

# Item Response Theory and Computerized Adaptive Testing

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# Outline

- Item Response Theory
  - versus Classical Test Theory
- Uses of IRT
  - Item Banking
  - Short Forms
  - Computerized Adaptive Tests

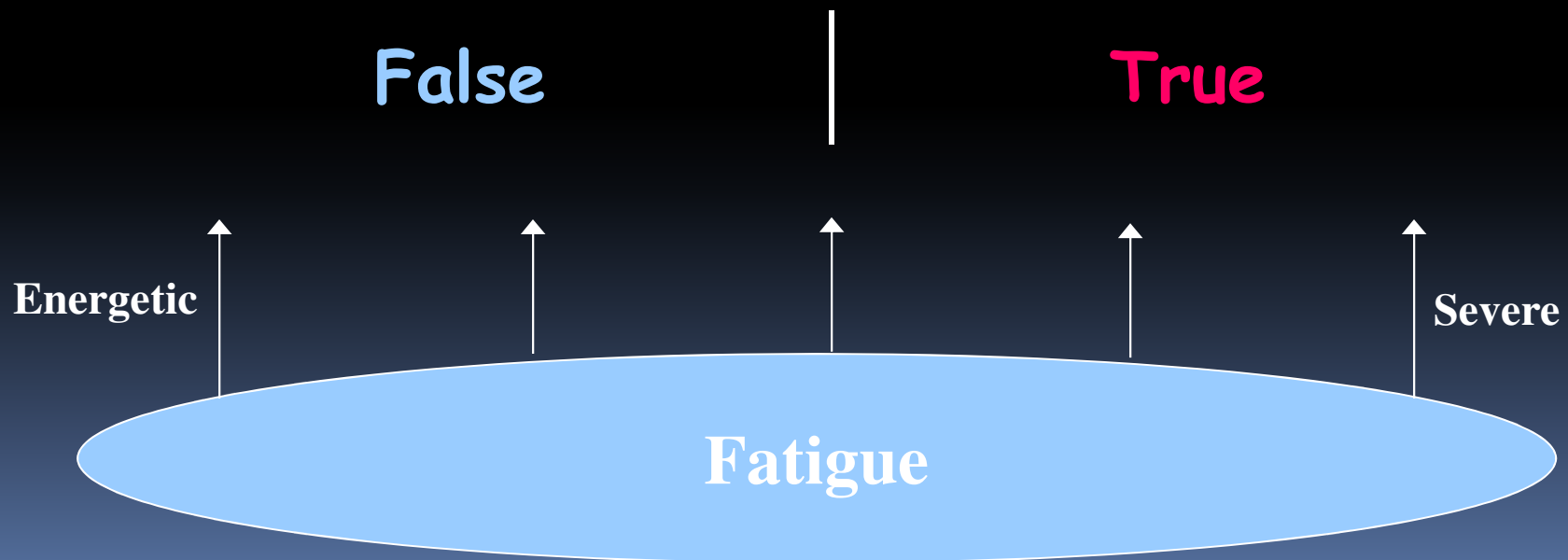
# Requirements for Measurement

- Measurement requires the concept of an underlying trait that can be expressed in terms of *more or less*
- Test items are the operational definition of the underlying trait
- Test items
  - can be ordered from easy to hard
- Test takers
  - can be ordered from less able to more able

# IRT Modeling is Latent Trait Modeling

- A latent trait is an unobservable latent dimension that is thought to give rise to a set of observed item responses.

**I am too tired to do errands**



## Latent Traits (cont.)

- These latent traits (constructs, variables,  $\theta$ ) are measured on a continuum of severity.

**I am too tired to do errands?**

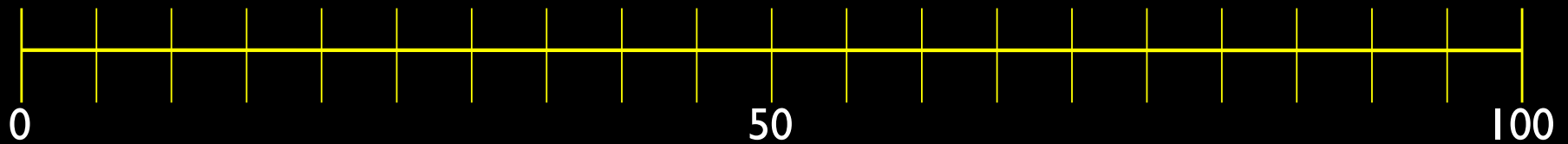
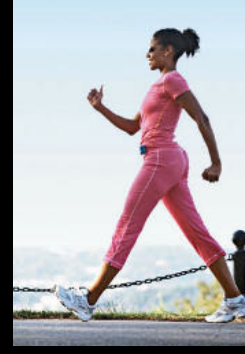


# Advantages of Using IRT

- Equal Interval Measure
- Test-takers and items are represented on the same scale
- Item calibrations are independent of the test-takers used for calibration
- Candidate ability estimates are independent of the particular set of items used for estimation
- Measurement precision is estimated for each person and each item

# Test-takers and Items are Represented on the Same Scale

- Item Difficulty = Severity = Measure = Theta = Item Calibration = Location
- Person Ability = Measure = Theta = Person Calibration = Location



## Physical Functioning Item Bank

- |        |        |        |        |        |        |        |        |        |         |         |         |         |         |         |         |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|---------|---------|
| Item 1 | Item 2 | Item 3 | Item 4 | Item 5 | Item 6 | Item 7 | Item 8 | Item 9 | Item 10 | Item 11 | Item 12 | Item 13 | Item 14 | Item 15 | Item 16 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|---------|---------|---------|---------|---------|

Are you able to get in and out of bed?

Are you able to walk a block on flat ground?

Are you able to run five miles?



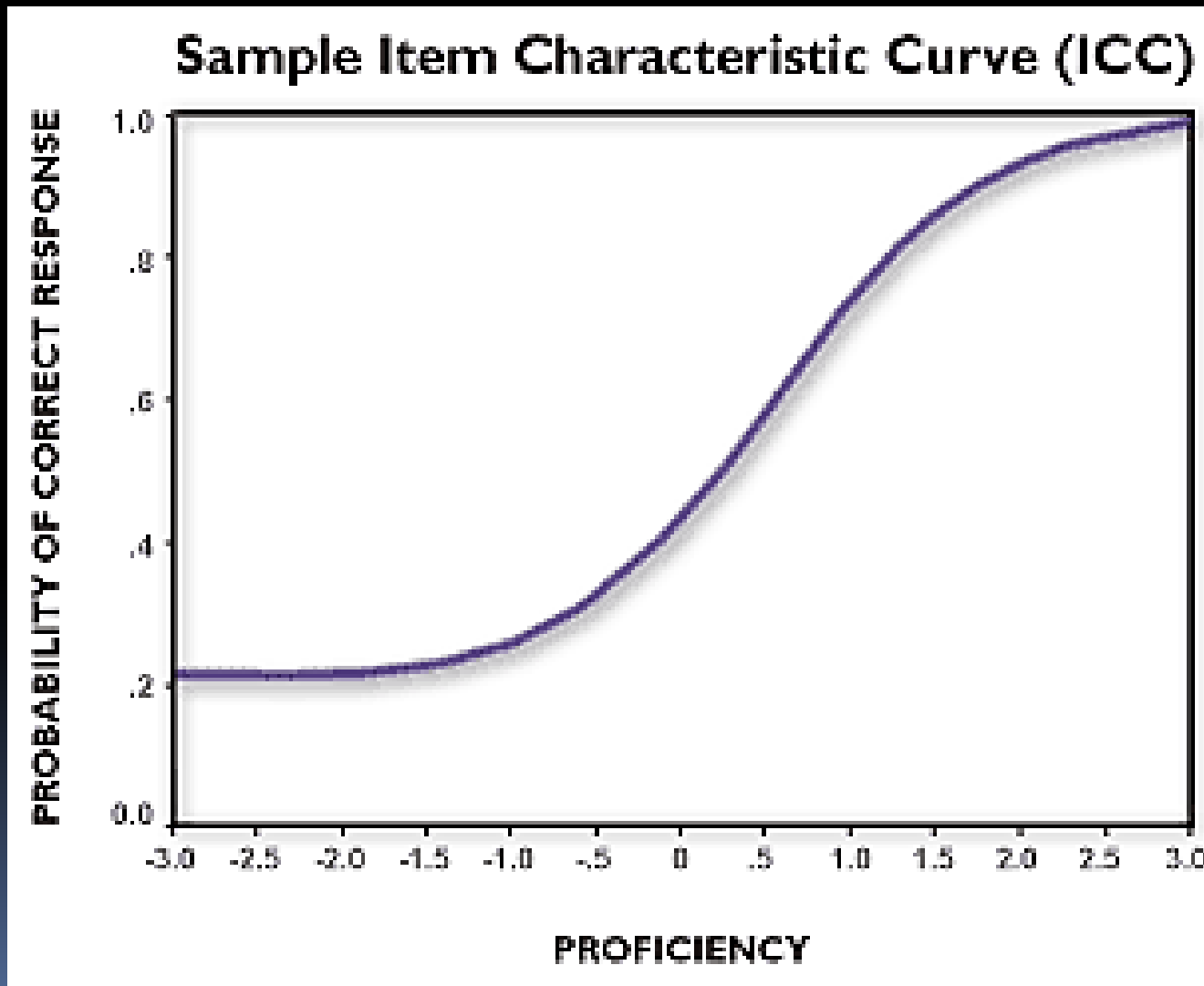
## More Basic Terms

- Discrimination = the degree to which an item discriminates person ability
- Item Information = the area where an item discriminates
- Test Information = the area where the test discriminates

# Item “Parameters”

- IRT statistics about an item
- Primary: Item Difficulty
- Often: Item Discrimination
- Sometimes: Guessing
- Lots of other “ugly looking numbers”

# The Item Characteristic Curve



# Differential Item Functioning (DIF)

- Does an item have different item parameters for different subgroups?
- Gender
- Race
- Age
- Disease

# The Three Main IRT Models

- Rasch model one parameter logistic model (1PL)
- Two parameter logistic model (2PL)
- Three parameter logistic model (3PL)

**How to choose an appropriate IRT Model**

**OR**

**My religion is better than your religion!**

**WARNING!**

You are about to see mathematical formulas!

# One Parameter Logistic Model

$$P_{1,0} = \frac{e^{(\text{ability} - \text{difficulty})}}{1 + e^{(\text{ability} - \text{difficulty})}}$$

When the difficulty of a given item exactly matches the Examinee's ability level, then the person has 50% chance of answering that item correctly:

$$P_{1,0} = \frac{e^{(0)}}{1 + e^{(0)}} = \frac{1}{2} = .50$$



# One Parameter Logistic Model

- Only option for small sample sizes
- Often the real model underlying a test labeled as three parameter
- Less costly
- “The simple solution is always the best”

# Two Parameter Logistic Model

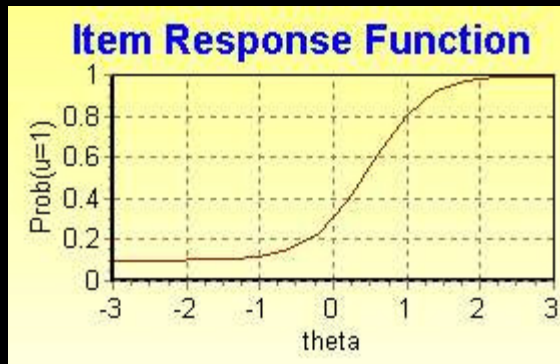
$$P_{1,0} = \frac{e^{a(\text{ability} - b)}}{1 + e^{a(\text{ability} - b)}}$$

Two parameters

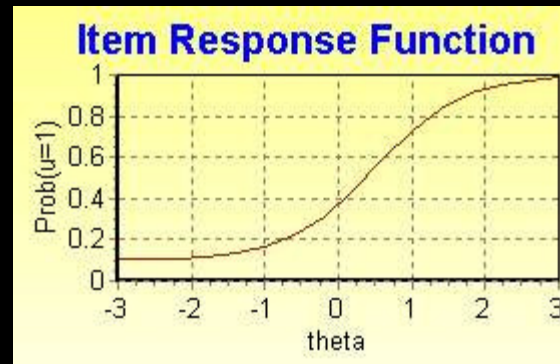
a=Discrimination

b=Item Difficulty

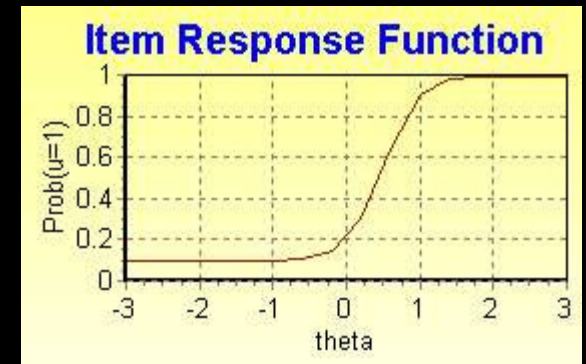
# Two Parameter Examples



$$a=0.5, b=0.5$$



$$a=1.5, b=0.5$$



$$a=2.5, b=0.5$$

# Three Parameter Logistic Model

$$P_{1,0} = c + (1-c) \frac{e^{a(\text{ability} - b)}}{1 + e^{a(\text{ability} - b)}}$$

Three parameters

a= Discrimination

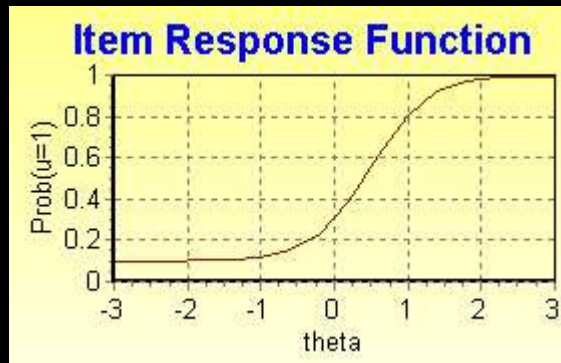
b= Item Difficulty

c= Guessing

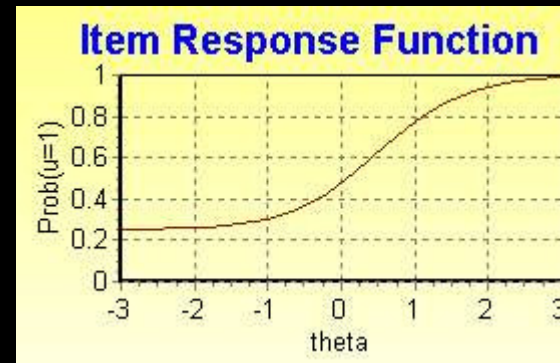
# Three Parameter Logistic Model (3PL)

- Requires a large sample size
- Significant research demonstrating that theoretically 3PL is better, but practically has little advantage over 1PL
- “Most accepted theoretical model”

# Three Parameter Examples



$$a=1.5, b=.5, c=.1$$



$$a=2.5, b=.5, c=.25$$

# Polytomous Models

## One Parameter

- Rating Scale Model
- Partial Credit Model

## Two Parameter

- Graded Response Model
- Generalized Partial Credit Model

# Multi-dimensional Models

There are also IRT models which consider more than one unidimensional trait at a time



How does IRT differ from conventional test theory?

# Classical Test Theory

- An individual takes an assessment
- Their total score on that assessment is used for comparison purposes
- High Score – The person is higher on the trait
- Low Score-The person is lower on the trait

# Item Response Theory

- Each individual item can be used for comparison purposes
- Person endorses better rating on “hard items” -  
The person is higher on the trait
- Person endorses worse rating on “easy items” -  
The person is lower on the trait
- Items that measure the same construct can be aggregated into longer assessments

# Reliability

## CTT

- Reliability is based upon the total test.
- Regardless of patient “ability”, reliability is the same.

