CODE-D

JEE(MAIN) – 2015 TEST PAPER WITH SOLUTION (HELD ON SATURDAY 04th APRIL, 2015)

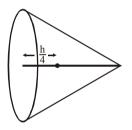
PART A - PHYSICS

1. Distance of the centre of mass of a solid uniform cone from its vertex is z_0 . If the radius of its base is R and its height is h then z_0 is equal to :-

(1)
$$\frac{5h}{8}$$
 (2) $\frac{3h^2}{8R}$
(3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

Ans. (4)

Sol.



for solid cone c.m. is
$$\frac{h}{4}$$
 from base

so
$$z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

2. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:-

(1) 5.48 V/m(2) 7.75 V/m(3) 1.73 V/m(4) 2.45 V/m

Ans. (4)

Sol. $I_{av} = \frac{1}{2} \epsilon_0 E^2 C = \frac{P}{4\pi r^2}$

$$E = \sqrt{\frac{2P}{4\pi r^2 \varepsilon_0 c}}$$

On putting value we get = 2.45 v/m. 3. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M . If the Young's modulus

of the material of the wire is Y then $\frac{1}{Y}$ is equal

to :- (g = gravitational acceleration)

$$(1) \left[1 - \left(\frac{T_{M}}{T}\right)^{2} \right] \frac{A}{Mg}$$

$$(2) \left[1 - \left(\frac{T}{T_{M}}\right)^{2} \right] \frac{A}{Mg}$$

$$(3) \left[\left(\frac{T_{M}}{T}\right)^{2} - 1 \right] \frac{A}{Mg}$$

$$(4) \left[\left(\frac{T_{M}}{T}\right)^{2} - 1 \right] \frac{Mg}{A}$$

Ans. (3)

Sol.
$$T = 2\pi \sqrt{\frac{L}{g}}$$
 ...(1)

$$T_{\rm M} = 2\pi \sqrt{\frac{L + \Delta L}{g}} \qquad \dots (2)$$

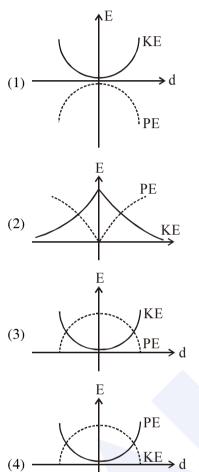
$$Y = \frac{F_{A}}{\Delta L_{L}}$$
FL

$$\Delta L = \frac{TL}{AY} \qquad ...(3)$$
Putting (3) in (2)

solving the equation we get the value of $\frac{1}{Y}$ as

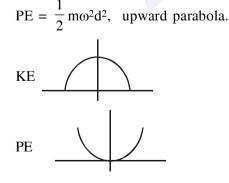
$$\left[\left(\frac{T_{M}}{T}\right)^{2}-1\right]\frac{A}{mg}$$

4. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale)



Ans. (4)

Sol. KE = $\frac{1}{2}$ m ω^2 [A² - d²], downward parabola



5. A train is moving on a straight track with speed 20 ms⁻¹. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :-

(1) 18% (2) 24% (3) 6% (4) 12% Ans. (4)

Sol.
$$f' = f\left(\frac{v}{v-v_s}\right)$$

 $f'' = f\left(\frac{v}{v+v_s}\right)$
 $\frac{f''}{f'} = \frac{v-v_s}{v+v_s}$
 $\frac{f''-f'}{f'} \times 100 = \frac{v-v_s-v-v_s}{v+v_s} \times 10$
 $= -\frac{2v_s}{v+v_s}$
 $= -\frac{2}{1+\frac{v}{v_s}} \times 100$

<u>~</u> 12%

6. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms⁻¹. If the electron density in the wire is 8×10^{28} m⁻³, the resistivity of the material is close to :-

> (1) $1.6 \times 10^{-6} \Omega m$ (2) $1.6 \times 10^{-5} \Omega m$ (3) $1.6 \times 10^{-8} \Omega m$ (4) $1.6 \times 10^{-7} \Omega m$

Ans. (2)

Sol.
$$V = iR = (neAV_d) \left(\frac{\rho \ell}{A}\right)$$

 $V = neV_d \ell$

$$\rho = \frac{V}{neV_d\ell}$$

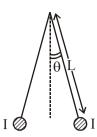
On putting values are got the answer = $1.6 \times 10^{-5} \Omega m$

JEE (Main + Advanced) Leader Course (Target-2016) for XII Passed / Appeared students Start on 15th April 2015 (English / Hindi Medium) and JEE (Main) Leader Course (Target-2016) for XII Passed / Appeared students Start on 15th April 2015 (English / Hindi Medium) at Kota Centre only.

