AP Physics **Energy and its Conservation – Part I**

Introduction: Energy is a term that most of us take for granted and use quite freely. We assume we know what we are talking about when speaking of energy. In truth, energy is probably the most complex and least understood of all physical quantities. Thus, in this unit energy will be defined as the capacity to do work. However, this definition states what energy does, not what it is. Still, as you learn the basic concepts related to energy, we will refer to it this way.

All forms of energy are interchangeable. When energy changes form the total energy of a system remains the same.

Performance Objectives: Upon completion of the activities and readings of this unit which is *Part I of the Energy Study*, and when asked to respond either orally or on a written test, you will:

- define work as the dot product of force and displacement and in terms of energy transfer.
- define kinetic energy. Calculate the kinetic energy of a body given the mass and velocity of the body. Calculate the change in kinetic energy of a body given the force applied and the displacement.
- explain what is meant by a conservative force. State the properties of a conservative force. Name the conservative forces studied in this unit. State their similarities and differences.
- define potential energy. Calculate the gravitational potential energy of a body near the surface of the earth.
- state the law of conservation of mechanical energy. Use the law of conservation of mechanical energy to solve problems.
- define elastic potential energy. Solve problems involving springs and moving objects.
- describe energy changes in a pendulum swing. Solve problems associated with a pendulum.

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Textbook Reference: Tipler Physics: Chapter 6, Section 4; Chapter 7
Glencoe Physics: Chapter 10, Section 1, Chapter 11

"A short demonstration of a remarkable error of Descartes and others (Newton and his disciples) concerning the Natural Law by which they think the Creator always preserves the same quantity of motion and by which the Science of Mechanics is totally perverted."

Beginning of a paper by Gottfried Wilhelm von Leibnitz and his followers denouncing the Law of Conservation of Momentum.

KINETIC ENERGY is the energy an object possesses because of its motion. Kinetic energy depends on mass and velocity. The net work done on a body can change its kinetic energy.

$$K = \tfrac{1}{2} mv^2 \qquad Work_{net} = \Delta K = K_f - K_i$$

Problems and Exercises:

1. An 8.0 kg mass moves at 30.0 m/s. What is its kinetic energy? *3600 J*

2. A 10.0 kg body has a kinetic energy of 500.0 J. What is the body's velocity? What is its momentum? *10 m/s 100kg-m/s*

3. A 10.0 kg body at rest on a frictionless table receives an impulse of 30.0 N-s. What is its final kinetic energy? *45J.*

4. A constant 20.0 N force acts for 10.0 s on a 5.0 kg object which is initially at rest.
a) What is the object's final momentum? *200 kg-m/s*
b) What is the object's final kinetic energy? *4000 J*
c) How much work was done in accelerating the body? *4000 J*

5. A 10.0 kg mass is moving with constant speed of 10.0m/s. a) How much work must be done to double the speed to 20 m/s? b) How much work must be done to halve its speed to 5.0 m/s? *1500 J -375 J*

6. A car of mass 1100 kg moves at 20 m/s. A breaking force of 1200 N is needed to bring the car to a halt. a) What is its stopping distance? b) If the same car is moving at 40 m/s, what is its stopping distance? *183 m 732 m*

7. What is the kinetic energy of a 1600 kg car which moves at a) 30.0 km/h? b) 60.0 km/h?
c) What is the ratio of (b) to (a)? d) How does the kinetic energy of a car change when its speed is doubled? *55000 J 220000 J 4:1*

8. Compare the kinetic energies of two objects, A and B, identical in every respect except one. Assume that the single difference is: a) Object A has twice the velocity of B. b) Object A moves north, B south. c) Object A moves in a circle, B in a straight line. d) Object A is a projectile falling vertically downward, B is a projectile moving vertically upward at the same speed.

9. Two bodies of unequal mass each have the same kinetic energy and are moving in the same direction. If the same retarding force is applied to each, how will the stopping distances of the bodies compare?

10. A force of 30.0 N accelerates a 2.0 kg object from rest for a distance of 3.0 m along a level, frictionless surface; the force then changes to 15 N and acts for an additional 2.0 m. a) What is the final kinetic energy of the object? b) How fast is it moving?
120 J   11 m/s

11. A 2.0 kg stone whirls around on the end of a 0.50 m string with a frequency of 2.0 revolutions per second. a) What is its kinetic energy? *39 J*
b) What is the centripetal force on it? *157 N*
c) How much work is done by the centripetal force in one revolution?

