Lecture 2.04: Multiple Mendelian Traits

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BIO 181, General Biology for Majors

Outline

1. Introduction: the question
2. Properties of probability
   - Multiplication rule
   - Addition rule
3. Application to Genetics

A more complex problem

Problem

What are the expected phenotypes and phenotypic ratios from a cross between spherical-, yellow-seeded, tall, purple-flowered F_1 (heterozygous) pea plants?

$$AaBbCcDd \times AaBbCcDd$$

- How big would the Punnett square have to be?
-
-

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- How big would the Punnett square have to be?
- $16 \times 16 = 256$ cells. (Yuck!)
- Luckily, there is a way to calculate the probabilities directly.

We need a little probability theory.

Properties of probability 1: Independent events

Concept: Random event

A **random event** is any event for which the result cannot be predicted with certainty.

"Multiplication Rule" (I.e., definition of independent events)

Suppose A and B are 2 independent random events; that is, the outcome of A has no effect on the outcome of B. Then the probability of both A and B is equal to the product of the probability of A and the probability of B. That is,

$$\Pr(A \text{ and } B) = \Pr(A) \times \Pr(B).$$

Examples of the multiplication rule

Example 1

What is the probability that two consecutive coin flips yields heads both times?

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What is the probability that two consecutive coin flips yields heads both times?

Solution:

$$\Pr(2\text{ heads}) = \Pr(1\text{st toss heads}) \times \Pr(2\text{nd toss heads})$$
$$= \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}.$$

Examples of the multiplication rule

Example 2

Joe and Amy plan to have two children. What is the probability that the first is a boy and the second is a girl?

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Joe and Amy plan to have two children. What is the probability that the first is a boy and the second is a girl?

Solution:

$$\begin{aligned}\Pr(\text{Boy then Girl}) &= \Pr(\text{1st kid boy}) \times \Pr(\text{2nd kid girl}) \\ &= 0.51 \times 0.49 \\ &= 0.2499.\end{aligned}$$

Examples of the multiplication rule

Example 3

What is the probability that, in a family with 5 children, all the kids are girls?

Example 4

What is the probability that a family of 5 children has at least 1 boy child?

Examples of the multiplication rule

Example 3

What is the probability that, in a family with 5 children, all the kids are girls?

Solution:

$$0.49^5 = 0.02825.$$

Example 4

What is the probability that a family of 5 children has at least 1 boy child?

Examples of the multiplication rule

Example 3

What is the probability that, in a family with 5 children, all the kids are girls?

Solution:

$$0.49^5 = 0.02825.$$

Example 4

What is the probability that a family of 5 children has at least 1 boy child?

Solution:

$$1 - 0.49^5 = 1 - 0.02825 = 0.97175.$$

Properties of probability 2: Disjoint events

"Addition rule"

Suppose that an outcome of a random event can occur in two distinctly different ways, A_1 or A_2. Then the probability of event A is the sum of the probabilities of A_1 and A_2. That is,

$$\Pr(A) = \Pr(A_1) + \Pr(A_2).$$

Note:

$$\text{“And”} = \times;$$
$$\text{“Or”} = +.$$

Examples of the addition rule

Example 1: Joe and Amy again

Joe and Amy plan to have two children. What is the probability that they **have one girl and one boy**?

What is the probability of each of these **events**?

or

or

HINT: They are not equally probable.

We have to consider the *identities* of the babies.

Examples of the addition rule

Example 1: Joe and Amy again

Joe and Amy plan to have two children. What is the probability that they **have one girl and one boy**?

Solution:

- $\Pr(\text{girl and girl}) = 0.49^2 = 0.2401;$
- $\Pr(\text{girl and boy}) = 0.49 \times 0.51 = 0.2499;$
- $\Pr(\text{boy and girl}) = 0.51 \times 0.49 = 0.2499;$
- $\Pr(\text{boy and boy}) = 0.51^2 = 0.2601;$

Therefore, $\Pr((\text{girl and boy}) \text{ or } (\text{boy and girl})$

$$\begin{aligned} &= \Pr(\text{girl and boy}) + \Pr(\text{boy and girl}) \\ &= 0.2499 + 0.2499 \\ &= 0.4998. \end{aligned}$$

Genetic examples

Example from Mendel's study

Mendel crossed two spherical-, yellow-seeded F_1 plants. What is the probability that the offspring is spherical-, green-seeded?