## IRT

- Reliability is calculated for each patient “ability” and varies across the continuum.
- Typically, there is better reliability in the middle of the distribution.

# Validity

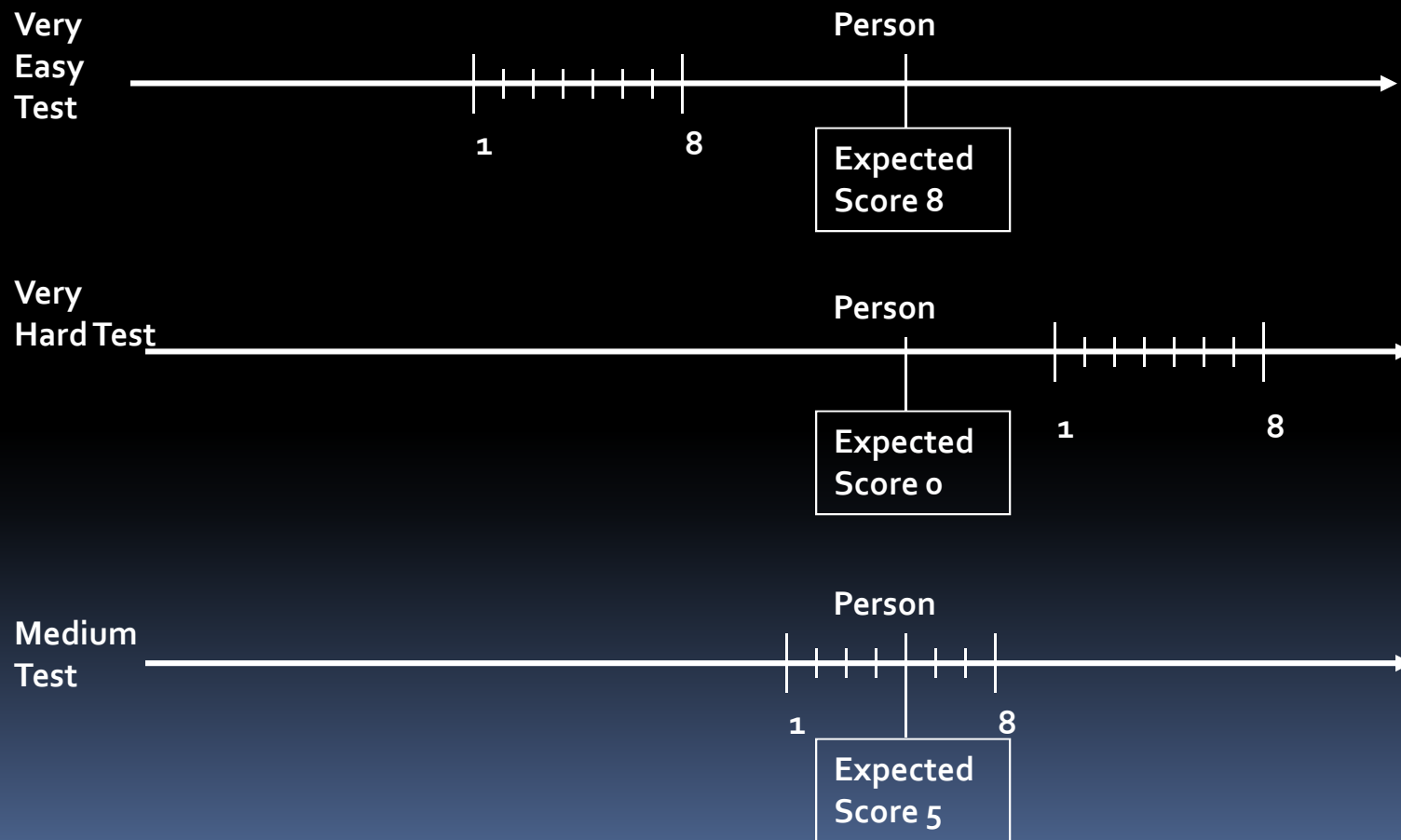
## CTT

- Validity is based upon the total test.
- Typically, validity would need to be re-assessed if the instrument is modified in any way.

## IRT

- Validity is assessed for the entire item bank.
- Subsets of items (full length tests, short forms and CAT) **all inherit the validity assessed for the original item bank.**

# How Scores Depend on the Difficulty of Test Items



# Raw Scores vs. IRT Measures

## IRT has Equal Interval Measurement

### 4 Item Test

Raw:



Logit Measures:



# I Have a Lack of Energy

## Traditional Test Theory



4 = Not at All

3 = A Little Bit

2 = Somewhat

1 = Quite a Bit

0 = Very Much



# I Have a Lack of Energy

## Traditional Test Theory



4 = Not at All    3 = A Little Bit    2 = Somewhat    1 = Quite a Bit    0 = Very Much

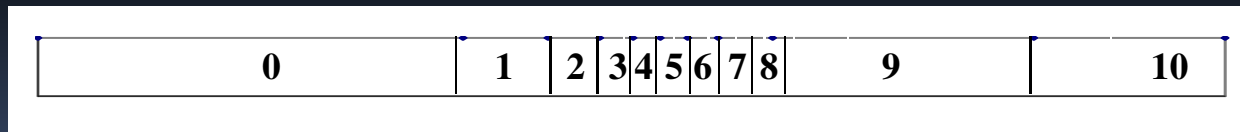
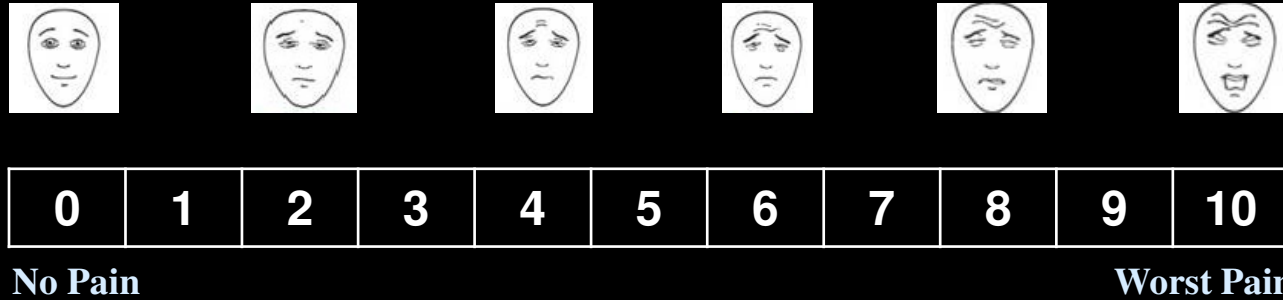
## Item Response Theory



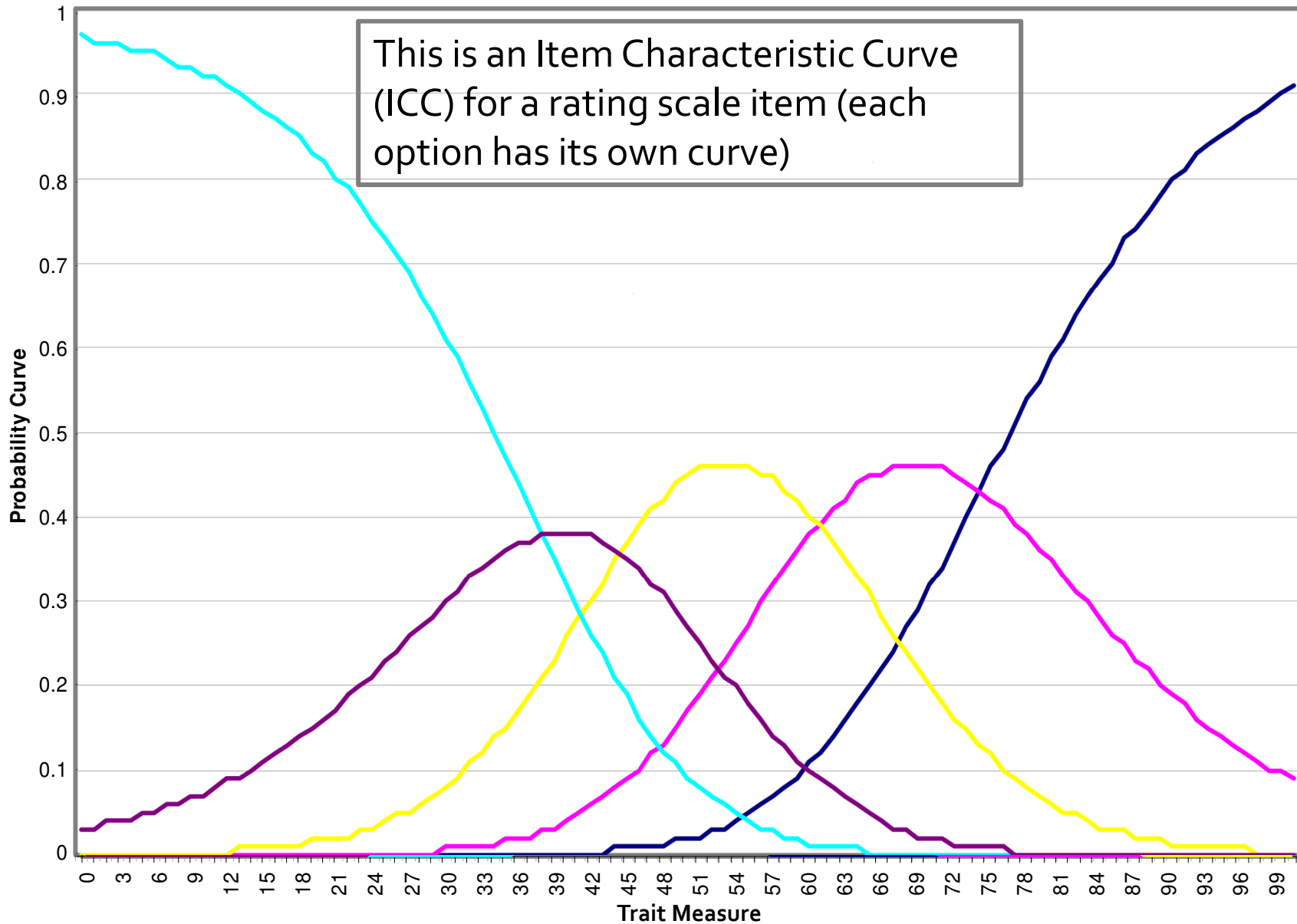
0    3    6    9    12    15    18    21    24    27    30    33    36    39    42    45    48    51    54    57    60    63    66    69    72    75    78    81    84    87    90    93    96    99