12. Is it possible to exert a force and yet not cause a change in kinetic energy?

13. The design of road dividers that separate opposing traffic lanes used to consist of metal rails mounted to make contact with the fenders and doors of the car. The newer design is of concrete with a cross-section very deliberately shaped to make contact with the tires. The frictional force between rubber and concrete is much larger than the frictional force between metal and metal. Why is this new design more effective in slowing down a car that side-swipes it?

Potential Energy is the energy an object has because of its position or state. The potential energy an object has due to its position above the surface of the earth is called its ***gravitational potential energy*, U_g.** When the distance above the earth is very small compared to the radius of the earth, we consider the gravitational field to be uniform and treat the gravitational force as a constant. The increase in the potential energy of any system is equal to the work done on the system by an outside force. For example if you lift a 5.0 kg box a distance of 2.0 meters above the ground, the work done on the box is 98 J. You are the outside force doing work on the box-earth system. The increase in the potential energy of the box-earth system is 98 J. You are actually working against gravity. The work done by gravity on the box is -98 J. The work done against gravity is stored as potential energy. $\boldsymbol{U_g = mg\Delta y}$, where m is the mass in kilograms, g is the acceleration due to gravity (the gravitational field strength) in m/s^2 and Δy is the vertical displacement in meters. $\Delta y = y_f - y_i$. If $y_i = 0$, Δy is called the height, h. You may see the potential energy formula written as $U = mgh$.

14. A 5.0 kg bowling ball is lifted from the floor to a height of 1.5 m. How much work is done on the bowling ball by the person lifting the ball? What potential energy does the ball possess at this height? *73.5 J   73.5 J*

15. A 50.0 kg sky diver falls toward the earth a distance of 500.0 meters before opening her parachute. What was the change in her potential energy? What force was doing work on her to change her potential energy? From your own experience or your study of physics thus far, what happens to her speed as she falls toward the earth? If her speed is increasing, what can be said about her kinetic energy? *-245 kJ*

The **Law of Conservation of Mechanical Energy** states that the total energy of a system cannot change, unless work is done on the system. Within an isolated system, the amount of energy always remains the same. Within an isolated system, energy can change forms - however energy can never be "lost" within the system. Energy can be transferred between systems when one system does work on another.

Work done by a conservative force conserves mechanical energy. If only conservative forces are working in a system, mechanical energy is conserved. $E = K + U$. Where E is the total mechanical energy, K is the kinetic energy and U is the potential energy of the system. This means that at any instant, the total mechanical energy (potential and kinetic) is a constant.

We will study two conservative forces in this unit – gravitational and elastic forces. The work done by a conservative force always has these properties: *1. It is equal to the difference between the initial and final values of a potential energy function. 2. It is reversible. 3. It is independent of the path of the body and depends only on the starting and ending points. 4. When the starting and ending points are the same - that is when the body travels a closed path – the total work done by the conservative force is zero.*

In a closed system with only conservative forces working, the total energy at the beginning of a process is equal to the total energy at the end of the process.

$$\text{Work}_{\text{done by conservative force}} = \Delta K = -\Delta U \quad \text{and} \quad U_i + K_i = U_f + K_f$$

You would be considered an outside force acting on a box-earth closed-system.

$$\text{Work}_{\text{done by outside force}} = \Delta U$$

16. An 8.0 kg flower pot falls from a window ledge12.0 m above a sidewalk. a) What elements comprise the closed system in this problem and what is the conservative force acting on the flower pot?
b) What is the potential energy of the flower pot just before it falls? *940 J*
c) What is the kinetic energy of the flower pot just before it strikes the sidewalk? *940 J*
d) Determine the speed of the pot just before it strikes the walk. *15 m/s*

17. A 15.0 kg model plane flies horizontally at 12.5m/s. The plane goes into a dive and levels off at 20.4 m closer to the earth. a) What elements comprise the closed system in this problem and what is the conservative force acting on the model plane?
b) What is its new kinetic energy? *4171 J*
c) Neglecting frictional effects, what is its new horizontal velocity? *23.6 m/s*

18. A bag of cement having a mass of 16.0 kg falls 40.0 m into a river from a bridge. a) What elements comprise the closed system in this problem and what is the conservative force acting on the bag of cement?
b) If air resistance is negligible, what is the vertical speed of the bag as it hits the water? *28 m/s*

