JEL classification: C13, C22, L94
Keywords: electricity, Hurst exponent, persistence, cycles

Long-term Memory in Electricity Prices: Czech Market Evidence*

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Abstract

We analyze the long-term memory properties of hourly prices of electricity in the Czech Republic between 2009 and 2012. Various statistical properties of these prices are studied, and as the dynamics of electricity prices is dominated by cycles—in particular intraday and daily—we opt for detrended fluctuation analysis, which is well suited to such specific series. We find that electricity prices are non-stationary but strongly mean-reverting, which distinguishes them from prices of other financial assets, which are usually characterized as unit root series. Such behavior is attributed to specific features of electricity, in particular its non-storability. Additionally, we argue that the rapid mean-reversion is due to the principles of electricity spot prices. These properties are shown to be stable across all the years studied.

1. Introduction

Electricity is a flow commodity with unique characteristics that influence the way it is traded and thus the behavior of spot and futures prices in the market. Electricity cannot be effectively stored (with the minor exception of pumped-storage hydro power plants, which are scarce), so the adjustment of demand and supply must be instantaneous. Electricity consumption reflects human behavior and the temporal patterns of human life with its daily and weekly routines. This is reflected in the daily pattern (with its single or double-peak structure) and weekly pattern of consumption (Simonsen et al., 2004). On a larger scale, there are seasonal fluctuations caused mainly by the weather, and in particular by the temperature and the number of hours of daylight (Lucia and Schwartz, 2002). The seasonal patterns are strongly geographically dependent—in northern countries, the highest consumption is usually observed during the winter months due to heating, and in southern countries, air-conditioning increases consumption during the summer (Zachmann, 2008).

Electricity prices on the spot market are very sensitive to temperatures and especially to sudden weather changes, which are expected up to a point. However, the weather forecast is never perfect, causing spot prices of electricity to be much more volatile than those of other financial assets (Asbury, 1975). Moreover, the electricity supply side is also weather dependent, especially in the case of renewable sources of energy such as wind turbines and photovoltaic power plants (von Bremen, 2010).

Demand for electricity is highly inelastic. In the short run, it is absolutely inelastic, so that the price is completely determined by the supply curve (merit order

* Support from the Czech Science Foundation under Grants 402/09/0965 and P402/11/0948 and project SVV 267 504 is gratefully acknowledged.

curve, marginal cost curve). The curve resembles an upward-sloping stairway, with each step approximately representing a different type of power plant and thus a different level of marginal costs. The price on the market rises until it reaches the marginal cost of the power plant on the next level, after which the supply rises. This is why the merit order curve is not smooth. In order to produce an additional MWh, more expensive power sources (plants) are activated, and as the supply increases, the price increases as well (Geman and Roncoroni, 2006; Sensfuss et al., 2008).

High (or excess) volatility is another typical feature of electricity prices and is mainly due to the non-storability of electricity itself. There are no reserves that can be used in the event of a sudden increase in demand or change in the weather (Janczura et al., 2013). Not only are the prices volatile, but the volatility also has a tendency to cluster. Apart from this clustering, the volatility also displays an inverse leverage effect—positive shocks increase the price volatility more than negative ones (Knittel and Roberts, 2005). In addition, electricity prices tend to "jump" very frequently. These jumps, usually referred to as "spikes", are typified by a sharp increase followed by a slower decrease, causing pronounced asymmetry. Due to the properties described above, electricity prices are often treated as non-stationary.

Unlike other financial time series, specifically prices of various assets, electricity prices are mean-reverting (Simonsen, 2003; Weron and Przybylowicz, 2000). According to Barlow (2002), estimates of the mean reversion time range from two to six days. Geman (2005) states that with constant or slightly increasing demand, the supply side is able to adjust the pattern so that prices remain close to their mean value. However, the strength of the mean reversion varies from study to study—some studies report electricity prices to be stationary (Park et al., 2006), while others find weak mean reversion close to the unit root (Simonsen, 2003), with many results lying in between. A more detailed description of these results is provided in the next section.

Electricity prices are also influenced by factors that are unthinkable for other "typical" financial assets: technical constraints. A power plant which is out of order due to either technical problems or regular maintenance can influence the price because the number of power plants is small and limited. Electricity can be easily and quickly transported, but transmission lines have capacity constraints which must not be exceeded. That is the main reason why electricity prices differ in neighboring areas, but it can also cause high levels of volatility due to potential instability of the whole system (Borenstein et al., 1997).

Last but not least, electricity demand and thus also prices depend on the business cycle, economic activity, and growth. Electricity consumption and economic growth are linked; different studies suggest different directions of the causality—from electricity consumption to GDP, vice versa, or both (Soytas and Sari, 2003; Lee, 2005; Squalli, 2007; Ciarreta and Zarraga, 2010).

In our study, we focus on various properties of electricity prices in the Czech Republic, paying special attention to the long-term memory of spot prices. To the best of our knowledge, this is the first such study of the Czech electricity market. The market was fully deregulated in 2006 and the network of power plants consists of less expensive hydro and nuclear power plants, and more expensive hard coal and gas power plants, with lignite plants somewhere in between (Sensfuss et al., 2008).