Trait Measure

# The IRT “Reality” of a 10 Point Rating-Scale Item

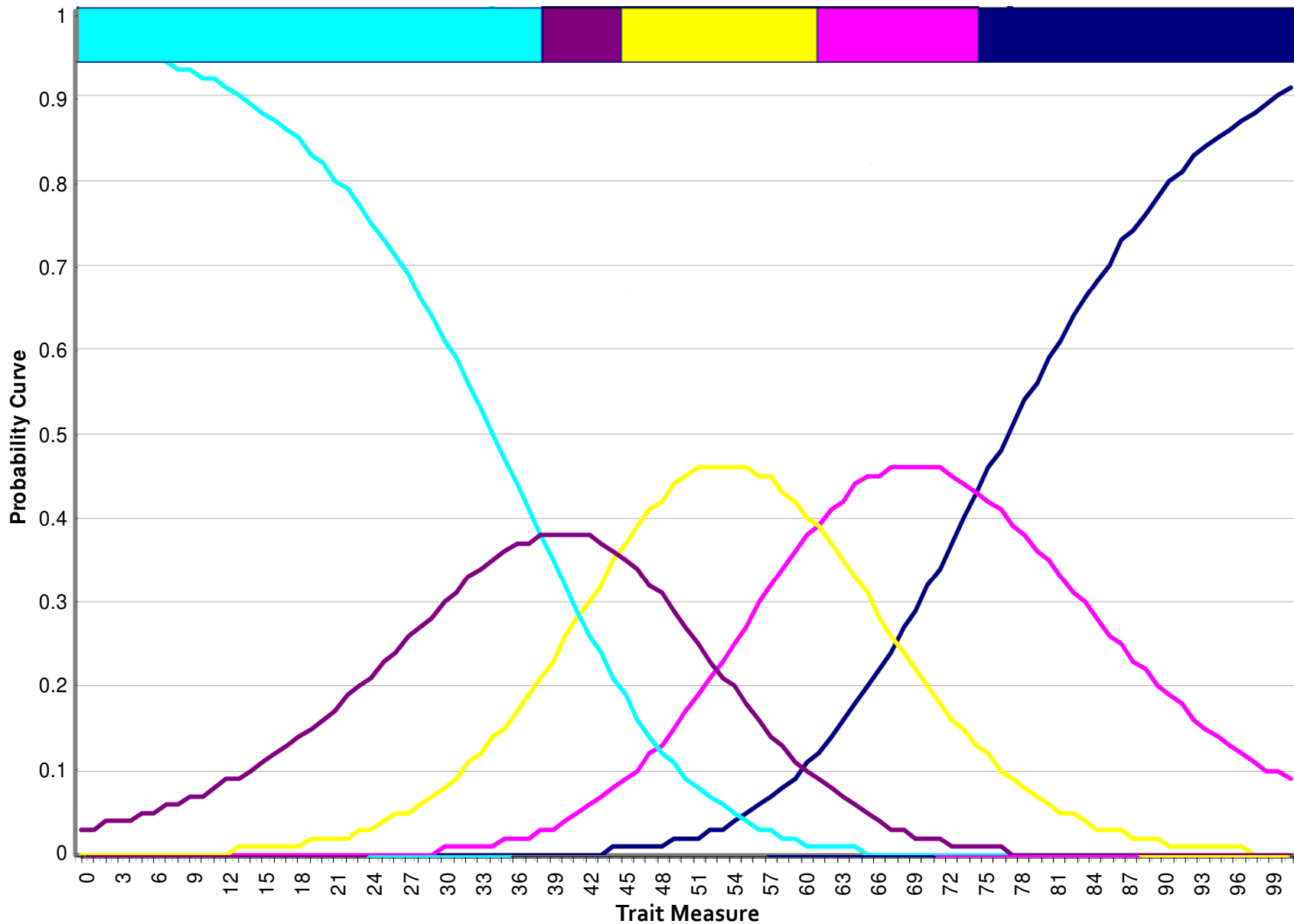


# I have a lack of energy



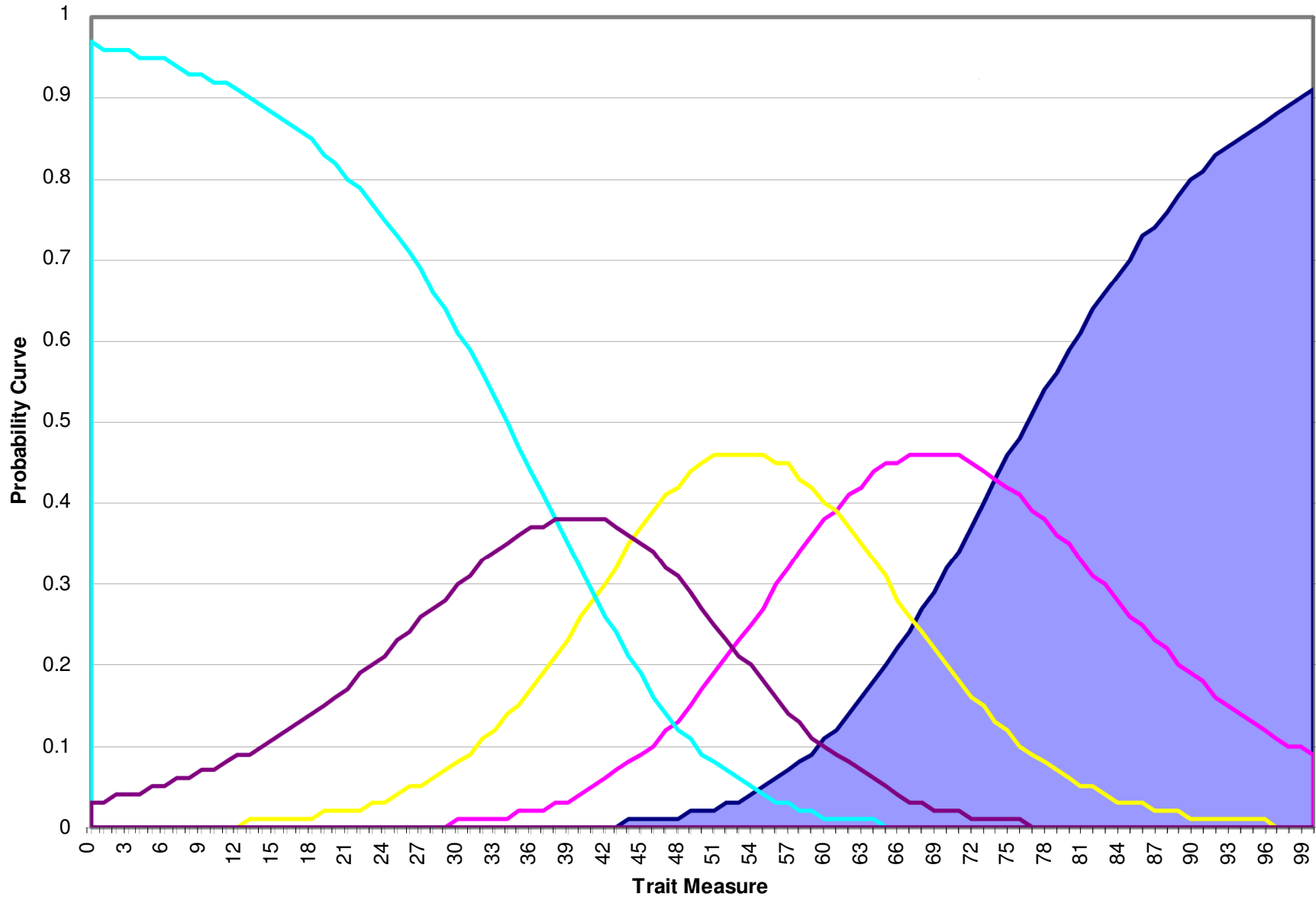
4 = Not at All 3 = A Little Bit 2 = Somewhat 1 = Quite a Bit 0 = Very Much

# I have a lack of energy



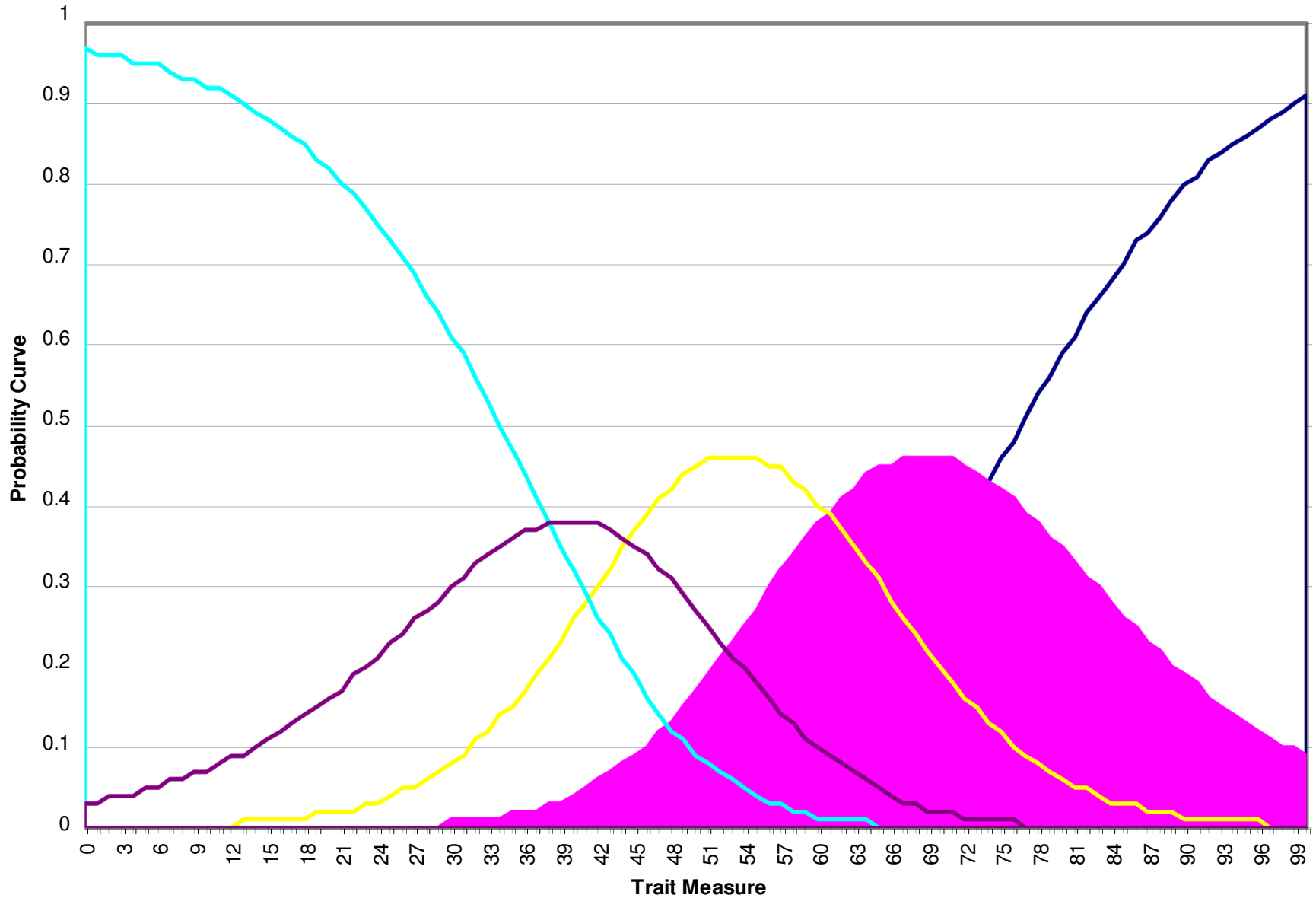
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# I have a lack of energy



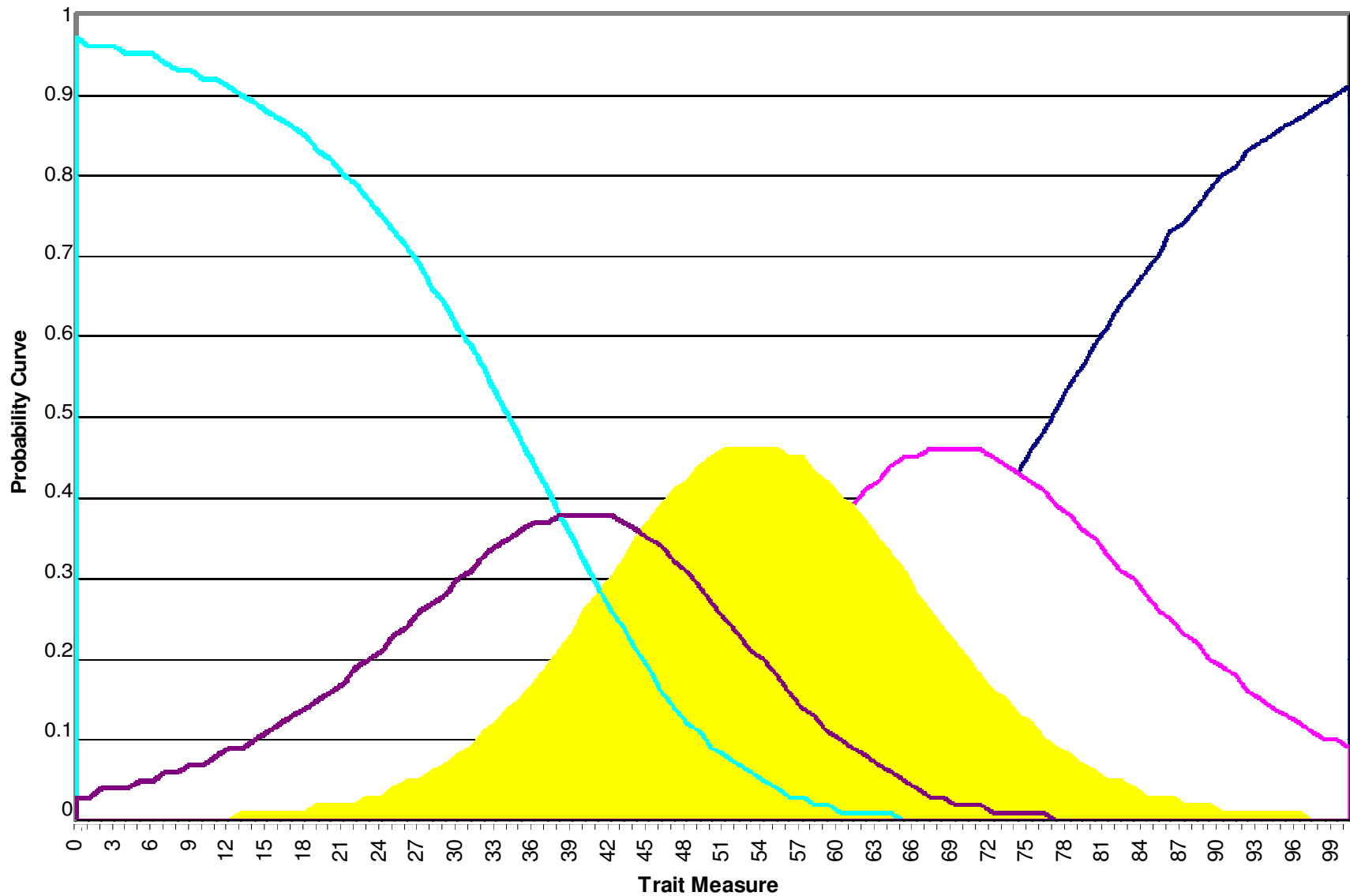
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# I have a lack of energy



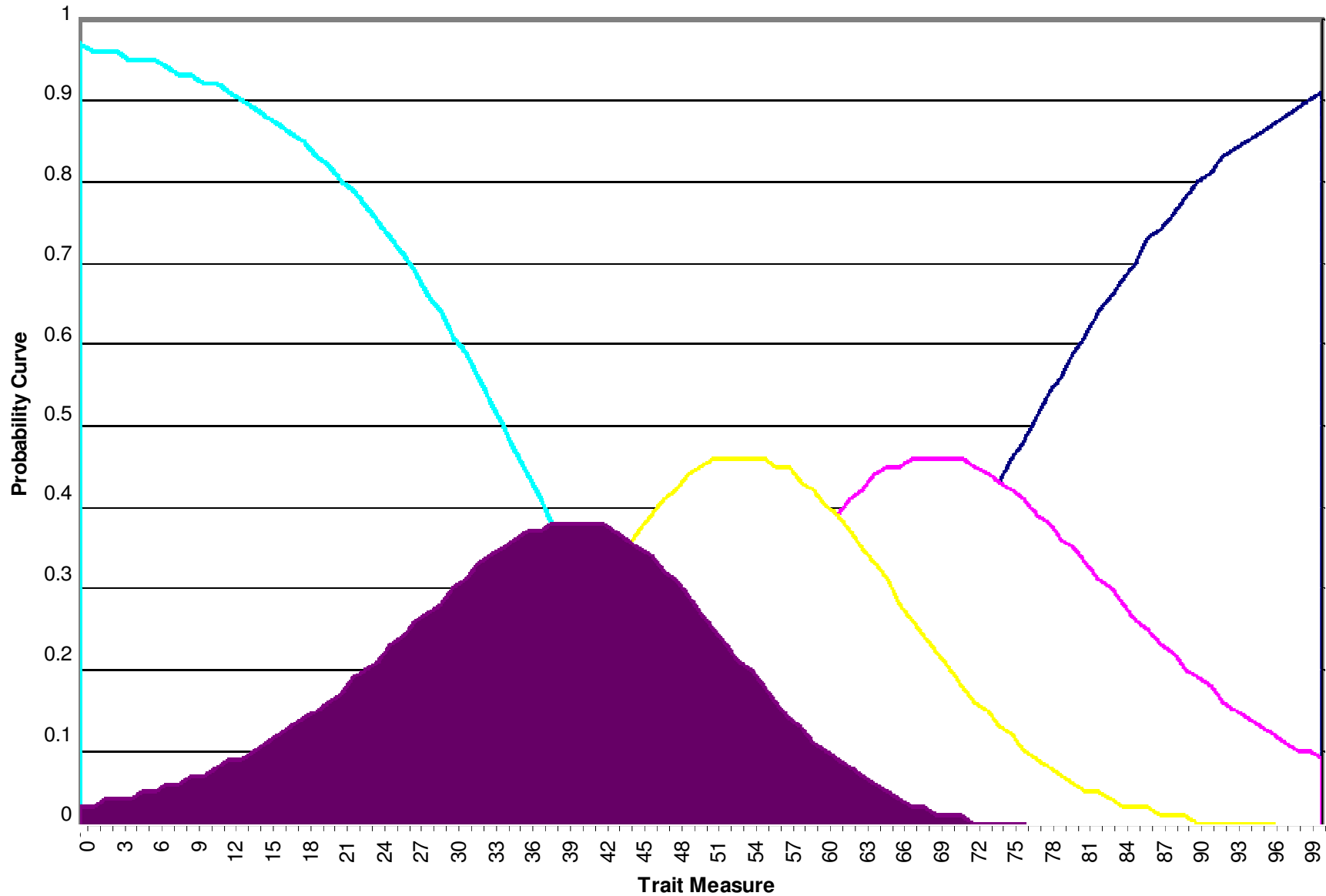
4 = Not at All 3 = A Little Bit 2 = Somewhat 1 = Quite a Bit 0 = Very Much

