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On the propagation of streamers in electrical discharges

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Abstract
Streamers are non-thermal filamentary plasmas developing in insulating mediums under the influence of strong external electric fields. The present knowledge, based on a widely-accepted concept of critical or stability field introduced half a century ago, indicates the existence of a unique minimum electric field in which streamers could propagate stably with constant radius and velocity. In this work, we present a new understanding indicating that growing, decaying, and stable propagation of streamers is controlled not solely by the external field but also by the physical dimensions of streamers. Stable propagation is demonstrated to be achievable in a wide range of electric fields, with a lower limit of ~5 kV cm^{-1} for positive streamers and ~10 kV cm^{-1} for negative streamers in air at atmospheric pressure, up to the breakdown field ~28.7 kV cm^{-1}. In these field ranges, the streamer radius required for stable propagation is inversely proportional to the external field, with larger and smaller initial radii, respectively, leading to growing and decaying streamers. The new mechanism suggests possible ways to flexibly control the streamer parameters in applications, such as changing the size and potential of the point electrode to obtain the required initial streamer dimensions for the desired propagation pattern.

Keywords: streamers, ionization, propagation, electrical discharges

(Some figures may appear in colour only in the online journal)

1. Introduction

The streamer theory was originally developed by Raether [1], Loeb and Meek [2] in the 1930s as an alternative to the Townsend theory to explain the experimental observations of spark discharges. Those pioneer works were followed by decades of extensive research on streamer physics and applications [3, 4]. It is now well-known that streamers are fundamental components in many types of gas discharges, such as the dielectric-barrier discharges between two electrodes [5], the lightning discharges in thunderstorms [6], and the transient luminous events in the upper atmosphere [7]. Streamers are also essential for the electrical breakdown phenomena in liquid dielectrics (e.g. transformer oil) and solid insulators [4, 8]. Moreover, it has been recently suggested that the thermal runaway electrons produced by streamers could be responsible for the terrestrial gamma ray flashes (TGFs) observed by satellites [9]. Those thermal runaway electrons with energies as high as ~100 keV can be further accelerated up to several MeVs energies by the electric field of lightning stepped leaders during the stage of negative corona flash, leading to the production of TGFs through bremsstrahlung radiation [9]. As for applications, streamers have long been used for industrial ozone production [5], plasma-assisted combustion [10], pollution control [11], and recently have shown a great medical potential [12, 13], for example, in wound healing and treatment of skin diseases [14, 15]. Understanding of streamer propagation mechanism is of essential importance for the studies of electrical breakdown phenomena and their related applications. It has been generally believed since the 1970s that streamers are propagating in such a way that their dynamics solely depends on the applied electric field. The external field that is equal to the critical field leads to stable propagation of streamers with constant radius and velocity. The fields with larger magnitudes lead to exponentially growing streamers, and those with smaller ones lead to decaying streamers that could propagate only for a short distance [16, 17]. In the framework of that theory, some fundamental concepts in streamer physics such as the critical fields and minimal streamer radius could not be explained, and flexible control of the streamer parameters in applications seemed difficult. In this paper, we present a new mechanism by deriving analytical criteria for decaying, stable and growing streamers, and by simulating streamer propagation in a wide range of electric fields in air at atmospheric pressure using a plasma fluid model. The results demonstrate that streamer dynamics, more specifically, the growing, decaying and stable propagation of streamers are controlled not solely by the external field but also by the physical dimensions of streamers.

2. Analytical criteria for streamer propagation

Consider a few free electrons moving in a gas (or a liquid dielectric) immersed in an external electric field E_0. The electrons are first accelerated by the electric force and then collide with neutral molecules, losing a part of their kinetic energy during the collision, after which the electron acceleration resumes. This process repeats such that on a macroscopic scale an ensemble of electrons propagates with an average velocity, referred to as the electron drift velocity. If the external electric field is stronger than the conventional breakdown field E_k (defined by the equality of ionization and dissociative attachment frequencies [18], $E_k \simeq 28.7$ kV cm^{-1} in air at atmospheric pressure [19]), electrons will gain enough energy between collisions to ionize the neutral molecules and create secondary electrons, which drift together with the primary ones. As this electron cloud moves forward, the number of secondary electrons increases exponentially, which is known as the electron avalanche phenomenon. Once the total number of electrons in the avalanche is so large that their space charge field becomes comparable to E_0, the avalanche-to-streamer transition occurs [18].

