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A Review of Dry Particulate Lubrication: Powder and Granular Materials

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Research efforts related to dry particulates in sliding contacts are reviewed. In the tribology community, there are primarily two types of dry particulate lubricants that are studied—granular and powder. Granular lubricants usually refer to dry, cohesionless, hard particles that transfer momentum and accommodate surface velocity differences through shearing and rolling at low shear rates, and collisions at high shear rates. Powder lubricants refer to dry, cohesive, soft particles that accommodate surface velocity differences mostly by adhering to surfaces and shearing in the bulk medium, in a manner similar to hydrodynamic fluids. Spanning the past five decades, this review proposes a classification system for the scientific works in the dry particulate tribology literature in terms of theory, experiments, and numerical simulations. It also suggests that these works can be further categorized based on their tribosystem geometry—annular, parallel, and converging. [DOI: 10.1115/1.2647859]

Keywords: dry particulates, granular and powder lubricants, cohesionless and cohesive, momentum, classification system, theory, experiments, numerical simulation

1 Introduction

Dry particulate materials have been proposed as viable candidates for lubrication in extreme environments (i.e., temperature and/or loads), where conventional lubricants cannot perform adequately [1–22]. For example, the increased capacity of turbine engines will result in high temperatures on the order of 800°C, posing serious problems for modern cooling technology. At temperatures greater than 500°C, conventional liquid lubricants are unable to sustain loads, hence, the advent of solid/particulate lubrication [12]. Nanoscale solid lubrication has also come into fruition, as nanopowder lubricants have recently demonstrated excellent lubrication capabilities at extreme temperatures [23–27]. Additionally, there are huge technological gains that can be made by understanding the behavior of dry particulates in sliding contacts under load in industries such as pharmaceutical processing, planetary rover exploration [28], coal-based gasification [29], attrition of granular salt [30], and food processing.

Researchers have proposed innovative forms of dry particulate lubrication, namely powder and granular. Powder lubricants are classified as dry, "cohesive," soft particles that radically deform under load and accommodate surface velocity differences mostly by adhering to surfaces and shearing in the bulk medium, similar to hydrodynamic fluids. Granular lubricants are classified as dry, "cohesionless," hard particles that adequately maintain their spherical geometry under load and accommodate surface velocity differences through sliding and rolling at low shear rates, and largely through collisions at high shear rates. Both powder and granular lubrication mechanisms have demonstrated that the particulates in the sliding contact can enhance lubrication and lower friction below boundary lubrication levels. At low speeds (i.e., low nominal shear rates), powder and granular lubricants may appear to take on similar velocity accommodation behavior. However, one distinguishing phenomenological factor is their behaviors at the boundary, where powders usually adhere and coat surfaces, and granules usually slip, roll, and/or collide with surfaces. Figure 1(*a*) shows a powder lubricant in a converging sliding contact [31], while Fig. 1(*b*) shows a high-speed image of granular particles colliding in an annular sliding contact [28].

Dry particulate lubrication schemes have been proposed for innovative bearing technologies. For example, Kaur and Heshmat developed an oil-free journal bearing [9], capable of supporting significant rotor loads operating at 815°C and 30,000 rpm, that was lubricated by *in situ* powder film transfer [6,7,10]. Dampers in gas turbine engines have also been lubricated with powder [17]. McKeague and Khonsari proposed the development of a granular lubricated bearing by formulating a model that predicts the behavior of colliding granules in a slider-bearing configuration [32]. Several important studies on granular lubrication have also been conducted recently by a number of authors [21,33–38].

Although there is a general agreement on the needs and motivation for an oil-free particulate lubrication mechanism, opinions still vary on issues such as classification (i.e., granular or powder lubrication), modeling approaches, and the effect of interface geometry. There is also a classification quandary: Do dry triboparticulate materials in sliding contacts exist as powder or granular lubricants? Can a unified dry particulate theory be applied to analyze them?

In the works described in this review, theoretical, experimental, and numerical simulation studies have been conducted either singly (i.e., powder or granular) or dually (i.e., powder and granular) to study the behavior of dry particulates. Furthermore, from the variety of geometrical configurations in the different studies, this

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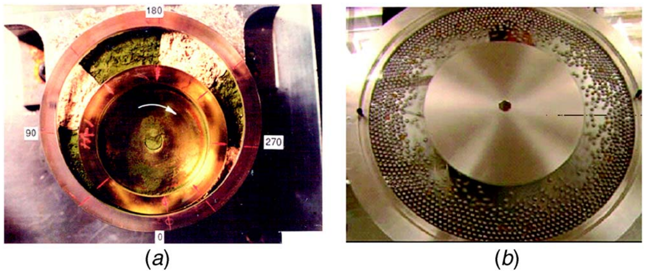


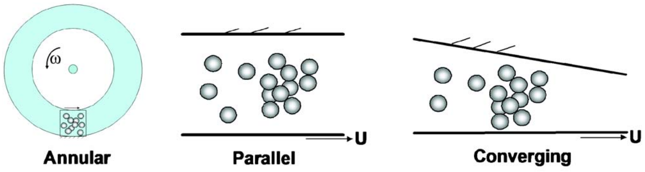
(*a*) (*b*)

Fig. 1 (*a*) Powder lubrication [31] and (*b*) Granular lubrication [103]

review paper focuses on dry particulate materials in annular, parallel, and converging geometry sliding contacts as shown in Fig. 2. Shown in Fig. 1(*b*), annular geometries are usually the simplest to fabricate. Parallel geometries are usually formed by focusing on a parallel section of a "race track" shaped shear cell geometry as shown in Fig. 3.

Converging geometries are usually used as bearing configurations where they commonly transmit load, as in eccentrically positioned (or displaced) journal bearings (see Fig. 1(*a*)). Thus, granular flows in hoppers, bins, and on inclines would not be appropriately suited for this work. The authors make an attempt to categorize the dry particulate body of tribology literature into a simple and clear classification system. For example, Fig. 4 is a catalog of representative papers from the dry particulate community that are either tribology related or forerunner papers to tribology-based work. While Fig. 4 does not highlight every work discussed in this review, it is representative of the major contributors to dry particulate lubrication literature. The interested reader should ultimately look to use this review as a primer of the dry particulate tribology literature and as a useful means for elucidating the interchangeable use of "granular lubrication" and "powder lubrication." Hopefully, this work will also be useful in aiding the increasing number of scientists who are studying the behavior of dry particles in sliding contacts to understand them from a tribological perspective.

Figure 5 shows a plot of the number of papers that have appeared in the tribology literature since the 1960s that deal with solid lubrication. One can observe that roughly the last decade has produced an unsteady but definite increase in papers, with 2005 representing the pinnacle of scholarly activity on dry particulate-based lubrication. One of the critical factors influencing the recent acclivity has been the advent of nanopowder lubrication. These nanosize lubrication materials suggests the potential for providing solid lubrication, not only macroscale, but nanoscale sliding contacts, where liquids films in devices such as microelectromechanical systems (MEMS) lead to show-stopping stiction [39,40].



