2\Pr A_1 t_4 m_3 t_{13} m_1}{A_2(m_3 + m_3)^2 - \Pr(m_1 + m_3) - \Pr \omega}$$

$$t_{29} = \frac{\Pr t_{10}(m_5 + m_1) - 2\Pr A_1 B_1 m_5 t_{11} m_1 + 2\Pr A_1 t_4 m_1 t_{15} m_5}{A_2(m_5 + m_1)^2 - \Pr(m_5 + m_1) - \Pr \omega}, \quad t_{30} = \frac{-2\Pr A_1 B_1 B_3 m_5 m_6}{A_2(m_5 + m_6)^2 - \Pr(m_5 + m_6) - \Pr \omega},$$

$$t_{31} = \frac{-2 \operatorname{Pr} A_1 B_1 t_{12} m_5 m_2}{A_2 (m_5 + m_2)^2 - \operatorname{Pr} (m_5 + m_2) - \operatorname{Pr} \omega}$$

$$t_{32} = \frac{-2 \operatorname{Pr} A_1 B_1 t_{14} m_5 m_4}{A_2 (m_5 + m_4)^2 - \operatorname{Pr} (m_5 + m_4) - \operatorname{Pr} \omega}, t_{33} = \frac{-2 A_1 B_1 \operatorname{Pr} t_{15} m_5^2}{4 A_3 m_5^2 - 2 \operatorname{Pr} m_5 - \operatorname{Pr} \omega}$$

$$t_{34} = \frac{2 \operatorname{Pr} A_1 B_3 t_3 m_3 m_6}{A_2 (m_3 + m_6)^2 - \operatorname{Pr} (m_3 + m_6) - \operatorname{Pr} \omega}, \quad t_{35} = \frac{2 \operatorname{Pr} A_1 t_3 t_{12} m_3 m_2}{A_2 (m_3 + m_2)^2 - \operatorname{Pr} (m_3 + m_2) - \operatorname{Pr} \omega}$$

$$t_{36} = \frac{2A_1t_3 \operatorname{Pr} t_{13}m_3^2}{4A_2m_3^2 - 2\operatorname{Pr} m_3 - \operatorname{Pr} \omega}, \quad t_{37} = \frac{2\operatorname{Pr} A_1t_3t_{14}m_3^2}{A_2(m_3 + m_4)^2 - \operatorname{Pr}(m_3 + m_4) - \operatorname{Pr} \omega}$$

$$t_{38} = \frac{2 \operatorname{Pr} A_1 t_4 B_3 m_1 m_6}{A_2 (m_1 + m_6)^2 - \operatorname{Pr} (m_1 + m_6) - \operatorname{Pr} \omega}, \quad t_{39} = \frac{2 A_1 t_4 \operatorname{Pr} t_{11} m_1^2}{4 A_2 m_1^2 - 2 \operatorname{Pr} m_1 - \operatorname{Pr} \omega}$$

$$t_{40} = \frac{2 \operatorname{Pr} A_1 t_4 t_{12} m_1 m_2}{A_2 (m_1 + m_2)^2 - \operatorname{Pr} (m_1 + m_2) - \operatorname{Pr} \omega}, \quad t_{41} = \frac{2 \operatorname{Pr} A_1 t_4 t_{14} m_1 m_4}{A_2 (m_1 + m_4)^2 - \operatorname{Pr} (m_1 + m_4) - \operatorname{Pr} \omega}$$

$$t_{42} = \frac{-GrB_5}{A_1m_8^2 - m_8 - R_2}, t_{43} = \frac{-Grt_{23} + m_3t_{16}}{A_1m_3^2 - m_3 - R_2}, t_{44} = \frac{-Grt_{24} + 2m_5t_{17}}{4A_1m_5^2 - 2m_5 - R_2}, t_{45} = \frac{-Grt_{25} + 2m_3t_{18}}{4A_1m_3^2 - 2m_3 - R_2}, t_{45} = \frac{-Grt_{25} + 2m$$

$$t_{46} = \frac{-Grt_{26} + 2m_1t_{19}}{4A_1m_1^2 - 2m_1 - R_2}, \quad t_{47} = \frac{-Grt_{27} + (m_5 + m_3)t_{20}}{A_1(m_5 + m_3)^2 - (m_5 + m_3) - R_2}, \quad t_{48} = \frac{-Grt_{28} + (m_3 + m_1)t_{21}}{A_1(m_1 + m_3)^2 - (m_1 + m_3) - R_2},$$

$$t_{49} = \frac{-Grt_{29} + (m_1 + m_5)t_{22}}{A_1(m_5 + m_1)^2 - (m_5 + m_1) - R_2}, t_{50} = \frac{-Grt_{30}}{A_1(m_5 + m_6)^2 - (m_5 + m_6) - R_2}, t_{51} = \frac{-Grt_{31}}{A_1(m_5 + m_2)^2 - (m_5 + m_2) - R_2}$$

$$t_{52} = \frac{-Grt_{32}}{A_1(m_5 + m_4)^2 - (m_5 + m_4) - R_2}, t_{53} = \frac{-Grt_{33}}{4A_1m_5^2 - 2m_5 - R_2}, t_{54} = \frac{-Grt_{34}}{A_1(m_3 + m_6)^2 - (m_3 + m_6) - R_2},$$

$$t_{55} = \frac{-Grt_{35}}{A_1(m_3 + m_2)^2 - (m_3 + m_2) - R_2} t_{56} = \frac{-Grt_{36}}{4A_1m_3^2 - 2m_3 - R_2}, t_{57} = \frac{-Grt_{37}}{A_1(m_3 + m_4)^2 - (m_3 + m_4) - R_2},$$

$$t_{58} = \frac{-Grt_{38}}{A_1(m_1 + m_6)^2 - (m_1 + m_6) - R_2}, t_{59} = \frac{-Grt_{39}}{4A_1m_1^2 - 2m_1 - R_2}, t_{60} = \frac{-Grt_{40}}{A_1(m_1 + m_2)^2 - (m_1 + m_2) - R_2},$$

$$t_{61} = \frac{-Grt_{41}}{A_1(m_1 + m_4)^2 - (m_1 + m_4) - R_2}$$