Streamers are narrow filamentary plasmas driven by highly nonlinear space charge waves [18]. Unlike an electron avalanche that propagates in a drift manner and has negligible space charge effect, the dynamics of a streamer is mainly controlled by a highly-enhanced field region in the tip of the filament, known as the streamer head. This head region, depicted as a crescent shape in figure 1(*a*), contains a large amount of net positive or negative space charge, that respectively corresponds to positive and negative streamers. The space charge strongly enhances the electric field to values about 3–7E_k in the region just ahead of the streamer, while screening the ambient field out of the streamer channel. The intense electron impact ionization in the high field region rapidly raises the electron density from an ambient value to the level in the streamer channel, leading to the extension of the streamer into a new region. Therefore, streamers are often referred to as space charge waves, which can penetrate into neutral gas with a velocity much higher than the electron drift velocity, up to a fraction of the speed of light. It is apparent that during the propagation of streamers, an external field E_0 must be applied to supply energy for such processes as ionization, excitation, and dissociative attachment. It is found experimentally that the critical field $E_{\rm cr}^+$ required for the propagation of positive streamers is close to the value 4.4 kV cm^{-1} in air at atmospheric pressure [20], and the $E_{\rm cr}^-$ for negative streamers is estimated to be a factor of 2–3 higher, about 8–12.5 kV cm^{-1} [18, 21]. It was also believed that these were the electric fields required for stable propagation of streamers, and with a larger or a smaller value, respectively, leading to growing or decaying streamers.

To achieve a better understanding of the streamer dynamics, it should be useful to derive an analytical criterion for streamer propagation. However, since streamer discharges are highly nonlinear processes, it is difficult to derive a simple criterion by solving analytically the continuity equations of charged species coupled with Poisson's equation. Hence, we simplify a streamer as a thin charge layer and derive a simple criterion for streamer propagation using the law of energy conservation on a macroscopic level following approach described on page 355 by Raizer [18]. The conclusions derived from the simple criterion will be demonstrated using plasma fluid modeling in later sections.

As shown in a cross-sectional view in figure 1(*b*), a streamer head and its extension in a short time Δt are represented as two thin cylindrical charge layers with two slightly different radii $R_{\rm s}$ and $R_{\rm s}'$. In this approach, growing, stable and decaying streamers, respectively, correspond to $R_{\rm s}' > R_{\rm s}$, $R_{\rm s}' = R_{\rm s}$, and $R_{\rm s}' < R_{\rm s}$. As a new section of the streamer head is created in the time period of Δt, part of the previous streamer head is neutralized by the secondary electrons drifted backward from the new section of the positive streamer (or by the electrons drifted forward from the streamer channel in the case of negative streamers), effectively leading to the propagation of the thin charge layer, namely the streamer head, for a short distance ΔL. The work done by the external field E_0 during this process can be written as

$$\Delta W = q_{\rm e} N_0 E_0 \Delta L \tag{1}$$

where $q_{\rm e}$ is the absolute value of electron charge and $q_{\rm e}N_0$ is the total amount of charge in the streamer head. The energy ΔW is mainly spent on creating the new section of the streamer, more specifically, on the chemical processes such as ionization, attachment, vibrational and electronic excitation of the neutral molecules in the streamer head. Having assumed that the total average energy expended on the above-mentioned processes in the discharge during creation of one electron-ion pair is $w_{\rm e}$, the law of energy conservation requires that

$$\Delta W = N_{\rm e} w_{\rm e} \tag{2}$$

where $N_{\rm e}$ is the total number of secondary electrons created in the new section of the streamer, so that

$$q_{\rm e} N_0 E_0 \Delta L = N_{\rm e} w_{\rm e} \tag{3}$$

The electron density in the new section can be approximated as

$$n_{\rm e} \approx N_0 / (\pi R_{\rm s}^2 L_{\rm s}) \tag{4}$$

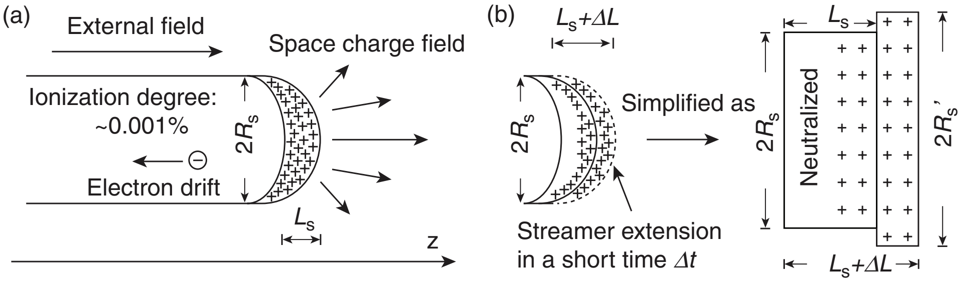


Figure 1. Simplified model of streamer propagation. (*a*) Cross-sectional view of a positive streamer propagating in an external field. (*b*) Schematics of a crescent-shaped streamer head and representation of its extension in a short time interval using two cylinders.