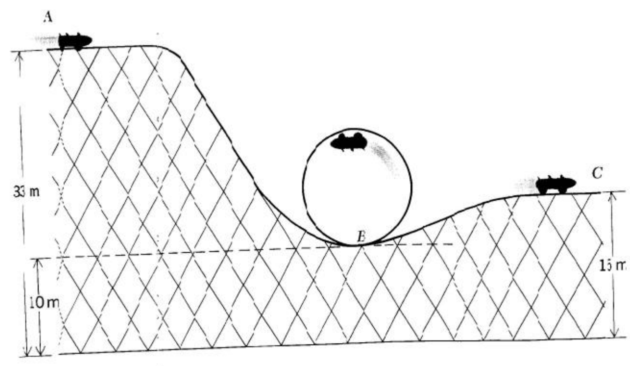
19. A 10.0 kg experimental rocket is catapulted from Kennedy Space Center. It leaves the launch pad with kinetic energy of 1960 J. a) What elements comprise the closed system in this problem and what is the conservative force acting on the experimental rocket?
b) How high will the rocket rise? *20 m*

20. In an electronics factory, small cabinets slide down a 30˚ frictionless incline for a distance of 16 m to reach the next assembly stage. The cabinets have a mass of 10.0 kg each. a) What elements comprise the closed system in this problem and what is the conservative force acting on the cabinets?
b) What kinetic energy would a cabinet have at the bottom of the incline? *784 J*
c) Calculate the speed each cabinet would acquire at the bottom of the incline. *12.5 m/s*

21. A roller coaster of mass 3000.0 kg starts from rest at Point A, which is 33 meters above the bottom of the coaster and proceeds to point C, which is 15 m above the bottom of the coaster.
a) What is its speed at C?
b) Recall that the formula for critical velocity is:

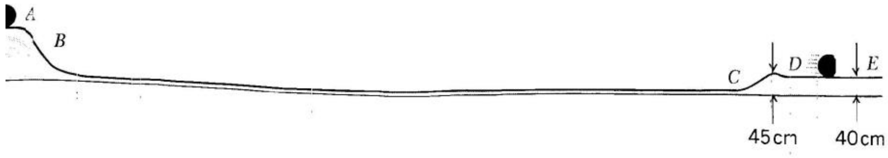
$$v_c = \sqrt{rg}$$

If the radius of the loop is 5.0 m, will the roller coaster have enough kinetic energy at the top of the loop to satisfy the critical velocity criteria?
$v_{@\ point\ C} = 18.78$ m/s $\quad v_c = 7.0$ m/s $\quad v = 16$ m/s *yes*



22. A ping-pong ball of mass m rolls off the edge of a table that is 1.0 m high. When the ball strikes the floor its speed is5.0 m/s. How fast was it rolling when it left the table. *2.3 m/s*

23. A bowling ball is returned to the bowler by an automatic pin-setter that transfers it to the return ramp 100.0 cm above the return tunnel below the alley as shown in the diagram at the bottom of the page. If the ball is given an initial speed of 0.6 m/s at point A, what is its speed when it reaches the bowler on the short level stretch DE? Ignore friction. *3.48 m/s*



Elastic Potential Energy: Hooke's Law for springs is written as $F = -kx$. This is from the spring's perspective. When a spring is compressed or stretched the force the spring exerts is in the opposite direction of the stretch or compression. If you are compressing or stretching a spring, the force you are exerting is in the same direction as the compression or stretch. The equation describing your action would be $F = kx$.

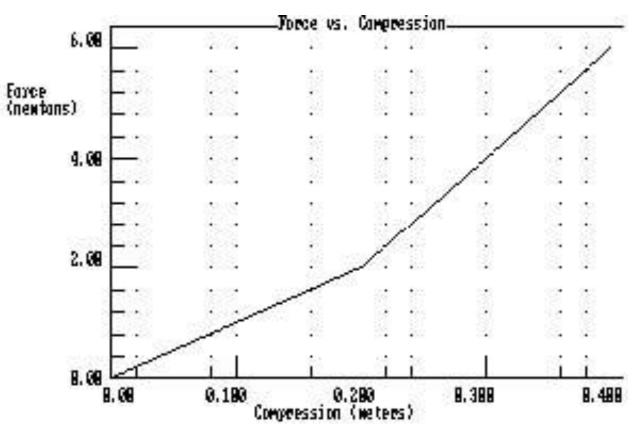
At the beginning of the year you participated in a lab where force was plotted as a function of stretched position of a spring. You were asked the physical significance of the slope of that graph. k is the slope of a force-position graph for a spring or $k = \Delta F/\Delta x$. The work you did to stretch that spring was stored in the spring.

