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Contents

Mathematical Content Knowledge

Investigative Task – Geometry (Year 5)

Teachers: Background information

Students have been:

- learning how to measure angles using a protractor
- viewing and comparing maps and 2D images of different scales
- converting measurements on maps using scale

Students are given two aerial images of the same suburban area [Figures 1 and 2]. The photo was taken in 1943; the map in 2016. The old photo has a scale of 1:2 000. The new map has a scale of 1: 1 000. Both images are marked with a scale, and contain grid markings on the outer edges. Some landmarks in the old photo are visible in the new map, such as buildings A, B and C. In the old photo is a building marked D. It is not on the new map because it was demolished in 1960. Students must use both images to identify and explain the exact location of Building D on the new map. Students have multiple copies of both images for rough working. They work in mixed-ability groups of three to solve the problem.

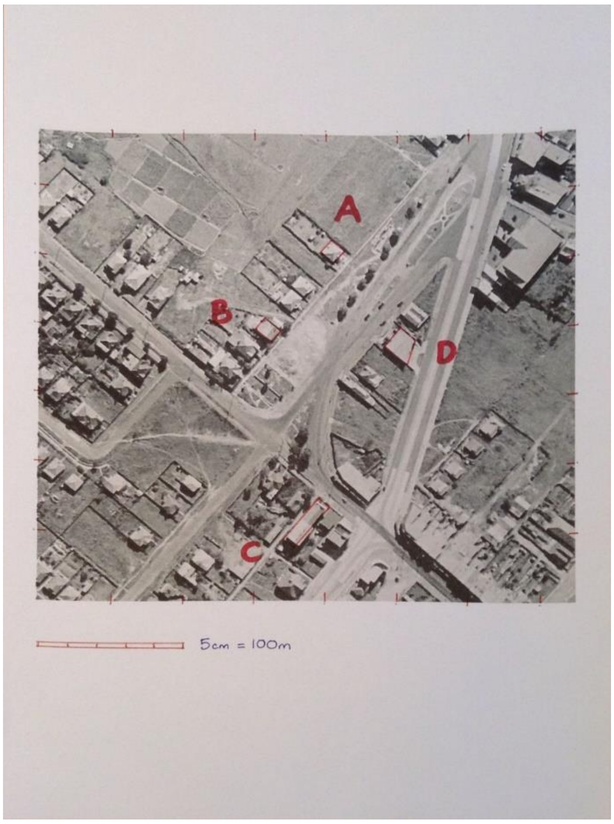


Figure 1

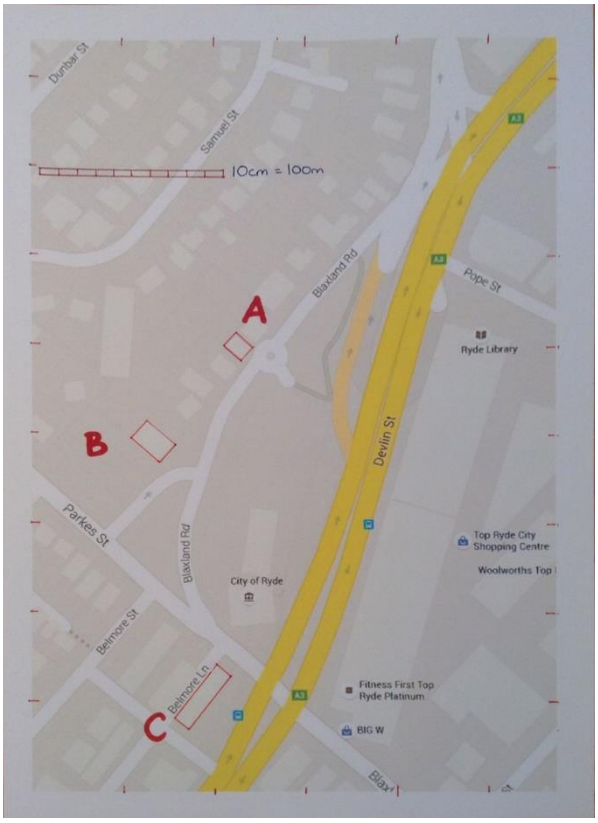


Figure 2

Students: Investigative Task

You are the Town Planner for the City of Ryde. You have two aerial images showing the same part of the City. The old photo was taken in 1943. The new map was created in 2016. You will see that Buildings labelled A, B and C are in both images. Building D, marked on the old photo, was demolished in 1960 and is not on the new map. The building was an important landmark at the time, and now the City Council wants to rebuild it on the same spot. Your job, as Town Planner, is to use the information available in the two aerial images to identify the exact location of Building D.

You will need to:

- Explain how to get to Building D from Buildings A, B and C. Your explanation must be clear and precise enough for someone else to follow. You will need to provide the distance between the buildings as well as the angles. Remember, you need two rays to measure an angle. How will you do this?
- Measure the length of the four outside walls of Building D. Pay careful attention to scale.
- Draw a scaled 'bird's eye' view of Building D, then cut it out and glue it accurately onto the new map. This needs to show its exact location. What scale will you use? How will you know how to position your drawing at the correct angle?
- Determine if it would be possible to reconstruct Building D on its old site. Why or why not?
- Work in groups of three.
- Consider the *Starting question* and *Thinking questions*.
- Record thoughts, questions, findings and progress in your Maths journal.
- Explain and justify your findings to the class at the conclusion of the investigation.

Starting question

- What are the first three things you should do to get started?

Thinking questions

- How might the grid markings and the scale on each image help you?

- When you give directions to Building D, will you give directions to the centre of the building, the northern corner of the building, or some other spot? Why should you be very specific about your start and end points?
- How can the protractor help you?
- When you have measured the three distances and the three angles, how will you know that they are all correct?

Australian Curriculum links (Year 5)

- Apply the enlargement transformation to familiar two dimensional shapes and explore the properties of the resulting image compared with the original (ACMMG115) (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2016a)
- Estimate, measure and compare angles using degrees. Construct angles using a protractor (ACMMG112) (ACARA, 2016a)
- Choose appropriate units of measurement for length, area, volume, capacity and mass (ACMMG108) (ACARA, 2016a)
- Use a grid reference system to describe locations. Describe routes using landmarks and directional language (ACMMG113) (ACARA, 2016a)

First Steps in Mathematics Key Understandings

- Represent transformation 3 (Department of Education WA [DEWA], 2013a, p. 130)
- Represent location 1 (DEWA, 2013a, p. 12)
- Represent location 2 (DEWA, 2013a, p. 26)
- Represent location 3 (DEWA, 2013a, p. 38)

Student demonstration during the task

- Determine a specific ‘start’ point on Buildings A, B and C, and a specific ‘end’ point for Building D
- Measure the three distances using a ruler, convert the measurements to metres, and convert the measurements to the scale on the map
- Discuss and decide upon how to measure the three angles, including the consistent selection of either a vertical or horizontal ray
- Correct placement of the vertex for each angle measurement (Wolfram Research Incorporated, 2016)