and

$$N_e = n_e \pi R_s'^2 \Delta L \tag{5}$$

Hence, we have

$$\frac{R_s'^2}{R_s^2} = \frac{q_e E_0 L_s}{w_e} = \frac{w_s}{w_e} \tag{6}$$

where $w_s = q_e E_0 L_s$ is the amount of energy gained by an electron propagating over a distance of L_s along an electric field of E_0. The condition $w_s > w_e$, $w_s = w_e$, and $w_s < w_e$ respectively, represents the requirement for the formation of a growing, a stable, and a decaying streamer. If the energy w_s can be used to produce exactly one secondary electron, the streamer will create a new streamer head identical to its previous one at the next moment of time. Note that identical streamer heads at different moments of time lead to constant potential drop (i.e. constant electric field) in the streamer head, which is the most important characteristic of stable streamers as discussed in previous literature [9, 21]. Otherwise with $w_s > w_e$, the extra secondary electrons lead to a radial expansion of the streamer and a slight increase of the electron density in the streamer head. On the contrary, for $w_s < w_e$, the streamer has to reduce its radius and the electron density in the streamer head decreases slightly as well. The above-derived relation clearly indicates that the streamer propagation is not only controlled by the strength of the external field E_0, but is also highly dependent on L_s, namely the size of the streamer head.

Since the thickness L_s of the streamer head is proportional to the streamer radius R_s [22], the equation (6) can be rewritten as

$$\frac{R_s'^2}{R_s^2} = \frac{q_e E_0 R_s}{C_0^{\pm} w_e} \tag{7}$$

where $C_0^+ \approx 1.0$ for positive streamers and C_0^- is a function of the external field for negative streamers in air at atmospheric pressure, as will be demonstrated below. Note that in the present work the streamer radius R_s is defined as the radial distance at which the electron density decreases to half of its value on the axis of symmetry of the streamers. The requirement for stable propagation of positive streamers can be therefore approximated as

$$E_0 R_{st} = w_e \,/\, q_e \tag{8}$$

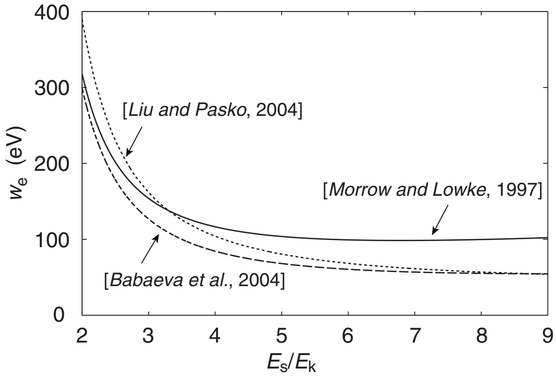


Figure 2. Effective energy required for producing a secondary electron in air at atmospheric pressure. The results shown by the solid and dashed lines are, respectively, calculated using the ionization rate and electron mobility as functions of the reduced electric field provided in the references [19, 23, 24].

Note that this stable condition is identical to that presented on page 355 by Raizer [18] (with a different interpretation in the present work). For negative streamers

$$E_0 R_{st} = C_0^-(E_0)\, w_e \,/\, q_e \tag{9}$$

where R_{st} represents the stability radius required for the stable propagation of streamers. Moreover, it can be estimated that

$$w_e(E_s) = \frac{\vec{J_s} \cdot \vec{E_s}}{\nu_i n_e} = \frac{q_e \mu_e E_s^2}{\nu_i} \tag{10}$$

where $\vec{J_s}$, $\vec{E_s}$, ν_i and μ_e are, respectively, the current density, the electric field, the ionization frequency, and the electron mobility in the streamer head. Note that ν_i and μ_e are functions of the reduced electric field E/N, where N is the neutral density. Figure 2 shows the values of $w_e(E)$ calculated in air at atmospheric pressure using three different sets of $\nu_i(E/N)$ and $\mu_e(E/N)$ commonly adopted in previous streamer modeling [19, 23, 24]. Although the results are slightly different, they all show almost constant w_e for $E > 4E_k$, which are the field values usually observed in positive streamer heads. For lower fields from $4E_k$ to $2E_k$ that commonly exist in negative streamer heads, the w_e value varies up to a factor of 4. This indicates that the stability radius R_{st} required for stable propagation of positive streamers is almost linearly proportional to $1/E_0$, but the relation between R_{st} and E_0 in negative streamers deviates from such a linear fashion due to the significant variation of w_e. For a streamer propagating in an external field E_0 with a