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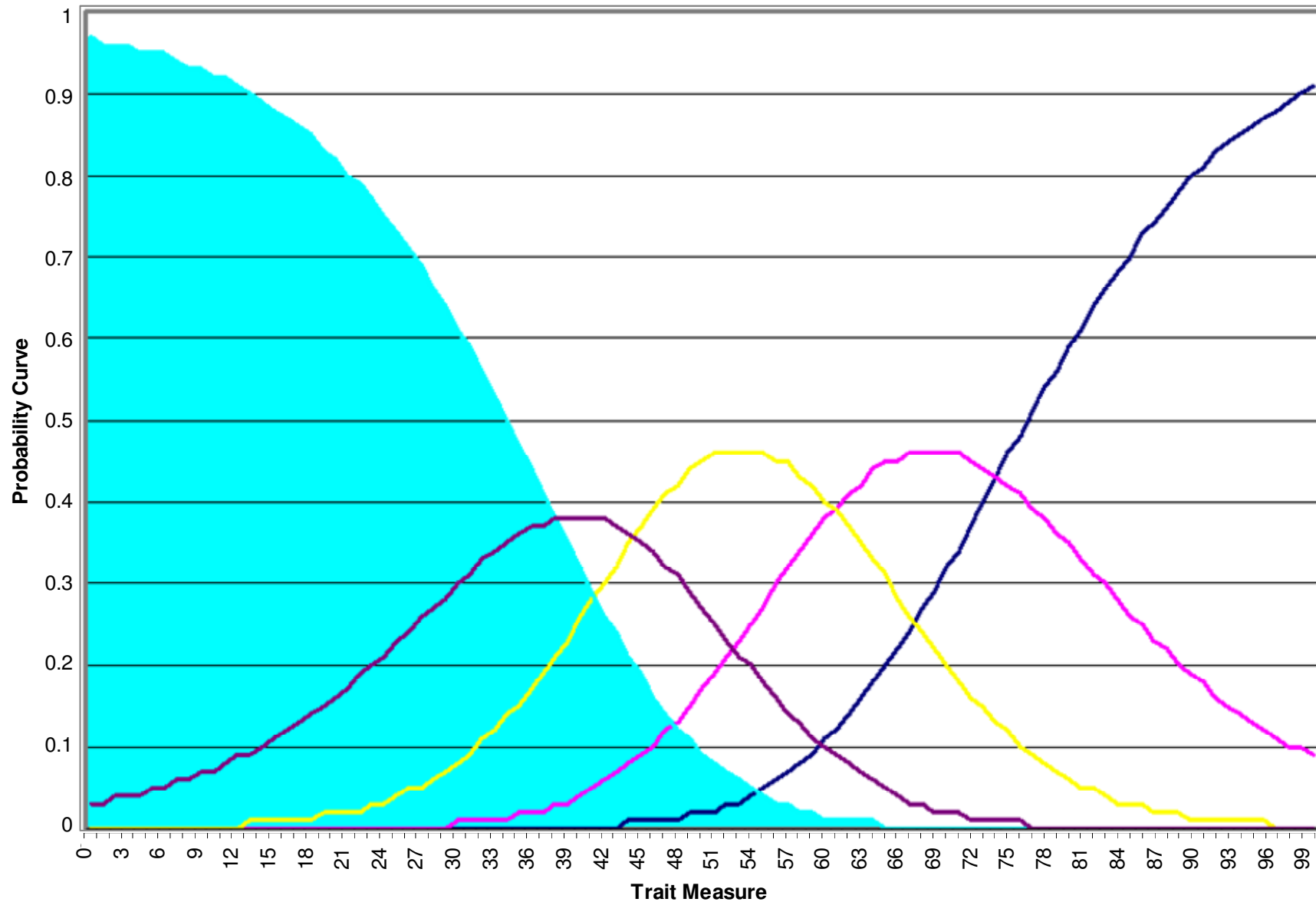
# I have a lack of energy



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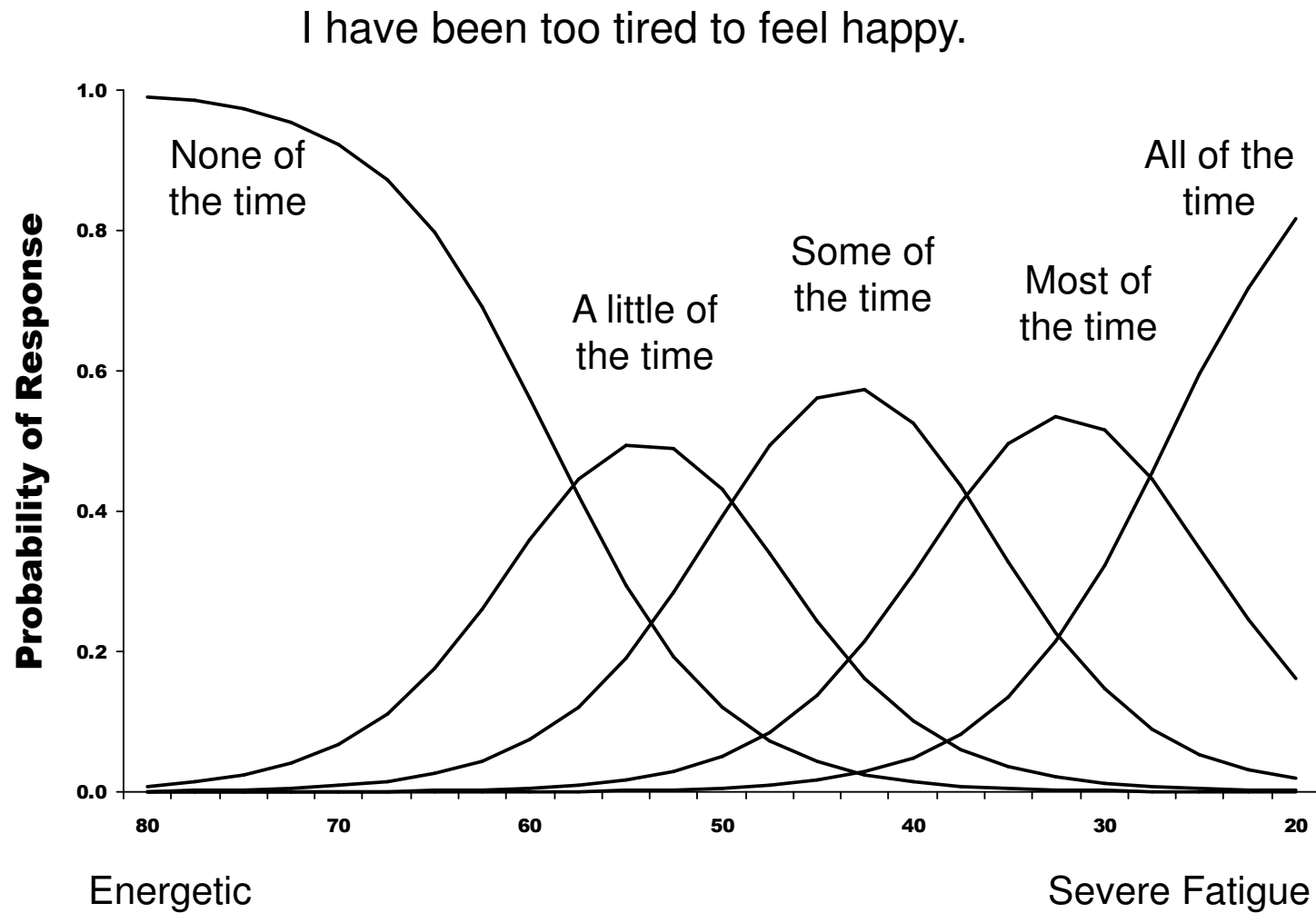