(*a*) (*b*) (*c*)

Fig. 2 Sliding contact geometries: (*a*) annular, (*b*) parallel, and (*c*) converging

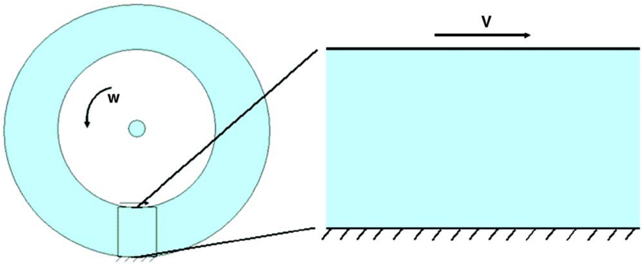


Fig. 3 Parallel section of an annular-shaped shear cell

2 Nature of Cohesive and Cohesionless Particulate Lubricants

Dry particulates (powders and granules) perform the most essential function of a lubricant, which is to reduce friction and/or wear between two surfaces during relative motion and hence reduce the damage to the surfaces. Unlike conventional hydrodynamic fluids, dry particulates can sustain load in static contacts when there is no entrainment flow or parallel geometries or when there is no "wedge" effect. Similar to hydrodynamic fluids, lubrication can be achieved by gradual velocity accommodation through the layers of discrete dry particles. The phenomenon of dry particulate lubrication has been known for centuries, but its use in the modern sense dates back approximately five to six decades. The nature of dry particulate materials is quite complex, as can be seen in Fig. 6, where Higgs and Heshmat [6] present the important characteristics that determine the state of the powder materials. Other important exposition on the nature of triboparticulates has been given by Heshmat and Brewe [41]. Their dual nature is also well known since they act as solid materials yet can flow similar to liquids when experiencing an external shear force that exceeds the material's yield or flow stress [42].

From experiments in the tribology and tribology-related literature, Higgs and Tichy [12] have identified some key differences and similarities between granular and powder flow lubricants [43], as follows:

Key *differences* between powder and granular flow lubricants:

- Powder particles are generally on the order of 1 μm, whereas granules are on the order of 1 mm.
- In loaded sliding contacts, all powder particles undergo completely inelastic collisions, whereas granular particles can undergo nearly elastic collisions.
- Powders can coalesce and transfer a thin lubricating film that can protect the tribosurfaces.

Key *similarities* between powder and granular flow lubricants

- Both have been shown to generate lift in sliding contacts.
- In sliding contact geometries, both have density distributions, where the density is larger away from the boundaries near the center of the film.
- In sliding contact geometries (see Fig. 2), both provide lubrication more favorable to dry or boundary lubrication.
- Unlike hydrodynamic fluids, both exhibit a load carrying capacity in static contact regions.
- Unlike hydrodynamic fluids, both exhibit a load carrying capacity in parallel sliding contacts.
- Unlike hydrodynamic fluids, both exhibit slip at the boundaries in macroscale geometries.
- Mixture properties, such as solid fraction (i.e., density), are dependent on pressure.

Since the parameters characterizing the particulate flow lubricants are sometimes similar, powder and granular flows have been compared directly in the literature. Additionally, terms such as "powder lubrication" and "granular lubrication" are used interchangeably. This review seeks to provide clarity to these types of issues.

2.1 What Makes Some Powders Exhibit Lubrication Behavior?

Powders cannot be completely classified as solids, liquids, or gases since they can (i) withstand some deformation when not pressed too hard, (ii) flow under certain circumstances, or (iii)