Questions that need to be answered:

- Are the traits independent? (Can we use the multiplication rule?)
- What are the genotypes of the parents?
- What is the probability the offspring is spherical?
- What is the probability the offspring is green?

Simple genetic example

Example from Mendel's study

Mendel crossed two spherical-, yellow-seeded F_1 plants. What is the probability that the offspring is spherical-, green-seeded?

Questions that need to be answered:

- Mendel already showed the traits were independent.
- $AaBb \times AaBb$.
- Probability the offspring is spherical $= 3/4$.
- Probability the offspring is green $= 1/4$.

Therefore,

$$\Pr(\text{spherical and green}) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}.$$

Back to the original problem

Problem

What are the expected phenotypes and phenotypic ratios from a cross between spherical-, yellow-seeded, tall, purple-flowered F_1 (heterozygous) pea plants?

$$AaBbCcDd \times AaBbCcDd$$

- What is the probability it's wrinkled?
- What is the probability it's green?
- What is the probability it's tall?
- What is the probability it's purple?

Back to the original problem

Problem 1

What are the expected phenotypes and phenotypic ratios from a cross between spherical-, yellow-seeded, tall, purple-flowered F_1 (heterozygous) pea plants?

$$AaBbCcDd \times AaBbCcDd$$

- Probability it's wrinkled $= 1/4$.
- Probability it's green $= 1/4$.
- Probability it's tall $= 3/4$.
- Probability it's purple $= 3/4$.

Therefore, Pr(wrinkled and green and tall and purple)

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{256}.$$

A more challenging problem

Problem 2

Consider the same cross as before, namely:

$$AaBbCcDd \times AaBbCcDd.$$

What is the probability that the offspring is heterozygous for at least 3 of the 4 traits?

How many ways can this criterion (heterozygous for at least 3 of 4 traits) be met?

A more challenging problem

Problem 2

Consider the same cross as before, namely:

$$AaBbCcDd \times AaBbCcDd.$$

What is the probability that the offspring is heterozygous for at least 3 of the 4 traits?

Solution: Here are all the possibilities.

AaBbCcDD	AaBbCcdd
AaBbCCDd	AaBbccDd
AaBBCcDd	AabbCcDd
AABbCcDd	aaBbCcDd

AaBbCcDd

A more challenging problem

Solution (cont.): Here are their probabilities.

$$\begin{aligned} \text{AaBbCcDD} &= (1/2)^3 \times 1/4 & \text{AaBbCcdd} &= (1/2)^3 \times 1/4 \\ \text{AaBbCCDd} &= (1/2)^3 \times 1/4 & \text{AaBbccDd} &= (1/2)^3 \times 1/4 \\ \text{AaBBCcDd} &= (1/2)^3 \times 1/4 & \text{AabbCcDd} &= (1/2)^3 \times 1/4 \\ \text{AABbCcDd} &= (1/2)^3 \times 1/4 & \text{aaBbCcDd} &= (1/2)^3 \times 1/4 \end{aligned}$$

$$\text{AaBbCcDd} = (1/2)^4$$

Now, combine them.

A more challenging problem

Solution (cont.): Here are their probabilities.

$$\begin{aligned} \text{AaBbCcDD} &= (1/2)^3 \times 1/4 & \text{AaBbCcdd} &= (1/2)^3 \times 1/4 \\ \text{AaBbCCDd} &= (1/2)^3 \times 1/4 & \text{AaBbccDd} &= (1/2)^3 \times 1/4 \\ \text{AaBBCcDd} &= (1/2)^3 \times 1/4 & \text{AabbCcDd} &= (1/2)^3 \times 1/4 \\ \text{AABbCcDd} &= (1/2)^3 \times 1/4 & \text{aaBbCcDd} &= (1/2)^3 \times 1/4 \end{aligned}$$

$$\text{AaBbCcDd} = (1/2)^4$$

Here's how they combine:

$$8\left(\frac{1}{2}\right)^3 \frac{1}{4} + \left(\frac{1}{2}\right)^4 = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}.$$