# I have a lack of energy

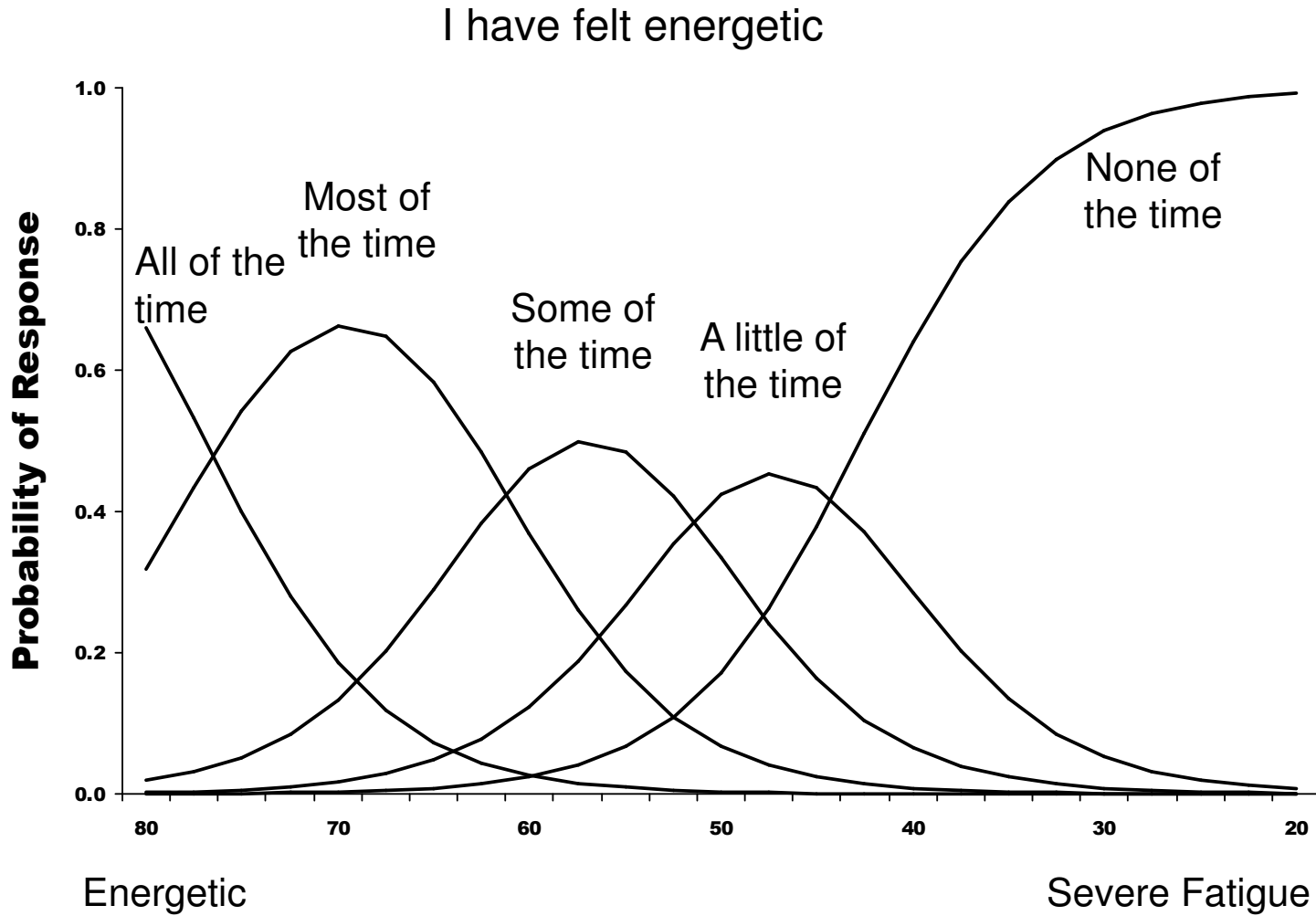


4 = Not at All 3 = A Little Bit 2 = Somewhat 1 = Quite a Bit 0 = Very Much

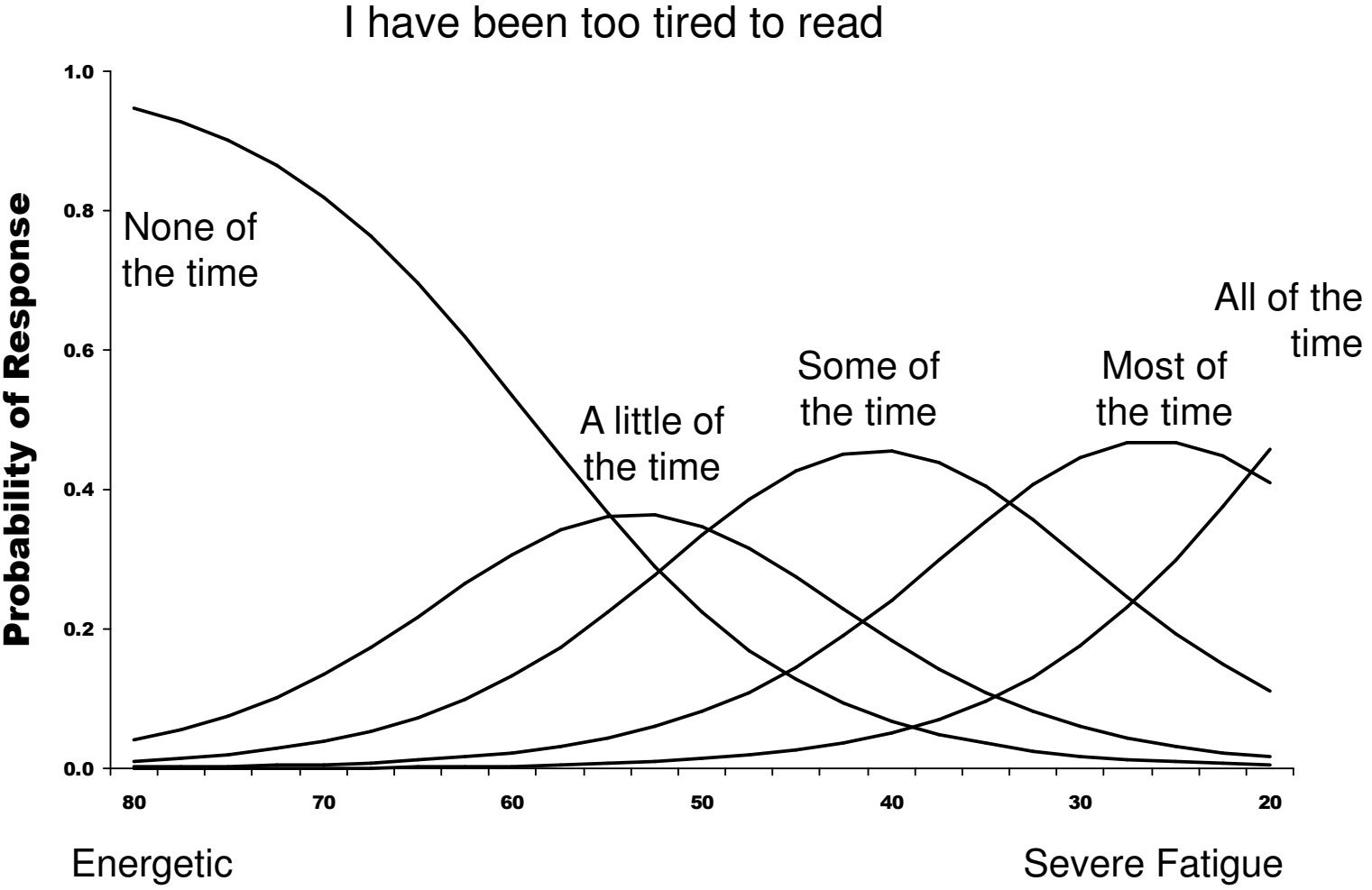
# IRT Polytomous Responses



# IRT Polytomous Responses

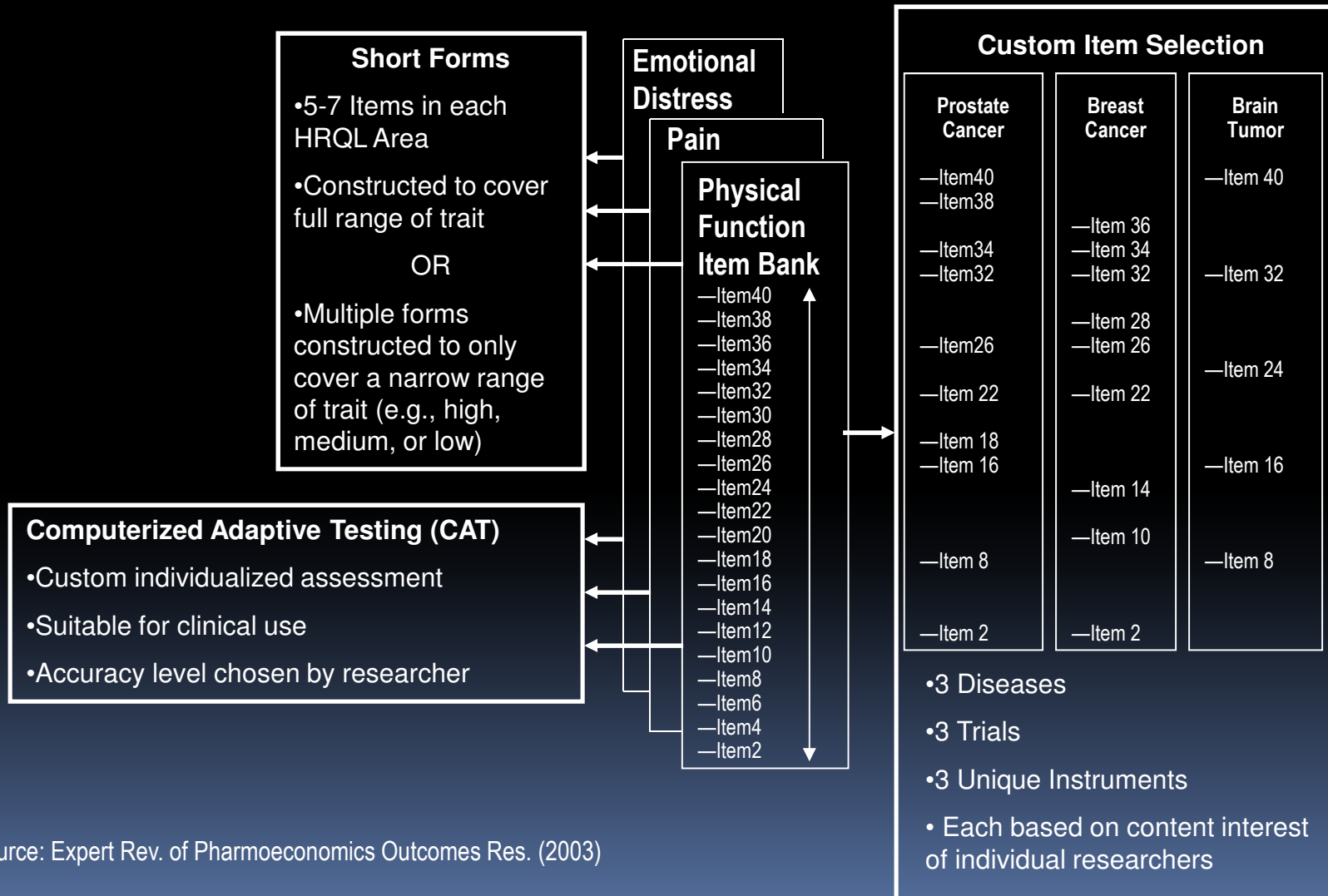


# IRT Polytomous Responses



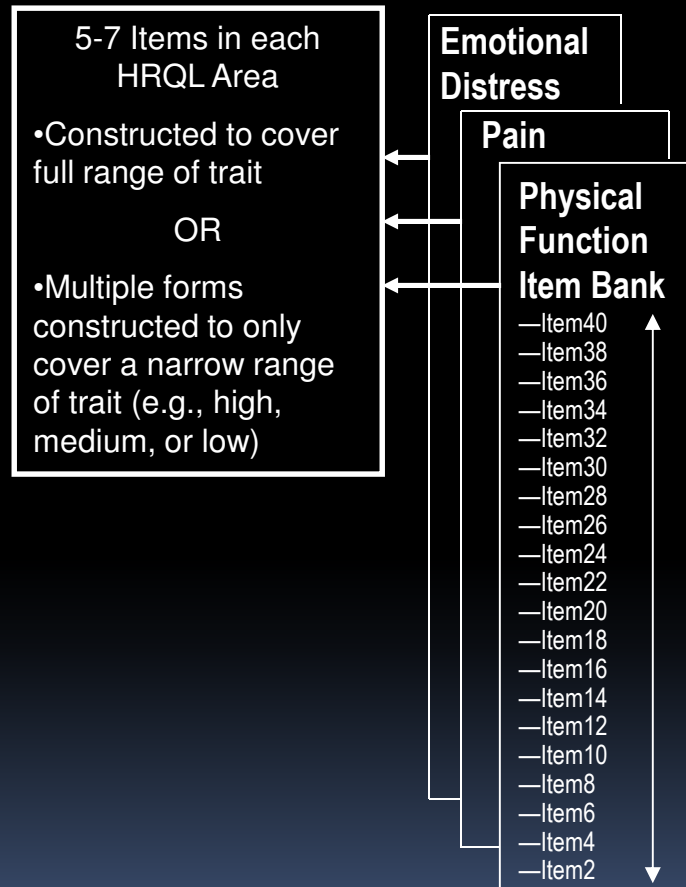
# Item Banking

# Calibrated Item Banks can be used to Create Numerous Instrument Types



Source: Expert Rev. of Pharmacoeconomics Outcomes Res. (2003)

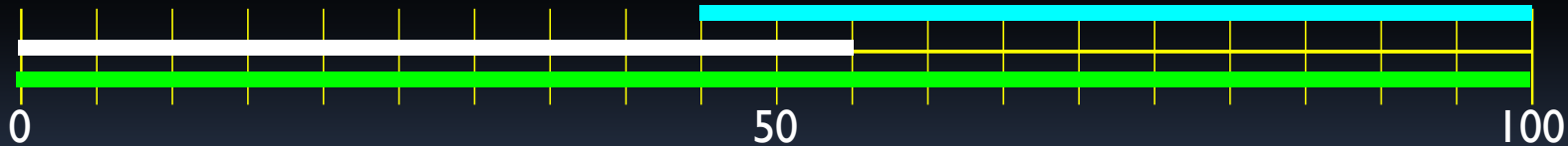
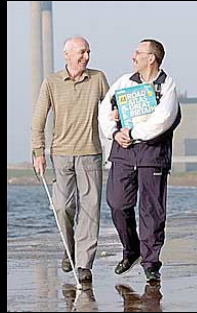
# Short Forms



Physical Function  
Form C

Physical Function  
Form A

Physical Function  
Form B

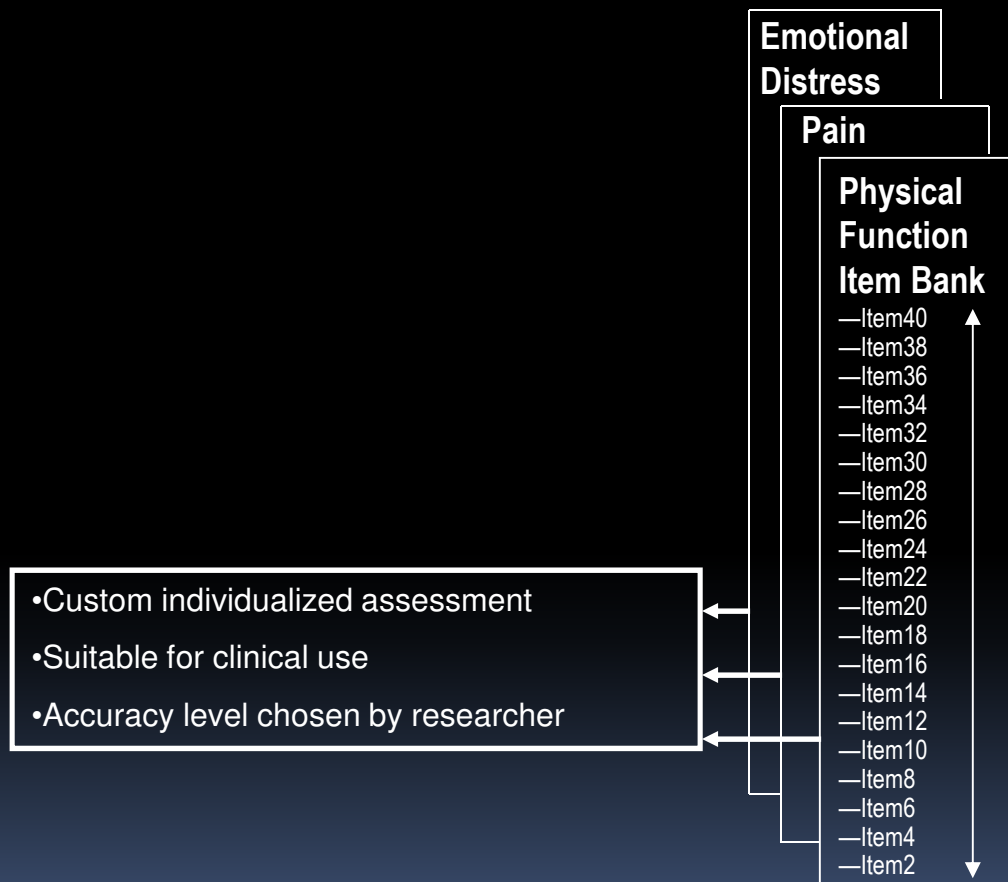


Physical Functioning Item Bank

- Item 1
- Item 2
- Item 3
- Item 4
- Item 5
- Item 6
- Item 7
- Item 8
- Item 9
- Item 10
- Item 11
- Item 12
- Item 13
- Item 14
- Item 15
- Item 16

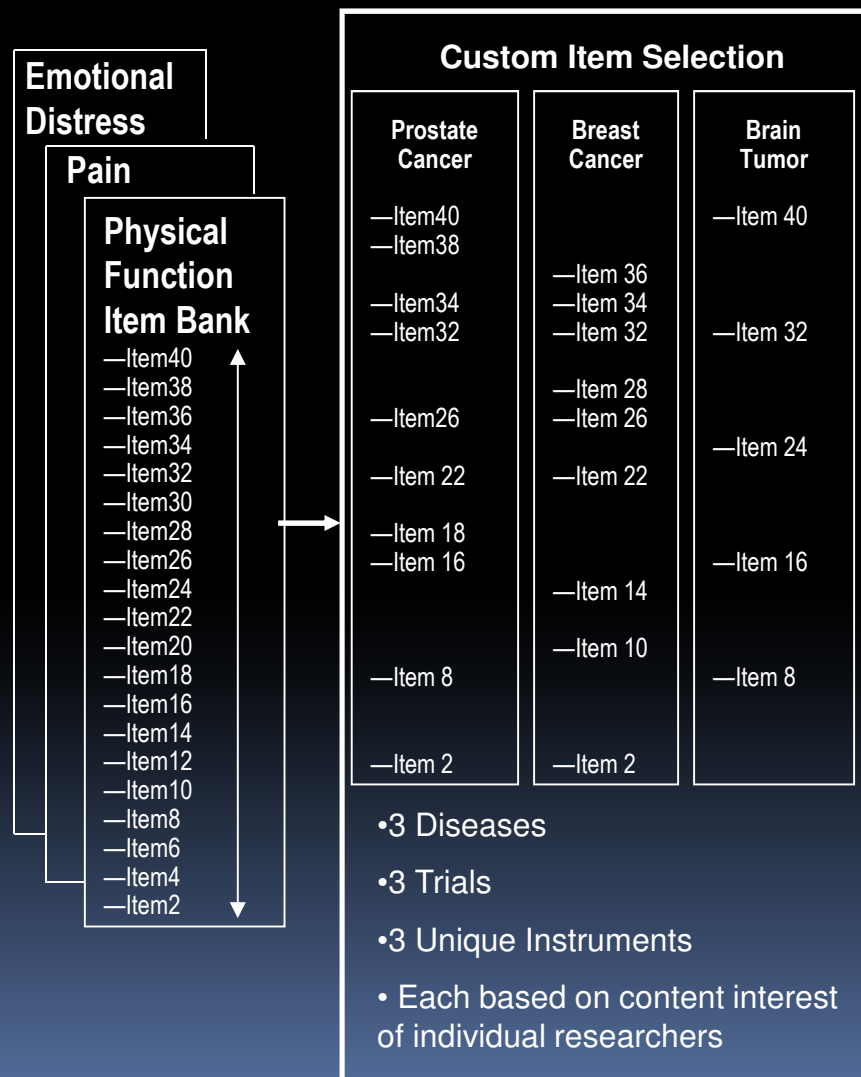


# Computerized Adaptive Testing (CAT)



Source: Expert Rev. of Pharmacoeconomics Outcomes Res. (2003)

# Custom Item Selection



Source: Expert Rev. of Pharmacoeconomics Outcomes Res. (2003)

In Summary,

Calibrated Item Banks can be used to:

- Create a standard static instrument
- Construct short forms
- Enable CAT
- Select items based on unique content interests and formulate custom short-form or full-length instruments

In every case, using a validated, pre-calibrated item bank allows any of these instruments to be pre-validated and produce standardized scores on the same scale

Computerized  
Adaptive  
Testing

# What is Computerized Adaptive Testing?

- Shorter
- Targeting
- Computerized Algorithm

# CAT in the Military

- Armed Services Vocational Aptitude Battery (ASVAB)



# CAT for Certification

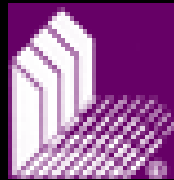
The logo for the American Association of Nurse Anesthetists (AANA), featuring the letters "AANA" in a teal, serif font.

American Association of Nurse Anesthetists

The logo for CompTIA, featuring the word "CompTIA" in white on a blue background with a stylized blue swoosh.The Microsoft logo, featuring the word "Microsoft" in white on a blue background.



# CAT for Licensure



AMERICAN DIETETIC ASSOCIATION



National Council  
of State Boards of Nursing, Inc.


# CAT for College Entrance

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GRADUATE MANAGEMENT ADMISSION COUNCIL

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ACCUPLACER OnLine

toefl.org  
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Graduate Record Examinations®