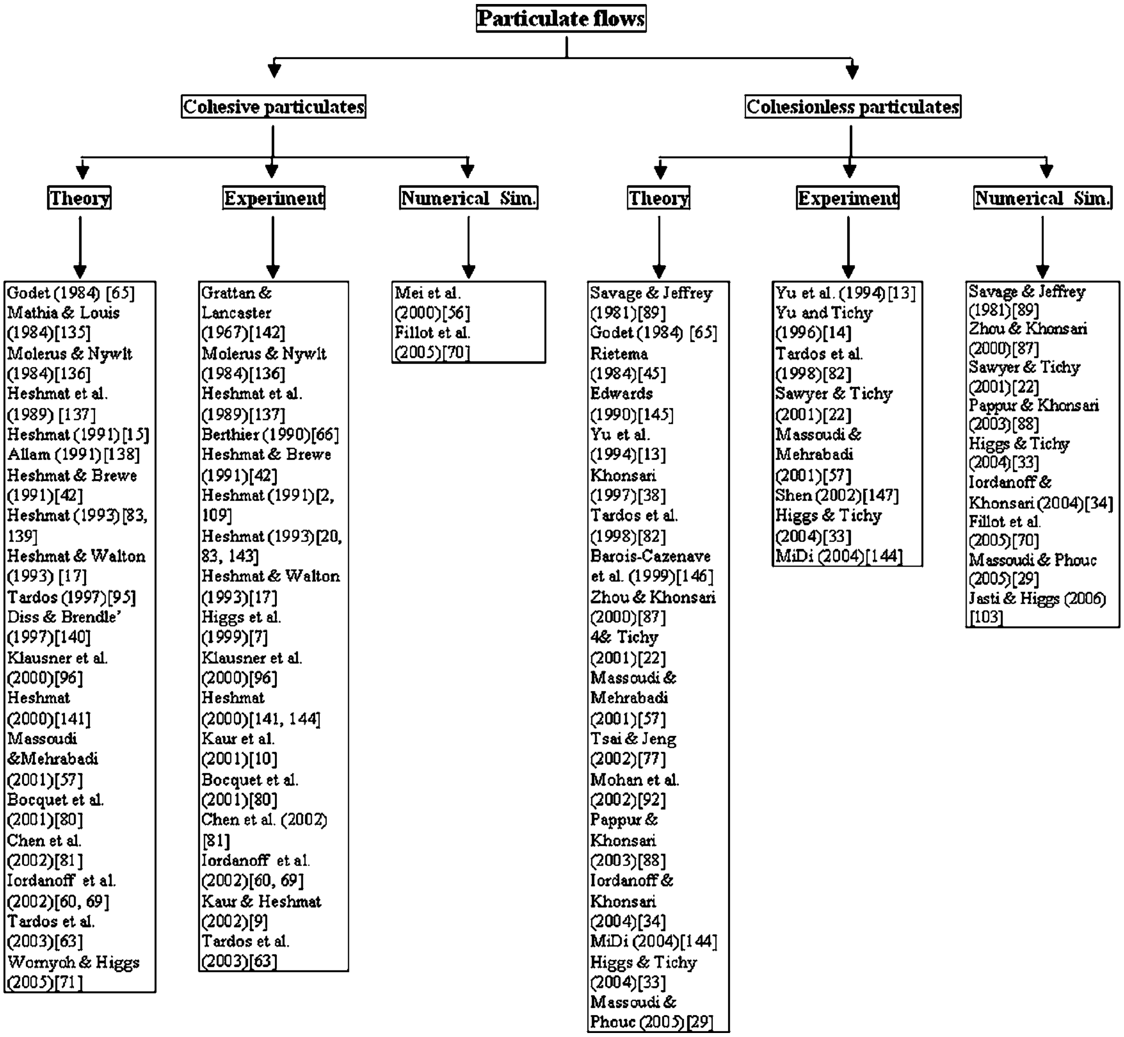


Fig. 4 Catalog of *representative* papers on dry particulate lubrication

be compressed to a certain degree, respectively [44,45]. Powders (as a third body) can exist between two surfaces to reduce wear and reduce friction [46]. Adhesion between these third-body particulates enables them to coalesce and provide the lubricating capabilities of friction, wear reduction, and velocity accommodation. As shown in Fig. 7, surface 1 and surface 2 are the first bodies, and the powder lubricant is the third body. V_1 and V_2 represent the velocities of the two first bodies, and δ_1 and δ_2 are the surface roughness, respectively.

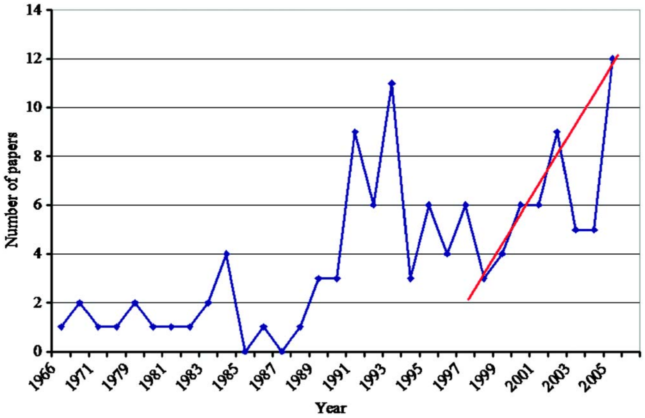


Fig. 5 Graphical representation of solid lubrication papers

An example of the creation and evolution of adhesion that occurs in wheel-rail natural third-body particulates have been described by Niccolini and Berthier [47]. According to their work, three factors that influence rheological changes as a function of sliding and rolling velocities are: (i) at very low sliding there is a local plastic flow of adhering third body, (ii) at transient adhesion load, there is a detachment of the adhering third body associated with the plastic flow of adhering third body, and (iii) at maximal adhesion, the different local strip of sliding particles more or less adheres to the contacting surfaces.

Some crystalline monochalcogenides, such as tin selenide (SnSe) and gallium selenide (GaSe), exhibit very good lubrication capabilities [48]. Additionally, the lamellar nature of powders such as graphite, molybdenum disulfide (MoS_2), titanium dioxide (TiO_2), and tungsten disulfide (WS_2), also makes them viable candidates as solid lubricants. Other lamellar powders include boron nitride [49], silicon nitride [49,50], and boric acid powder [51–53], which is an environmentally benign powder lubricant. Since lamellar materials are solid lubricants with inherently low shear strength, they provide velocity accommodation, reduction in interfacial friction, and load-carrying capacity.

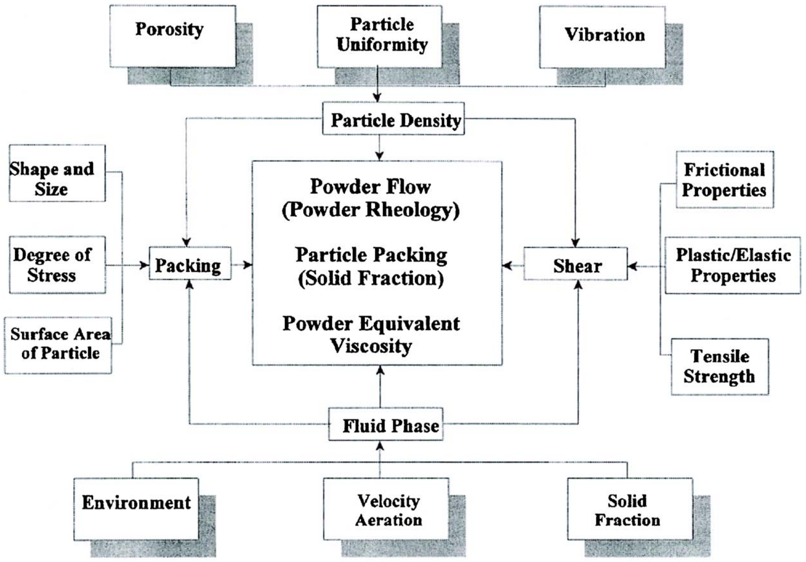


Fig. 6 Variables and properties that affect powder lubricants [6]

2.2 What Makes Some Granular Materials Exhibit Lubrication Behavior? In granular flow lubrication, cohesionless, partially inelastic particles imposed between two surfaces accommodate differing surface velocities and sustain loads. Unlike conventional liquid lubricants, granular flows have demonstrated an ability to sustain loads in static and dynamic contacts. It has been observed that two modes of operation exist in granular lubrication. At lower shear rates or high loads, the load is supported by strong contact forces between the compacted beads (i.e., granules). This regime is known as granular *contact* lubrication. Global frictional forces are due to the continuous shearing of the beads, and the load carrying capacity is due to elastic and plastic deformation of the granules in contact. At increased shear rates or small loads, the granules are more agitated and lubrication in this secondary regime is known as granular *kinetic* lubrication. There is also a *transition* regime which may be quasi-static [34,54]. Load carrying capacity in this mode is due to the shear and normal forces created by the colliding particles against the upper surface.

2.3 Macroscopic and Microscopic Interactions of Powder Lubricants. Typically, some microscopic quantities that affect powder flow behavior are particle size, friction, cohesion, interaction forces between particles, and porosity [55,56]. These microscopic characteristics contribute largely to some important macroscopic properties, such as hardness and compaction. Particulate size has also been adopted by Massoudi and Mehrabadi [57] as a criterion for classifying powders: powders have been described as composed of particles up to 100 μm in diameter with further subdivision into ultrafine (0.1–1 μm), superfine (1–10 μm), or granular (10–100 μm). Friction data, crystal-chemical form, and evidence from electron microscopy indicate that interlayer bonding affects the lubrication mechanisms of solid lubricants [48]. In powder lubrication, cohesive powders coalesce, shear, and coat surfaces to provide enhanced lubrication performance. Our focus is on such cohesive particulates in sliding contacts, which we denote as "powder lubricants."