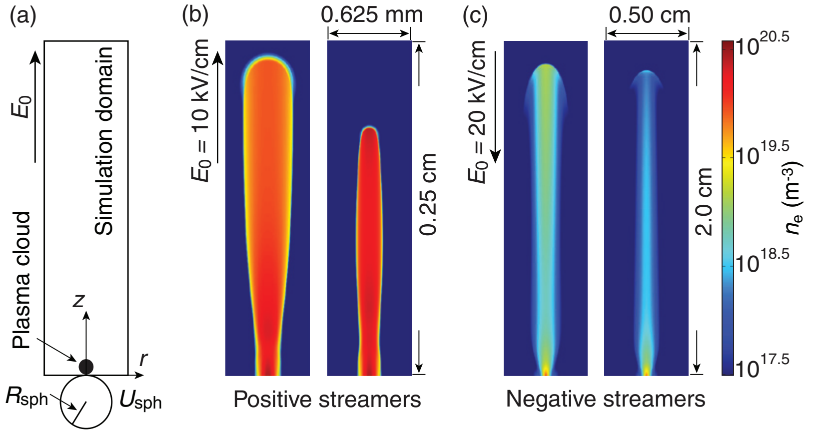


Figure 3. (*a*) Geometry of the simulation domain. (*b*) Propagation of growing and decaying positive streamers in an external field of 10 kV cm^{-1}. Both positive streamers are initiated from a Gaussian distributed plasma cloud with a peak density of 10^{20} m^{-3} and a characteristic size σ_0 of 0.05 mm. The radius of the spherical electrode R_{sph} is 0.5 mm. The only difference is that in the left panel the spherical electrode has a potential $U_{sph} = 3.5$ kV, whereas in the right panel $U_{sph} = 3.2$ kV. (*c*) Propagation of negative streamers in an external field of 20 kV cm^{-1}. For both negative streamers, the initial plasma cloud has a peak density of 10^{18} m^{-3} and a characteristic size of 0.10 mm. The electrode radius $R_{sph} = 1.0$ mm, and in the left panel $U_{sph} = 4.0$ kV, whereas in the right panel $U_{sph} = 3.4$ kV.

radius $R_s \neq R_{st}$, which depends on its initiation conditions, the streamer will experience an exponential growth if $R_s > R_{st}$ or decay if $R_s < R_{st}$. We note that in the above analytical derivation of the simple criterion, we simplified a streamer as purely a thin charge layer and neglected the variations in the streamer channel, which affect slightly the dynamics of the streamer head [25]. The effectiveness of the criterion, therefore, needs to be demonstrated using numerical simulations that take into account the contributions of the streamer channel.

3. Streamer propagation in numerical simulations

Streamer propagation is simulated using a plasma fluid model accounting for the electron impact ionization of N_2 and O_2, the electron dissociative attachment to O_2, and the electron detachment process $O^- + N_2 \rightarrow e + N_2O$. Photoionization processes are included using the three-group SP$_3$ model [26]. The electron mobility, electron diffusion coefficient, the ionization frequency, and the two-body and three-body attachment frequencies are defined as functions of the reduced electric field E/N using modified formulations of *Morrow and Lowke* [19]. The motion of charged species is simulated by solving the drift-diffusion equations for electrons and ions coupled with the Poisson's equation, and open boundary conditions are used in all simulations [27, 28].

We use a technique introduced by Babaeva and Naidis [21] to model point electrode configurations of previous experiments [20]. A small spherical electrode of radius R_{sph} and potential U_{sph} was placed in the uniform electric field E_0, as shown in figure 3(*a*). The sphere enhances the electric field in the region near the sphere but leaves the electric field far away from the sphere almost unaffected. In a cylindrical system of coordinates, the potential of Laplacian field (in the absence of space charge) at a given location (r, z) is

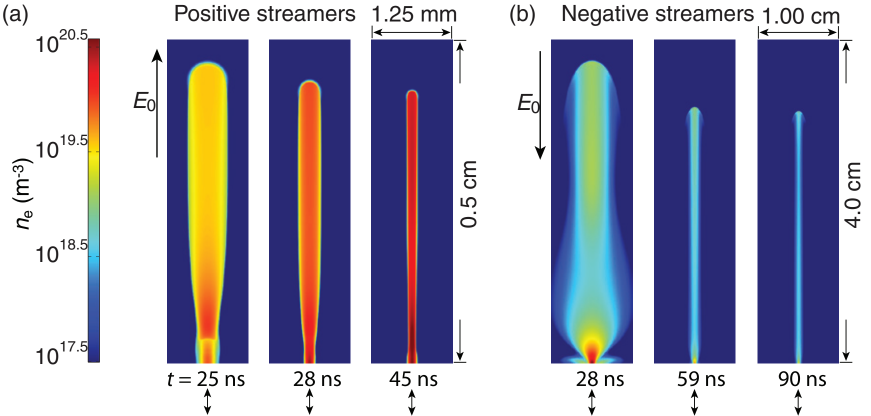
$$U_L(r,z) = U_{sph}\frac{R_{sph}}{\rho} - E_0\left(1 - \frac{R_{sph}^3}{\rho^3}\right)\left(z + R_{sph}\right) \qquad (11)$$

where $\rho = [(z + R_{sph})^2 + r^2]^{1/2}$. Note that for $z \gg R_{sph}$, $U_L(r, z) = -E_0 z$. For computational efficiency, a plasma cloud formed by equal amounts of electrons and positive ions with a Gaussian spatial distribution, i.e. $n_e = n_p = n_0\exp\{-(r/\sigma_0)^2 - [(z - 2\sigma_0)/\sigma_0]^2\}$, represented by a black sphere in figure 3(*a*), was placed near the spherical electrode on the axis of symmetry of the simulation domain to rapidly initiate a streamer.