# CAT for Education

Northwest Evaluation Association



Renaissance Learning™

Low  
Able

Pass  
Point

High  
Able



**PASS!**

Low  
Able

Pass  
Point

High  
Able

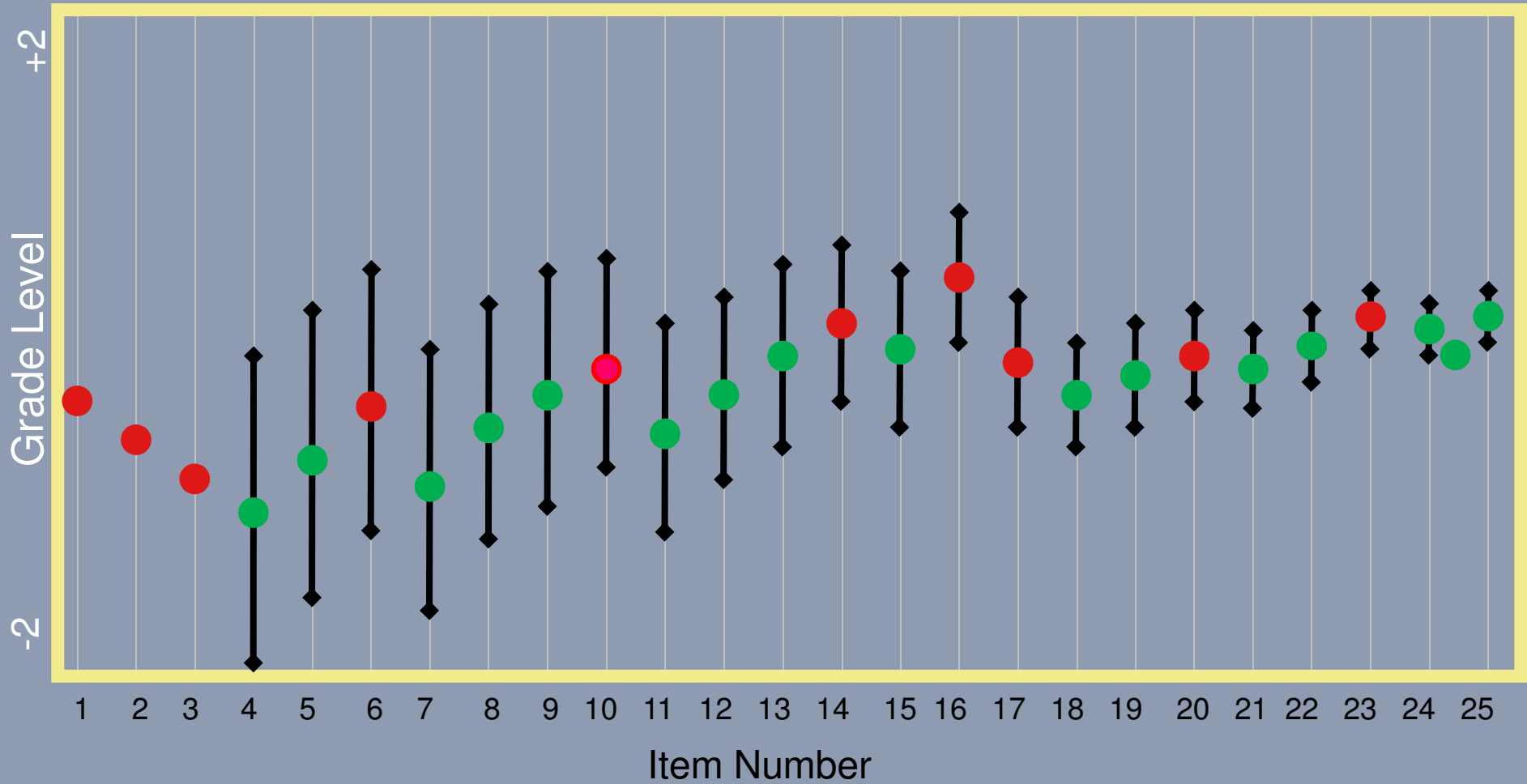


**FAIL**

# Example – Binary Search

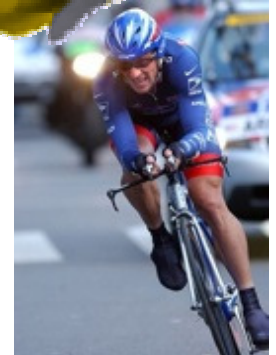
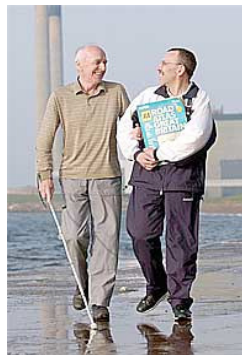
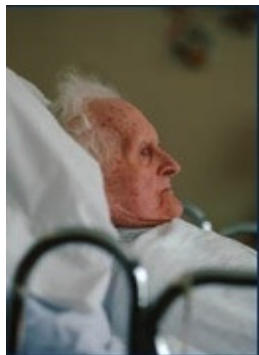
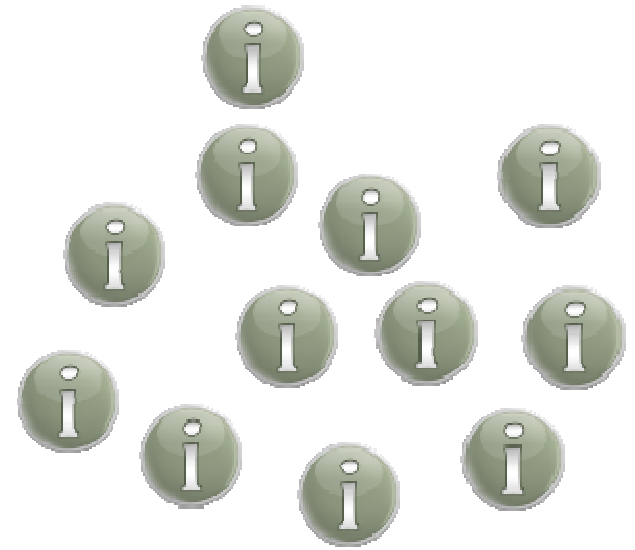
- Binary search

With each successive item,  
Standard error decreases

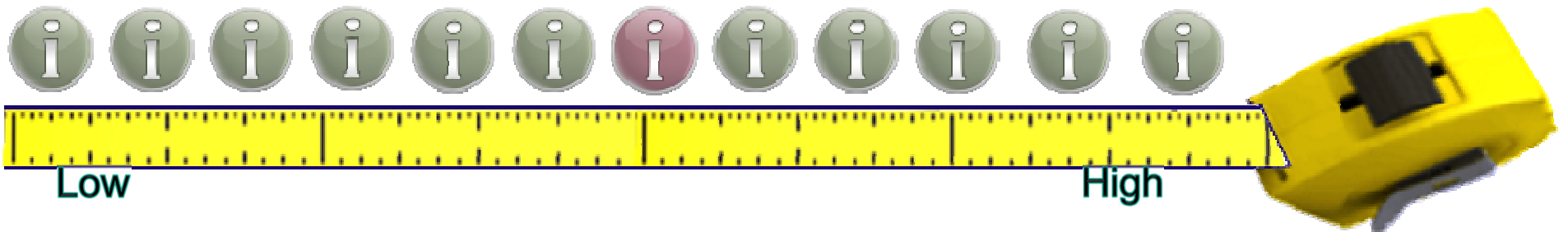




Physical Function

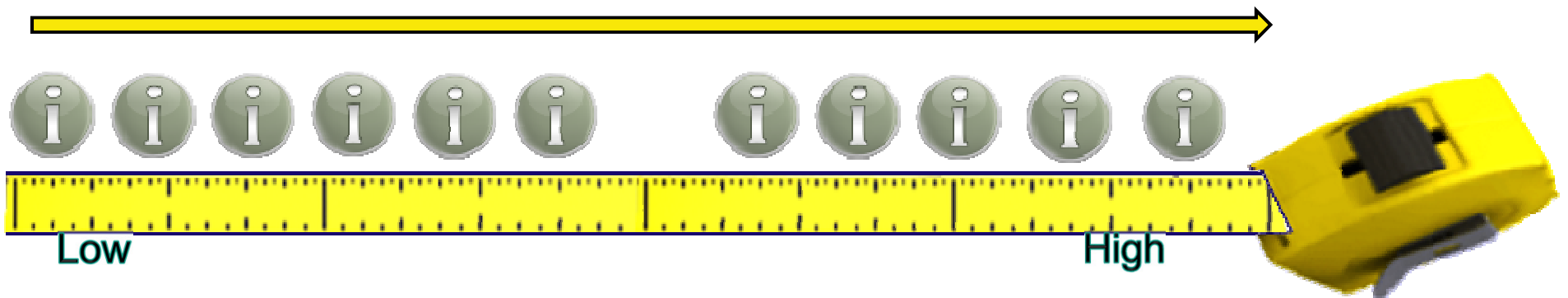








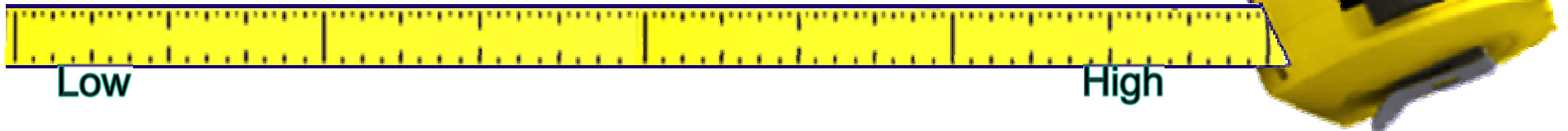
$$\ln L(u|\theta) = \sum [u_i \ln P_i + (1-u_i) \ln Q_i]$$

