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Non-Integer Order Controller Based Robust Performance Analysis of a Conical Tank System

By

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ABSTRACT

The design of robust controller for any non linear process is a challenging task because of the presence of various types of uncertainties. In this paper, various design methods of robust PID controller for the level control of conical tank are discussed. Uncertainties are of different types, among that structured uncertainty of 30% is introduced to the nominal plant for analysing the robustness. As a first step, the control of level is done by using conventional integer order controller for both nominal and uncertain system. Then, the control is done by means of Fractional Order Proportional Integral Derivative (FOPID) controller for achieving robustness. With the help of time series parameters, a comparison is made between conventional PID and FOPID with respect to the simulated output using MATLAB and also analyzed the robustness.

Keywords

Conical tank, Level Control, Robust controller, Fractional order PID controller

1. INTRODUCTION

While designing the controller, the nominal properties related to the characteristics of the system include the model of the controlled process should be the exact replica of the real process behavior. Generally, the single conical tank system exhibits non-linear behavior. A non-linear process has to be linearised first before an automatic controller can be applied effectively. Using Taylor series expansion method, non linearity present in conical tank system is to be linearised. That linearised transfer function is used in this paper and controller is designed for the same. For checking the robustness of the controller, uncertainty is to be introduced to the modeled conical tank system transfer function.

Uncertainties occur when system gain or some other parameters vary over a particular range. There are two types of uncertainties [4] and among that, structured uncertainty is introduced to the nominal system and robustness is analyzed. This paper deals with the design to achieve the robustness against uncertainties and a fractional order controller is designed for the system having uncertainties and also simulated using MATLAB software. The simulated output of Fractional Order Proportional Integral Derivative (FOPID) [11] controller has compared with conventional PID based on step response characteristics.

2. REPRESENTATION OF UNCERTAINTY

Uncertainty in a system can occur in various forms and from various sources. Examples of real parameter uncertainties include uncertain gain and uncertain time constant. One source of uncertainty is neglected high-frequency dynamics. Then uncertainty is modeled by one unknown transfer function with some weights and is called unstructured uncertainty. Another type of uncertainty arises if one or more parameters of the plant are not known precisely. This leads to structured uncertainty.

2.1 Unstructured Uncertainty

Uncertainty caused by non- modeled dynamics and which is obtained by combining the nominal plant G(s) with a transfer function of the kind W(s) Δ(s). Such kind of uncertainty is denoted as unstructured uncertainty from which the transfer function W(s) is known and Δ(s) is unknown. Then the feedback system consists of the extended plant, the uncertainty Δ and the controller K. Normally the high frequency dynamics of the plant is inaccurate. This means that at high frequency, gain and phase lie within a band that gets broader if the frequencies increase.

2.2 Structured Uncertainty

Structured uncertainty arises when system gain or other parameters are not precisely known, or can vary over a given range. Examples of real parameter uncertainties include uncertain pole and zero locations and uncertain gains. The unstructured uncertainty is useful in describing non-modeled or neglected system dynamics. These complex uncertainties usually occur in the high-frequency range and may include non-modeled lags (time delay), parasitic coupling, hysteresis and other nonlinearities. However, dynamic perturbations in many industrial control systems may also be caused by inaccurate description of component characteristics, torn-and-worn effects on plant components, or shifting of operating points, etc. Such perturbations may be represented by variations of certain system parameters over some possible value ranges (complex or real). They affect the low-frequency range performance and are called "parametric uncertainties".

3. MODELING OF SYSTEM WITH UNCERTAINTIES

The mathematical model of the conical tank is determined by considering two assumptions: One is the level as the control variable and the second is inflow to the tank as the manipulated variable. They can be achieved by controlling the input flow into the tank. Fig 1 shows the single conical tank system.