It was found in those simulations that for a given uniform electric field E_0, lowering the potential U_{sph} of the sphere (that results in a smaller initial streamer radius) leads to slower growth of the streamer in the region of uniform electric field, and with a critically low U_{sph}, the streamer decays even if it propagates in a uniform electric field that is only slightly weaker than E_k. Four representative examples of simulations under those conditions are shown in figure 3. For the two positive streamers, the only difference in their initiation conditions is that the potential of the charged sphere is lowered from 3.5 kV to 3.2 kV, leading to the formation of a growing streamer with a large radius and the formation of a decaying streamer with a smaller radius in the same uniform field E_0 = 10 kV cm^{-1}. Similarly, as shown in figure 3(*c*), lowering the sphere potential from 4.0 kV to 3.4 kV leads to two negative streamers that are respectively growing and decaying in the external field E_0 = 20 kV cm^{-1}. These modeling results indicate that there should exist a stability U_{sph} that initiates a streamer with the stability radius R_{st}. By intentionally initiating streamers with small initial radii using small spherical electrodes, the above finding is confirmed in a wide range of electric fields, with the modeling results shown in figure 4.

In our simulations, stable propagation of streamers is confirmed by constant value of the integral $\int_0^\infty 2\pi r n_e(r)\,\mathrm{d}r$ in the streamer head at different moments of time. This criterion is more accurate when compared to the criteria of constant radius or constant velocity, because

$$\int_0^\infty 2\pi r n_e(r)\,\mathrm{d}r \propto n_e R_s^2 \qquad (12)$$



E_0 (kV/cm)	5.0	10.0	20.0	10.0	20.0	25.0
R_s (mm)	0.278	0.110	0.052	1.387	0.387	0.267
E_s / E_k	4.05	5.11	7.12	2.97	2.78	2.29
v_s ($\times 10^5$ m/s)	1.59	1.59	0.93	11.7	5.20	3.47
w_e (eV)	115.6	103.0	98.6	155.9	170.6	237.0
$C_0 = q_e E_0 R_s / w_e$	1.20	1.07	1.05	8.90	4.54	2.82

Figure 4. (*a*) Stable positive streamers in external fields of 5, 10, and 20 kV cm^{-1}. The plasma cloud is identical to that used in figure 3(*b*). The radius and potential of the spherical electrodes are, respectively, 1.0 mm and 6.1 kV, 0.5 mm and 3.40 kV, 0.5 mm and 1.302 kV for the cases from left to right. Note that according to simulations, a smaller streamer needs a more accurate sphere potential to propagate stably for a given distance (e.g. 0.5 cm). (*b*) Stable negative streamers in external fields of 10, 20, and 25 kV cm^{-1}. The plasma cloud is identical to that used in figure 3(*c*). The radius and potential of the spherical electrodes are, respectively, 2.0 mm and 25.0 kV, 1.0 mm and 3.75 kV, 1.0 mm and 0.23 kV for the cases from left to right.

where n_e on the right hand side represents the average density in the streamer head, such that a slight variation of R_s or n_e can lead to a significant variation of the integral. It is also confirmed in our simulations that all stable streamers, regardless of the external field, propagate with a constant velocity and have a constant potential drop in the streamer heads, which is consistent with the stable streamer propagation in critical fields $E_{cr}^{\pm}$ observed in previous literature [21]. Constant potential drop in stable streamer heads is due to identical streamer heads at different moments of time, which, as indicated in our analytical derivation, can be achieved if $q_e E_0 L_s = w_e$. Moreover, constant potential drop in the streamer head leads to the fact that the total potential drop in the stable streamer channel should be approximately equal to $E_0 L$, where L is the length of the streamer channel, which has also been confirmed in our modeling study with some of the results shown in figure 5. Note that stable propagation of streamers are better demonstrated in the case of negative streamers, which show perfectly identical streamer heads at different moments of time and that the electric field in the streamer channel is equal to the applied field (see figures 5(*c*) and (*d*)). The positive streamer shown in figures 5(*a*) and (*b*), which is stable according to the criterion of a constant value of the integral $\int_0^\infty 2\pi r n_e \mathrm{d}r$, still exhibits slight variation in the streamer head due to a relatively small simulation domain used in the positive case as a very high spatial resolution is required for the modeling of thin positive streamers. Nevertheless, we expect that once propagating over a longer distance the positive streamer shown in figures 5(*a*) and (*b*) could propagate in a stable fashion similar to that of the stable negative streamer shown in figures 5(*c*) and (*d*). In addition, it is known that the electric potential differences in the heads of growing streamers increase exponentially [9]. In our simulations, we observed that for decaying and growing streamers, the potential drop in the streamer heads is, respectively, decreasing and increasing, and the total potential drop in the streamer channels is, respectively, larger and smaller than $E_0 L$.