A small increase in demand can thus put into operation considerably more expensive power plants and the occurrence of spikes is potentially high.

OTE (the Czech electricity and gas market operator, established in 2001) has been organizing the day-ahead spot electricity market since 2002. The Czech market has been coupled through implicit auctions with the organized day-ahead electricity market in Slovakia since 2009 and with the day-ahead electricity market in Hungary since 2012. It has the form of a daily auction, with a traded period of 1 hour and a minimum tradable volume of 1 MWh, and with the euro as its trading currency. The market always closes at 11 a.m. the day before. The volume of electricity registered in the OTE system for the day-ahead market in 2012 was 10,971 GWh for sale and 10,562 GWh for purchase.[1]

In this paper, we describe the temporal patterns, distributional properties, and, above all, the correlation structure, paying special attention to the long-term memory of prices. To do so, we use detrended fluctuation analysis, which is well suited to time series with such a complicated structure as electricity spot prices. We show that the prices are non-stationary and strongly persistent, but remain strongly mean-reverting, which distinguishes them from other financial prices such as stock prices and exchange rates, which follow a random walk pattern (Cont, 2001). To the best of our knowledge, this is the first detailed analysis of Czech electricity prices and their dynamics. The paper is structured as follows. Section 2 focuses on recent studies on the long-term memory properties of electricity prices. Section 3 presents the data. Section 4 describes the methodology. Section 5 discusses the results, and Section 6 concludes.

2. Brief Literature Review

The correlations and memory characteristics of electricity prices have been the subject of many studies in recent years. Weron and Przybylowicz (2000) analyze California Power Exchange (CalPX) and Swiss Electricity hourly prices using re-scaled range analysis and find mean-reverting characteristics. This analysis is then broadened by Weron (2002), who studies four electricity markets (CalPX, Nord Pool, Entergy, and UK spot) with three different methods (rescaled range analysis, de-trended fluctuation analysis, and periodogram methods) and confirms that the returns of electricity prices are anti-persistent. Simonsen (2003) analyzes Nord Pool prices using the multi-scale wavelet approach and compares it with standard rescaled range analysis to show that the returns of electricity prices are weakly anti-persistent. The author stresses that the choice of an appropriate technique for long-term memory estimation is crucial. Park et al. (2006) examine 11 U.S. electricity markets using the vector autoregression methodology and importantly find several price series to be stationary. This contradicts the standard understanding of prices of financial assets, which typically form a unit root series and are thus strongly non-stationary.

Koopman et al. (2007) develop an adjusted fractionally integrated autoregressive moving average model with generalized autoregressive conditional hetero-skedasticity (ARFIMA-GARCH), which is able to capture day-of-the-week patterns and extreme price movements, specifically for electricity prices on three European markets (the German EEX, the French Powernext, and the Dutch APX). They show

[1] Details are available at https://www.ote-cr.cz/statistics.

that the weekly patterns are indeed crucial in daily price analysis. Norouzzadeh et al. (2007) study the long-term memory and multi-fractality of the Spanish spot market and find persistent yet strongly mean-reverting prices. Erzgraber et al. (2008) focus on long-term memory in Nord Pool markets and find the returns to be weakly (compared to the previous study) anti-persistent. They also find that the strength of the memory depends on the time of day of the measurement, i.e., the prices are correlated not only from hour to hour, but also in the same hour from day to day. Moreover, they show that the memory parameter varies strongly over time. Uritskaya and Serletis (2008) examine Alberta and Mid-C electricity prices using detrended fluctuation analysis and spectral exponents and find that both Alberta and Mid-C prices are persistent and mean-reverting. However, the former remain stationary, whereas the latter do not.

Malo (2009) combines various properties of electricity prices and uses a Markov-switching multifractal model with conditional copulas to construct a model for risk minimization of the Nord Pool markets. Comparing various methods of long-term memory estimation, the author finds anti-persistent returns for electricity prices. Conditional value at risk is also discussed in detail using various copula specifications. Alvarez-Ramirez and Escarela-Perez (2010) analyze the Ontario and Alberta electricity markets with detrended fluctuation analysis and the Allan factor model to show that the long-term memory properties of both prices and demand strongly vary over time. Haugom et al. (2011) model Nord Pool electricity prices using a long-term memory mimicking heterogeneous autoregressive model with realized variance (HAR-RV) and show that incorporating the strongly persistent realized variance improves the predicting power of the model. And Rypdal and Lovsleten (2013) model the Nord Pool data using a multifractal random walk model adjusted for mean-reversion and volatility persistence to capture the most important characteristics of electricity prices. Using the model, the authors show that the characteristics of electricity prices are very different from those of stock market prices. In our analysis, we apply detrended fluctuation analysis to the hourly spot prices of Czech electricity. Specifically, we utilize its ability to separate cycles and seasonalities from the long-term memory.