- Consistently place the protractor along the horizontal or vertical ray to correctly measure each angle
- Explain the route from the ‘start’ points at Buildings A, B and C to the ‘end’ point on Building D, using precise measurements (in appropriate units) and angles
- Measure the perimeter of Building D, convert the measurements to the scale of the map, draw and cut out Building D to its correct scale, and glue it onto the new map to show its precise location
- Observe Building D’s location on the new map and decide if it would be possible to reconstruct the building on its former site based on the position of any obstructions such as other buildings or roads
- Keep coherent and accurate notes in their Maths journals throughout the investigation
- Explain and justify their answer

Specific maths outcomes

By the end of this investigative task the students will be able to:

- Accurately convert measurements between two different scales
- Use a protractor to measure angles between objects
- Combine distance and angle measurements to accurately locate objects on a map (DEWA, 2013a, p. 51)
- Use measurement terminology to explain the location of a place on a map in relation to objects on the same map
- Justify their answers (Jones, 2003, p. 93)

Likely range of outcomes

<u>Satisfactory response</u>

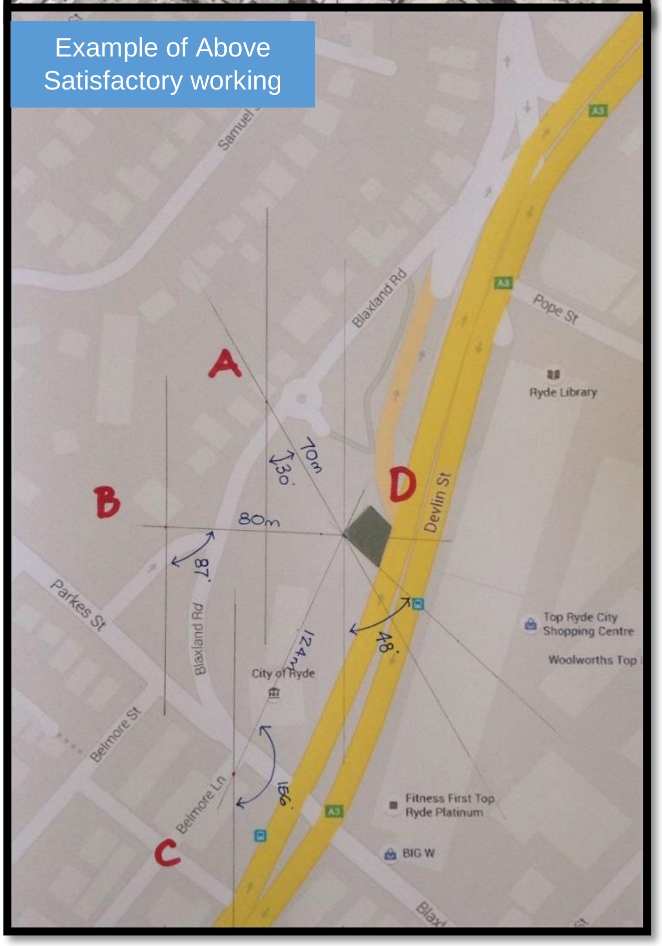
- Gives approximate measurements from Buildings A, B and C
- Converts millimetres to metres
- Converts measurements according to scales
- Includes angles for each measurement

- States start and end points for the three explanations
- Draws Building D to correct scale
- Shows Building D in the correct location
- Makes notes in Maths journal
- Justifies decisions

Above Satisfactory response

- Gives precise measurements from Buildings A, B and C
- Converts millimetres to metres
- Converts measurements according to scales
- Includes precise angles from each starting point
- States start and end points clearly and consistently for the three explanations
- Draws Building D to correct scale
- Shows Building D in the correct location
- Maths journal notes are detailed and coherent
- Findings are justified using notes and measurements

Example of Above Satisfactory working



Example of Above Satisfactory working

Below Satisfactory response

- Gives imprecise or inaccurate measurements from Buildings A, B and C
- Dimensions incorrectly or inconsistently converted

- Angles inconsistently measured and/or incorrect
- Directions from A, B and/or C lead to different end points
- Start and end points vague or not stated
- Building D is not drawn to scale
- Building D in incorrect location

Correct answer

Buildings A, B and C were all measured for their distance from the western corner of Building D.

The distance from the southern corner of Building A to the end point of Building D is 70 metres, measured at an angle of 30 degrees from vertical.

The distance from the southern corner of Building B to the end point of Building D is 80 metres, measured at an angle of 87 degrees from vertical.

The distance from the northern corner of Building C to the end point of Building D is 124 metres, measured at an angle of 156 degrees from vertical.

The outside walls of Building D were measured on the old photo. The western wall is 9mm, the southern wall is 11 mm, the eastern wall is 10 mm and the northern wall is 7mm. The scale of the old photo is 1cm (10mm) is equal to 20m. This means that 1mm is equal to 2m. The full scale measurements of the outside walls of Building D were calculated. The western wall is 18m, the southern wall is 22m, the eastern wall is 20m, and the northern wall is 14m.

A bird's eye view of Building D was drawn to the scale of the new map. The scale of that image is 1cm is equal to 10m. To position the scale drawing accurately on the new map, the angle of Building D was measured on the old photo. The southern wall of the building was measured as 48 degrees from horizontal. This angle was transferred to the 'end point' on the new map, and the scale drawing was glued in place.

Building D can be reconstructed on the old site because the land is free of buildings, roads or other structures.

Misconceptions and potential points of difficulty

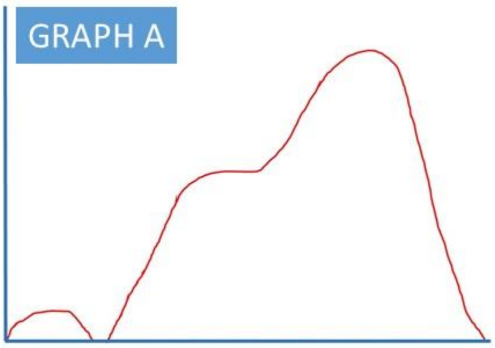
- Interpreting the scale on each image, and making allowances for the different scales (Reys et al., 2012, p. 312).
- Converting measurements from millimetres to metres (Reys et al., 2012, p. 420).

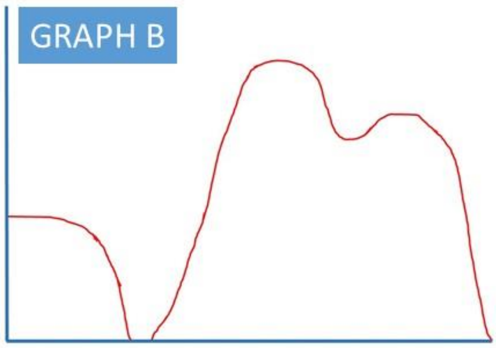
- Using consistent start and end points for the three measurements. This could mean Building D is placed incorrectly on the map.
- Using a protractor (Booker, Bond, Sparrow & Swan, 2014, p. 431) to accurately determine angles from each building, particularly deciding upon the position of the second ray against which to measure the angle.
- If students do not measure angles and distances successfully on the old photo, this might result in multiple ‘end points’ when drawn onto the new map.
- Drawing Building D to the correct scale. If students use the photo scale, Building D will be too small for the map and be in an inaccurate position.