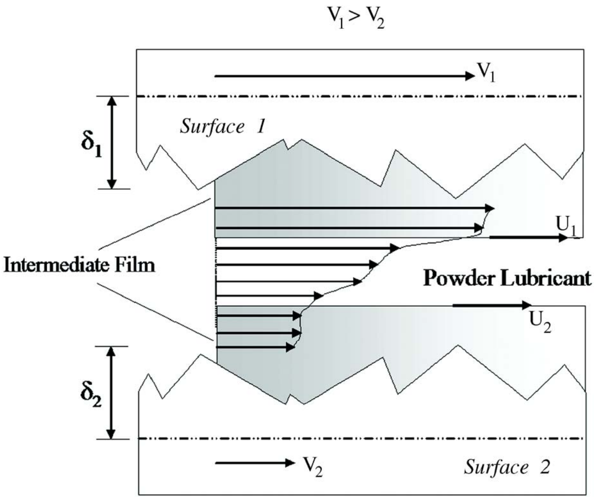


Fig. 7 Powder lubricant as a third body [2]

2.4 Macroscopic and Microscopic Interactions of Granular Lubricants. Granular properties, such as particle size, friction between granules, particle-particle coefficient of restitution, wall-particle coefficient of restitution, and hardness, influence the granular flow characteristics, such as velocity, spin, solid fraction, and granular temperature. Typically, the size of the granules is on the order of 1 mm, although size is not the sole delineating criteria. For example, rigid 100 μm steel granules between sliding tribosurfaces could accommodate velocity differences through collisions or rolling and sliding. These granular properties couple with external parameters, such as surface speed and surface roughness control lubrication characteristics of granular flows. Important lubrication characteristics are load carrying capacity or normal shear and shear stress at the walls, which is a measure of friction.

3 Theoretical Modeling of Dry Powder and Granular Materials in Lubrication

During powder and granular lubrication, the particulates provide a load-carrying capability and reduce the friction and wear of the interacting surfaces. To successfully model these materials in the interface, governing equations must be developed. A common approach adopted by several authors has been the use of conservation equations for mass, momentum, and modified or pseudo energy, which takes into account velocity fluctuations and the inelastic collisions of particles in the case of granular lubrication. Once obtained, the governing equations are solved for parameters such as velocity, solid fraction (or density), friction coefficient, and load-carrying capacity. The solution form is largely influenced by the complexity of the particular geometry used. Constitutive relations are also needed to describe the behavior of dry particulates, in addition, to describing the behavior of the particulates at the surface boundaries.

3.1 Governing Equations

3.1.1 Governing Equations for Powders. The nonexistence of a clear-cut fundamental equation of motion for powder lubrication led researchers to adopt a variety of forms. For example, some authors have favored rheological studies as a viable means. Bingham defined rheology as "study of the deformation and flow of matter" [58]. Rheology combines the theories of continuum mechanics with ideas obtained by considering the microstructure of the objects being studied (a terminology invented by Bingham in 1929) [58]. Heshmat [1] used this approach in the development of a semi-empirical model to predict the behavior and performance of powders he called "quasi-hydrodynamic" lubricant films. Soil mechanics principles, such as Coulomb's laws, have also been used by other authors, such as Arkers [59], who studied the microscopic phenomena that largely determines bulk properties of powder and granular materials. For simple shear flow of cohesive powders, Mei et al. [56] developed a method to quantify particle concentration non-uniformity for both dilute and dense conditions. The quantitative measurement of particle concentration nonuniformity was used to identify particle cluster structures at high bulk concentration under a varying range of shear rates and to understand the run-monotonic behavior of the stress-strain rate in the presence of strong cohesion. Iordanoff et al. [60] outlined limitations for using a continuum model to describe powders by citing two works by Berthier et al. [61,62]. The limitations arise from: (i) the main mechanism responsible for the macroscopic mechanical and physicochemical properties around the contact, (ii) the first bodies that influence the geometry of the contact and the natural source flow, and (iii) the third body, which refers to the triboparticulates in the interface.

Tardos et al. [63] studied the rheological behavior of powders in the "intermediate" regime lying between the slow and rapid flow regime. Although they did study the intermediate flow of frictional bulk powder in an annular couette geometry, such flows were not directed at lubrication studies. These studies were aimed at powders moving relative to solid walls in hoppers, bins, inserts, and moving paddles.

The phenomenological insights of Kohen et al. [64] and Godet [65] have had useful impacts in the analysis of sliding contacts. Using these insights, Berthier et al. [61,62] and Berthier [66] advanced experimental and theoretical evidence to clarify the third-body concepts, especially as it relates to velocity accommodation and other key tribological phenomena. Additionally, Fillot et al. [67], and Descartes et al. [68] have studied the third-body concept in greater detail. Furthermore, Iordanoff et al. [69] and Fillot et al. [70] have also used the third-body approach to perform useful numerical simulations that have yielded fundamental mass balance laws. More recently, Wornyoh and Higgs [71] have adopted these mass balance laws to formulate the governing equations for a pellet-on-disk with slider arrangement. In their control volume fractional coverage (CVFC) model, the governing equation of motion was solved and applied to the linear-rule of mixtures from Dickrell et al. [72,73], resulting in the prediction of important tribological parameters, such as friction coefficient and wear factor.

3.1.2 Governing Equations for Granular Materials. Similar to conventional fluid mechanics for gases or liquids, the governing equations for granular flows consist of the conservation equations for mass, momentum, and energy. However, the gases and liquids are composed of colliding molecules, whereas a granular flow would consist of inelastically colliding granules. The only adjustments made to the conservation equations so that they could be applied to granular flows were (i) the internal energy of the flow is characterized as the pseudothermal energy, and (ii) the granular particle collisions are not elastic as assumed in gas and liquid molecules. In recent years, the granular tribology community [21,74–77] has employed granular forms of the conservation equations as described by either Haff [78] or Lun et al. [79]. The conservation of mass equation for granular flows is of the form:

$$\frac{D\rho}{Dt} = -\rho(\vec{\nabla}\cdot\vec{U}) \tag{1}$$

where the granular flow density ρ and granular mixture velocity U are the key parameters. The granular conservation of momentum equation is

$$\rho\frac{D\vec{U}}{Dt} = \rho\vec{g} - \nabla\cdot\overleftrightarrow{\pi} \tag{2}$$

where $\overleftrightarrow{\pi}$ is the stress tensor and $\vec{g}$ is the body force vector. The granular conservation of energy equation is also known as the pseudoenergy equation. It is similar to the conventional energy equation for fluids except that the rate of change of the granular temperature is balanced against the energy added and dissipated from the system due to friction and inelastic particle collisions. The granular temperature is a measure of the fluctuating component of the granular particles relative to the mean granular velocity field [43]. Thus, it is written as

$$\frac{3}{2}\frac{D(\rho T)}{Dt} = -\vec{\nabla}\cdot\vec{q} - \varphi^f - \varphi^c \tag{3}$$

where $\vec{q}$ is the molecular energy transport, ϕ^f is the work rate of momentum, and ϕ^c is the inelastic work rate (dissipation due to inelastic particle collisions). Details on Eqs. (1)–(3) can be found in Higgs and Tichy [12].