3. Methodology

3.1 Long-term Memory

Long-term memory is traditionally linked with slowly decaying autocorrelation functions. For an autocorrelation function $\rho(k)$ with lag *k*, the decay is described as asymptotically hyperbolic, so that $\rho(k) \propto k^{2H-2}$, where $k \to +\infty$. The autocorrelation function thus follows an asymptotic power law. A characteristic parameter of long-term memory is the Hurst exponent *H*, which ranges between 0 and 1 for stationary processes. The breaking value of 0.5 is connected with a short-term correlated process (usually characterized by exponential or more rapid decay of the autocorrelation function). For $H > 0.5$, the underlying process is positively correlated and locally trending and is traditionally labeled as a persistent process. For $H < 0.5$, the process is anti-persistent and switches direction more frequently than a random process would (Beran, 1994; Samorodnitsky, 2006).

For non-stationary processes, the definition of long-term memory via the autocorrelation function is inappropriate, as the process has infinite variance and there are no correlations. For this case, and also in the general case, a spectrum-based definition is used. Assuming that the spectrum or pseudo-spectrum of an underlying process exists near to the origin, i.e., $f(\lambda)$ exists for $\lambda \to 0+$, we define long-term memory via a power law at the origin of the spectrum, i.e., $f(\lambda) \propto \lambda^{1-2H}$ for $\lambda \to 0+$. For persistent processes, $f(\lambda)$ diverges at the origin, whereas for anti-persistent processes, it collapses to zero (Samorodnitsky, 2006).

Historically, there have been two major streams of Hurst exponent estimators —time-domain estimators and frequency-domain estimators. Time-domain estimators are based on the autocorrelation definition of long-term memory and its implications for the scaling of the variance of partial sums. The most frequently used ones include rescaled range analysis (Hurst, 1951; Mandelbrot and Wallis, 1968; Mandelbrot and van Ness, 1968), detrended fluctuation analysis (Peng et al., 1993, 1994; Kantelhardt et al., 2002), the generalized Hurst exponent approach (Alvarez-Ramirez et al., 2002; Di Matteo et al., 2003; Di Matteo, 2007), and detrending moving average (Alessio et al., 2002). The frequency domain estimators are based on the spectrum definition and the most popular ones include the GPH estimator (Geweke and Porter-Hudak, 1983), the average periodogram estimator (Robinson, 1994), the log-periodogram estimator (Beran, 1994; Robinson, 1995b), and the local Whittle estimator (Künsch, 1987; Robinson, 1995a). Due to the very specific statistical properties of electricity prices, which have been mentioned in the previous sections and are also discussed in the following section, we opt for detrended fluctuation analysis, which has desirable properties for this type of analysis. As a control estimator, we choose the GPH estimator.

3.2 Detrended Fluctuation Analysis

The detrended fluctuation analysis (DFA) of Peng et al. (1993, 1994) is a special case of multifractal detrended fluctuation analysis (MF-DFA) introduced by Kantelhardt et al. (2002). For a better understanding of the procedure, we present the more general MF-DFA as an initial step.

Let us have a time series $\{x_t\}$ with $t = 1, \ldots, T$, where T is a finite time series length. The profile $X(t)$ is constructed as

$$X(t) = \sum_{i=1}^{t} (x_i - \bar{x}) \tag{1}$$

where $\bar{x} = \frac{1}{T}\sum_{t=1}^{T} x_t$ is the time series average. The profile is then divided into $T_s \equiv T / s$ non-overlapping windows with length s (scale), where $\lfloor\,\rfloor$ is a lower integer part operator. As T_s is not necessarily equal to T/s, part of the time series is left at the end of the series. In order not to lose the information of this segment, the profile is also divided from the opposite end and both sets of blocks of length s are further utilized (we thus get $2T_s$ windows of length s).

In each of these $2T_s$ segments, we calculate the mean squared deviation from the trend of the series in this particular window. This means that for the k-th window, the mean squared deviation $F^2(k, s)$ is obtained as

$$F^2\left(k,s\right)=\frac{1}{s}\sum_{i=1}^{s}\left(X\left(s\left[k-1\right]+i\right)-\hat{X}_k(i)\right)^2 \quad (2)$$

where $\hat{X}_k(i)$ is a polynomial fit of a time trend at position I in window k. In our application, we use a linear fit obtained via ordinary least squares regression, which is standard for the DFA and MF-DFA procedures (Hu et al., 2001; Grech and Mazur, 2005; Kantelhardt, 2009; Kristoufek, 2010). This is applied for windows $k=1,\ldots,T_s$, and then for windows $k = T_s + 1, \ldots, 2T_s$ we obtain

$$F^2\left(k,s\right)=\frac{1}{s}\sum_{i=1}^{s}\left(X\left(T-s\left[k-T_s\right]+i\right)-\hat{X}_k(i)\right)^2 \quad (3)$$

The multifractal analysis stems in the scaling of the q-th order fluctuations, so we need to find the behavior of the fluctuations at scale s for different values of order q. To do so, we construct the q-th order fluctuation function

$$F_q\left(s\right)=\left(\frac{1}{2T_s}\sum_{k-1}^{2T_s}\left[F^2(k,s)\right]^{\frac{q}{2}}\right)^{\frac{1}{q}} \quad (4)$$

For $q = 0$, the zeroth-order fluctuation function is defined as

$$F_0\left(s\right)=exp\left(\frac{1}{4T_s}\sum_{k=1}^{2T_s}\log\left[F^2(k,s)\right]\right) \quad (5)$$

Order q can take any real value. For $q = 2$, the MF-DFA procedure reduces to DFA and it is used to analyze the long-term memory properties of series $\{x_t\}$. Later in the text, we label $H \equiv H(2)$. For other values of q, the interpretation is not so straightforward, but the scaling behavior dependence on q is the basis of the multifractal analysis, which we do not discuss here. In practice, minimum and maximum scales s_{min} and s_{max} need to be set, as for finite series the averaging and trend-fitting procedures can become unreliable. Usually, the minimum scale is set as $s_{min} \approx 10$ and the maximum scale as $s_{max} = T/4$ to avoid inefficient trend fitting for low scales and imprecise averaging at high scales.