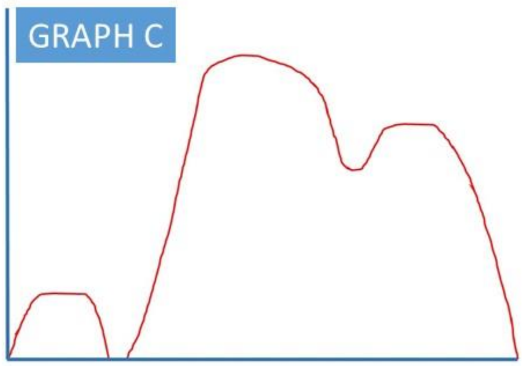
Multiple choice questions: Statistics and probability (Year 6)

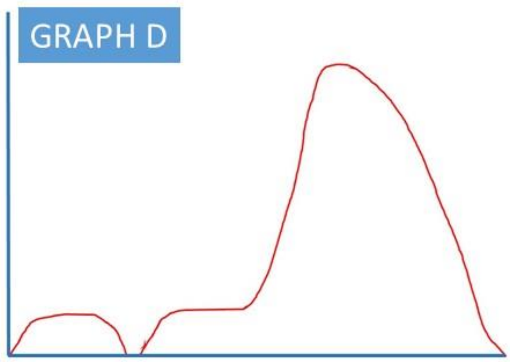
<u>Question 1</u>

A graph has been prepared to represent a train journey. The train started from Allonville station. It travelled slowly a short distance to Brookford where it stopped to collect some passengers. It then travelled quickly before slowing to climb a big hill. The train increased speed for a short distance as it got close to Corona, and then quickly slowed to stop at the station.









Which of the above graphs best represents the train's journey from Allonville to Corona via Brookford?

a) Graph A
b) Graph B
c) Graph C
d) Graph D

Question 1 rationale

Australian Curriculum link (Year 6 Mathematics)

- Interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables (ACMSP147) (ACARA, 2016b)

First Steps in Mathematics Key Understandings

- Interpret Data 2 and 3 (DEWA, 2013b, p. 213)

Correct answer

- Graph C

Answer choices

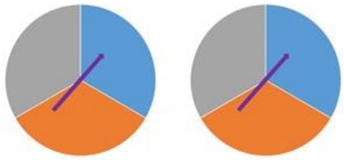
- Graph B is not supported by the text because it shows the train starting at speed.
- Graphs A and D visually match part of the question wording in relation to the train climbing the hill.
- These answers could highlight a student's poor conceptual understanding of what a graph is communicating because the graphs do not contain labels and students might therefore make a visual, rather than numerical, interpretation of the graphs (Reys et al., 2012, p. 441).

Question 2

You have created the following three probability activities.

ACTIVITY ONE

ACTIVITY TWO



ACTIVITY THREE

Activity One: roll a regular six-sided die and flip a coin at the same time. The goal is to roll the die to get a 3 and flip the coin to get tails.

Activity Two: flip two coins at the same time. The goal is to get two heads.

Activity Three: spin two identical spinners at the same time. The goal is for both spinners to land on blue.

Which of the following answers has the activities listed in order from highest probability to lowest probability?

a) Activity One, Activity Three, Activity Two
b) Activity Three, Activity Two, Activity One
c) Activity Two, Activity One, Activity Three
d) Activity Two, Activity Three, Activity One

Question 2 rationale

Australian Curriculum link (Year 6 Mathematics)

- Describe probabilities using fractions, decimals and percentages (ACMSP144) (ACARA, 2016b)

First Steps in Mathematics Key Understandings

- Understand Chance 3, 5 and 6 (DEWA, 2013b, p. 11)

Correct answer

- (d)

Answer choices

- Answer choice (a) would order the probability of the activities as one-twelfth, one-ninth and one-quarter; that is with the denominators in descending order. This could indicate poor understanding of ordering fractions and decimals (Booker et al., 2014, p. 169)
- Answer choices (b) and (c) give no consideration to order. Students selecting either option might have inefficient strategies for identifying the probabilities of the activities (Booker et al., 2014, p. 535)
- Answer choices (b) and (c) could also highlight a student's lack of perceptual understanding of probability relating to "unconnected" events (Lakin, 2011, p. 135)

Question 3

Rani buys tickets in the '5-from-25' lottery to give to her friends. When the lottery is drawn, five numbers between one and 25 will be drawn at random. If anyone has the five selected numbers marked on their card, they win the cash prize. The friends' cards look like this:

1	**2**	**3**	**4**	**5**
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

1	2	3	4	5
6	**7**	8	9	10
11	12	**13**	14	15
16	17	18	**19**	20
21	22	23	24	**25**

1	2	3	**4**	5
6	7	**8**	9	10
11	12	13	14	15
16	17	**18**	19	**20**
21	22	23	24	25

Who is right?

a) Peta
b) Sara
c) Graham
d) Rani

Question 3 rationale

Australian Curriculum link (Year 6 Mathematics)

- Describe probabilities using fractions, decimals and percentages (ACMSP144) (ACARA, 2016b)

First Steps in Mathematics Key Understandings

- Understand Chance 3, 5 and 6 (DEWA, 2013b, p. 11)

Correct answer

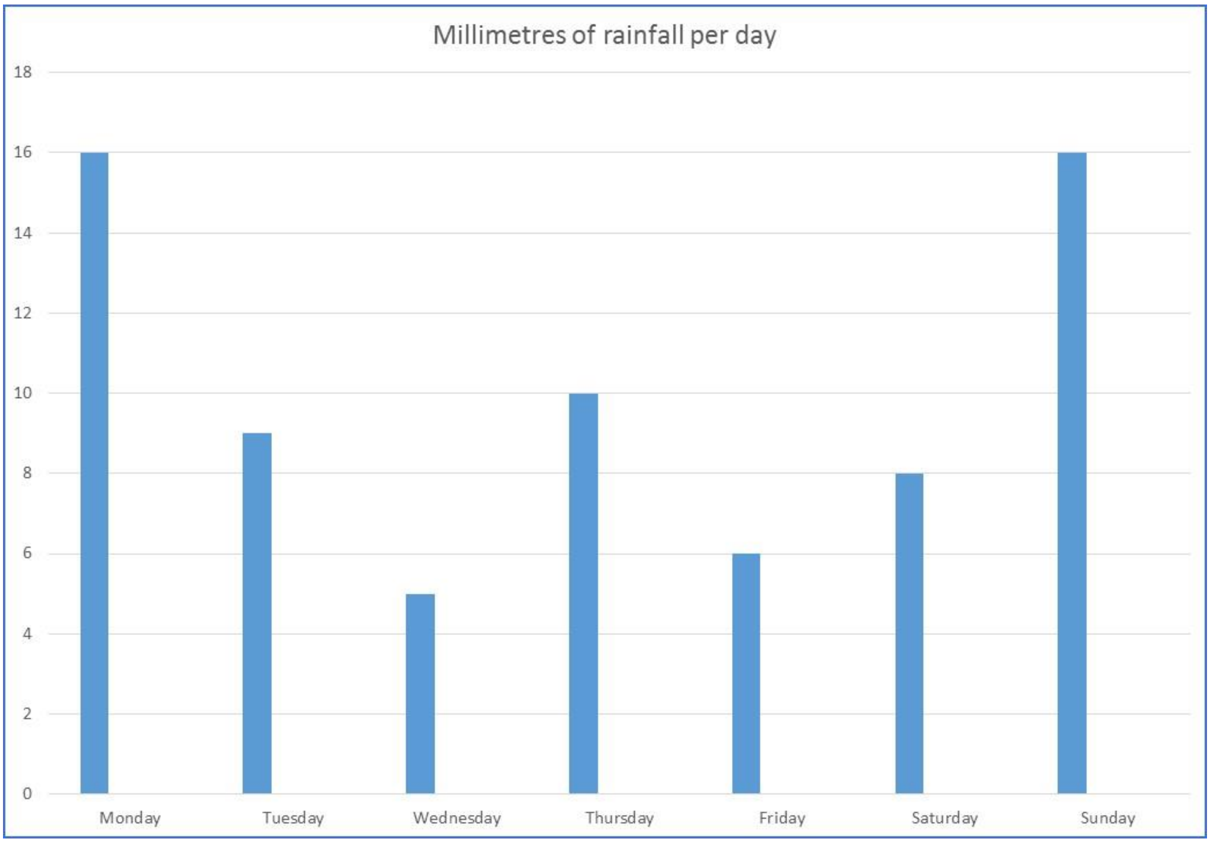
- (d)