Tribologists have employed these dry particulate conservation equations in novel ways. Tsai and Jeng [77] developed a governing lubrication equation (i.e., a "granular Reynolds equation") for granular flows using Haff's conservation equations. They subsequently applied their average lubrication equation for grain flow to powder-lubricated journal bearings [21], yet compared it to the powder bearing experiments of Heshmat and Brewe [4]. Because of the dearth of granular (dry hard cohesionless particles) tribology experiments, it is understandable that granular tribologists make comparisons between the granular tribology models and the plethora of powder (dry soft cohesive particles) lubrication experiments developed by Heshmat and his collaborators (see powder experiments in Sec. 4).

3.2 Constitutive Relations. Constitutive relations typically show the relation between shear stress and strain rate. The need for a constitutive relation for the stress tensor has been demonstrated by both theory and experiment [2,80]. Together with the governing equations and or boundary and or initial conditions, any dry lubrication problem can be completely formulated. The difficulty has been the divergence of opinions on the appropriate forms. For instance, Heshmat and Brewe [3] developed an equivalent viscosity model based on experimental results fitted to a computer program for predicting the quasi-hydrodynamic behavior of powder lubrication. This continuum-based rheological approach describes the flow of powders that occurs between a critical yield stress and a limiting shear stress. However, Chen et al. [81] proposed alternate constitutive equations to describe the rheology of powders. In modeling granular kinetic lubrication, one applies the proper ("invariant" or "admissible") rheological constitutive equations for stress, conduction, and dissipation to thin shearing flows.

3.2.1 Constitutive Relations for Powders. To develop their constitutive models for powder flows, Chen et al. [81] experimented with an annular shear cell-powder rheometer and confirmed some important observations by Tardos et al. [82] concerning the behavior of cohesive powders. For example, (i) the shear stress is invariant with shear rate in the frictional regime of a powder plug, and (ii) "the shear-to-normal stress initially increases with shear rate," and when a critical shear rate is reached, the shear-to-normal stress decreases with increasing shear rate until collisional stresses become stronger. Using a simple quadratic model, they approximated cohesive powder rheological behavior as

$$\frac{\tau_{xr}}{\sigma_o} = a_o + b_o\left(\frac{du}{dr}\right) + c_o\left(\frac{du}{dr}\right)^2 \tag{4}$$

where a_o, b_o, and c_o, were unknown coefficients to be determined by experiment. a_o is the tangent of the internal angle of friction, $a_o\sigma_o$ is a viscous coefficient, and $c_o\sigma_o$ is a second-order correction that accounts for decreasing dynamic friction with increasing shear rate.

Heshmat [1] compares the stress-strain rate behavior of powder lubricants to the rheology of a hydrodynamic liquid using the relationship

$$\frac{\partial u}{\partial y} = \frac{1}{\mu_o}[\tau + \alpha\tau^3 + \gamma\tau^5] \tag{5}$$

where τ is the shear stress, μ_o is the viscosity, and α, γ are rheological parameters that characterize the non-Newtonian rheology of the powder. He showed that a powder layer has two shear stress limits, typical of a typical pseudoplastic material [16,83]. Consequently, powder will not flow until the shear stress exceeds its yield value τ_y, and it is incapable of withstanding a shear stress above a limiting value of a shear stress τ_L, known as the powder's limiting shear stress above a limiting value of τ_y. Each of these is bulk properties of the powder film and must be obtained from experiment for any powder lubricant candidate.

3.2.2 Constitutive Relations for Granular Materials. Kinetic theory uses molecular models and the methods of statistical mechanics to create constitutive relations that predict the behavior of the bulk flow instead of that of the individual particles. For dense gases, the kinetic theory was originally used to characterize the stress on the surface as being caused by the transport of momentum across the gas by the colliding molecules as described by Elrod [11]. Haff used the kinetic theory approach to modeling granular flows noting that the individual grains are treated as a "molecules of granular fluid" [21]. As stated previously, Haff's continuum theory and constitutive relations [78] for describing the motion of granular material are used frequently by granular tribologists ever since Elrod took the "first look" in his granular tribology review paper [11], which focused largely on granular flows. Adopting Haff's constitutive relations, Dai et al. [84] worked to determine the capability of granular flows to be viable mechanisms for lubrication in slider bearings. Subsequently, McKeague and Khonsari [32] used his theories to perform parametrical studies with granular flows and also to predict the hydrodynamic pressure profiles from the well-known powder lubrication experiments of Heshmat [85]. Tribologists have also used constitutive relations by Lun et al. to model granular flows in parallel sliding contacts under load [12,79,86,87] and more recently, granular slider bearings [88].

There have been useful works outside of the tribology community that are useful for understanding tribological phenomena observed in granular flow. For example, Savage and Jeffrey [89] developed constitution relations for stress at high shear rates, which is likely the regime that granular tribologists call "granular kinetic lubrication" [12,13] as described in Sec. 2.2. They also developed their constitutive relations based on the Carnahan and Starling spatial distribution [90] for granules, which was utilized in several granular lubrication papers [12,79,86,87]. Massoudi and Mehrabadi [57] developed a constitutive relation for the stress tensor to observe the "dilatancy" effect in granules, where the granular material expands during shearing. They used a nonkinetic theory-based continuum mechanics approach as opposed to the kinetic approach pioneered by Johnson and Jackson [91]. It is interesting to note that granular tribologists would likely interpret dilatancy as some form of the granular material's "lift" or load-carrying capacity, which relates to its ability to support load. Mohan et al. developed the Cosserat model for slow couette shearing granular flows and omitted the granular energy equation since collisional interactions are assumed negligible in slow shearing [92].

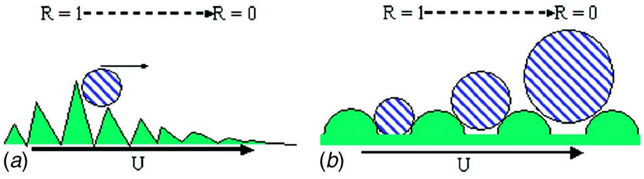


Fig. 8 Schematic of roughness factors. Roughness factors *R* are defined as (*a*) the fraction of lateral momentum imparted by the surface and (*b*) the fraction of granular particles that fits between wall disks

Developing relations to describe the behavior of granular materials around the boundaries is also important. The boundary equations in the granular tribology community are often derived from the work of Jenkins and Richman [93] and Hui et al. [94]. In these works, the roughness factor R varies as $0 \leq R \leq 1$, where $R=0$ is a smooth surface and $R=1$ is a perfectly rough surface. It has been characterized in Fig. 8 as follows:

1. the fraction of lateral momentum transferred to the granular flow by the walls [30]
2. the fraction of granules that fit exactly between the cylindrical wall disks [31]

The roughness factor affects the slip at the boundary, which ultimately affects the granular film's ability to accommodate velocity and carry load. For example, a smooth ($R=0$) surface would have high slip at the boundaries, which would radically reduce the velocity variation across the interface. Reduced velocity variation and high slip would lead to higher friction coefficients between the granular flow and the boundaries.