3.3 Useful Properties of MF-DFA

Estimation of the long-term memory parameters H has a long history, starting with Hurst (1951). Since then, many methods have been developed to study the power-law scaling of the autocorrelation function and the connected phenomena of the divergent-at-origin spectrum and the power-law scaling of the variance of the partial sums. Estimators have been developed in both the time and frequency domains (see Taqqu et al., 1995; Taqqu and Teverovsky, 1996, and Di Matteo, 2007, for reviews of the various methods).

As the MF-DFA method can be labeled as the most frequently used method of multifractal analysis, its strengths and weaknesses have also been given an appropriate focus in the literature. None of the other methods have been studied in such detail. For our purposes, we are mainly interested in the ability of MF-DFA to deal with cycles and heavy-tailed distributions.

Hu et al. (2001) discuss the effect of trends on the properties of detrended fluctuation analysis and give special attention to periodic cycles. For long-term memory processes combined with a sinusoidal trend, they show that the scaling function $F^2(s)$ undergoes several cross-overs (changes in scaling rules) due to interaction between the long-term memory and the sinusoidal trend. For both persistent and anti-persistent series, the scaling passes through three cross-overs and the scaling laws connected to the long-term memory effects are observed for scales s below the first and above the third cross-over scales. In this way, it is possible to distinguish between the effect of long-term memory and sinusoidal trends. Importantly, the authors show that for anti-persistent processes, the third cross-over scale is frequently higher than $T/4$ or even T, so that the long-term memory scaling needs to be obtained only from the scales below the first cross-over.

Barunik and Kristoufek (2010) study the effect of heavy tails on the most frequently used heuristic methods of Hurst exponent estimation. They show that DFA is unbiased regardless of how heavy the tails are. For MF-DFA, they are interested in the case of $q = 1$ and find that for reasonable tails (with a tail parameter between 1.5 and 2, where the value of 2 is connected with the Gaussian distribution and the value of 1 with the Cauchy distribution), the estimates of $H(1)$ are practically unbiased as well.

In the original study, Kantelhardt et al. (2002) also discuss the possibility of highly anti-persistent processes with H close to 0. In such situations, practically all the estimators become severely upward-biased. However, the MF-DFA methodology is constructed for both asymptotically stationary and non-stationary processes. In practice, the series $\{x_t\}$ can be integrated into a new series $\{y_t\}$ defined as $y_t = \sum_{i=1}^{t} x_i$ for $t = 1, \ldots, T$ and MF-DFA can be applied to $\{y_t\}$. Labeling the generalized Hurst exponent of the series $\{x_t\}$ as $H_x(q)$ and the generalized Hurst exponent of the integrated series $\{y_t\}$ as $H_y(q)$, it holds that $H_x(q) = H_y(q) - 1$. Therefore, if $\{x_t\}$ possesses properties resembling strong anti-persistence, the generalized Hurst exponent for the series can be obtained by running MF-DFA on the integrated series and reducing the estimate by 1.

As a special case of MF-DFA, DFA is thus an ideal candidate for long-term memory analysis of electricity prices, as the above-mentioned properties match the properties of electricity prices discussed in the previous sections as well as in the next section dealing with the specific properties of Czech electricity spot prices. Before we turn to the dataset description and results, we introduce the control estimator.

3.4 Alternative Estimators

Probably the most severe disadvantage of the MF-DFA and DFA estimators is their lack of asymptotic properties. To cover this issue, we also include two frequency-based estimators—the original GPH and GPH with a smoothed periodogram. Apart from the fact that both have well defined asymptotic properties, we also use them to stress the superiority of the DFA approach in such a complex matter as electricity prices. We expect that frequency-based estimators will not be able to deliver reliable results as their parametric specification is too strict.

The GPH estimator (Geweke and Porter-Hudak, 1983) is based on a full functional specification of the underlying process as an ARFIMA(0,d,0) process with a specific spectral form:

$$f\left(\lambda\right) \propto \left|1-\exp\left(-i\lambda\right)\right|^{-2(H-0.5)} = \left(4 sin^2(\lambda/2)\right)^{-(H-0.5)} \tag{6}$$

The spectrum $f(\lambda)$ is estimated using the periodogram and the Hurst exponent is estimated using ordinary least squares on

$$\log I\left(\lambda_j\right) \propto -(H-0.5)\log\left(4 sin^2\left(\lambda_j/2\right)\right) \tag{7}$$

The estimator is consistent and asymptotically normal (Beran, 1994), specifically

$$\sqrt{T}\left(\hat{H}-H^0\right) \rightarrow_d N\left(0,\pi^2/6\right) \tag{8}$$