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7.



Two long current carrying thin wires, both with current I, are held by the insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle ' θ ' with the vertical. If wires have mass λ per unit length then the value of I is :-

(g = gravitational acceleration)

(1)
$$2\sqrt{\frac{\pi g L}{\mu_0} \tan \theta}$$
 (2) $\sqrt{\frac{\pi \lambda g L}{\mu_0} \tan \theta}$

(3)
$$\sin\theta \sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$$
 (4) $2\sin\theta \sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

Ans. (4)

$$f_{B}$$

 θ
 λg
 $2\ell \sin \theta$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi (2\ell \sin \theta)}$$

$$\tan\theta = \frac{f_B}{\lambda g}$$
 where f_B is force per unit length(Bi)

$$\lambda g \tan \theta = \frac{\mu_0 I}{2\pi (2\ell \sin \theta)} \times I$$

on solving

$$I = 2 \sin\theta \sqrt{\frac{\pi \lambda g \ell}{\mu_0 \cos\theta}}$$

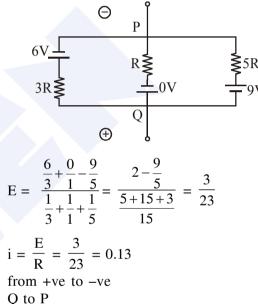
8.
$$\begin{bmatrix} 6V & p & 2\Omega \\ P & m \\ 3\Omega & Q & 3\Omega \end{bmatrix} 9V$$

In the circuit shown, the current in the 1Ω resistor is :-

(1) 0.13 A, from Q to P (2) 0.13 A, from P to O

(3) 1.3 A, from P to Q (4) 0A

Ans. (1) **Sol.**



9. Assuming human pupil to have a radius of 0.25cm and a comfortable viewing distance of 25cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is :-

(1) 100 µm	(2) 300 µm
(3) 1 μm	(4) 30 µm

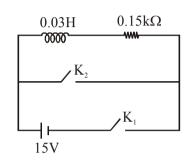
Ans. (4)

Sol.
$$\frac{d_0}{d_1} = \theta = \frac{1.22\lambda}{d}$$

 $d_0 = \frac{1.22 \times 500 \times 10^{-9}}{0.5 \times 10^{-2}} \times 25 \times 10^{-2}$
 $d_0 = 1.22 \times 25 \times 10^{-6}$
 $d_0 \approx 30 \ \mu m$



10. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K₁ has been kept closed for a long time. Then at t = 0, K₁ is opened and key K₂ is closed simultaneously. At t = 1ms, the current in the circuit will be (e⁵ ≈ 150) :-



(1) 6.7 mA	(2) 0.67 mA
(3) 100 mA	(4) 67 mA

Ans. (2)

Decay of current

 $I = I_0 e^{-\frac{tR}{L}}$

$$=\frac{15}{150}e^{-\frac{10^{-3}\times150}{0.03}}$$

$$=\frac{1}{10}e^{-5}$$

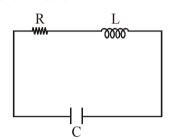
$$=\frac{1}{1500}$$

$$= 6.66 \times 10^{-4}$$

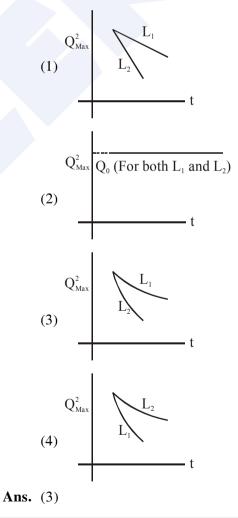
 $= 0.666 \times 10^{-3}$

= 0.67 mA

11. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below :-



If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 $(L_1 > L_2)$ of L then which of the following represents this graph correctly ? (plots are schematic and not drawn to scale)