Note that in most simulations except those related to very thin streamers, the actual simulation domains are larger than zoom in views shown in figures 3 and 4. The spatial resolution for positive streamers is 0.5 cm/2000 = 2.5 μm and for negative streamers is 4 cm/3000 = 13.3 μm. It is also important to emphasize that in our simulations the transport coefficients and rate constants of kinetic processes are taken from the work of *Morrow and Lowke* [19]. Additional tests not included here for the sake of brevity demonstrate that using coefficients and rate constants taken from different sources in refereed literature, the potential of the spherical electrodes need to be adjusted slightly to produce stable streamers. For example, with the coefficients taken from the work of *Babaeva et al* [24] or *Liu and Pasko* [23], the potential applied to the sphere electrode needs to be, respectively, ~2.6 kV and ~4.0 kV to produce stable streamers in an external field of 10 kV cm^{-1} using otherwise identical conditions to those of figure 4(*a*) (middle panel). Other factors such as the high-order scheme used to calculate the electron flux and the photoionization model might also slightly affect exact conditions for stable streamers but do not change fundamentally new conclusions of the present work.

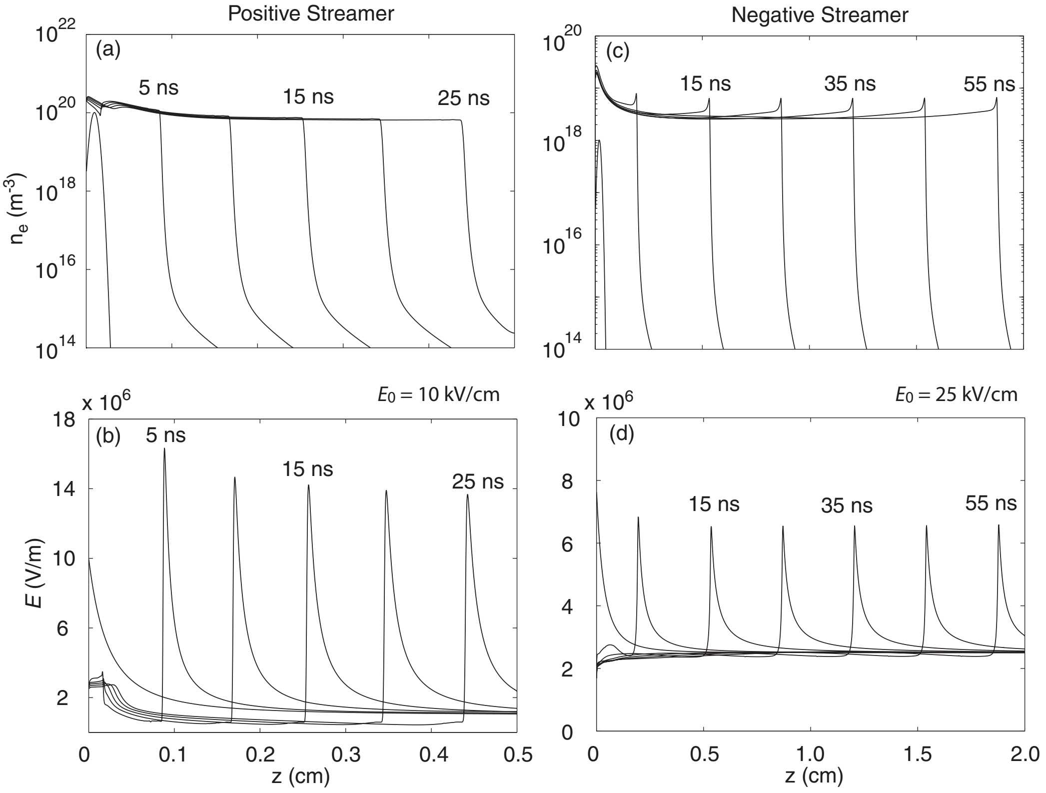


Figure 5. (*a*, *b*) Electron density and electric field on the axis of symmetry of the stable positive streamer shown in the middle panel of figure 4(*a*). (*c*, *d*) Electron density and electric field on the axis of symmetry of the stable negative streamer shown in the right panel of figure 4(*b*).

4. The lower and upper limits of the stability fields

It is necessary to clarify the concepts of critical field E_{cr}, stability field E_{st}, and stability radius R_{st}. Identical to its previous definition, the critical field E_{cr} is defined as the minimum electric field required for the propagation of streamers in a given medium. However, we emphasize that the stability field E_{st} is not identical to E_{cr}, but represents a wide range of electric fields, with its lower limit equal to the critical field E_{cr} and its upper limit confined by the breakdown field E_k. Each stability field E_{st} is related to an (almost) unique streamer radius, referred to as the stability radius R_{st}, with which streamers can propagate stably.