As the periodogram is not a consistent estimator of the spectrum, Reisen (1994) and Reisen et al. (2000) propose to apply the smoothed periodogram for the estimation procedure (see both references for more details on smoothing). We use both methods. A major issue with frequency-based estimators such as GPH is the fact that the underlying series do not necessarily follow the assumed process specification. In the case of GPH, this is a simple ARFIMA(0,d,0). In the following sections, however, we show that the correlation structure of the electricity series is very complicated and it is thus an oversimplification to assume this specification. Moreover, assuming such specification incorrectly yields biased estimates as expected. To at least partly overcome this issue, Robinson (1995a) and Phillips and Shimotsu (2004) propose to utilize only a part of the periodogram for the estimation of the Hurst exponent in Eq. 7. The part of the periodogram taken into consideration, m, is usually taken as a root of the time series length T, so that $m = T^{\eta}$, where parameter η varies between 0 and 1. The asymptotic properties then change to

$$\sqrt{m}\left(\hat{H}-H^0\right) \rightarrow_d N\left(0,\pi^2/6\right) \tag{9}$$

The estimator thus becomes less efficient, but is less sensitive to bias at high frequencies. The estimates of the Hurst exponent are then drawn against varying m to see whether the estimates stabilize at some point so that the correct estimate can be identified.

4. Data Description

We analyze hourly day-ahead spot prices of electricity[2] in the Czech Republic between January 1, 2009 and November 30, 2012, with a total of 34,316 observa-

Table 1 Descriptive Statistics

	Price	Change in price
mean	43.7700	0.0001
SD	16.2551	6.6093
skewness	0.1040	0.4136
excess kurtosis	1.6686	7.4503
Shapiro-Wilktest	14.9120	19.6280
p-value	< 0.01	< 0.01
Jarque-Bera test	4041	80315
p-value	< 0.01	< 0.01
ADF-test (50)	-13.9594	-36.1585
p-value	< 0.01	< 0.01
KPSS (50)	8.7456	0.0012
p-value	< 0.01	>0.1

tions.[3] The prices are denominated in EUR per MWh and negative prices were not allowed before 2013. In *Figure 1*, we show the evolution of prices during the period analyzed. It is evident that prices jump frequently in both directions. The first differences of the prices strongly resemble the standard returns of stocks or exchange rates, with volatility clustering and extreme movements. The first differences are far from being normally distributed, as shown in *Figure 2*. However, the original price series are close to normally distributed if we omit the fact that the prices are censored from below. Overall non-normality of the distributions is supported by the Shapiro-Wilk (Shapiro and Wilk, 1965) and Jarque-Bera (Jarque and Bera, 1980, 1981) tests in *Table 1*, which show strong rejection of normality for both series. Standard descriptive statistics support only mild heavy tails for prices, but heavy tails for the first differences. Both series are positively skewed, so more extreme upward movements are more likely. However, the skewness of prices is very close to zero, hinting at symmetry, which is again in line with the histograms in *Figure 2*.

To analyze the dynamics of the series, it is crucial to distinguish between stationary and non-stationary series. To this end, we use the ADF (Dickey and Fuller, 1979) and KPSS (Kwiatkowski et al., 1992) tests. The null hypothesis of the former is a unit root against no unit root, whereas for the latter, the hypothesis of stationarity against non-stationarity is tested. This provides an ideal combination of tests. The results, presented in *Table 1*, provide evidence that the price series are non-stationary but do not contain a unit root, whereas the first difference series are stationary. In terms of long-term memory notation, the prices of electricity are in the interval $1 < H < 1.5$ and thus the first differences lie in the range $0 < H < 0.5$. We thus follow with an analysis of prices, and not first differences, for three reasons. Firstly, we do not want to lose information about the dynamics of the prices, which we would lose by first differencing. Secondly, the price series are much more inter-

[2] OTE—the electricity market operator in the Czech Republic—runs four trade platforms: a block market, a day-ahead spot market, an intra-day market, and a balancing market in regulating energy. For more information, see the Product Sheet at http://www.ote-cr.cz/about-ote/main-reading.

[3] The data were obtained from OTE's Yearly Reports, available at http://www.ote-cr.cz/statistics.

Figure 1 Time Series Plots
Hourly electricity prices (left) and changes therein (right) are shown. The changes resemble the returns of various financial assets, whereas the dynamics of prices are much further from such returns.

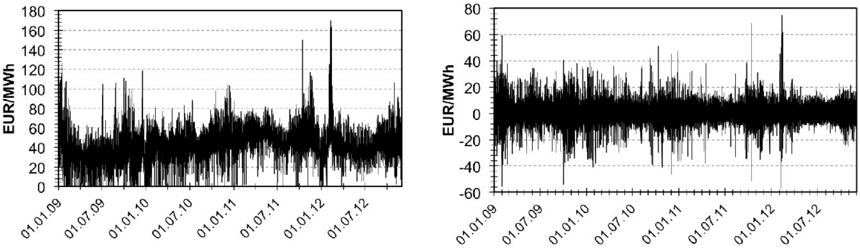
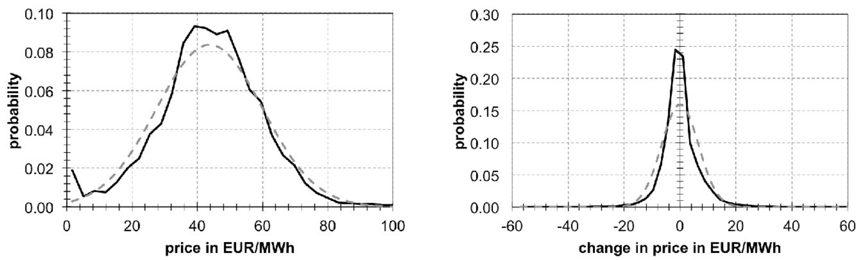