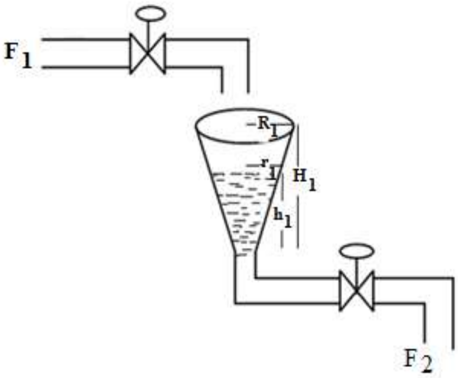

Fig 1: Single Conical Tank System

The shape of the conical tank gives rise to its non-linearity that need to be linearised for designing the controller. Here the process delay is measured by calculating the time taken by the flowing fluid to reach the input control valve from the reservoir tank.

$$\frac{H_1(s)}{F_1(s)} = \frac{K}{\tau s+1} * e^{-ts} \tag{1}$$

Where,

$$K = 2\left(\frac{\alpha}{\beta}\right) h_{1s}^{\frac{1}{2}} \quad ; \quad \tau = \frac{2}{\beta} h_{1s}^{\frac{1}{2}}$$

$$\alpha = \frac{H_1^2}{R_1^2 \pi} \quad ; \quad \beta = \alpha c_v \sqrt{2g}$$

t- Process delay

c_v -Valve co-efficient

g- Specific gravity of liquid inside the tank

3.1 Nominal Plant Model

The prototype hardware of the conical tank system is shown in Fig 2 and the hardware specifications of the system are given below:

H - Total height of the conical tank = 60 cm

R - Top radius of the conical tank =15 cm

h - Steady state of height of the tank = 15cm

L - Process delay = 4 seconds

The model of nominal plant is obtained by substituting the specifications in Equation 1.

Fig 2: Hardware implementation of conical tank

The transfer function of nominal plant is obtained as,

$$\frac{H_1(s)}{F_1(s)} = \frac{1.095}{48.395s+1} * e^{-4s} \tag{2}$$

3.2 Uncertain Model

In the nominal plant model, uncertainty of 30% is introduced. Maximum and minimum uncertainties are obtained and corresponding transfer function is calculated. The uncertainty ranges are shown in Table-1.

Table-1.Uncertainty Ranges

Para meter	**Value**	**% Uncertainty**	**Uncertainty value**	
			Min	**Max**
K	1.095	30%	0.7665	1.4235
τ	48.395	30%	33.875	62.915

The transfer function for +30% uncertainty,

$$G(s) = \frac{1.4235}{62.915s+1} \tag{3}$$

The transfer function for -30% uncertainty,

$$G(s) = \frac{0.7665}{33.875s+1} * e^{-4s} \tag{4}$$

4. PADE APPROXIMATION

In some situations, as in frequency response based analysis of control systems containing a time-delay, it is necessary to substitute time delay with an approximation in form of a rational transfer function. The most common approximation is the Pade approximation. The Pade approximation is based on a minimization of the truncation errors in a finite series expansion. For a conical tank system having a time delay of 4 seconds as given in Equation-2, Pade approximation is performed and a controller is to be designed for the obtained transfer function. The equation governing the time delay approximation is,

$$e^{-4s} = \frac{1-2s}{1+2s} \qquad (5)$$

After performing first order pade approximation for Equation 2, the transfer function obtained is,

$$G_p(s) = \frac{-1.095445s + 0.5477}{48.395s^2 + 25.1976s + 0.5} \qquad (6)$$

Similarly, pade approximation has also to be performed for both maximum and minimum uncertainties. For the obtained transfer function, controller needs to be designed.

5. CONTROLLER DESIGN

5.1 Design of Conventional PID Controller

The design of the conventional PID controller is in need for the exact values of the controller tuning parameters K_P , K_I and K_D in accordance with the system transfer function. By using Ziegler-Nichols (Z-N) and Cohen-Coon (C-C) methods, the values of the PID controller tuning parameters are calculated and obtained values are given in Table-2.

Table-2 K_P, K_I, K_D Values of PID Controller

Tuning Method	**Tuning Parameters**		
	K_P	**K_I**	**K_D**
Z-N	10.563	1.3649	20.4369
C-C	14.91	1.5678	21.377

By substituting the tuning values obtained by the Z-N method and C-C methods in the PID controller equation, the controller transfer function is then obtained.