The lower limits of the stability fields, namely the critical fields $E^+_{\mathrm{cr}} = 4.4$ kV cm^{-1} for positive streamers and $E^-_{\mathrm{cr}} \simeq 8.0 - 12.5$ kV cm^{-1} for negative streamers in air at atmospheric pressure, have been measured and simulated extensively in the literature [16, 17, 20, 21]. However, physical explanations for the existence of such specific values were not given: why streamers cannot propagate in a sub-critical field? The answer naturally follows from the present analysis, as it shows that the propagation of streamers in a weaker field requires a larger radius. The streamer branching puts a limit on how large streamer radii could be realized. The critical field E_{cr} is determined by the maximum radius $R_{\max}$ that a streamer could possess in weak external fields. Evaluation of the $R_{\max}$ requires knowledge of the streamer branching mechanism which is not well understood yet. Nevertheless, it has been generally accepted that the streamer branching phenomenon is related to the process of photoionization produced by the photons emitted from the streamer head [23, 29]. This is particularly true for positive streamers which propagate in the opposite direction of the electron drift and require ambient seed electrons ahead of them for their spatial advancement [30]. Photoionization is the main process that supplies these seed electrons [29, 31]. As for negative streamers, they are generally more difficult to branch because the electrons in the streamer channel drift in the same direction as the streamer propagation and, together with the photoelectrons, serve as the seed electrons ahead of the negative streamers [23, 32]. It should be emphasized that in those previous studies of streamer branching, the external fields are assumed to be stronger than the breakdown field E_k. How streamers branch in weak external fields is poorly understood. We find that streamers are more easily to reach numerical branching in weaker external fields, most likely because of the relatively weak space charge fields in the streamer head (see figure 4) that lead to less effective photoionization. However, since the plasma fluid model can not fully reproduce the stochastic streamer branching phenomenon as it is a deterministic model, we could only hypothesize that the maximum streamer radius $R_{\max}$ should decrease with decreasing external field E_0, in contrast to R_{st} which increases with decreasing E_0. The critical field E_{cr} is determined by the relation $R_{\max}(E_0) = R_{\mathrm{st}}(E_0)$. If the hypothesis is correct, it is expected that in the external field E_{cr}, streamers could only propagate in a stable manner with the radius R_{st}, because a smaller radius leads to a decaying streamer, and a larger radius leads to streamer branching. It is also expected that in the case of lower gas pressure (or at higher altitudes with lower air density N in the Earth's atmosphere), the critical field $E_{\rm cr}$ required for streamer propagation is lower than $E_{\rm cr0}N/N_0$, where $E_{\rm cr0}$ and N_0, respectively, represent the critical field and air density at atmospheric pressure. This is because photoionization is more efficient at lower pressure due to weaker quenching of the excited states responsible for the photoionizing radiation [23], such that streamers can have larger radii than those expected from a linear scaling of N_0/N.

The upper limit of the stability fields is naturally confined by the conventional breakdown field $E_k \simeq 28.7$ kV cm^{-1}, as in a stronger external field a single seed electron can lead to an exponentially growing electron avalanche and then transform into a growing streamer. This has been confirmed by numerical simulations of negative streamers in which stable propagation can occur in external fields up to ~28 kV cm^{-1}. It should be noted that in such a strong external field, the peak electric field $E_{\rm s}$ in a (quasi) stable streamer head is as weak as $\sim 1.2E_k$, with only $\sim 0.2E_k$ contributed from the space charge in the streamer head, and the streamer velocity is $\sim 1.50 \times 10^5$ m s^{-1}, which is almost equal to the electron drift velocity $\sim 1.45 \times 10^5$ m/s in an electric field of $1.2E_k$. In other words, in the upper limit of the stability field, stable negative streamers are in fact electron avalanches. For positive streamers, stable propagation has been verified by practical calculations in external fields up to ~20 kV cm^{-1}. In stronger external fields, streamers with small radii can easily develop into a stage when the peak electric field $E_{\rm s}$ in the streamer head is higher than $\sim 9E_k$, that leads to extremely slow computational advancement.