Figure 2 Histograms of Electricity Prices and Changes in Prices
The probability distribution function of prices (left, in black) is quite close to the normal distribution (dashed grey line), with the exception of the censored left tail (no negative prices in the sample). Changes in prices (right) are much further from normality.



esting in the electricity context—there are no actual returns to the series, as it is not possible to buy a MWh of electricity and sell it in the following period. And thirdly, the expected anti-persistence of the first-differenced series might cause the estimates of the Hurst exponent to be biased, whereas for the price series, DFA provides more reliable results.

The memory properties of electricity prices are further illustrated by the sample autocorrelation function and periodogram[4] in *Figure 3*. There, we observe that the dynamics of prices are very cyclical, with a dominating frequency of 24 hours. Both the autocorrelation function and the periodogram are well in line with the definitions of long-term memory. However, we can see that both the power-law decay of the autocorrelation function and the power-law divergence at the origin of the periodogram are disturbed by the aforementioned cyclical properties. Due to this fact, we use DFA to analyze the long-term memory properties of the series, as discussed in the previous section. The complex cyclicality is further illustrated in *Figure 4*, where we show how the average price and average traded volume depend on the hour of the day, the day of the week, and the week and month of the year. This again calls for a robust method of Hurst exponent estimation, as discussed previously.

[4] The periodogram is based on Bartlett weights with a bandwidth of 370, i.e., approximately 0.1 of the time series length.

Figure 3 Correlation Structure of Prices
Both the autocorrelation function (left) and the periodogram (right) show a strong seasonal component with a dominating scale of 24 hours.

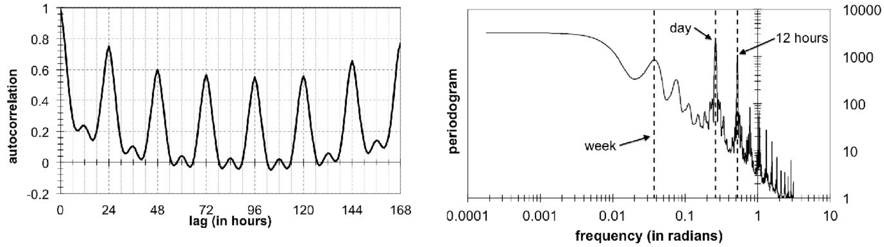
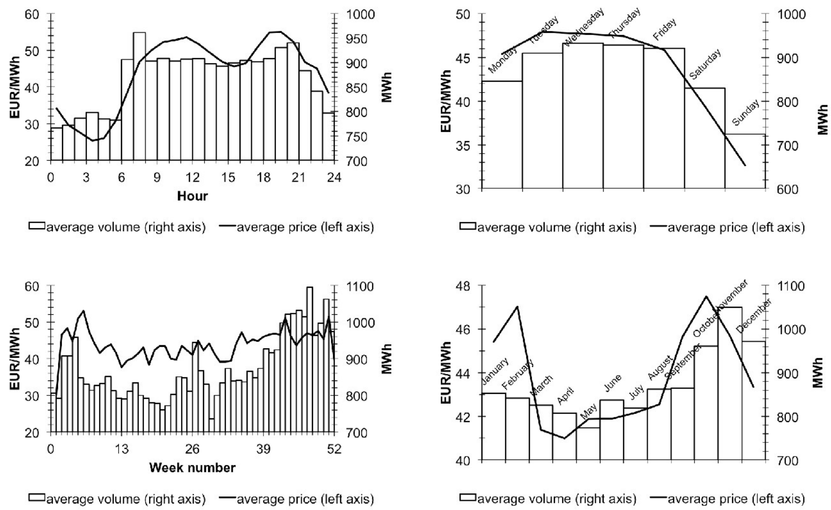


Figure 4 Cyclical Properties of Electricity Prices and Volumes
Seasonal patterns are shown for the intraday (upper left), daily (upper right), weekly (lower left), and monthly (lower right) scales. Apart from the weekly scale, both prices and volumes show pronounced seasonal patterns.