5.2 Design of FOPID Controller

In order to design the FOPID controller, it is essential to obtain five controller tuning parameters like K_P , K_I , K_D , λ and μ. For real time implementation, by using Ziegler Nichols tuning method, K_P, K_I, and K_D values are chosen and the values of λ and μ are taken in accordance with the optimum setting. FOPID Controller parameters are given in Table-3.

Table-3 FOPID Tuning Parameters

S.No.	K_P	K_I	K_D	λ	μ
1.	10.563	1.36489	20.4369	0.5	0.6

Then by substituting the values given in Table- 3, the transfer function of the FOPID controller is derived as,

$$G_c(s) = \frac{20.4369s^{1.1} + 10.563s^{0.5} + 1.3649}{s^{0.5}} \qquad (7)$$

6. RESULTS AND ANALYSIS

The different characteristics of the system response are analyzed by giving step input. Simulation is performed using MATLAB for both conventional PID controller and FOPID Controller.

6.1 Response for Nominal Plant

Without introducing uncertainty, the simulation output response obtained for conventional PID controller using MATLAB by giving step input to the conical tank system is shown in Fig 3.

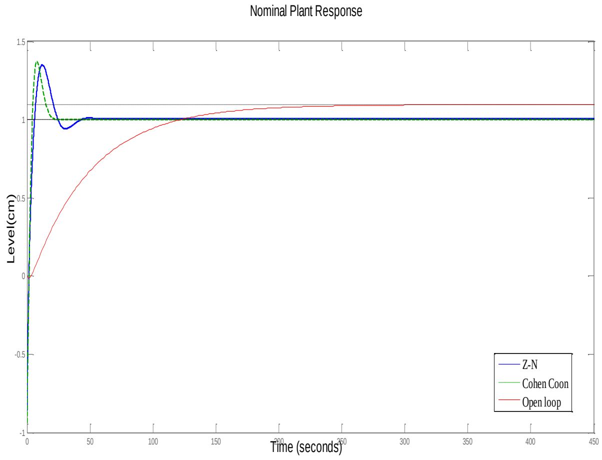

Fig 3: Step Response without Uncertainty using Z-N and Cohen Coon Tuning method

6.2 Response with Maximum Uncertainty

After introducing uncertainty, system performance gets affected. Here uncertainty is introduced in system gain and time constant. The obtained step response with maximum uncertainty to the plant is shown in Fig 4.

Structured uncertainty of +30% is added to the modeled plant transfer function which affects the response so that the settling time is increased. As the settling time increases, the time taken to reach the set point is high which indicates that the control action is quite insignificant. It is found that the robustness cannot be achieved by using conventional PID controller and hence it is necessary to go for other type of controller which should give robust performance.

From the transient parameters listed in Table-4, there is a need to use a suitable controller. From the literature survey made, it is concluded that the use of a non-integer based controller such as a fractional order controller is to be used.

Table-4 Transient Response Parameters for maximum uncertain plant

Tuning Method	**Settling Time (sec)**	**Rise Time (sec)**	**Peak Amplitude**	**Overshoot (%)**
Z-N	52.213	3.728	1.370	39.097
C-C	19.819	2.202	1.392	39.257