Answer choices

- The incorrect answer choices were selected to challenge students' concepts of randomness as it relates to fairness (Reys et al., 2012, p. 453)
- Answer (a) (Peta's card) has five numbers that children will often consider unlucky or impossible (Reys et al., 2012, p. 455)
- Answer (b) (Sara's card) also highlights probability misconceptions by providing numbers that students might think are more probable (Reys et al., 2012, p. 455)
- Some students might consider answer (c) (Graham's card) as more random and therefore more likely to win than the others, highlighting a lack of "quantitative reasoning" (Booker et al., 2014, p. 527)

Question 4

This bar graph represents the millimetres of rain measured by the class rain gauge over the wettest week of the term.



Which days recorded rainfall that was equal to or greater than the mean average rainfall for the week?

a) Monday, Tuesday, Thursday and Sunday
b) Monday, Thursday and Sunday
c) Monday and Sunday
d) Monday, Tuesday, Thursday, Saturday and Sunday

Question 4 rationale

Australian Curriculum link (Year 6 Mathematics)

- Interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables (ACMSP147) (ACARA, 2016b)

First Steps in Mathematics Key Understandings

- Interpret Data 2 and 3 (DEWA, 2013b, p. 213)

Correct answer

- (b)

Answer choices

- The incorrect answer choices might display an inability to interpret data, and/or confusion over the mean, median and mode forms of average (Reys et al., 2012, p. 445)
- Answer (a) might be selected by a student who calculates the median average (9)
- Students who calculate the mode average (16) would be expected to answer (c)
- Answer (d) might indicate the student lacks both procedural and conceptual understanding of averages (Booker et al., 2014, p. 516)

Assessment 2: ePortfolio: Knowledge for Teaching

1.0 Introduction

Statistics, probability and geometry are difficult to learn and teach. They are also highly relevant to children's lives away from school, and provide challenging opportunities for meaningful learning. This report outlines the knowledge essential to teach these content strands. It begins by describing the importance of the strands, the stages through which learning progresses, and many of the concepts children need to learn. This report prescribes constructivist pedagogy and calls on published research to outline some of the experiences, resources, challenges and terminology that constitute learning and teaching statistics, probability and geometry. Accordingly, it is a valuable indicator of the understanding required to teach these areas of mathematics to primary school children.

2.0 The importance of statistics, probability and geometry

Statistics, probability and geometry hold prominent places in Australian mathematics classrooms. Interpreting statistics and understanding probability are skills that empower children to engage fully in a data-rich world. "Statistical literacy" (Australian Bureau of Statistics [ABS], 2013, p. 1) means students can assess and synthesise data, and make informed decisions based on critical analysis. Significantly, the study of statistics and probability can stimulate critical thinking skills (Reys et al., 2012, p. 431) that help children make sense of the data they encounter in everyday life. Without such knowledge, students are prone to misinterpret data (Reys et al., 2012, p. 432) or be misled (ABS, 2013, p. 1). Geometry is eminently practical. It has roots in the ancient world and relevance in the technological age. It helps children understand, measure, navigate through, and explain the world around them (Booker, Bond, Sparrow & Swan, 2014, p. 450). Geometry helps children construct and interpret the visual world by fostering "spatial reasoning" (Reys et al., 2012, p. 393). Geometry supports other areas of mathematics and develops problem-solving skills (Booker et al., 2014, p. 450). The skills developed through the study of statistics, probability and geometry are central to the objectives of the national Mathematics curriculum (ACARA, 2016c) and meet the second goal of the Melbourne declaration (Ministerial Council on Education, Employment, Training and Youth Affairs, 2008, p. 8). Their status in the curriculum is warranted.

3.0 Learning stages

Children learn statistics and probability in increasingly-sophisticated steps. As reflected in the Australian Curriculum (ACARA, 2016c), they begin by "organising and representing data"

(Reys et al., 2012, p. 436) and making inferences. Children then develop "descriptive statistics" skills (Reys et al., 2012, p. 442), posing their own questions, and comparing theoretical and experimental probabilities (Booker et al., 2014, p. 530). The Curriculum is mirrored by the First Steps program (Department of Education WA [DEWA], 2013b) which escalates learning from the simple to the intricate; and from the "subjective" to the "numerical" (Nisbet, 2006, p. 7). Students graduate from procedural to conceptual knowledge (Reys et al., 2012, p. 448) as their learning progresses.

Geometry learning begins with children's everyday experiences and increases in complexity during early schooling. This is evident in the national curriculum (ACARA, 2016c) which takes children from recognising and describing shapes, for example, to making prisms and using the Cartesian coordinate system. The visual components of geometry are ever-present, however more cognitively-demanding "formal geometry" (Booker et al., 2014, p. 452) involving theory and proof, emerge in later primary. "Geometric reasoning" (Booker et al., 2014, p. 456) commences from Year Three and employs spatial awareness and the study of angles. The progression that students make through learning geometry supports and reflects their increasing "geometric thought" (Reys et al., 2012, p. 376) as they advance from observing shapes to the more formal aspects of proof and justification.

4.0 Important Understandings

To successfully study statistics, probability, and geometry, children must understand at least the following elements:

Statistics

Ask suitable questions. Questions should be posed which address the problem being considered (Booker et al., 2014, p. 510). For example, in early primary school, students can pose yes/no questions.

Bias. Ensure bias does not impact the way data is collected (Lakin, 2011, p. 106). Results can be influenced by the wording of questions and the sample surveyed.

Collecting data. A collection method should be chosen which suits the problem (Reys et al., 2012, p. 434). For example, a survey is more suitable than an experiment for canvassing opinions.

Data categorisation and analysis. Findings might need to be sorted into key categories that help accurately answer the question (Booker et al., 2014, p. 511). Categories can be narrowed or broadened as the data is analysed.

Representation. An appropriate visual display will help others clearly read the results (Lakin, 2011, p. 105). A pie chart is a good way to represent children's favourite sports, but not effective for representing sunset times measured over a week.

Interpretation. Inferences and conclusions can be drawn from the findings (Booker et al., 2014, p. 511). Children can calculate the mean average, for example, of a set of results, which might help to make predictions.