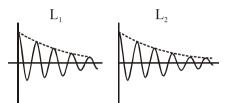
Sol. As damping is happening its amplitude would vary as



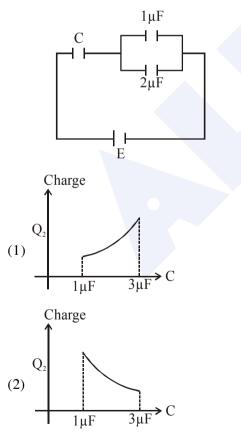
The oscillations decay exponentially and will be proportional to $e^{-\gamma t}$ where γ depends inversely on L.

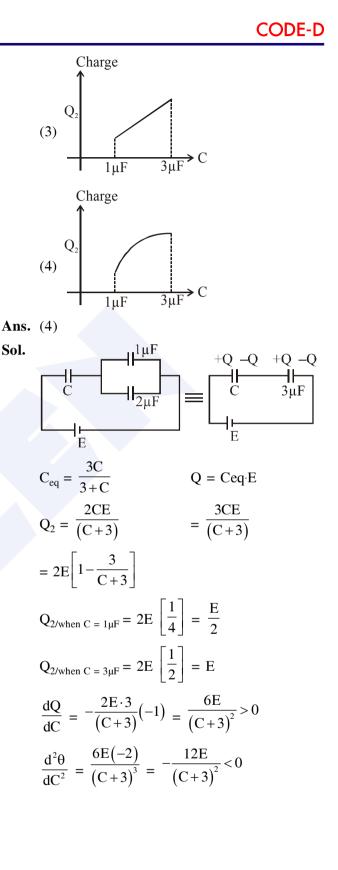
So as inductance increases decay becomes slower

∴ for



12. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale) :-





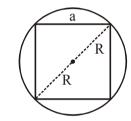
13. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:-

(1)
$$\frac{4MR^2}{9\sqrt{3}\pi}$$
 (2) $\frac{4MR^2}{3\sqrt{3}\pi}$

(3)
$$\frac{MR^2}{32\sqrt{2}\pi}$$
 (4) $\frac{MR^2}{16\sqrt{2}\pi}$

Ans. (1)

Sol.



Let mass and side of cube be M' and a $\sqrt{3} a = 2R$

$$M' = \frac{M}{\frac{4}{3}2R^3}a^3$$

Moment of Inertia of cube = $\frac{M'a^2}{6}$

$$=\left(\frac{M}{\frac{4}{3}2R^3}a^3\right)\frac{a^2}{6}=\boxed{\frac{4MR^2}{4\sqrt{3}\pi}}$$

14. The period of oscillation of a simple pendulum

is $T = 2\pi \sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm

known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is :

$$(1) 1\% (2) 5\%$$

Ans. (4)

Sol.
$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

 $g = \frac{4\pi^2 \ell}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$$
$$\frac{\Delta g}{g} = \frac{1 \times 10^{-3}}{20 \times 10^{-2}} + 2 \times \frac{1}{100 \times \frac{90}{100}}$$
$$\therefore \qquad \frac{\Delta g}{g} \times 100 = 2.722\% \approx 3\%$$

- 15. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light
 - beam : (1) bends downwards
 - (2) bends upwards
 - (3) becomes narrower

(4) goes horizontally without any deflection **Ans.** (2)

Normal
$$\mu_2$$
 μ_1 increases

 $\mu_2 > \mu_1$

: light bends towards normal

- \therefore light beam bends upwards (as μ^{\uparrow} with height)
- 16. A single of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are -

(1) 2005 kHz, 2000 kHz and 1995 kHz

(2) 2000 kHz and 1995 kHz

- (3) 2 MHz only
- (4) 2005 kHz and 1995 kHz

Ans. (1)

Sol. Frequency present after modulation

$$f_{c}, f_{c} \pm f_{s}$$

 \Rightarrow 2000 KHz, 2005 KHz and 1995 KHz

17. A solid body of constant heat capacity
$$1 J^{\circ}/C$$





is being heated by keeping it in contact with reservoirs in two ways -

- (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
- (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is -