4 Experiments With Dry Powder and Granular Materials

Several experiments have been conducted using powder particulates and granules in various tribosystems. From the variety of geometries used in the literature, three fundamental geometries (see Fig. 2) emerged from the literature: (i) annular-type geometry, (ii) parallel-type geometry, and (iii) converging-type geometry. Annular geometry refers to the concentric cylinder setup. Parallel geometry, often called "parallel type," refers to horizontal plates or "race-track" geometries, which are commonly studied for first-order predictions and as idealistic conditions. Converging-type geometry, also called bearing-type, usually refers to a geometry that has a converging region that the particulate materials are entrained in during operation to produce a lubrication effect.

4.1 Annular-Type Geometry

4.1.1 Powders. Heshmat [31] developed powder flow visualization experiments to present a "visual documentation" of the fluidlike flow of certain powders. The test rig assembly purposely designed for the flow visualization experiment included a variable speed electric motor and a cup-shaped transparent journal bearing. Additionally, Heshmat and Walton [17] extended the quasi-hydrodynamic model of powder lubrication to develop a novel powder-lubricated rotor bearing damper system for use in high-temperature, high-speed rotating machinery, such as advanced aircraft gas turbine engines. This experiment also provided guidelines for selection of proper geometries, materials, and powders suitable for these tribological processes.

To analyze the powder flow in the frictional flow regime described by Tardos [95], Klausner et al. [96] fabricated an annular shear-type cohesive powder rheometer. This was used to experimentally study the behavior of a 30 μm spherical silica powder and a 40 μm industrial polymer powder. Even though the silica powder was found to be cohesionless while the industrial powder appeared to be cohesive, they were both found to display similar rheological behavior in the frictional flow regime. This experiment also demonstrated the need for a constitutive model relating the stress tensor to the deformation rate for the powder assembly. Recall, at low shear rates, powder and granular materials may behave similarly if their sizes are similar. However, powders deform greatly, often losing their sphericity.

4.1.2 Granular Materials. The pioneering experiments of Bagnold [97] were the first to study the flow of granules under shear. He sheared dispersion of uniform-sized, solid spherical grains in Newtonian fluid in annular space between two concentric drums. Two distinct regions, grain-inertia region and macroviscous region, were identified, and empirical relations for shear stress were formulated. He also established a third transition region. Tardos et al. [82] sheared fluidized bed of fine glass beads between concentric rough vertical cylinders and measured the shear and normal stresses. Veje et al. [98] used a two-dimensional annular setup with rough rotating inner wheel. They sheared photoelastic disks at slow rates. Velocity, spin, and solid fraction distributions were measured by tracking particles using imaging techniques. They also used the same setup to measure the stress fluctuations using photoelasticity [99].

Mueth et al. [100] combined three noninvasive techniques: magnetic resonance imaging (MRI), x-ray tomography, and high-speed-video particle tracking to obtain the particle velocity, spin, and solid fraction data in three dimensions. They sheared mustard and poppy seeds between concentric rough cylinders. Losert et al. [101] sheared black glass beads between rotating inner cylinder and outer cylindrical frame. They measured the mean particle velocities and the velocity fluctuations to develop constitutive relations for them as a function of distance from the rotating wall. The inner and the outer cylinder walls were coated with glass beads to make them rough. These experimental results were compared to a continuum model they developed in [80].

MiDi [102] summarized experiments and simulations conducted on granular flows and classified them into six geometric configurations, some of which are similar to the three described in Fig. 2. The aim of their paper was to identify robust features and relevant nondimensional parameters for each configuration. Though some of the sections are relevant for tribology community, they do not focus on tribological issues and hence vastly differ from the current work. Higgs et al. [28] recently presented results for granular materials in an annular shear cell. Their granular shear cell (GSC) in Fig. 1(*b*) is the first granular experimental configuration to operate at different speeds and with varying roughness factor to match with the boundary conditions specified in Fig. 8(*b*). They presented experimental data for granular velocity, solid fraction, and slip, which resembled the modeling results for a parallel geometry using the granular kinetic lubrication continuum [12] and lattice-based cellular automata modeling approaches [103]. Figure 9 shows a photograph of the varying roughness *R* imposed on the inner driver wheels in the GSC developed in the author's laboratory [28].

4.2 Parallel-Type Geometry

4.2.1 Powders. Tardos et al. [63] studied the rheological behavior of powders in the "intermediate" regime lying between the granular contact lubrication and granular kinetic lubrication regimes using frictional bulk powder in a parallel-type device. Although they studied the intermediate flow of frictional bulk powder in a parallel geometry, such flows are not necessarily for lubrication studies. They occur when powders move relative to solid walls in hoppers, bins, and inserts or are mixed in high and low shear mixer, using moving paddles. The material described also fits cohesionless bulk powder. The powder was sheared between two tall rotating cylinders that form a shear gap where gravity, acting perpendicular to the shear field cannot be neglected.

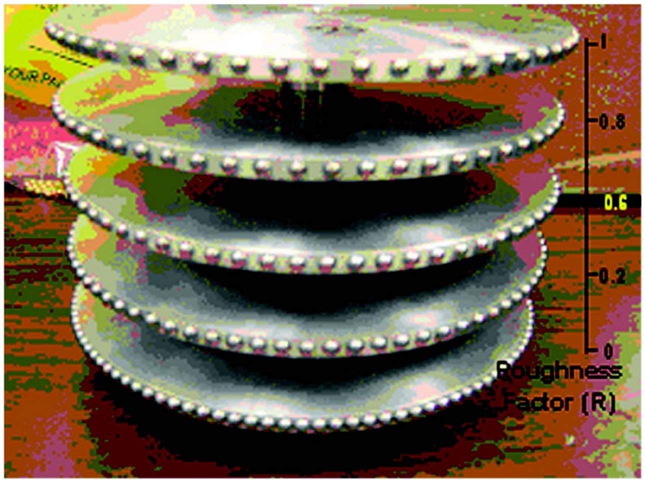


Fig. 9 Wheel roughness on in annular GSC: from top to bottom, $R=0-1$ [28]

4.2.2 Granular Materials. Savage and Sayed [104] developed and performed experiments on annular shear cells with various granular materials, at rapid shear rates. Though the setup was annular, the shear zone was parallel type. They determined the effects of shear rates and solid fraction on shear and normal stresses. Similar to the annular shear cell, a parallel-plate shear cell was used to shear glass spheres mixed with water or air at high speeds by Hanes and Inman [105]. They observed two types of flows. At high shear rates, all the granules in the shearing gap were mobile. On the other hand, in second type of flow some granules remained stationary and others were sheared rapidly creating an internal boundary.