Probability

Likelihood. The probability of an event occurring ranges on a scale from zero to one (Booker, et al., 2014, p. 526). Children can convert probability into decimals, fractions or percentages.

Theoretical and experimental. Many events can be measured theoretically, such as the probability of landing on heads when tossing a coin. Such events can also be measured by children conducting hands-on experiments and recording results (Booker et al., 2014, p. 530). Some events such as measuring how far a paper plane can travel over 20 'flights' can only be measured experimentally.

Randomness and fairness. These two concepts are inter-related (Reys et al., 2012, p. 453). A game, for example, is fair if every player has an equal chance of winning. Drawing equal-sized, well-mixed different-numbered balls from an opaque bag ensures the single winning ball is selected at random, and therefore the draw is fair.

Independence of events. The probability of an event is often not influenced by a previous outcome (Reys et al., 2012, p. 454). An example is rolling a regular six-sided die. Every roll provides a one-sixth chance of rolling a three no matter how many times a three was rolled previously.

Geometry

2D shapes and 3D objects. Children need to recognise, classify, describe and draw 2D shapes and 3D objects, and make 3D objects (Reys et al., 2012, p. 375). This should include everyday as well as less familiar shapes and objects, for example septagons and nonagons, to extend children's understandings.

Transformations. Reflections, translations and rotations do not change an object's shape or size. An enlargement alters its size. Other transformations, such as distortion, can change both shape and size (Booker, et al., 2014, p. 490).

Spatial sense. Children with spatial sense can visualise and build a 3D object from its 2D representation. This extends to children manipulating 2D shapes and 3D objects in their heads. The ability to ‘see’ what is unseen is an essential geometric understanding (DEWA, 2013a, p. 84).

Symmetry. Some shapes can be bisected through their line of symmetry. The line or lines of symmetry may be vertical, horizontal or diagonal. “Rotational symmetry” (Booker et al., 2014, p. 494) is less commonly understood and is important for expanding children’s geometric thinking.

Angles. This is introduced in Year Three due to its complexity (Booker et al., 2014, p. 429). Understanding, measuring and drawing angles, and using protractors, are grouped within “geometric reasoning” (ACARA, 2016c) as it calls on problem-solving abilities.

Reasoning. Children are required to justify their geometric explanations when solving problems (DEWA, 2013a, p. 155). They think about the relationships and arrangements of shapes, objects and angles, and support conclusions with evidence.

5.0 Constructivist pedagogy

Statistics, probability and geometry are suited to a constructivist setting. When children are engaged in authentic investigations and activities (Nisbet, 2006, p. 2) they have opportunities to solve problems that build new knowledge (Sharma, 2015, p. 80). In later primary school, this can be achieved by, for example, inviting children to develop their own statistics and probability questions (Reys et al., 2012, p. 434). In geometry, ask children to make as many polygons as possible using tangram pieces (Dr Paul Swan, 2013) and share their discoveries. This way, students take greater ownership of their learning (Sparrow, 2008, par 14). Regardless of year level, children are able to “make sense of mathematics” (Reys et al., 2012, p. 23) when teachers adopt constructivist pedagogy.

6.0 Experiences for understanding

Children need exposure to a range of experiences to develop their conceptual understandings. Experiences can be varied and repeated (Brousseau, Brousseau & Warfield, 2002, p. 405) to build and strengthen understandings. The learning environment should focus on process over product, and understanding over memorisation (Reys et al., 2012, p. 17). Experiences, in the form of problems to be solved, can be either created by the teacher or by the students.

Teacher-developed experiences should be “non-routine” in nature (Reys et al., 2012, p. 114). Fermi problems (Peter-Koop, 2005, p. 4) are an ideal example. That is, the methodology necessary for the solution is not apparent at the outset. This way, children must stretch their cognitive abilities (Jones, 2003, p. 89) as they search for strategies to solve abstruse challenges. The importance is to engage students with surprising and diverse tasks (Sparrow, 2008, par 2) not found in a didactic classroom.

Student-created challenges can be most engaging. This approach requires children to first consider how to pose a problem (Reys et al., 2012, p. 120) before engaging in solution strategies. Children who want to know, for example, which way a leaf will land when dropped from a height of two metres, must carefully consider the problem and whether it can be solved theoretically. The “PCAI” cycle (Graham, 2006, p. 88) keeps children focused, while the process lends itself to increased understanding of probability concepts (ABS, 2013, p. 1). In geometry, children work in pairs to develop ‘treasure hunt’ maps for others to follow. To make their maps effective the children need to create geometric, not just topological, diagrams (Salisbury, 1987, p. 472). Tasks such as these conclude with class reflections (Sharma, 2015, p. 80) and give children ownership of their learning.

7.0 Resources

Resources are plentiful in the constructivist classroom. Activities and manipulatives are engaging and strengthen learning (Booker et al., 2014, p. 21). They should be carefully selected and purposeful to be effective. Some resources that can build children’s conceptual understanding are outlined here.

Statistics

Online graph creators. Children can use the “Create a Graph” online tool (National Center for Education Statistics, 2016) to understand the importance of selecting a graph type which appropriately represents specific data (Reys et al., 2012, p. 436). (Appendix A).

Averages. Children calculate the mean, median and mode of distances students live from the school, and decide which average is most suitable and why (Reys et al., 2012, p. 446). (Appendix B).

Interpretation. Matching a story to its graphical representation (DEWA, 2013b, p. 187) builds conceptual understanding (Reys et al., 2012, p. 448). Children match several stories to their graphs.

Probability

Likelihood. Playing the “Greedy Pig” game (Nisbet, 2006, p. 7) expands children’s understanding of randomness and independence.

Independence. Coin-tossing activities extend understanding of independence (DEWA, 2013b, p. 44). Children calculate theoretical and experimental probabilities of tossing heads (Booker et al., 2014, p. 530). (Appendix C).

Fairness. Children make ‘fair’ and ‘unfair’ spinners (Booker et al., 2014, p. 538), and match spinners to game results to investigate what makes a game fair.

Geometry

Online tangrams. In the “Tangram Game” (Thirteen Productions, 2015) students manipulate online tangrams to make specific shapes. Such activities develop problem-solving and reasoning skills (Dr Paul Swan, 2013).

Maps. Compare a floorplan of a home with a photo taken from inside. Children use their “map sense” to identify the photographer’s location (Salisbury, 1987, p. 475-7). (Appendix D).

Triangles. Children identify the number of triangles in a diagram (Quinlan, 2000, p. 140). “Cognitive conflict” is created when children have different answers, thereby creating challenges which extend understanding (Borg, 1998, p. 25). (Appendix E).

Journals are universally-valuable mathematics tools. They help children with metacognition (Burns & Silbey, 2001, p. 19), build “reasoning” proficiencies (ACARA, 2016c), strengthen understanding, and record progress.

8.0 Challenges: Statistics and probability

Three difficulties with learning statistics and probability, and suggested teaching approaches, are outlined here.

- *Challenge: Representing data appropriately* (Ryan & Williams, 2007, p. 121). *Teaching*: Move students from concrete to pictorial and then symbolic displays of collected data (Reys et al., 2012, p. 436). This ensures they understand the purpose of representation and the structure of graphs (Booker et al., 2014, p. 518). Further, Ryan and Williams (2007, p. 134) insist that when data collection is meaningful, children are more inclined to suitably represent findings.