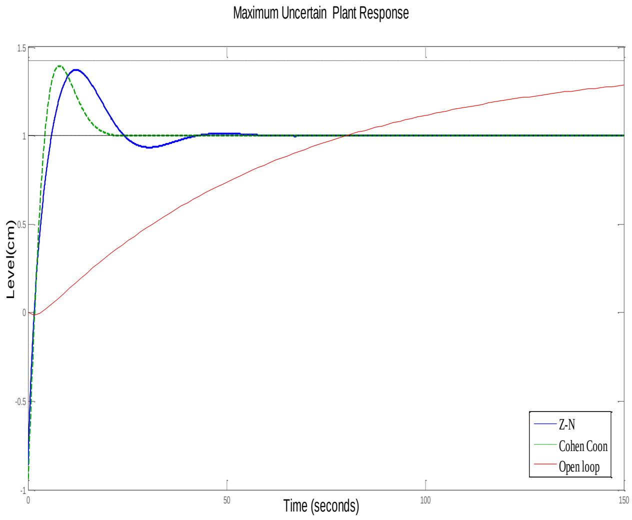

Fig 4: Step Response with Maximum Uncertainty using Z-N and Cohen Coon method

6.3 Response with Minimum Uncertainty

Similarly, the uncertainty of -30% is introduced to the conical tank system, which is a minimum uncertainty range and for which the step response is obtained by using Z-N based controller settings. Fig 5 shows the step response of the plant with minimum uncertainty.

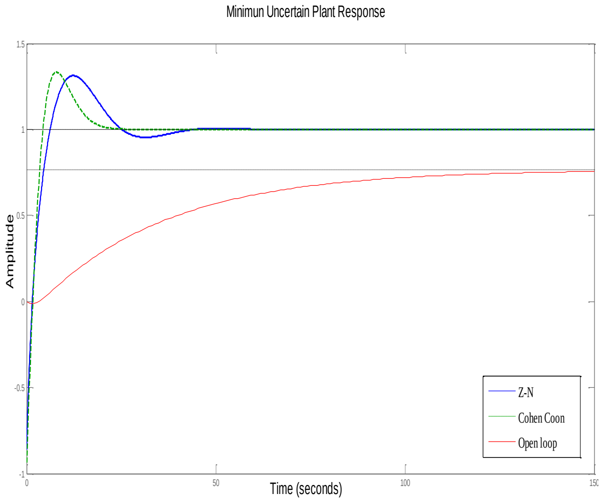

Fig 5: Step Response with Minimum Uncertainty using Z-N method

Instead of taking entire uncertainty ranges, just maximum and minimum uncertainties are given to the single conical tank system.

Table-5 Transient Response Parameters for minimum uncertain plant

Tuning Method	**Settling Time (sec)**	**Rise Time (sec)**	**Peak Amplitude**	**Overshoot (%)**
Z-N	57.079	3.690	1.370	37.270
C-C	21.378	2.004	1.332	33.23

Uncertainties in real time include parameters in the linear model identified from noisy input/output data and non-linearity in actuators and sensors such as actuator/sensor saturation, actuator/sensor failure, and hardware deterioration over time.

6.4 Response of FOPID Controller

FOPID controller greatly increases the robustness and eliminates the overshoot and time delay problems. It describes a real object more accurately than classical integer order methods. When comparing the closed loop transfer function of PID controller, the FOPID controller equation is in order of 2.5. But in case of PID controller third order equation is obtained. In FOPID, different fractional values in first and second order will increase the efficiency and there is no need to go for higher orders. Fig 6 shows the simulated response for single conical tank system with and without uncertainty using FOPID controller.

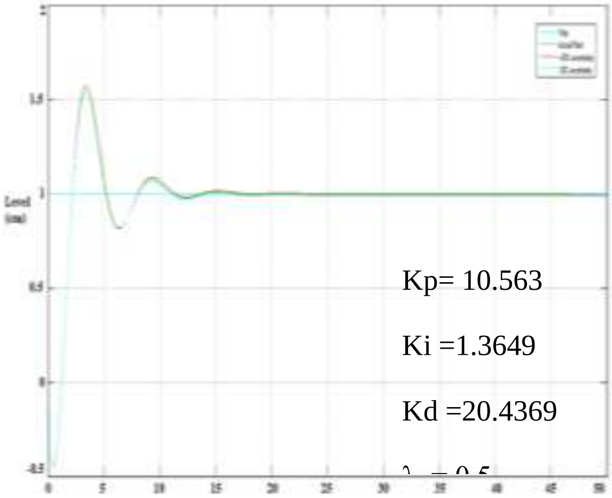

Fig 6: Response of FOPID Controller with and without uncertainty

Table-6 Transient characteristic parameters for Conventional PID and FOPID Controllers with and without uncertainty