5. The concept of minimal streamer radius

The stability radius $R_{\rm st}$ is also the minimum radius that a streamer could have in the external field $E_{\rm st}$, as streamers with smaller radii decay rapidly in a short distance and then stop propagating. In other words, the minimum streamer radius is a function of the external field. In previous studies, the minimum streamer radius for positive streamers in air at atmospheric pressure was reported to be ~0.20 mm [22, 33]. When this value is compared with the modeling results shown in figure 4, it appears that it corresponds to the stability radius of positive streamers in an external field of ~5.0 kV cm^{-1}. The new mechanism predicts that with appropriate initiation conditions, positive streamers can propagate in a stronger external field with a radius as small as ~0.05 mm. The absolute minimum radius $R_{\rm abs}$ of positive streamers can be estimated by taking $E_k = 28.7$ kV cm^{-1} as the upper limit of the stability field and assuming $E_{\rm st}R_{\rm st}$ as a constant for positive streamers

$$R_{\rm abs} \approx \frac{E_{\rm st}R_{\rm st}}{28.7} = \frac{20.0 \times 0.052}{28.7} = 0.036\,\text{mm} \qquad (13)$$

Although in air at atmospheric pressure streamers with such small radii have not been previously reported, spectroscopic measurements of the streamer radius in oxygen at a pressure of 300 Torr have been reported which lead to a radius as small as 0.02 mm [34]. Similar estimates are not applicable to stable negative streamers, which are essentially electron avalanches in strong stability fields.

6. Differences between positive and negative streamers

The intrinsic difference in the radii of positive and negative streamers observed in previous simulations and experiments can be consistently explained using the analytical criteria for streamer propagation obtained in the present work. It is known that the electric fields in positive and negative streamer heads are different due to different directions of the flow of charged particles in positive and negative streamers, that leads to the requirement of a stronger electric field in the negative streamer channel (thus a weaker one in its head) to maintain its radial density profile [35]. Typical values of the electric fields in negative streamer heads are $\sim 2\text{–}4E_k$, whereas those in positive streamer heads are $\sim 4\text{–}8E_k$ (see figure 4). Weaker electric fields in negative streamer heads lead to a larger energy expenditure $w_{\rm e}$ required for production of one electron ion pair. Since for stable propagation, the thickness of a streamer head needs to be $L_{\rm s} = w_{\rm e}/(q_{\rm e}E_0)$, a larger $w_{\rm e}$ leads to a larger value of $L_{\rm s}$, that is, a larger streamer. Furthermore, it appears that positive streamers have an edge over negative streamers for propagating in weak external fields because of their smaller radii. In identical external fields, stability radii of negative streamers are ~7–12 times larger than those of positive streamers according to the results shown in figure 4. It can be therefore estimated that negative streamers need to have a radius of at least ~2.5 mm to propagate in an external field of 5 kV cm^{-1}. In practice this radius is not likely to be realizable due to branching.

7. Conclusions

In the present work, we studied the propagation mechanism of streamers in weak external fields (below the breakdown field E_k), and refined some fundamental concepts in streamer physics. The principal contributions can be summarized as follows:

(a) The growing, decaying and stable propagation of streamers is demonstrated to be controlled not solely by the external field but also by the physical dimensions of streamers. Stable propagation is shown to be achievable in a wide range of electric fields. In this field range, the streamer radius required for stable propagation is inversely proportional to the external field, with larger and smaller initial radii respectively, leading to growing and decaying streamers. The new mechanism suggests that streamer propagation can be manipulated by adjusting the streamer initiation conditions. For example, stable streamers can be produced in electric fields higher than the critical field by intentionally initiating thin streamers using a small point electrode, which could be useful in streamer applications.

(b) The concept of stability field for streamer propagation is refined, which is not identical to the critical field, but represents a wide range of electric fields, with its low limit equal to the critical field and its upper limit confined by the breakdown field. The concept of stability radius is introduced to define the radius with which a streamer can propagate stably in a given external field.

(c) An explanation for the existence of specific values of the critical field required for streamer propagation, which is ~4.4 kV cm^{-1} for positive streamers and ~8.0–12.5 kV cm^{-1} for negative streamers in air at atmospheric pressure, is proposed. It is suggested that since stability radius is inversely proportional to the external field, stable or growing propagation of streamers in a sub-critical field requires an excessively large streamer radius, which cannot be realized due to streamer branching in those sub-critical fields.

(d) The concept of minimum streamer radius is refined, which is now defined as a function of the external field. For a given external field, the corresponding minimum streamer radius is identical to the stability radius, as streamers with smaller radii decay rapidly in a short distance and then stop propagating. Streamers propagating stably with a minimal streamer radius can be achieved when the external field is approaching the breakdown field.

(e) The differences between positive and negative streamers are discussed in the context of the new streamer propagation mechanism. Relatively high electric fields in the positive streamer heads lead to smaller physical dimensions of the positive streamers when compared to their negative counterparts. In identical external fields in air at atmospheric pressure, stability radii of negative streamers are ~7–12 times larger than those of positive streamers. This intrinsic difference between the stability radius of positive and negative streamers leads to the fact that positive streamers can propagate in electric fields as low as ~4.4 kV cm^{-1} whereas negative streamers cannot, because in such lower fields they require an excessively large radius to propagate, which cannot be realized due to streamer branching.

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