- *Challenge: Understanding averages* (Booker et al., 2014, p. 515). For example, a mean average might not appear in the original data or be skewed by outliers. *Teaching*: Varied and repeated activities develop deeper understandings (Booker et al., 2014, p. 516). Children explore data (MacGillivray & Pereira-Mendoza, 2011, p. 109) to decide which 'average' is appropriate (Reys et al., 2012, p. 446).

- *Challenge: Independence in probability*. In die-rolling or coin-tossing, for example, students often believe in luck or that results take turns (Booker et al., 2014, p. 527). *Teaching*: Facilitate hands-on activities (Sharma, 2015, p. 79) that create “cognitive conflict” (Borg, 1998, p. 25). Games involving independent and dependent events (Nisbet, 2006, p. 2) strengthen students’ concepts of probability.

9.0 Challenges: Geometry

Three difficulties with learning geometry, and suggested teaching approaches, are described here.

- *Challenge: Identifying properties of irregular shapes and objects* (Booker et al., 2014, p. 462). *Teaching*: Exposure to a wide variety of shapes and objects in different transformations (Booker et al., 2014, p. 464). Internet-based tools that allow for deeper geometric exploration are encouraged (Prieto, Juanena & Star, 2014, p. 413) as is requiring students to justify their geometric definitions (Reys et al., 2012, p. 377).

- *Challenge: Mentally manipulating shapes and objects*. *Teaching*: Display models of shapes and objects in the classroom (Reys et al., 2012, p. 379). This allows children to strengthen their geometric concepts by, for example, scrutinising and explaining the models. Booker et al (2014, p. 454) further recommend children draw and make their own models. Engaging with shapes and objects strengthens understanding (Reys et al., 2012, p. 395).

- *Challenge: “Geometrical awareness”* (Salisbury, 1987, p. 477). *Teaching*: Children interpret optical illusions, such as Gestalt diagrams (Woolfolk & Margetts, 2013, p. 254). They engage in “problem-based learning” (Scheffino, 2011, p. 347) involving geometry in the world around them, thereby extending conceptual understanding.

10.0 Language

Language in the statistics, probability and geometry classroom can impact children's understanding and address misconceptions. Terms such as *possible*, *likely* and *uncertain*, for example, build children's concepts associated with probability (Booker et al., 2014, p. 529) as do the terms *randomness*, *fairness* and *independence* (Reys et al., 2012, pp. 453-4). Such language is essential for the Key Understandings of chance (DEWA, 2013b, p. 11). Further, children need to be familiar with *range*, *outliers* and *quartiles* (Booker at al., 2104, p. 517) when explaining statistics (Sharma, 2015, p. 80). The geometry lexicon is vast and extends beyond the naming of shapes and objects. It includes words like *sides*, *faces*, *edges* and *vertices* (Booker et al., 2104, p. 476) and increases in complexity during the primary years. Knowledge of words including *parallel*, *perpendicular*, *convex* and *concave* must be understood when children represent locations and shapes (DEWA, 2013a, p. 6). Accordingly, for children to develop problem-solving and reasoning proficiencies (ACARA, 2016c) in these areas, correct use of statistics, probability and geometry terminology is essential.

11.0 Conclusion

The relevance of statistics, probability and geometry in the world beyond the classroom awards them deserved prominence in the national curriculum. These important areas are simultaneously challenging and rewarding for children to master. Through their primary school journey children must travel from basic understandings of statistics, probability and geometry, to highly-developed problem-solving and reasoning skills. This demands classrooms rich in investigations that challenge misconceptions, extend understandings, cultivate concepts and allow children to construct robust mathematical knowledge. Teachers cognisant of the difficulties inherent in these complex fields equip themselves with the knowledge, skills, language and a proliferation of valuable resources to teach them successfully. Describing every essential element necessary for the effective teaching of these content strands is beyond the scope of this report. It does however offer comprehensive insight into the complexities of these vast mathematical fields. Teachers can build on this platform to help children be inspired by and make sense of these three engaging areas of the Mathematics curriculum.

(Words count = 2 736)

References

Australian Bureau of Statistics (2013). Statistical literacy background paper. Retrieved from http://www.abs.gov.au/websitedbs/CaSHome.nsf/home/downloadable+files.es/$FILE/Statistical_Literacy_Paper.pdf

Australian Curriculum, Assessment and Reporting Authority (2016a). Mathematics: Curriculum – Year 5. Retrieved from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?y=5&s=NA&s=MG&s=SP&layout=1

Australian Curriculum, Assessment and Reporting Authority (2016b). Mathematics: Curriculum – Year 6. Retrieved from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?y=6&s=NA&s=MG&s=SP&layout=1

Australian Curriculum, Assessment and Reporting Authority (2016c). Mathematics: Curriculum (v 8.2). Retrieved from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Booker, G., Bond, D., Sparrow, L., & Swan, P. (2014). *Teaching primary mathematics* (5th ed.). Frenchs Forest, N.S.W.: Pearson.

Borg, J. (1998). Teaching Probability from a Constructivist Perspective. *Australian Mathematics Teacher*, 54 (4), pp. 25-28. Retrieved from http://search.informit.com.au.dbgw.lis.curtin.edu.au/documentSummary;dn=432222755299496;res=IELHSS

Brousseau, G., Brousseau, N., & Warfield, V. (2001). An experiment on the teaching of statistics and probability. *Journal of mathematical behavior*, 20 (3), pp. 363-411. DOI: 10.1016/S0732-3123(02)00078-0

Burns, M. & Silbey, R. (2001). Maths journals boost real learning: How words can help your students work with numbers. *Scholastic Instructor,* April 2001, pp. 18-20.Retrieved from https://lms.curtin.edu.au/bbcswebdav/pid-4031048-dt-content-rid-22882941_1/courses/EDPR3006-DVCEducatio-1155390448/BURNS%20SILBEY%20Maths%20Journals%281%29.pdf

Department of Education WA (2013a). First steps in mathematics: Space – Developing geometric understandings. Retrieved from http://det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/

Department of Education WA (2013b). First steps in mathematics: Chance and data – Developing probability and statistics. Retrieved from http://det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/

Dr Paul Swan (2013, March 8). Pswan tangrams 3: Flip, slide, turn [video file]. Retrieved from https://youtu.be/sKYqsTvZR0w

Google (2016). Map of Ryde NSW [image]. Retrieved from https://www.google.com.au/maps/place/Ryde+NSW+2112/@-33.8077696,151.0964813,14z/data=!3m1!4b1!4m5!3m4!1s0x6b12a5be90fc1aad:0x5017d681632c9f0!8m2!3d-33.815278!4d151.101111

Graham, A. (2006). *Developing thinking in statistics*. London, GBR: Sage Publications

Jones, L. (2003). The problem with problem solving. In I. Thompson (ed.), *Enhancing primary mathematics teaching*, Chapter 8, pp. 96-97. Open University Press. Retrieved from https://lms.curtin.edu.au/bbcswebdav/pid-4031057-dt-content-rid-22883010_1/courses/EDPR3006-DVCEducatio-1155390448/JONES%20Problem%20solving.pdf

Lakin, S. (2011). *How to improve your maths skills.* Essex, England: Pearson.