(2) 2ln2, 8ln2

 $(1) \ln 2, 2\ln 2$

Ans. (4)

Sol.
$$\Delta S = \int_{T_1}^{T_2} nc \left(\frac{dT}{T}\right)$$
$$\Delta S_1 = \int_{100}^{200} l \left(\frac{dT}{T}\right) = \ell n 2$$
$$\Delta S_2 = \int_{100}^{200} l \left(\frac{dT}{T}\right) = \ell n 2$$

18. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume

$$u = \frac{U}{V} \propto T^4$$
 and pressure $p = \frac{1}{3} \left(\frac{U}{V} \right)$. If the shell

now undergoes an adiabatic expansion the relation between T and R is -

1

(1)
$$T \propto \frac{1}{R}$$

(2) $T \propto \frac{1}{R^3}$
(3) $T \propto e^{-R}$
(4) $T \propto e^{-3R}$

Ans. (1)

Sol.

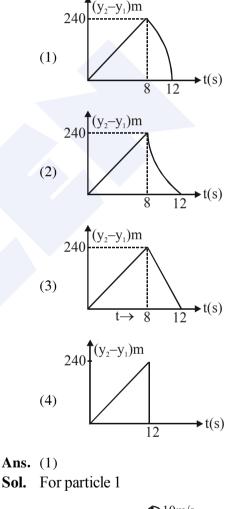
$$P = \frac{1}{3} \left(\frac{U}{V} \right)$$
$$P \propto T^{4}$$
using PV = nRT

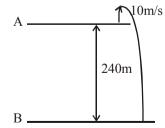
$$\frac{1}{V} \propto T^{3}$$
$$\Rightarrow T \propto \frac{1}{R}$$

19. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?

> (Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)

> (The figure are schematic and not drawn to scale)





$$-240 = +10t - \frac{1}{2} gt^{2}$$

$$5t^{2} - 10t - 240 = 0$$

$$t_{1} = 8 \sec$$
For particle 2
$$A - 40m/s$$

$$B - 240 = 40t - \frac{1}{2} gt^{2}$$

$$5t^{2} - 40t - 240 = 0$$

$$t_{2} = 12 \sec$$
for $0 < t < 8 \sec \rightarrow a_{rel} = 0$
straight line x-t graph
for $8 < t < 12 \sec \rightarrow a_{rel} = -g$
downward parabola
for $t > 12 \sec \rightarrow a_{rel} = -g$
downward parabola
for $t > 12 \sec \rightarrow a_{rel} = -g$
downward parabola
for $t > 12 \sec \rightarrow a_{rel} = -g$
downward parabola
for $t > 12 \sec \rightarrow a_{rel} = 0$
straight line x-t graph
for $t > 12 \sec \rightarrow a_{rel} = -g$
downward parabola
for $t > 12 \sec \rightarrow Both$ particles comes to rest
A uniformally charged solid sphere of radius
R has potential V_{0} (measured with respect to ∞)
on its surface. For this sphere the equipotential

surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R1, R2, R3 and R4 respectively. Then (1) $R_1 = 0$ and $R_2 < (R_4 - R_3)$ (2) $2R < R_4$ (3) $R_1 = 0$ and $R_2 > (R_4 - R_3)$ (4) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$ **Ans.** (1 or 2)

for r < RSol.

20.

$$V = \frac{kQ}{2R} \left(3 - \frac{r^2}{R^2} \right)$$

respect to ∞) equipotential

& for
$$r > R$$
 $V = \frac{kQ}{r}$
at $r = R$ $V_0 = \frac{kQ}{R}$
at $r = 0$ $V_0 = \frac{3}{2}\frac{kQ}{R} = \frac{3}{2}V_0$
 $\therefore R_1 = 0$

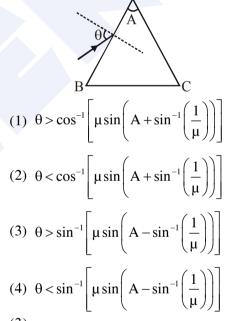
for
$$V = \frac{5}{4}V_0$$
 $\frac{kQ}{2R} = \left(3 - \frac{r^2}{R^2}\right) = \frac{5}{4} \cdot \frac{kQ}{R}$
 $R_2 = \frac{R}{\sqrt{2}}$
for $V = \frac{3}{4}V_0$ $\frac{kQ}{R_3} = \frac{3}{4}\frac{kQ}{R}$
 $R_3 = \frac{4R}{3}$
for $V = \frac{V_0}{4}$ $\frac{kQ}{R_3} = \frac{kQ}{4R}$