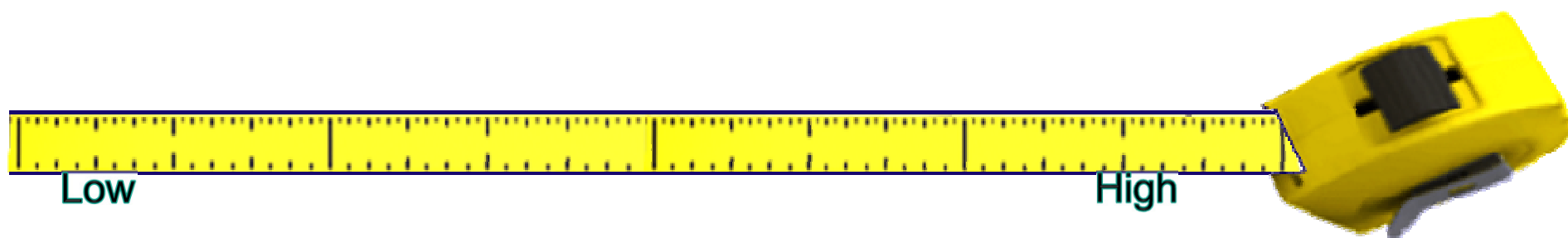


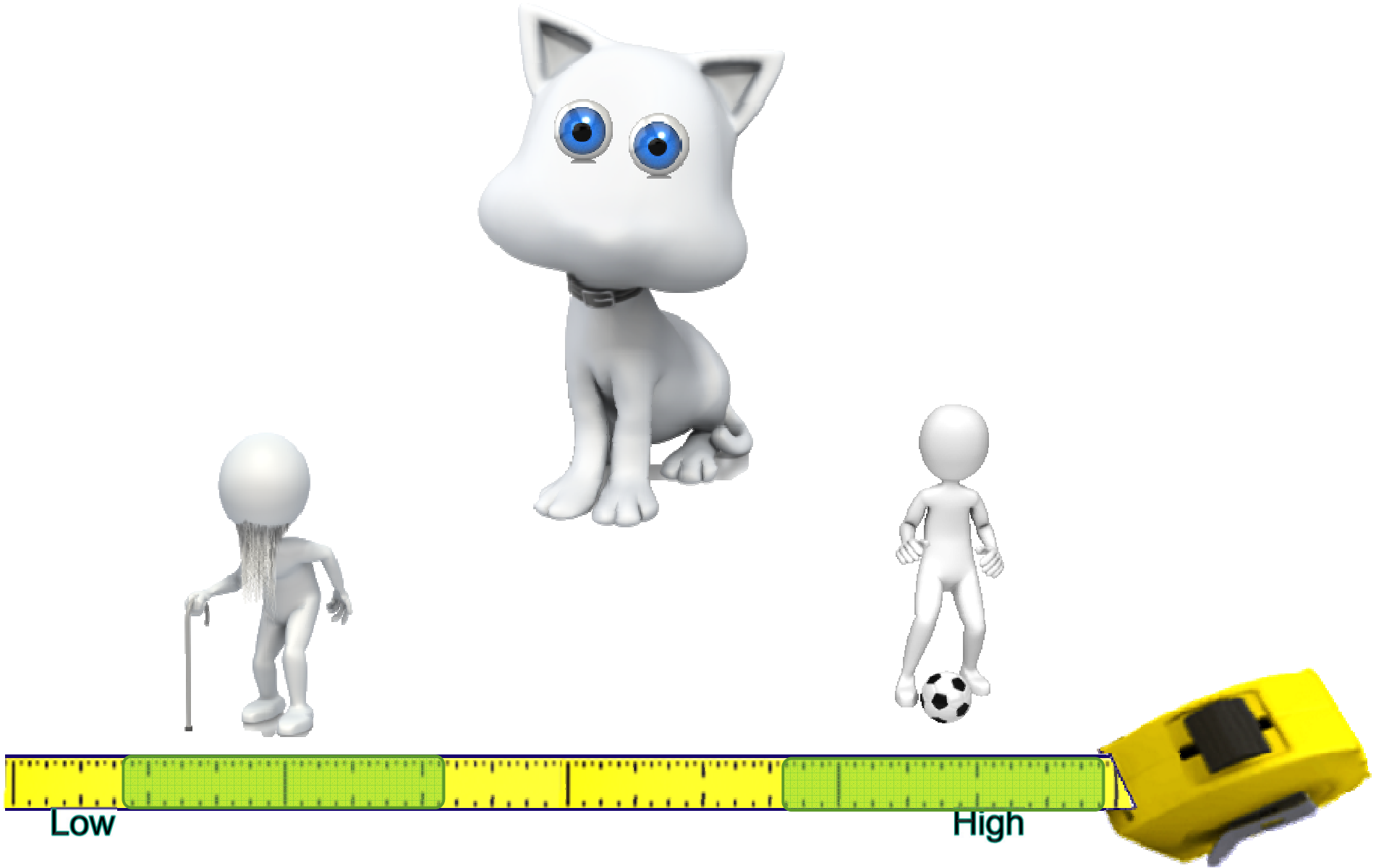


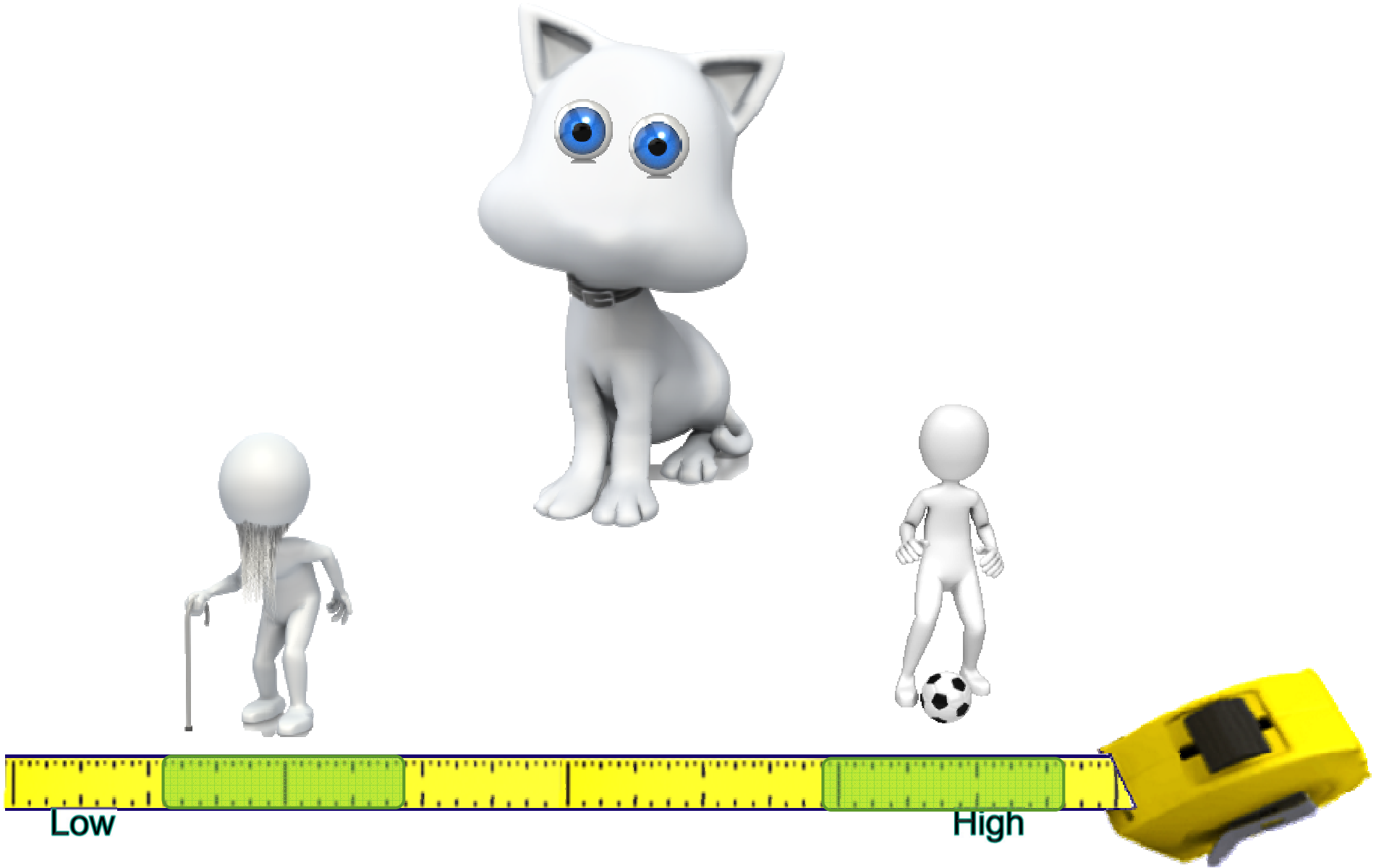
Low

High



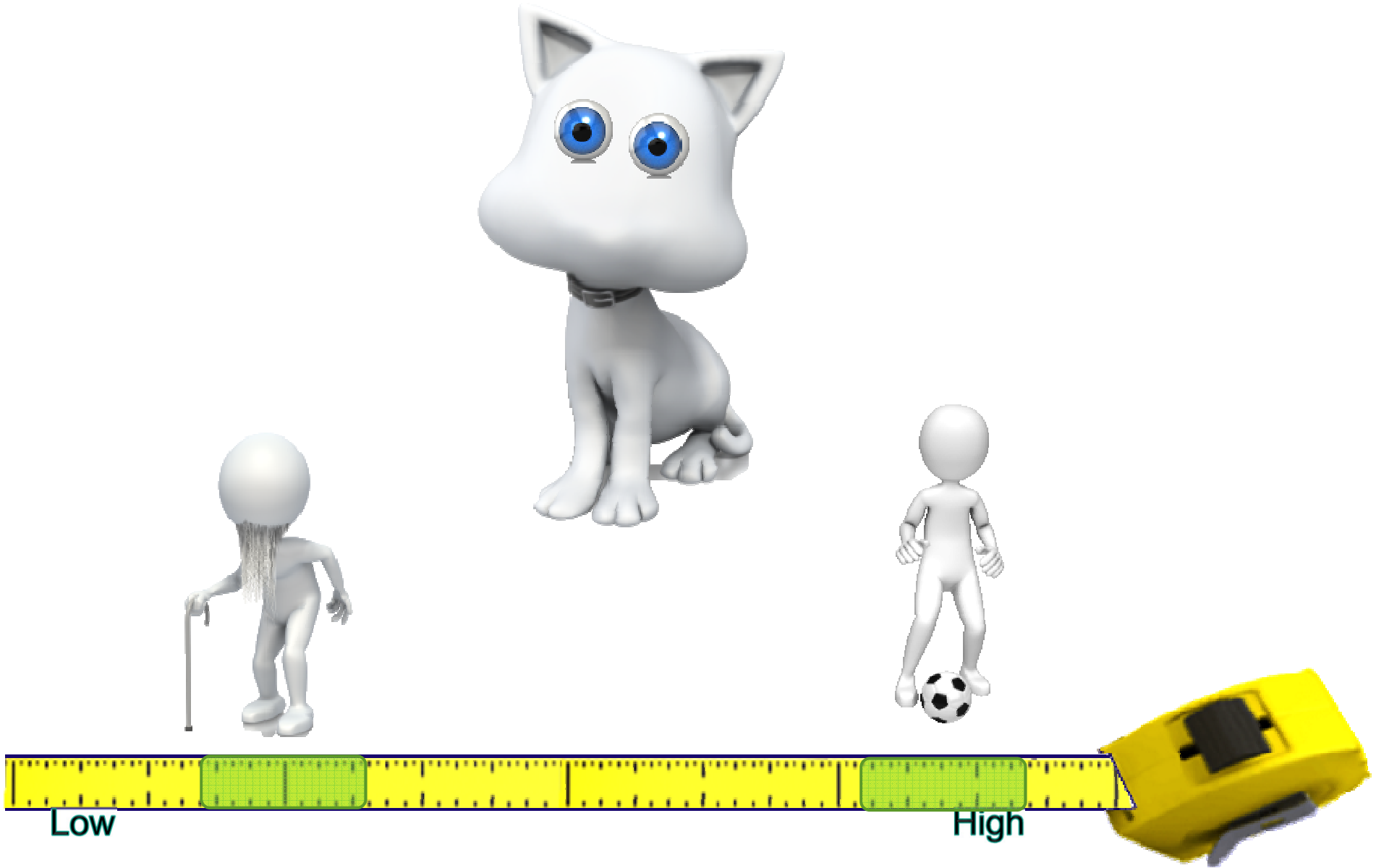






Low

High



Low

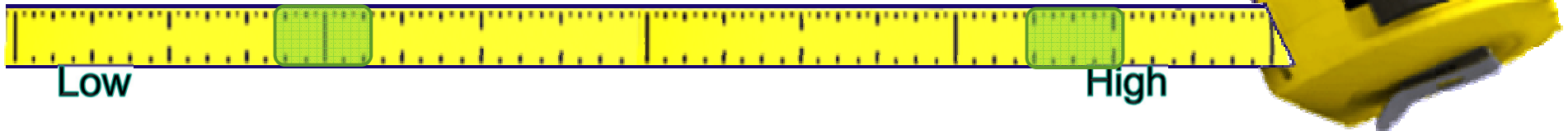
High





Specified # of items

Specified level of precision



Why bother?



Reduce burden of responding



Make room for measuring more domains



# CAT Requirements

- Calibrated item bank
- Administration software

# Test Specifications

- Starting rule
- With item which provides maximum information
- At cut point

# Test Specifications

- Stopping Rule
  - Fixed length
  - Variable length
  - By Total Test/Subtest
  - Calculated
    - Specified precision of measure
    - Specified confidence in a pass/fail decision
  - Maximum item count
  - Minimum item count

# Adaptive Algorithm

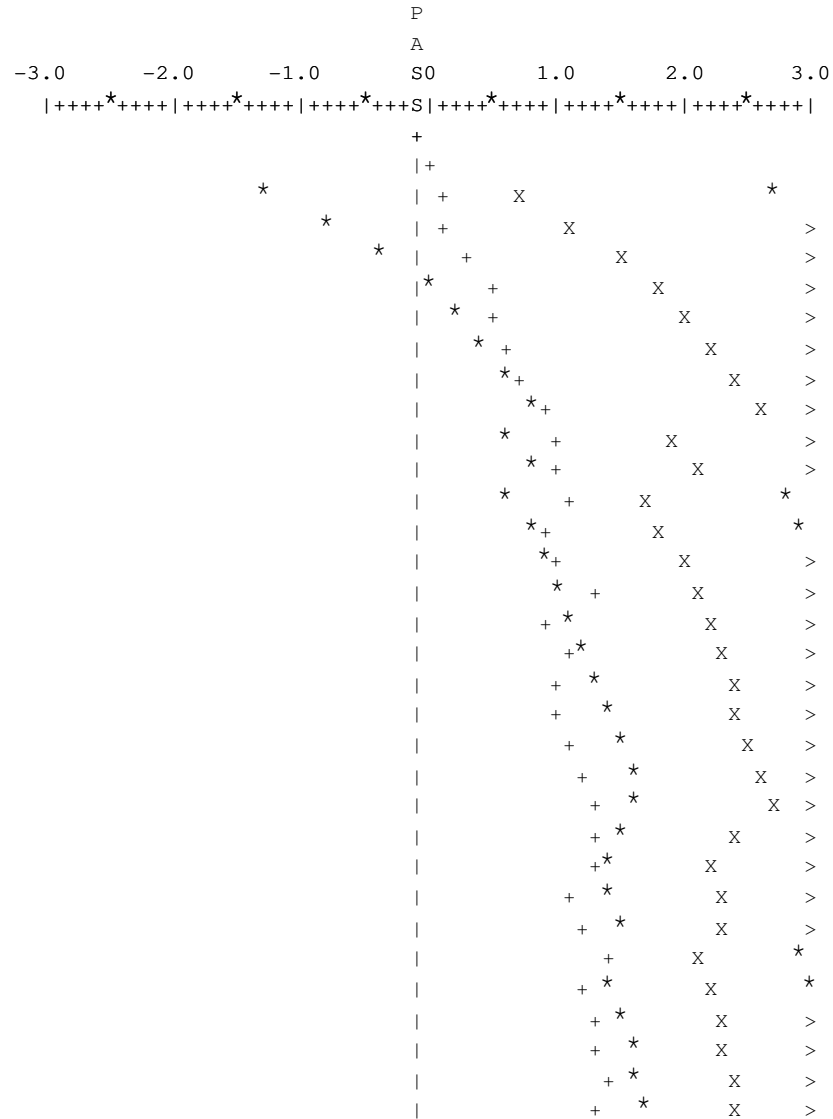
- Person ability algorithm
- Item selection algorithm
  - Test difficulty
  - Maximum jump size
  - Content issues
  - Item exposure control
  - Option to not allow same items to be used during retesting
  - Overlapping items (items that cue other items)

# Clear Pass

339585909 Entry= 1  
MLT Ver: 10/01/01

Tested: 01/28/02  
Status: 2

Item	AN	Cont	Diff	Ans	⊙ =	Time !	Meas	SE
1	21151	BBN	-0.09	1	1	0	2'30	9.99 9.99
2	22805	CHE	0.03	2	1	0	2'56	9.99 9.99
3	22479	HEM	0.13	4	0	0	0'36	0.72 1.22
4	21986	MIC	0.13	3	1	0	0'29	1.15 1.15
5	22397	IMM	0.26	1	1	0	0'10	1.48 1.12
6	21793	UA	0.46	4	1	0	0' 9	1.76 1.10
7	22504	BBN	0.50	3	1	0	0'56	1.99 1.08
8	22083	CHE	0.57	4	1	0	0'22	2.19 1.07
9	22641	HEM	0.74	4	1	0	0'59	2.38 1.06
10	20194	MIC	0.90	2	1	+	3'17	2.56 1.05
11	22032	BBN	1.00	4	0	0	1'26	1.92 0.78
12	20344	CHE	1.00	4	1	0	1' 0	2.08 0.77
13	22261	HEM	1.12	4	0	0	1' 9	1.72 0.66
14	21851	MIC	0.94	4	1	0	1'15	1.85 0.65
15	21511	IMM	1.02	1	1	0	2'14	1.97 0.65
16	21450	UA	1.27	1	1	+	1'17	2.09 0.64
17	20537	BBN	0.93	3	1	0	0'35	2.18 0.64
18	22330	CHE	1.12	2	1	+	2'32	2.28 0.63
19	21218	HEM	1.02	1	1	0	0'37	2.36 0.63
20	21628	MIC	0.96	3	1	0	1' 3	2.44 0.63
21	22748	BBN	1.07	1	1	0	2'10	2.51 0.62
22	22553	CHE	1.22	3	1	0	0'31	2.59 0.62
23	22639	HEM	1.28	1	1	0	0'57	2.66 0.62
24	22646	MIC	1.35	2	0	=	2'44	2.40 0.55
25	22663	IMM	1.27	1	0	0	1'17	2.19 0.50
26	22557	UA	1.06	2	1	0	0'41	2.25 0.50
27	20686	BBN	1.15	1	1	0	0'27	2.31 0.50
28	22634	CHE	1.37	3	0	0	1'19	2.15 0.46
29	21646	HEM	1.16	2	1	0	0'15	2.20 0.46
30	22387	MIC	1.31	4	1	0	0'23	2.26 0.46
31	20018	BBN	1.27	3	1	0	0'34	2.31 0.45
32	22059	CHE	1.40	1	1	0	0'48	2.37 0.45
33	22471	HEM	1.34	1	1	0	0'41	2.42 0.45

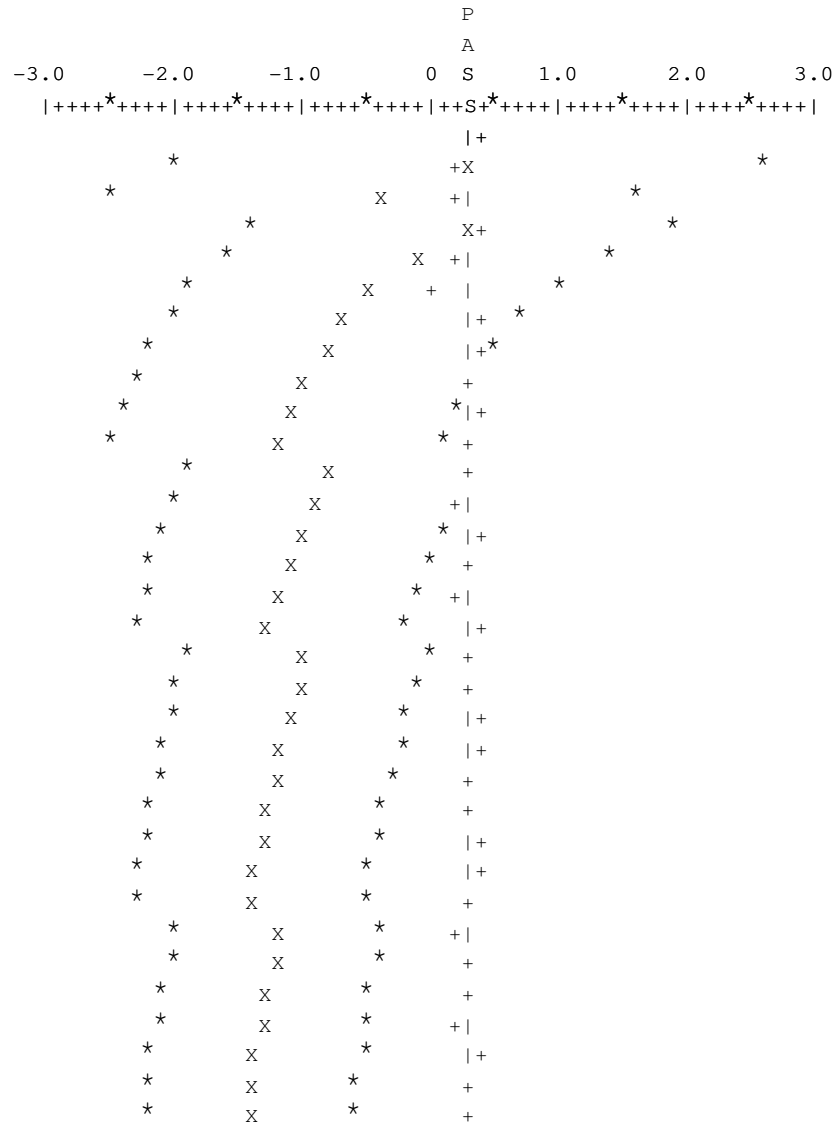


434843789 Entry= 1  
 HT Ver: 01/01/02

Clear Fail

Tested: 01/28/02  
 Status: 1

Item	AN	Cont	Diff	Ans	=	Time	!	Meas	SE
1	31384	ST	0.35	1	0	0'34		-9.99	9.99
2	31009	FIX	0.22	3	1	0'18		0.29	1.41
3	31113	LO	0.18	1	0	0'26		-0.44	1.22
4	30385	MIC	0.36	3	1	0'33		0.28	1.00
5	30873	ST	0.24	3	0	0'31		-0.14	0.91
6	30533	PRO	0.05	2	0	0'30		-0.46	0.87
7	30525	ST	0.35	2	0	0'16		-0.67	0.84
8	31008	FIX	0.37	4	0	0'31		-0.83	0.82
9	30664	ST	0.30	2	0	0'32		-0.98	0.80
10	31086	LO	0.35	4	0	0'12		-1.11	0.79
11	31626	ST	0.34	2	0	0'23		-1.22	0.78
12	31356	MIC	0.32	4	1	0'41		-0.81	0.67
13	31210	PRO	0.21	2	0	0'35		-0.92	0.66
14	31148	ST	0.39	1	0	0'20		-1.01	0.65
15	31620	FIX	0.25	4	0	0'10		-1.10	0.65
16	30224	ST	0.20	4	0	0'25		-1.19	0.64
17	30940	FIX	0.40	2	0	0'32		-1.25	0.64
18	31288	ST	0.25	3	1	1'14		-0.97	0.57
19	31529	LO	0.28	1	0	0'58		-1.04	0.56
20	31120	ST	0.40	2	0	0'11		-1.10	0.56
21	31355	MIC	0.36	2	0	0'59		-1.15	0.56
22	31207	PRO	0.33	2	0	0'34		-1.21	0.55
23	30745	ST	0.33	4	0	0'33		-1.26	0.55
24	31285	FIX	0.40	3	0	0'13		-1.31	0.55
25	30237	ST	0.39	3	0	0'22		-1.35	0.55
26	30179	ST	0.26	1	0	0'24		-1.40	0.54
27	31055	FIX	0.23	4	1	0'24		-1.18	0.50
28	31598	LO	0.29	2	0	0'33		-1.23	0.49
29	30384	MIC	0.27	3	0	0'11		-1.27	0.49
30	30524	ST	0.20	1	0	0'21		-1.31	0.49
31	31470	PRO	0.38	1	0	0'16		-1.35	0.49
32	30188	ST	0.31	3	0	0'21		-1.39	0.49
33	31402	FIX	0.28	3	0	1' 9		-1.42	0.49