Another mechanical system that is conveniently described using the concept of potential energy, or stored energy, is the spring. If work is done by an outside force on a spring to either stretch or compress it, energy is stored in the spring. If the work is done by a moving body the kinetic energy of the moving body is transferred to the stored energy in the spring. The stored energy of a spring is called *elastic potential energy* and denoted by U_S or U_E. The potential function for the spring is $U_S = \frac{1}{2} kx^2$, where k is the force constant for the spring in N/m and x is the position in meters of the stretched or compressed spring relative to its unaltered position. If there is no friction in the system, the force exerted by the spring can be considered a conservative force. Recall from the previous discussion that if only conservative forces are working on the system then the law of conservation of mechanical energy can be used to solve problems.

24. Consider a spring 4.0 m long with a spring constant of 500.0 N/m. a) What outside force is needed to compress the spring 3.0 meters to a new length of 1.0 m?
1500 N
b) What is the elastic potential energy stored in the spring if the spring is compressed to a length of 1.0 m?
2250 J

25. A spring is compressed 0.20 m by a force of 20.0 N and 0.30 m by a force of 30.0 N.
a) What is the force constant k for this spring?
100.0 N/m
b) What is equation for the potential energy stored by the spring as a function of its compression (its potential function)? $U_S = 50\, x^2$

26. For the force compression curve of a spring shown.



a) How much work is done in compressing the spring 0.3 m? b) What is the potential energy stored in the spring when compressed this amount? *0.5 J*

Systems Which Include a Spring And a Moving Object

If a moving block collides elastically with a spring and causes the spring to compress, then the kinetic energy of the block is converted to the elastic potential energy of the spring. The moving block has maximum kinetic energy wrt the system just before it collides with the spring. As soon as the block starts to interact with the spring, it loses kinetic energy and the spring gains potential energy. The spring will have maximum potential energy when it has maximum compression and the block is no longer moving relative to the spring.

27. A block of mass 5.00 kg is moving at 8.00 m/s along a frictionless horizontal surface toward a spring with force constant k = 500.0 N/m that is attached to the a wall. Find the maximum distance the spring is compressed. The mass of the spring is negligible. *0.8 m*

28. You are asked to design spring bumpers from the walls of a parking garage. A freely rolling 1200.0 kg car moving 0.50 m/s is to compress the spring no more than 0.080 m before stopping. What should be the force constant of the spring? *46,875 N/m*

29. A 1.2 kg block on a horizontal frictionless surface is attached to one end of a horizontal spring (k = 200 N/m) which has its other end fixed. The block is displaced 0.20 m horizontally from its equilibrium position and released from rest. What is the speed of the block when it is 0.10 m from its equilibrium position? *2.2 m/s*

30. A spring having a force constant k = 300.0 N/m rests on a frictionless horizontal surface. One end is in contact with a stationary wall, and a 4.00 kg block is pushed against the other end, compressing the spring 0.100 m. The block is then released on a frictionless horizontal surface with no initial velocity. a) What is the block's speed when the spring returns to its uncompressed length? b) What is the block's speed when the spring is still compressed 0.060 m? *0.866 m/s* *0.693 m/s*

Systems with Two Conservative Forces:
The Conservation of Mechanical Energy formula, $U_i + K_i = U_f + K_f$ *, can still be used to solve problems. Remember there are two kinds of potential energy.*

31. A 1.50 kg book is dropped from a height of 0.60 m onto a spring with force constant k = 1960 N/m. Find the maximum distance the spring will be compressed.
0.103 m

32. A spring (k = 600 N/m) is placed in a vertical position with its lower end supported by a horizontal surface. A 2.0 kg block that is initially 0.40 m above the end of the spring is dropped from rest onto the spring. What is the kinetic energy of the block at the instant it has fallen 0.50 m (compressing the spring 0.10 m)? *6.8 J*

33. A brick of mass m, initially at rest, is dropped from a height h onto a spring in a vertical position with its lower end supported by a horizontal surface. The force constant for the spring is k. Find the maximum distance y that the spring will be compressed?
Answer: positive root of

$$y = \frac{mg \pm \sqrt{(mg)^2 + 2kmgh}}{k}$$

.
To what does the negative root correspond?