Miller et al. [106] performed experiments on variable sized glass beads by shearing them in a couette parallel geometry. They measured the fluctuations in normal stress. Yu et al. [13] sheared glass beads in a shear cell apparatus to measure the normal and shear stresses. This is the first experimental paper that refers to normal stress as load and shear stress as a frictional stress. They also proposed the idea of granular kinetic lubrication in this paper. Effects of surface roughness, particle size, and solid fraction are further elaborated using the same setup as Craig et al. [107,108].

4.3 Converging Geometry

4.3.1 Powders. Heshmat performed powder flow visualization experiments with powder in an enlarged journal bearing setup [31]. The test rig assembly purposely designed for the powder flow visualization experiment included a variable-speed electric motor and a cup-shaped transparent journal bearing. His experiments showed evidence of the formation of hydrodynamic pressure profiles in powder lubricants. He also observed that the adhesion of a thin layer of powder to the two mating surfaces is responsible for the low-friction behavior of the tribological process. A series of work led to him developing his quasi-hydrodynamic theory for powders in converging contacts [1,31,85]. Heshmat and his collaborators [3,4,7,9,10,109] studied powders in converging contacts and also used pin-on-disk tests to evaluate the performance of molybdenum disulfide (MoS_2) in bearings. The "pellet-on-disk" tests were performed using a modified pin-on-disk tribometer where compacted MoS_2 powder was run against a titanium carbide (TiC) disk [6,9,10]. The tests helped to establish the optimum geometries and system parameters needed to make a MoS_2 lubricant pellet. From this work, a self-contained solid/powder lubricated auxiliary hydrodynamic bearing was developed [9]. This bearing was operated at 30,000 rpm and at loads up to 445 N (100 lb).

4.3.2 Granular Materials. The shear cell apparatus developed by Yu et al. [13] can have a flat shear surface or a surface containing three sloping regions with a step. This creates a converging geometry between shearing surfaces. They confirmed that lubrication wedge effect exists and that stress is roughly proportional to the square of the surface slope. Since there has not been much experimental work with granular materials in converging or "bearing" contacts, granular tribologists sometimes use granular lubrication models [21,87] to predict the data from Heshmat's powder experiments discussed in Sec. 6.

5 Numerical Simulations for Powder and Granular Flows

The Fig. 4 flowchart reveals the dearth of numerical simulations in the area of powder lubrication. In comparison, granular simulations seemed to have appeared much more frequently in the literature. Yet, there is a great need for accurate discrete element simulations (DES) for modeling granular materials in sliding contacts [28]. Simulations of granular materials are much more sophisticated, dating all the way back to the work of Campbell and Brennen [110]. However, simulating powders in sliding contacts has recently began to make some strides [69,70].

5.1 Simulations of Powders. Mei et al. [56] numerically studied the simple shear flow of cohesive powders and developed a method to quantify particle concentration non-uniformity for both dilute and dense conditions. They obtained the variance-to-square of mean ratio for particle number concentration, using a variety of subcells of different volumes that were based on a DES of the particle dynamics in a simple shear flow. They incorporated microscopic parameters, such as particle size, friction, cohesion, and interaction forces between the particles, in the DES. Here, each individual particle within the system is followed exactly as it moves and interacts with its immediate neighbors. By statistically averaging the individual particle dynamics, the macroscopic behavior of a particle assembly was obtained. In the face of experimental difficulties, Iordanoff et al. [60] suggest the use of a numerical modeling approach. They studied mechanisms operating in sliding contacts and outline the influence of external parameters. The proposed unified approach considers the following conventional modeling: (i) the quasi-hydrodynamic model developed by Heshmat and (ii) the kinetic model developed by Haff [78] and extensively modified by tribologists, such as Dai et al. [84], McKeague and Khonsari [32,111], Yu et al. [13], and Zhou and Khonsari [87]. Their discrete model was based on the principles of distinct element method (DEM) proposed by Cundall and Strack for geotechnical applications [112]. Recently, simulations involving solid third bodies, akin to pelletized powder, have been conducted by Fillot et al. [70]. By simulating the pellet wear process, they observed that the third-body behavior is akin to the description in Fig. 10, which shows a material being degraded as the third body (wear debris) is being deposited on the surface. The parameters L_x and L_y represent the confined walls of the simulation cell. Wornyoh and Higgs recently adopted and modified their mass wear laws to model experimental data from a pellet-on disk with slider tribometer [71].

5.2 Simulations of Granular Flows. Campbell and Brennen [110] contrasted their two-dimensional simulations of granular particle collisions to experiments by Savage and Sayed [104] and Bagnold [97]. The first paper to study granular flow from a tribological perspective was Elrod [11]. He performed two-dimensional, two-degree-of-freedom granule-granule simulations to predict the flow of particles in slider bearings. He showed that placing a slider, with a diverging leading edge, on a surface composed of granules could generate lift.

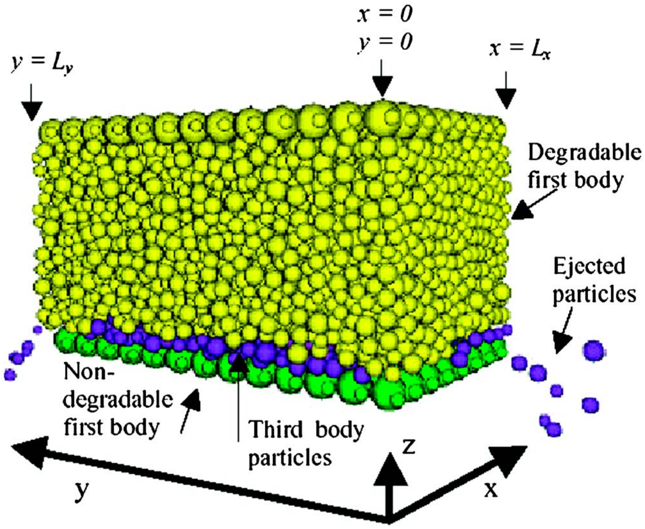


Fig. 10 Compacted powder wear simulation [70]

Babic et al. [113] developed a numerical model to simulate the effects of simple shear flow on uniform, inelastic, frictional, deformable disks. They concentrated on understanding mechanisms that operate in the transitional and quasi-static regimes. Later, Hopkins and Louge [114] developed a two-dimensional computer simulation for rapidly shearing uniform smooth inelastic disks. They compared the results to the theoretical model of Jenkins and Richman [115] with good agreement. Thompson and Grest [116] developed a two-dimensional molecular dynamics-type simulation for uniform sized granules under shear in gravity. Their model has rough walls and periodic boundary conditions. Stick-slip behavior was observed at low shear rates. They also identified two distinct static and flowing regions separated by a phase boundary. Yi and Campbell [117] developed a discrete particle computer simulation modeling the same problem. Their simulation modeled all regions of granular flows from a solid-type stagnation regime to a transition regime and finally a fluid-type granular flow regime. In tribology, these regimes under a finite load would be classified as granular contact lubrication, a transitional- or "two-phase"-type of granular lubrication [34], and granular kinetic lubrication.