Land and Property Information (2016). Aerial photograph of Ryde NSW 1943 [image]. Retrieved from https://maps.six.nsw.gov.au/

MacGillivray, H., & Pereira-Mendoza, L. (2011). Teaching statistical thinking through investigative projects. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (14), pp. 109-120. DOI: 10.1007/978-94-007-1131-0_14

Ministerial Council on Education, Employment, Training and Youth Affairs (2008). Melbourne declaration on educational goals for young Australians. Retrieved from http://www.curriculum.edu.au/verve/_resources/National_Declaration_on_the_Educational_Goals_for_Young_Australians.pdf

National Center for Education Statistics (2016). Create a graph [online tool]. Retrieved from https://nces.ed.gov/nceskids/createagraph/

Nisbet, S. (2006). Take a chance: Chance games and learning experiences for the classroom in relation to the years 1-10 mathematics syllabus. Retrieved from https://lms.curtin.edu.au/bbcswebdav/pid-4031054-dt-content-rid-22883030_1/courses/EDPR3006-DVCEducatio-1155390448/NISBET%20Chance%20Activities.pdf

Peter-Koop, A. (2005). Fermi problems in primary mathematics: Fostering children's mathematical modelling processes. *Australian Primary Mathematics Classroom*, 10 (1), pp. 4-8. Retrieved from https://lms.curtin.edu.au/bbcswebdav/pid-4031057-dt-content-rid-22883040_1/courses/EDPR3006-DVCEducatio-1155390448/PETER-KOOP%20Fermi%20Problems.pdf

Prieto, N.J., Juanena, J.M.S., & Star, J.R. (2014) Designing Geometry 2.0 learning environments: A preliminary study with primary school students. *International Journal of Mathematical Education in Science and Technology*, 45 (3), pp. 396-416. DOI: 10.1080/0020739X.2013.837526

Quinlan, C. (2000). Student-centred teaching ideas for angles in geometry. *Vinculum*, 37 (3), pp. 9-20. Retrieved from http://search.informit.com.au.dbgw.lis.curtin.edu.au/fullText;dn=103153;res=AEIPT

Reys, R.E., Lindquist, M.M., Lamdin, D.V., Smith, N.L., Rogers, A., Falle, … Bennett, S. (2012). *Helping children learn mathematics*. Milton, QLD: John Wiley & Sons.

Ryan, J. & Williams, J. (2007). *Children's mathematics 4 – 15: Learning from errors and misconceptions*. Maidenhead, UK: Open University Press.

Salisbury, A.J. (1987). Primary school geometry: Some unusual activities. *International Journal of Mathematical Education in Science and Technology*, 18 (3), pp. 471-478, DOI: 10.1080/0020739870180319

Schettino, C. (2011). Teaching geometry through problem-based learning. *The Mathematics Teacher*, 105 (5), pp. 346-351. Retrieved from http://www.jstor.org.dbgw.lis.curtin.edu.au/stable/pdf/10.5951/mathteacher.105.5.0346.pdf

Sharma, S. (2015). Teaching statistics: A socio-constructivist perspective. *Teaching Statistics: An International Journal for Teachers*, 37 (3), pp. 78-84. Retrieved from http://onlinelibrary.wiley.com.dbgw.lis.curtin.edu.au/doi/10.1111/test.12075/epdf

Skamp, K. (2012). Teaching primary science constructively. In K. Skamp (Ed.), *Teaching primary science constructively* (4th ed., pp. 1-54). South Melbourne, VIC: Cengage Learning.

Sparrow, L. (2008). Chips, cola, and chocolate biscuits: Purposeful mathematics in the classroom. *Cross Section* 18 (1), pp. 13-19. Retrieved from https://lms.curtin.edu.au/bbcswebdav/pid-4031048-dt-content-rid-22883062_1/xid-22883062_1

Thirteen Productions (2015). Games: Tangram game [online tool]. Retrieved from http://pbskids.org/cyberchase/math-games/tanagram-game/

US Puzzle Championship (2008). Triangles puzzle [image]. Retrieved from http://wpc.puzzles.com/history/tests/2000wpc/images/triangle.gif

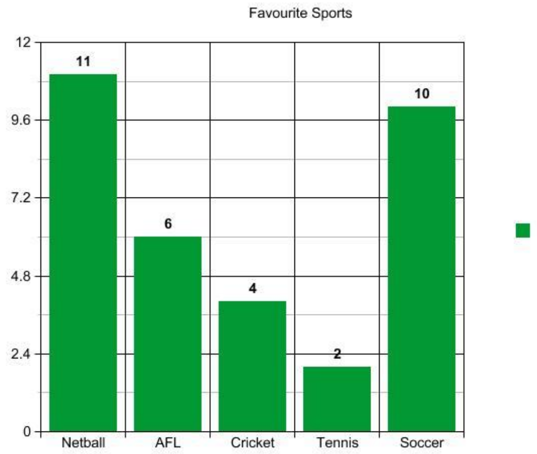
Woolfolk, A.H., & Margetts, K. (2013). *Educational psychology* (3rd ed.). Frenchs Forest, NSW: Pearson.

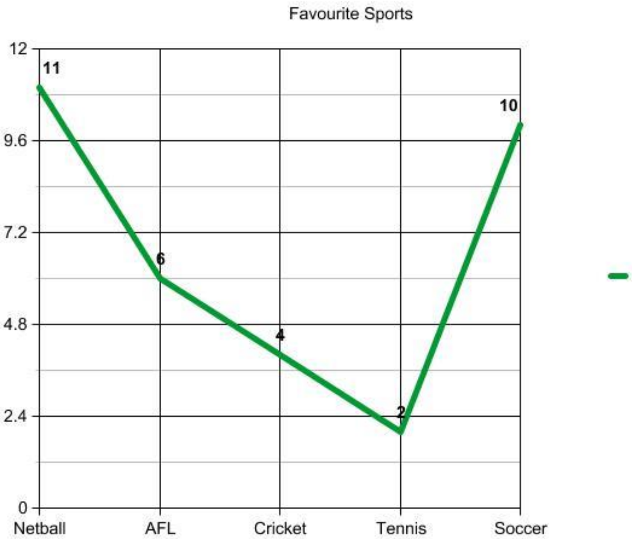
Wolfram Research Incorporated (2016). Vertex angle. Retrieved from http://mathworld.wolfram.com/VertexAngle.html

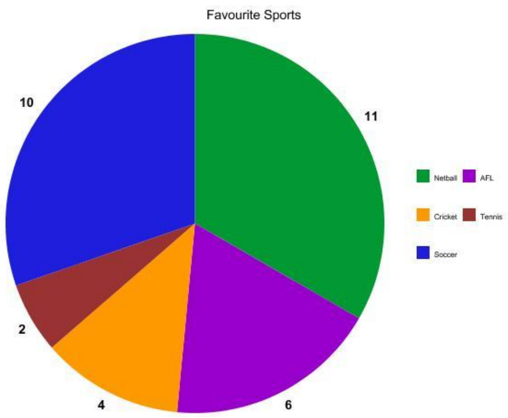
Appendix A

Developed using Create a graph (National Center for Education Statistics, 2016).