5. Results and Discussion

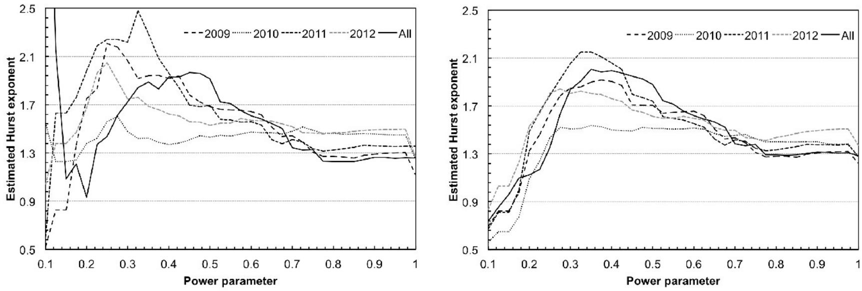
As shown in the previous section, the correlation structure of electricity prices in the Czech Republic is very complicated. To control for the most evident seasonalities, we analyze hourly prices, which control for intra-day patterns. Specifically, we standardize the first differences of the prices by subtracting the mean value for the given hour of the day and then dividing the difference by the standard deviation of the first differences for the given hour. The price series are then formed as an integrated series of these standardized first differences.

Before turning to the results of the detrended fluctuation analysis, we provide estimates of the Hurst exponent based on GPH and on the version of GPH based on a smoothed periodogram.[5] In *Figure 5*, we show how the estimates vary with η,

[5] For this, we use the functions *fdGPH* and *fdSperio* in the *fracdiff* package in R-project.

Figure 5 GPH Estimates of the Hurst Exponent

GPH (left) and GPH based on the smoothed periodogram (right) are shown for varying *m*, where $m = T^{\eta}$. The power parameter η is shown on the *x*-axis. The estimates for both methods vary widely between $H \approx 0.5$ (uncorrelated noise) and $H \approx 2$ (a strongly persistent non-stationary non-mean-reverting process).

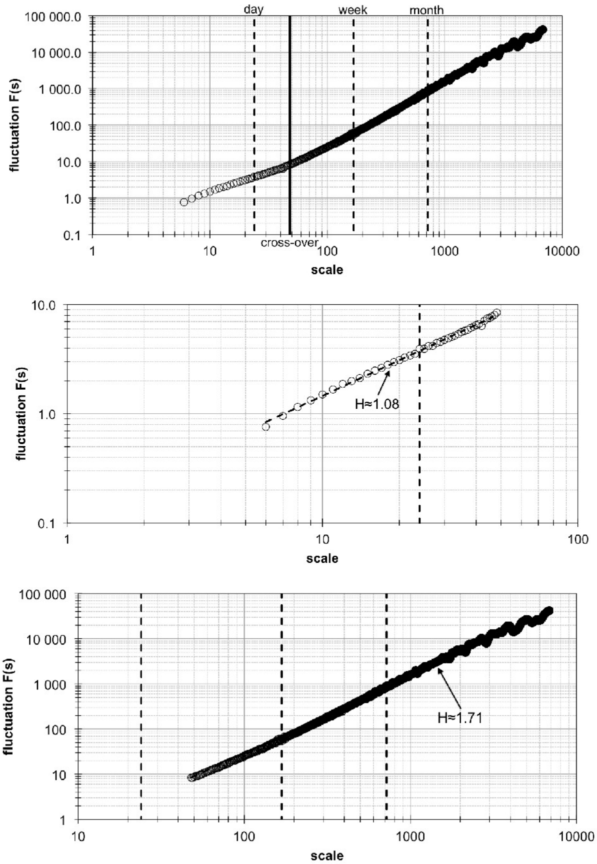


between 0.1 and 1 with a step of 0.025. Moreover, the results are presented for the whole time period as well as for the separate years. We observe that the estimates fluctuate widely with changing η. For the original GPH, the estimates vary between $H \approx 5.1$ and $H \approx 0.9$, while most of the estimates lie between the Hurst exponent of 1.2 and 2.1. The range is thus very wide and the estimates stabilize at least somewhat for $\eta > 0.7$. However, these estimates are based on almost the whole periodogram, where high frequencies (and thus low scales) dominate. Even though the estimates are much less erratic for the smoothed version of GPH, the range of the estimates does not narrow down enough—the estimates range between $H \approx 0.5$ and $H \approx 2.1$. The estimates again stabilize for $\eta > 0.7$. Even though the estimated Hurst exponents practically overlap for all years, the GPH approach can hardly be taken as reliable for this specific case of electricity prices, and the fact that the estimators have well defined asymptotic properties does not help our analysis at all. These results only stress the need for a more robust estimation technique—detrended fluctuation analysis.

For the detrended fluctuation analysis, i.e., the multifractal detrended fluctuation analysis with $q = 2$, we set $s_{min} = 6$ and $s_{max} = T/4$ to obtain the scaling of the fluctuation $F^2(s)$ illustrated in *Figure 6*. As the data frequency equals one hour, the minimum scale is set at a quarter of a day and the maximum scale is approximately one year. Based on the initial analysis of the series in the Data Description section, we assume that the series contain strong cycles but might also possess long-term memory. It is thus reasonable to assume that the scaling of $F^2(s)$ contains at least one cross-over. This is indeed true for the electricity prices analyzed, as shown in *Figure 6*. We observe one evident cross-over at $s_x \approx 48$. The cross-over splits the scaling chart into two laws, which strongly resemble a power-law scaling, as shown in the split charts in *Figure 6*. This gives two Hurst exponents: $H \approx 1.1$ for $s \leq 48$ and $H \approx 1.7$ for $s \geq 48$. Note that these Hurst exponents do not differ considerably for varying s_x between 36 (1.5 days) and 72 (3 days) and they are thus quite stable. This multi-scaling can be attributed to the competing effects of the long-term memory and periodic trends, which are both strong parts of the dynamics of electricity prices. As discussed in the previous section, the Hurst exponent based on scales below the first cross-over scale s_x can be used for interpretation of