Energy Considerations of a Pendulum: The swinging of a pendulum illustrates the way in which potential energy maybe transformed into kinetic energy and then back to potential energy. When a pendulum is pulled to one side, the mass has no velocity and no kinetic energy, but it does have potential energy. When the mass is released, it swings downward, acquiring kinetic energy and losing potential energy. At the bottom of the swing the pendulum bob has maximum kinetic energy. As it swings upward, this kinetic energy is transformed again into potential energy. Once again, if there is no friction, work is being done by only gravity, which is a conservative force. This means that the law of conservation of mechanical energy can be used to solve problems.

34. A pendulum bob is pulled to one side until its center of gravity has been raised 10.0 cm above its equilibrium position. Find the speed of the bob as it swings through the equilibrium position. *1.4 m/s*

35. The suspended bob of a 1.0 m pendulum is drawn back so as to allow the cord to make an angle of 10˚ with the vertical before being released. Calculate the speed of the bob as it passes through its lowest position. *0.54 m/s*

36. A pendulum bob has a mass of 0.5 kg. It is suspended by a cord 2.0 m long which is pulled back through an angle of 30˚.

a) Find its maximum potential energy relative to its lowest position. *1.31 J*
b) Find its potential energy when the cord makes an angle of15˚ with the vertical. *0.33 J*
c) Find its maximum speed *2.29 m/s*
d) Find its speed when the cord makes the angle of 15˚ with the vertical. *1.98 m/s*

A few questions for thought and discussion.

1. Why is it easier to stop a light truck than a heavier one that has an equal speed?

2. Which requires more work to stop, a light or a heavy truck moving with the same momentum?

3. Two arrows are fired into a bale of hay. If one has twice the speed of the other, how much farther does the faster arrow penetrate?

4. When a rifle with a long barrel is fired, the force of expanding gases acts on the bullet for a longer distance than in a short barrel gun. What effect does this have on the velocity of the emerging bullet? (Do you see why long-range cannons have such long barrels?)

5. With what force does a rock that weighs 10 newtons strike the ground if dropped from a rest position 10 meters high?
(Does this question have a straight forward answer?)

6. You and a flight attendant toss a ball back and forth in an airplane in flight. Does the kinetic energy of the ball depend on the speed of the airplane? Explain.

7. Can a body have energy without having momentum? Explain. Can a body have momentum without having energy? Explain.

8. A satellite orbits the earth. The point in orbit farthest from the earth is the apogee and the point nearest the earth the perigee. With respect to the earth, at which of these points does the satellite have the greatest gravitational potential energy? The greatest kinetic energy? The greatest total energy? Explain.

9. You're on a roof top and throw a ball downward to the ground below, and another upward whereupon after rising, it falls and also strikes the ground below. If your downward and upward initial speeds are the same, how do the speeds of the balls compare upon striking the ground? Explain in terms of energy.

10. In the absence of air resistance, a ball thrown vertically upward with a certain initial kinetic energy will return to its original level with the same kinetic energy. When air resistance is a factor affecting the ball, will it return to its original level with the same, less, or more kinetic energy? Does your answer contradict the conservation of energy? Defend your answer.

11. Two physics students are discussing the design of a roller coaster. One student says that each summit must be lower than the previous one. The other student says this nonsense for as long as the first one is the highest it doesn't matter what height the others are. What do you say? Do you agree with either student or is there another explanation?

12. A diver who weighs 500 N steps off a diving board that is 5.0 m above the water. How much kinetic energy does she have just before hitting the water? Would the kinetic energy be greater at this point if she had instead sprung upward from the board? Explain.

13. The northern hemisphere of the earth is approximately 6.7×10^6 km closer to the sun in winter than in summer. The earth moves along its orbit faster in winter than in summer. Explain these two statements in terms of the earth's potential and kinetic energy.

14. At what point in its motion is the kinetic energy of a pendulum bob a maximum? At what point is its potential energy a maximum? When its kinetic energy is half its maximum value, how much potential energy does it have?

15. A physics instructor demonstrates energy conservation by releasing a heavy pendulum bob as shown in the sketch and allowing it to swing to and fro. What would happen if in his exuberance he gave the bob a slight shove as it left his nose?