A class survey has identified the children's favourite sports. The data has been entered into the online graph creator. Children decide which graph most appropriately represents the data, and justify their reasons.







Appendix B

Are you average? (adapted from Reys et al., 2012, p. 446).

Problem question: On average, how far do we all live from school?

Every child measures the distance between the school and their home. Use these measurements, to the nearest metre, to calculate the mean, median and mode averages. Which measure of average is the most appropriate for answering the question above? Explain your reasons.

Distance in metres (do not use names)

Mean average = ______________________________

Median average = ______________________________

Mode average = ______________________________

Justification

Appendix C

Coin Tossing (adapted from Booker et al., 2014, p. 530)

A $1 Australian coin is flipped 100 times.

Predict how many times will it land on heads, and how many times will it land on tails. Provide justification for your explanation.

Prediction:

In pairs or groups of three, flip a $1 Australian coin 100 times and note how many times it lands on heads and how many times it lands on tails. What do you notice?

Observation:

Heads	**Tails**

Compare your results to the other groups in your class.

Explanation:

What did you notice about your results?	
What did you notice about the other groups' results?	
How can you explain these results?	
How do the results compare with your prediction, and how can you explain any differences?	

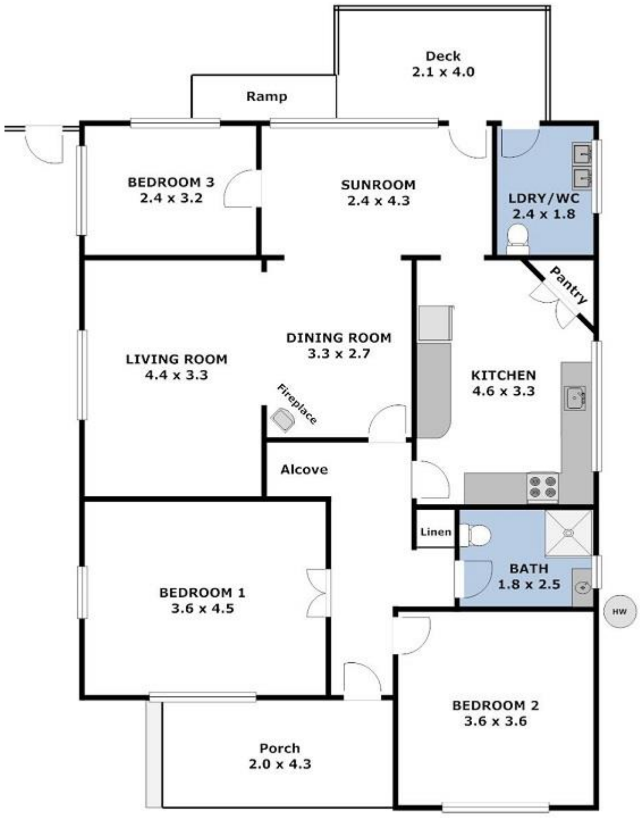
("Predict Observe Explain" adapted from Skamp, 2012, p. 28)

Appendix D

Map sense (adapted from Salisbury, 1987, p. 477)

This is the floor plan of a home. Below it is a photograph taken from inside the same home. Mark an X on the floorplan where you believe the photographer was standing. Justify your answer in the space provided.

Justification:

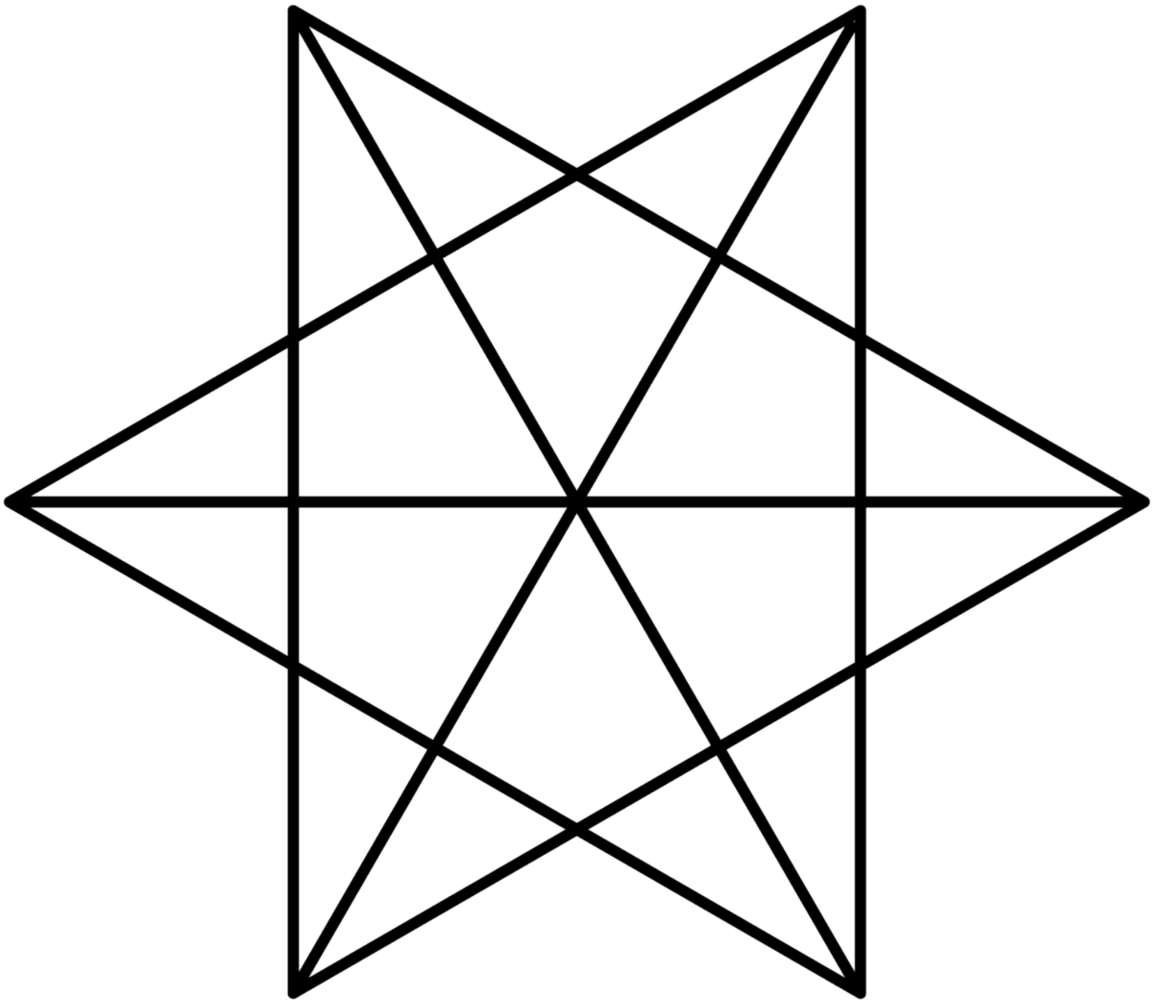


Appendix E

(Adapted from Quinlan, 2000, p. 140)

How many triangles can you see?

How will you know when you have found them all?



(Image from US Puzzle Championship, 2008)