Additional computational works involving simple shearing of monosized particles have appeared in literature [118–124]. Schwarz et al. [125] used a particle dynamics, such as the discrete mesodynamic method, to model frictional granules as well as perfectly smooth monosized particles in a gravity-free environment. They performed the simulations at high shear rates and high solid fractions and showed that the normal stresses have contributions from both, collisional forces of particles and formation of stress chains. They confirmed the existence of critical solid fraction, where transition occurs from the granular contact regime to the granular kinetic regime.

Schollmann [126] developed a two-dimensional molecular dynamics simulation for an annular-type shear cell. He modeled the experiments performed by Veje et al. [98]. Effects of coefficient of restitution, friction factor, and solid fraction on tangential and normal velocity profiles are studied in the model. The simulation reproduces the rapid shear zone near the moving boundary found in the experiment. Latzel et al. [127] developed a discrete element simulation to model the same geometry. Additionally, they developed averaging techniques useful for deriving macroscopic tensorial quantities, such as stress and strain, from microscopic quantities tabulated from the simulation.

Sawyer and Tichy [86] performed numerical and particle dy-

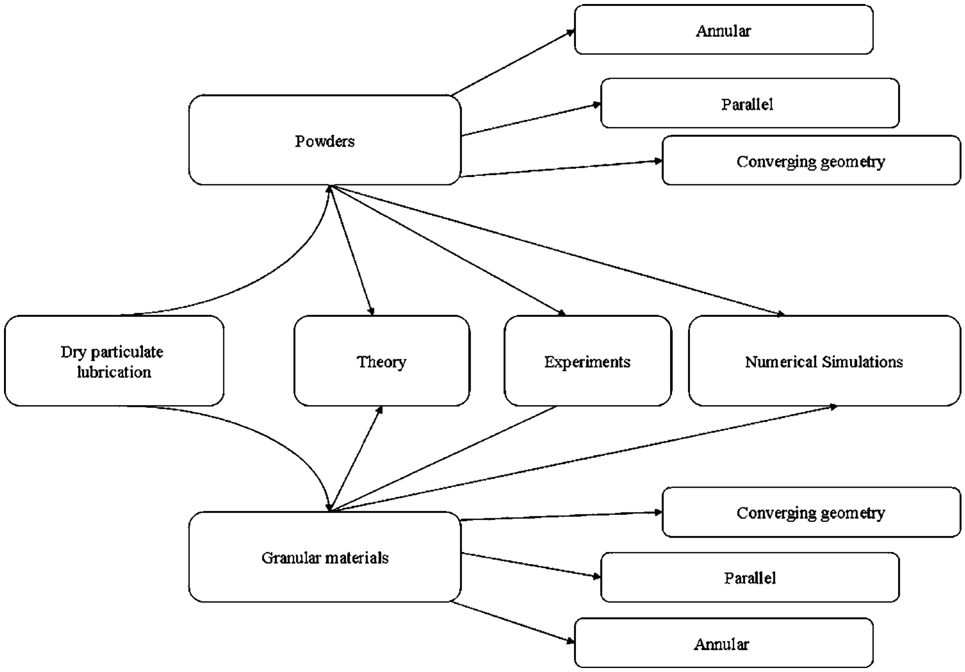


Fig. 11 Summary diagram of dry particulate tribology research for powder (cohesive) and granular (cohesionless) particles

namic simulations to generate results that were compared to the granular experiments of Yu and Tichy [14]. Their particle dynamic simulations presented a three-dimensional analysis and also gave a thoughtful overview of other particle simulation work [97,128–133] by authors whose efforts have been influenced by granular tribology. Aharonov and Sparks [134] propose a two-dimensional simulation for simple couette shear configuration. The top moving surface shears the granules and moves up and down due to applied pressure (load). They observed fluidlike and solidlike regions at low and high pressures, respectively. Jasti and Higgs [103] studied the validity of using cellular automata techniques to model granular flows is tribological contacts. Results were in agreement with the theoretical model of Higgs and Tichy [12].

6 Conclusion

Given the increasing scientific importance of dry particulates in sliding contacts, the absence of a rigorous classification system led the authors to develop this review. Figure 11 diagrammatically summarizes the type of work that has been done in the various areas of dry particulate lubrication, namely, powder and granular. Figure 11 shows that theory, experiments, and numerical simulations have been adopted as techniques for investigating lubrication by dry cohesive particulates. The papers reviewed thus far attempt to be exhaustive from a tribological perspective, but cursory with respect to the dry particulate areas outside of the community. Spanning the past five decades, this review introduces a classification system for the dry particulate tribology literature in terms of theory, experiments, and numerical simulations, in addition to suggesting that published research can be further categorized by the geometry of the tribosystem, such as the annular, parallel, or converging types.

The authors believe that solid/particulate lubrication mechanisms hold promise because of their quasi-hydrodynamic fluid nature and ability to provide solid lubricant films *in situ*. In the case of powders, a powder lubricant medium may start off as discrete particles, but their nature under load usually causes them to coalesce and coat surfaces. As a result, the frictional response of the tribosystem may be similar to solid lubricant coatings or transfer films. However, we still classify such a mechanism as powder lubrication because the deposition processes (e.g., magnetron sputtering, pulsed laser deposition, thermal spray, etc.) used for traditional solid lubricant films are much more complex than the simplistic methods presented in this paper. There is still a great need for numerical simulation work of powder films as Fig. 4 clearly indicates. Currently, granular lubrication studies appear to be more of an academic toolbox for understanding the behavior of hard, solid particles under load in various tribosystems. They are also useful for understanding wall/particle interactions in cutting-edge “wet” tribological processes, such as chemical mechanical polishing. Currently, the authors know of no commercialized dry granular triboapplications, but there is potential for studying granular systems, such as coal energy systems and detergent systems, which feature hard particles being sheared under load. As micro/nanoscale surface motion comes more into fruition, dry particulate lubrication could provide some unique opportunities since show-stopping stiction issues in devices, such as MEMS, suggest the need for dry lubrication alternatives. Therefore, the authors have included recent works done in nanopowder lubrication. As shown in Fig. 5, the advent of nanopowder lubrication has driven the numbers of papers published in dry particulate lubrication to its highest levels since this area of tribology began [135–147]. The future of viable, widely used, commercial dry particulate lubrication mechanisms is uncertain, but our review of the literature indicates that it is extremely promising as a solid lubrication alternative.

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