Level (cm)	**Settling Time (s)**		**Rise Time (s)**		**Peak Amplitude**	
	PID	FOPID	PID	FOPID	PID	FOPID
Actual	40.93	18.5	3.673	3.3	1.349	1.56
Max. Uncertainty	55.21	19.2	3.728	3.3	1.370	1.64
Min. Uncertainty	57.07	18.92	3.690	3.4	1.370	1.51

Transient characteristics parameters are determined without uncertainty and minimum and maximum uncertainty for the single conical tank system designed by using conventional PID controller and FOPID controller. They are given in Table-7 and from which it is found that FOPID gives robust performance even in the presence of uncertainties.

7. CONCLUSION

The primary task of the controller is to maintain the process under stable conditions, even at different kinds of disturbances occurs. As a result of this proposed work, structured uncertainty of ±30% is introduced to the single conical tank system and pade approximation is performed both for nominal plant model and uncertain plant model. Conventional PID and FOPID controllers are designed for the condition of with and without uncertainty and corresponding plots are also obtained using MATLAB software. By giving step input to the system with both of the controllers, the transient characteristic parameters based comparison is made. It is found that FOPID gives robust performance even in the presence of process model mismatch.

REFERENCES

[1] Giovanini L, “Robust Adaptive Control using Multiple Models, Switching and Tuning”, Control Theory and Applications, IET, Vol.52, No.18, 2011.

[2] Karad S G and Hingmire S G, “Analysis of Performance of Fractional Order PID Controller for Continuous Yeast Fermentation Process”, IOSR Journal of Electronics and Communication Engineering (IOSR-JECE), pp.01-06, 2012.

[3] Krishnamoorthy Natarajan, “Robust PID Controller Design for Hydroturbines”, IEEE Transactions on Energy Conversion, Vol.20, No.3, 2005.

[4] Mackenroth U, “Book on robust control systems: Theory and case studies”, School of Applied sciences, Published by Springer (India) Private Limited, 2004.

[5] Ming-Tzu Ho and Chia-Yi Lin, “PID Controller Design for Robust Performance”, IEEE Transactions on Automatic Control, Vol.48, No.8, .1404-1409, 2003.

[6] Ming Ge, Min-Sen Chiu and Qing-Guo Wang, “Robust PID Controller Design via LMI Approach”, ELSEVIER- Journal of Process Control, Vol.12, pp.3-13, 2002.

[7] Nithya S, Sivakumaran , Radhakrishnan T K and Anantharaman N, “Soft Computing Based Controllers Implementation for Non-linear Process in Real Time”, Proceedings of the World Congress on Engineering and Computer Science(WCECS), Vol.2, October 20-22, San Francisco, USA, 2010

[8] Olof Garpinger, Tore Hagglund and Karl Johan Astrom, “Performance and robustness trade-offs in PID control”, ELSEVIER- Journal of Process Control, Vol.24, pp.568-577, 2014.

[9] Pham Luu Trung Duong and Moonyong Lee, “Robust PID controller design for processes with stochastic parametric uncertainties”, ELSEVIER- Journal of Process Control, Vol.22, pp.1559-1566, 2012.

[10] Richard D. Braatz, Manfred Morari and Sigurd Skogestad , “Loopshaping for Robust Performance”, International Journal of Robust and Nonlinear Control, Vol.6, pp.805-823, 2006.

[11] Shivaji Karad, Chatterji S and Prasheel Suryawanshi , “Performance Analysis of Fractional Order PID Controller with the Conventional PID Controller for Bioreactor Control”, International Journal of Scientific & Engineering Research, Vol.3, Issue 6, ISSN 2229-5518, 2012.

[12] Tomislav B, Sekara and Miroslav Matausek, “Optimization of PID Controller Based On Maximization of the Proportional Gain Under Constraints on Robustness and Sensitivity to Measurement Noise”, IEEE transactions on automatic control, Vol.54, No. 1, 2009.