Figure 6 Scaling of Fluctuations *F*(*s*)

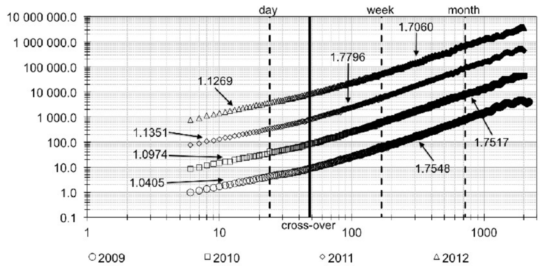
The upper panel shows the scaling of the fluctuation function and a pronounced cross-over at approximately two days. The other two panels show the scaling for both regimes in more detail. The middle panel, characterized by $H = 1.08$, is attributed to the long-term memory of the price process and the bottom panel shows the scaling for scales dominated by cyclical components.



the long-term memory. Therefore, the price dynamics is characterized by $H \approx 1.1$ and the prices are thus strongly persistent and non-stationary, but still remain well below the unit-root level of $H = 1.5$, so they remain mean-reverting. This is consistent with the basic description in *Table 1*.

The persistence of the series implies that prices follow rather long-lasting trends, which even exceed the standard long-term memory with $0 < H < 1$, making the prices non-stationary. Nonetheless, the dynamics are far from unit-root behavior and the prices return to their long-term levels. Such behavior is very different from other financial assets, which usually follow a random walk and their returns are thus unpredictable (or at least not systematically predictable). However, we need to keep in mind that such persistence of electricity prices cannot be easily exploited for profit. The persistence can also be seen as a product of incorrect expectations about

Figure 7 Scaling of Fluctuations *F*(*s*) for Separate Years
The estimated Hurst exponents are shown for two scaling regimes for separate years between 2009 and 2012. The scaling exponents are remarkably stable.



the future need for electricity among market participants. Remembering that the electricity spot market exists to cover unexpected demand for electricity (as the majority of the electricity supplied is based on medium- and long-term contracts), extreme price movements are caused mainly by unexpected external events (extremes of temperature or humidity, macroeconomic news, etc.). When an unexpected event occurs, it usually has a medium- or long-lasting effect (for example, a temperature above the long-term average usually lasts a whole day or even longer), but traders cannot "pre-buy" electricity quickly. To cover the increased demand, additional (and usually more expensive) power sources need to be connected to the network, and this increases electricity prices. The combined effect of non-storability and the connection of less efficient power sources pushes electricity prices toward persistent behavior.

To see whether these properties are stable over time, we analyze the long-term memory components in the same way but for the separate years 2009–2012. In *Figure 7*, we observe that the Hurst exponent linked with long-term memory is rather stable and approximately 1.1 for all the price series. For higher scales, we again see stability of the scaling exponent around 1.75. Non-stationary mean-reverting persistence is thus observed even for separate years. Note that only the very specific characteristics of the DFA method allow us to study the long-term memory without arriving at spurious results. Normally, the Hurst exponent linked with higher scales would be reported. However, the difference between having $H < 1.5$ and $H > 1.5$ is crucial. For the former, prices return to their long-term mean. But for the latter, prices would explode. Note that having $H \approx 1.1$ implies that the mean reversion is very rapid. These characteristics are very stable over time, mirroring the results for the Ontario and Alberta markets as reported by Alvarez-Ramirez and Escarela-Perez (2010). The results therefore lie somewhere between the stationary electricity prices in the USA reported by Park et al. (2006) and for Alberta by Uritskaya and Serletis (2008) and the almost unit-root prices found for the Nord Pool market by Simonsen (2003).

6. Conclusion

We analyzed the long-term memory properties of hourly spot prices of Czech electricity between 2009 and 2012. As electricity prices have very intriguing properties, such analysis is rather challenging. We showed that Czech prices follow patterns similar to those observed for other electricity prices, in particular intraday, daily, and monthly seasonality in both prices and volume. By applying detrended fluctuation analysis, we were able to separate these cyclical properties from the long-term memory. The results are in line with the majority of the relevant literature, as we show that electricity prices are non-stationary but mean-reverting, so their behavior is partly predictable. However, due to the specific features of electricity (mainly its non-storability), such predictable behavior cannot easily be exploited for profit. Electricity prices are thus very different from prices of standard financial assets such as stocks or exchange rates and they need to be treated accordingly. The patterns found in the behavior of electricity prices can be attributed to their structure, as spot prices were analyzed. These serve mainly to balance demand for electricity, which is not covered by futures contracts. As such, an unexpected change in demand for electricity is rather short-lived and the reversion to the long-term price is quite rapid, as represented by a Hurst exponent close to (but higher than) unity. The stability of the results in specific years and the correspondence to the results of more developed markets underline that the Czech electricity market has reached a similar level of development.

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