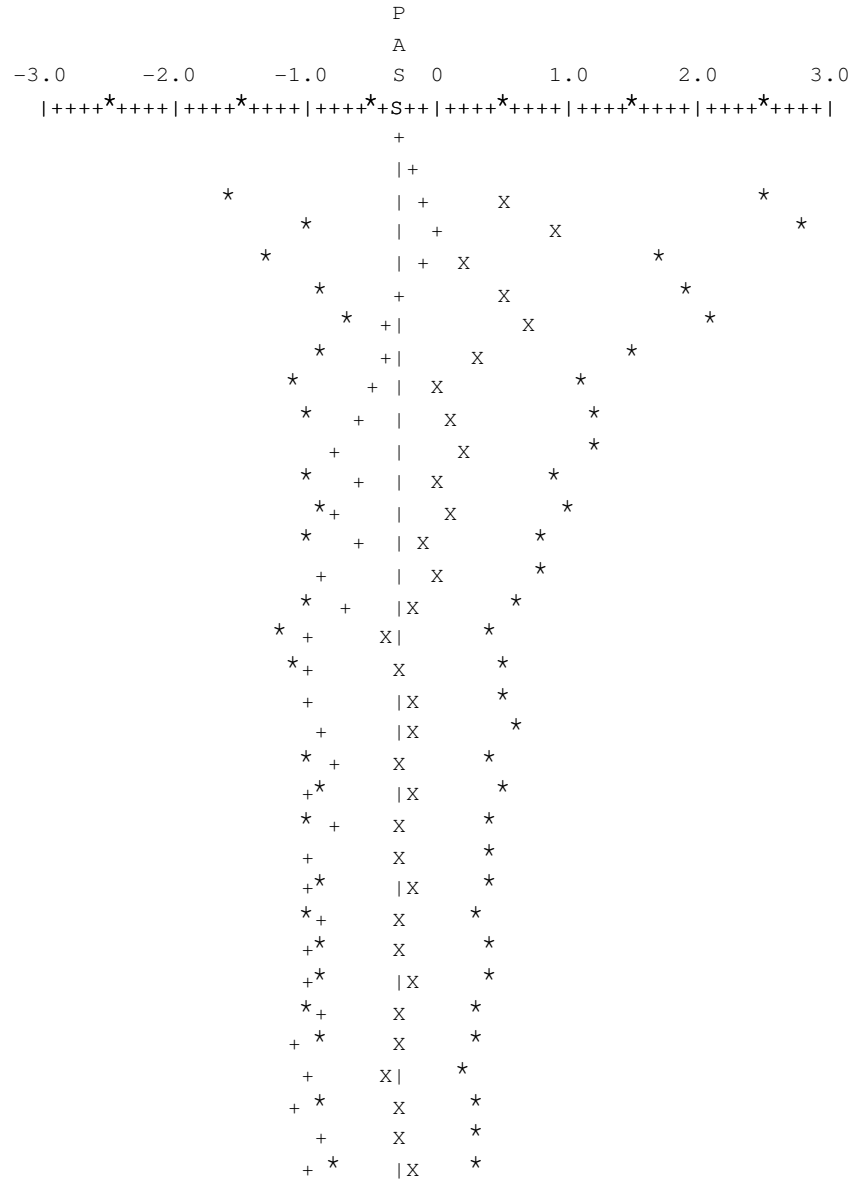


411433522 Entry= 1  
PBT Ver: 10/01/01

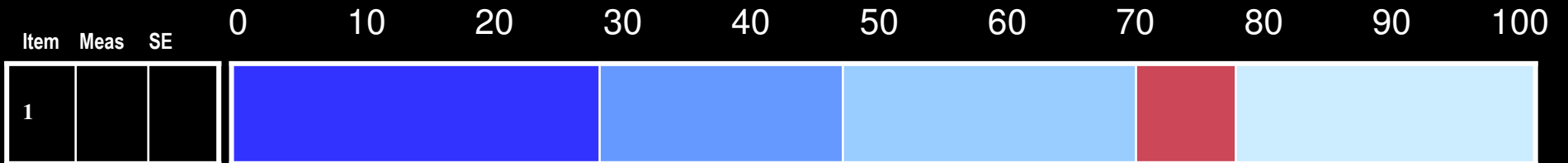
# Fence Sitter

Tested: 01/26/02  
Status: 1

Item	AN Cont	Diff	Ans	⊙ =	Time	!	Meas	SE
1	220576 SC	-0.33	3	1	○		0'37	9.99 9.99
2	220304 LO	-0.24	2	1	○		1'13	9.99 9.99
3	220935 SPH	-0.13	4	0	○		1' 3	0.46 1.22
4	220213 SC	-0.03	1	1	+		0'52	0.92 1.15
5	220378 AP	-0.11	3	0	=		0'40	0.24 0.91
6	220523 SC	-0.30	4	1	○		0'10	0.50 0.87
7	220611 LO	-0.37	2	1	○		0'17	0.70 0.84
8	220928 SC	-0.38	1	0	○		0'33	0.27 0.73
9	220218 SPH	-0.48	3	0	○		0'50	-0.04 0.67
10	220975 SC	-0.65	3	1	○		0'46	0.10 0.65
11	220709 SC	-0.79	1	1	○		0'35	0.21 0.63
12	220634 LO	-0.56	2	0	=		0'41	-0.03 0.59
13	220708 SPH	-0.81	1	1	○		0'22	0.07 0.57
14	220748 SC	-0.65	2	0	○		0'34	-0.13 0.54
15	220369 AP	-0.88	2	1	○		0'39	-0.04 0.53
16	220777 SC	-0.68	1	0	○		0'40	-0.21 0.50
17	220265 LO	-0.97	1	0	○		0'12	-0.37 0.49
18	220885 SC	-0.95	1	1	○		0'33	-0.29 0.47
19	220302 SPH	-0.98	2	1	○		0' 8	-0.22 0.46
20	220044 SC	-0.88	1	1	○		0'32	-0.15 0.46
21	220442 SC	-0.80	4	0	○		0'16	-0.28 0.44
22	220263 LO	-1.01	1	1	○		0'52	-0.22 0.43
23	220507 SPH	-0.79	1	0	○		0'30	-0.34 0.42
24	220037 SC	-1.00	4	1	+		0'43	-0.28 0.41
25	220317 AP	-1.05	3	1	○		0'11	-0.23 0.41
26	220535 SC	-0.92	3	0	=		0'51	-0.33 0.40
27	220987 LO	-1.02	4	1	○		0'25	-0.28 0.39
28	220342 SC	-0.99	3	1	○		0'49	-0.23 0.39
29	220089 SPH	-0.89	2	0	○		0'41	-0.33 0.38
30	220860 SC	-1.11	2	1	○		0'20	-0.29 0.37
31	220754 SC	-0.98	3	0	○		0'47	-0.38 0.36
32	220610 LO	-1.08	3	1	○		0'23	-0.33 0.36
33	220347 SPH	-0.91	1	1	○		0'49	-0.29 0.36
34	220856 SC	-1.01	2	1	+		1' 2	-0.25 0.35



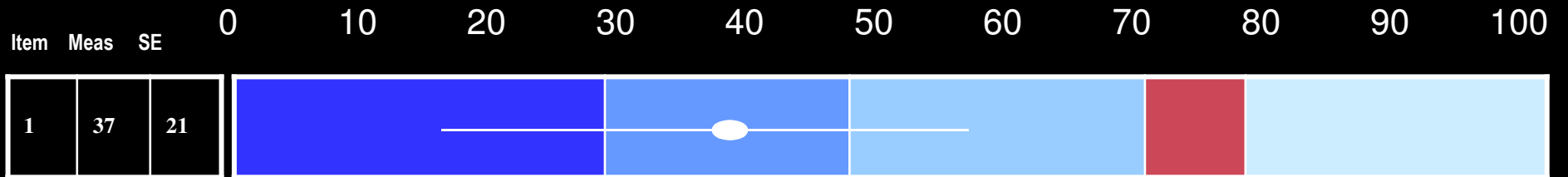
Simulate Measure = 48



GP1 – I have a lack of energy

0= Very Much 1= Quite a Bit 2= Somewhat 3= A Little Bit 4= Not at All

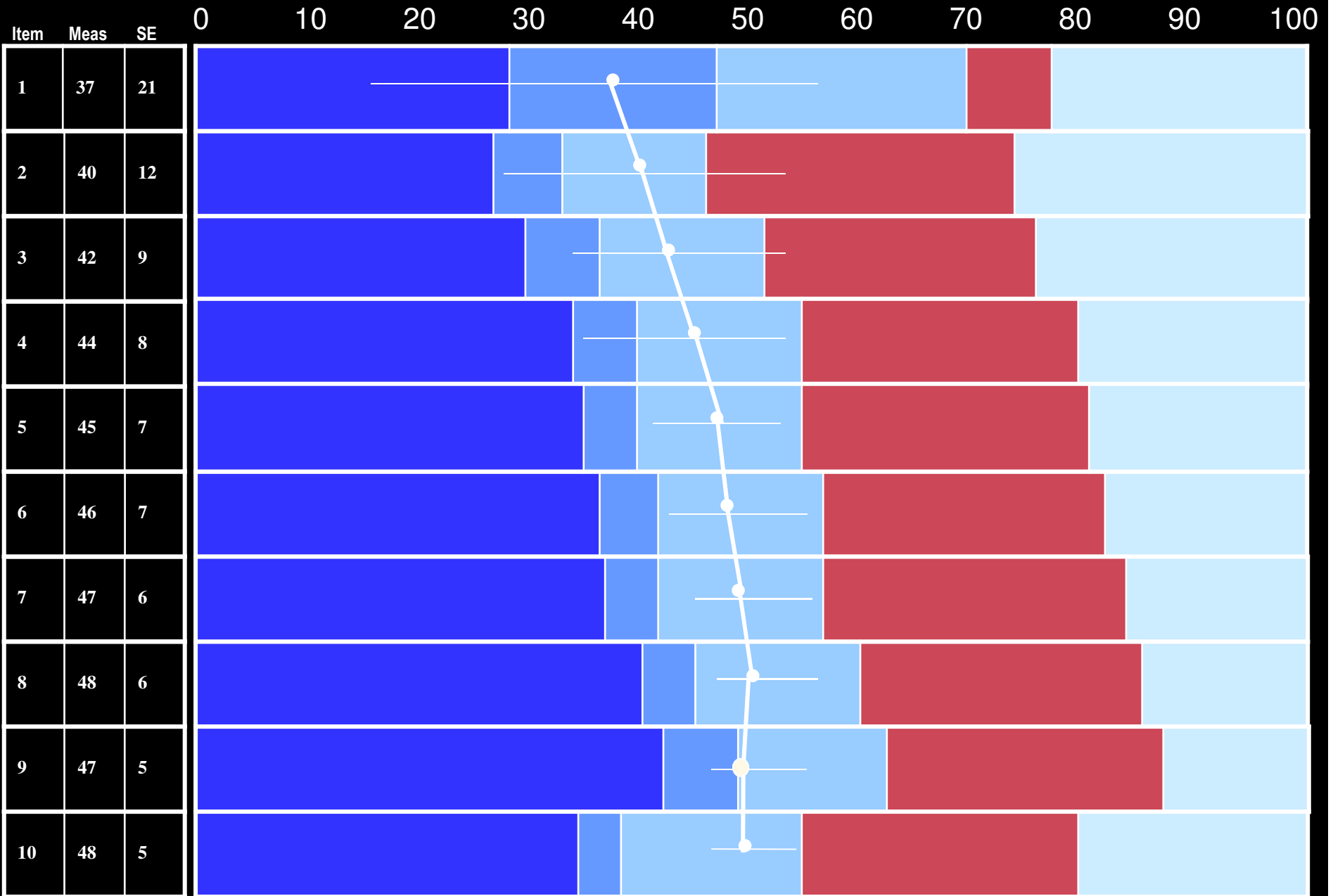
Simulate Measure = 48



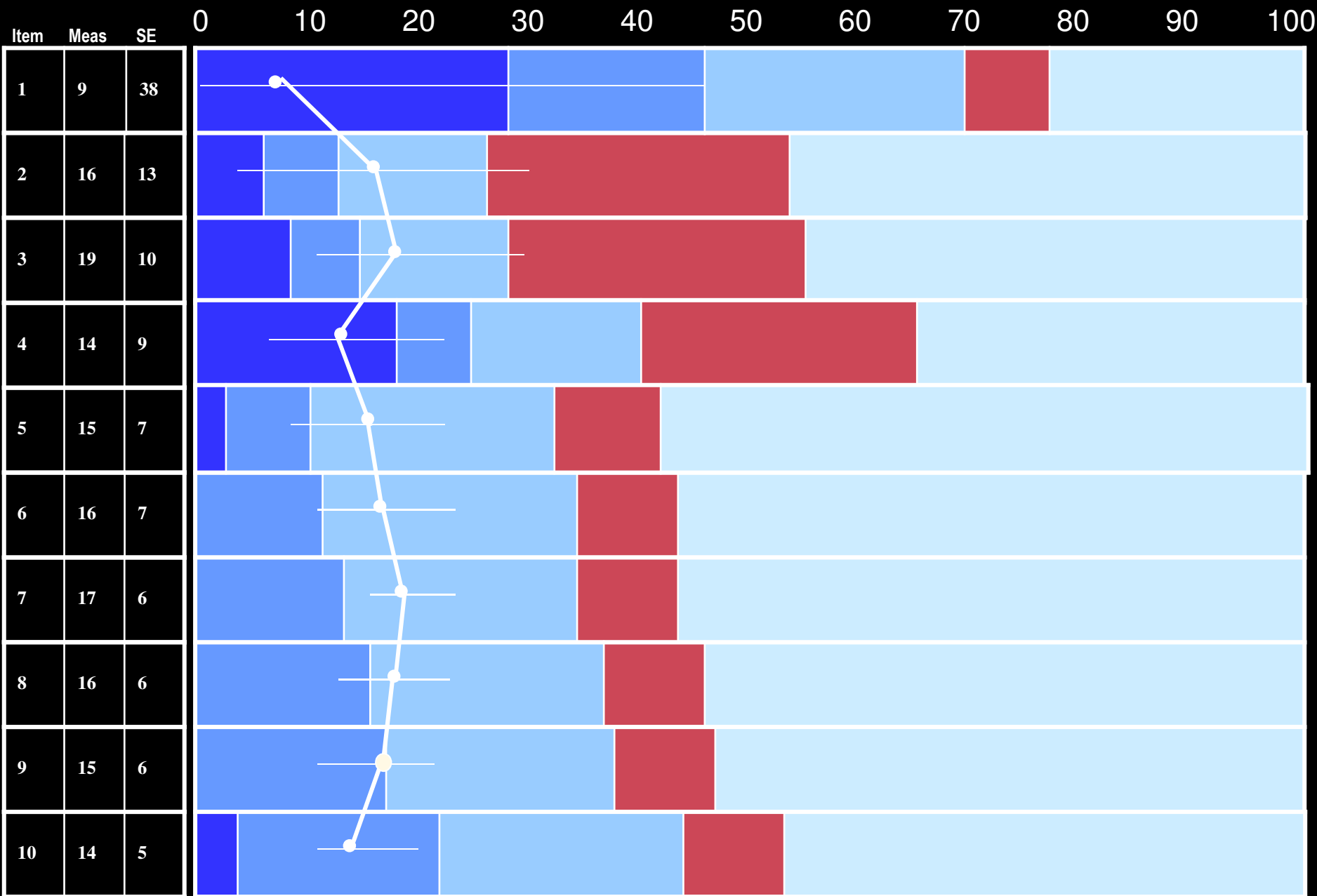
GP1 – I have a lack of energy

0 = Very Much; 1 = Quite a Bit; 2 = Somewhat; 3 = A Little Bit; 4 = Not at All

# Simulate Measure = 48



# Simulate Measure = 15



# Simulate Measure = 92