 $R_4 = 4R_0 .$ 21. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ , a ray, incident at an angle θ , on the face AB would get transmitted through the face AC of the prism provided :

4

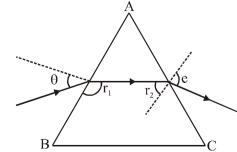
R₄

 $4R_0$









(Prism)



For all light to come out from face AC angle

of emergence e = 90

Apply Snell's Law at face AC $\mu \sin r_2 = 1 \sin e$

$$r_2 = \sin^{-1}\left(\frac{1}{\mu}\right)$$
 (if e = 90)

$$r_1 = A - \sin^{-1}\left(\frac{1}{\mu}\right)$$
 (: $r_1 + r_2 = A$)

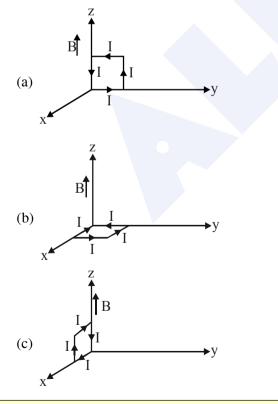
Apply Snell's law at face AB 1 $\sin\theta = \mu \sin(r_1)$

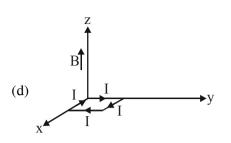
$$\theta = \sin^{-1}\left(\mu \sin(A - \sin^{-1}\left(\frac{1}{\mu}\right))\right)$$

for all light transmitted through AC, $e < 90^{\circ}$

$$\Rightarrow \theta > \sin^{-1}\left(\mu \sin(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)$$

22. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is place in different orientations as shown in the figures below :





If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

- (1) (b) and (d), respectively
- (2) (b) and (c), respectively
- (3) (a) and (b), respectively
- (4) (a) and (c), respectively

Ans. (1)

Sol. For stable equilibrium $\vec{B} \parallel \vec{M}$

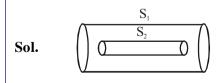
For unstable equilibrium $\vec{B} \downarrow \upharpoonright \vec{M}$ B = magnetic field M = magnetic moment

23. Two coaxial solenoids of different radii carry

current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then :

- (1) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$
- (2) \vec{F}_1 is radially outwards and $\vec{F}_2 = 0$
- (3) $\overrightarrow{F_1} = \overrightarrow{F_2} = 0$
- (4) \vec{F}_1 is radially inwards and \vec{F}_2 is radially outwards.

Ans. (3)



24.

Sol.

10 -

Field due to solenoid 2 outside the solenoid is 25. zero *.*.. $\vec{f}_{1} = 0$ Due to action reaction $\vec{f}_2 = 0$ or solenoid 2 behave as magnetic dipole $\vec{f}_2 = \frac{MdB}{dx}$ field of 1 is uniform $\therefore \frac{dB}{dx} = 0$ $f_2 = 0$ A particle of mass m moving in the x direction with speed 20 is hit by another particle of mass 2m moving in the y direction with speed v. If the collisions perfectly inelastic, the percentage loss in the energy during the collision is close to : So (1) 56 % (2) 62%(3) 44% (4) 50% **Ans.** (1) **Before collison** F 26. Kinetic energy = $\frac{1}{2}m(2v)^2 \times \frac{1}{2}2m(v)^2$ $= 3 \text{ mv}^2$ After collison Applying momentum conservation for inelastic collision $2mv\hat{i} + m2v\hat{i} = 3m\vec{v}_{f}$ $\left| \vec{\mathbf{v}}_{\mathrm{f}} \right| = \sqrt{\frac{8}{9}} \mathbf{v}$ $K_{f} = \frac{1}{2} \times 3m \times \left(v_{f}^{2}\right) = \frac{4mv^{2}}{3}$ % $\Delta K = \frac{K_i - K_f}{K_i} = \frac{5mv^2/3}{3mv^2} = \frac{5}{9} = 56\%$ **Ans.** (4)

Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as Vq. where V is the volume of the gas. The value of q is :-

$$\left(\gamma = \frac{C_p}{C_v}\right)$$
(1) $\frac{\gamma + 1}{2}$
(2) $\frac{\gamma - 1}{2}$
(3) $\frac{3\gamma + 5}{6}$
(4) $\frac{3\gamma - 1}{6}$

Ans. (1)

bl.
$$\tau \propto \frac{\text{Volume}}{v_{\text{avg}}}$$

$$\tau \propto V^1 T^{-\gamma/2}$$

For adiabatic process
 $T \propto V^{1-\gamma}$

$$\tau \propto V^{1}T^{-1/2} \propto V^{\frac{\gamma+1}{2}}$$

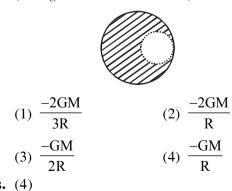
comparing

$$q = \frac{\gamma + 1}{2}$$

From a solid sphere of mass M and radius R,

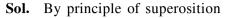
a spherical portion of radius $\frac{R}{2}$ is removed, as

shown in the figure. Taking gravitational potential V = 0 at $r = \infty$, the potential at the centre of the cavity thus formed is : $(G = gravitational \ constant)$





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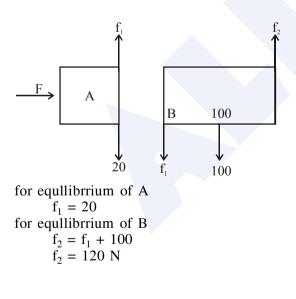
$$V = -\frac{GM}{2R^3} \left[3R^2 - \frac{R^2}{4} \right] + \frac{3G}{2} \frac{M}{8R^2}$$
$$= \frac{-11 GM}{8R} + \frac{3GM}{8R} = -\frac{GM}{R}$$
27.
 F A B

Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :-

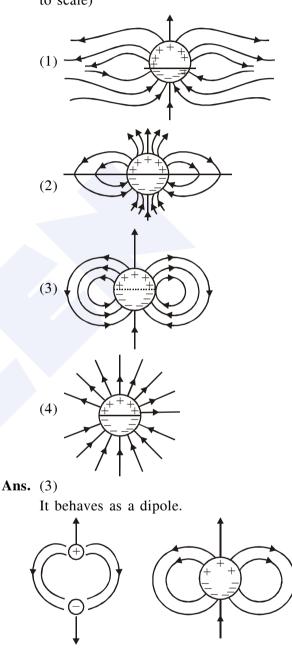
(2) 150 N(4) 80 N

(1)	120	Ν	
(3)	100	Ν	

Ans. (1) **Sol.**



28. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in : (figures are schematic and not drawn to scale)





- **29.** As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion :
 - (1) kinetic energy decreases, potential energy increases but total energy remains same
 - (2) kinetic energy and total energy decrease but potential energy increases
 - (3) its kinetic energy increases but potential energy and total energy decreases
 - (4) kinetic energy, potential energy and total energy decrease

Ans. (3)

Sol. $K = +13.6 \frac{z^2}{n^2}$ as n decreases k increases

$$U = -27.2 \frac{z^2}{n^2}$$
 as n decrease
$$\int U \& T \text{ decrease}$$
$$T = -13.6 \frac{z^2}{n^2}$$

30. Match List-I (Fundament Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list :

	List-I		List-II
(A)	Franck-Hertz Experiment.	(i)	Particle nature of light
(B)	Photo-electric experiment	(iii)	Discrete energy levels of atom
(C)	Davison-Germer Experiment	(iiii)	Wave nature of electroc
		(iv)	Structure of atom

(1) A-ii, B-i, C-iii (2) A-iv, B-iii, C-ii

(3) A-i, B-iv, C-iii (4) A-ii, B-iv, C-iii

Ans. (1)

- Sol. Self Explanatory/Theory
 - (A)Franck-Hertz experiment explains disrete energy levels of atom
 - (B) Photo-electric experiment explain particle nature of light
 - (C) Davison Germer experiment explain wave nature of electron.