K-Notes

Thermodynamics

Second Law of Thermodynamics

1st law does not give information whether a certain process will proceed or not. 2nd law provides information regarding feasibility of the process.

Hence 2nd law of thermodynamics is known as directional law or law of degradation of energy.

High grade energy (work) is fully convertible to low grade energy (heat) but low grade is not fully convertible to work.

Cyclic heat engine

A heat engine is a thermodynamic cycle in which there is a net heat transfer to the system and network transfer from the system.

$\sum Q = \sum W$ (1st law)

$$\left[\eta_{thermal} = \frac{W_{net}}{Q_{input}} = 1 - \frac{Q_{rejected}}{Q_{input}}\right]$$

TER (Thermal Energy Resevoir) is defined as a large body of infinite heat capacity $\left(C \rightarrow \infty,\ \Delta T = 0\right)$ which is capable of absorbing or rejecting an unlimited quantity of heat.

Kelvin Plank's Statement

It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.

Machine violating the Kelvin-plank statement is called PMM2. Hence PMM2 is impossible.

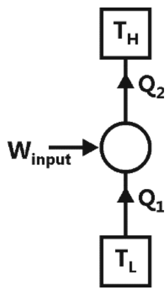
Clausius statement

It is impossible to construct a device working in cycle which will produce no effect other than the transfer of heat from a cooler to a hotter body without any work input.

$$\text{COP refrigerator or reverse carnot cycle} = \frac{Q_1}{W_{input}}$$

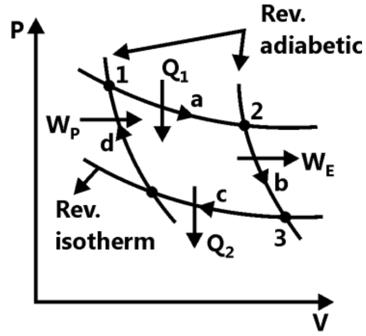
$$COP_{HP} = \frac{Q_2}{W_{input}}$$

$$\left[COP_{HP} = COP_{ref} + 1\right]$$



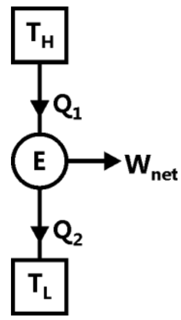
- Heat pump provides a thermodynamic advantage over electrical heater.

Carnot Cycle



(a) Reversible isothermal heat addition
(b) Reversible adiabatic expansion
(c) Reversible isothermal heat rejection
(d) Reversible adiabatic compression

- It states that of all the heat engine operating between constant source and sink temperature, none has higher efficiency than a reversible engine.
- The efficiency of a reversible engine is independent of the nature or the amount of the working substance undergoing the cycle.



- $\frac{Q_1}{Q_2} = \frac{T_H}{T_L}$
- $\eta_{carnot} = 1 - \frac{T_L}{T_H}$

Note: The temperatures appearing in the expression should be the temperatures of the working fluid, if both source and sink temperatures and working fluid temperatures are given.

Drawbacks of Carnot cycle

- All processes are reversible
- Isothermal process takes place at infinite slow speed where as adiabatic at very fast speed. So the combination of two process is practically not possible.

3rd law of Thermodynamics

It is impossible by any procedure no matter, how idealized to reduce any system to absolute zero temperature in a finite number of operations.

Entropy

The degree of randomness of a system is called Entropy. Entropy represents degradation of energy.

Note: Randomness should not be linked with velocity. It basically signifies the scattering of energy in different directions.

Clausius Inequality

The cyclic integral of $\frac{dQ}{T}$ for a reversible cycle is equal to zero. This is known as Clausius theorem.

Clausius Inequality

$\oint \frac{\delta Q}{T} \leq 0$

Cases:

(i) $\oint \frac{dQ}{T} < 0$; irreversible

(ii) $\oint \frac{dQ}{T} = 0$; reversible

(iii) $\oint \frac{dQ}{T} > 0$; impossible

- $ds = \int \frac{dQ}{T}$ only for reversible process
- $dS > \int \frac{dQ}{T}$ for irreversible process
- Entropy is a point function and does not depend on path
- Area under T-S plot gives heat.
- Since $dS > \frac{dQ}{T}$

  For isolated system or universe $dQ = 0$

  $\left[dS > 0\right]$

  $dS_{universe} = dS_{system} + dS_{surrounding}$
- When system is in equilibrium, change in entropy would be zero.

Applications of Entropy

(i) To find the direction of flow
(ii) Transfer of heat through finite temperature difference
(iii) Mixing of two fluids
(iv) Maximum work obtainable from two finite bodies at temperature T_1 and T_2
(v) Maximum amount of work obtainable from a finite temperature body and a TER

Entropy transfer mechanism

(a) Heat transfer
(b) Mass transfer

- No transfer of entropy is associated with work
- Heat flow increases the disorder hence entropy increases
- Work may increase internal energy due to which entropy may increase but is as such no entropy transfer to it

Entropy generation in a closed system

(a) By heat interaction
(b) By internal irreversibility

$$dS = \frac{\delta Q}{T} + \delta S_{generation}$$

Fixed mass entropy analysis

1. Reversible process $(\delta S_{gen} = 0)$

(a) Heat addition $(\delta Q = +ive)$

$\therefore dS = +ive$

(b) Heat rejection $(\delta Q = -ive)$

$\therefore dS = -ive$

(c) Adiabatic $(\delta Q = 0)$

$\therefore dS = 0 \Rightarrow$ Isentropic

2. Irreversible process $(\delta S_{gen} = +ive)$

(a) Heat addition $(\delta Q = +ive)$

$$dS = \frac{\delta Q}{T}(+ive) + \delta S_{gen}(+ive)$$

$\therefore dS = +ive$

(b) Heat rejection $(\delta Q = -ive)$

$$dS = \frac{\delta Q}{T}(-ive) + \delta S_{gen}(+ive)$$

$\therefore dS = -ive,\ +ive,\ zero$

(c) Adiabatic $(\delta Q = 0)$

$$dS = zero + \delta S_{gen}(+ive)$$

$\therefore dS = +ive$

Some important points

- A reversible adiabatic process is always isentropic.
- Entropy of an isolated system can never decrease.

- Entropy is not a conserved property.
- Entropy generation represents degradation of energy.
- Isentropic process is not always reversible adiabatic.
- If an irreversible process is to be isentropic, it must be non-adiabatic.
- Universe is an isolated system.

Entropy generation in an open system

$\dot{S}_{gen} = \dot{S}_{exit} - \dot{S}_{inlet}$ and $\dot{S}_{gen} \geq 0$

Important results

(a) $dQ = dE + dW$
Valid for all processes, reversible or irreversible, open system or closed system

(b) $dQ = dU + dW$
Every process but closed system

(c) $dQ = dU + PdV$
Closed and reversible (Quasi-static)

(d) $dQ = TdS$ reversible only

(e) $TdS = dU + PdV$
Valid for all process and system as it contains only properties.

(f) $TdS = dH - VdP$
Valid for all process and system as it contains only properties.

- $s_2 - s_1 = c_v \int \frac{dT}{T} + R \int \frac{dV}{V}$
- $s_2 - s_1 = c_p \int \frac{dT}{T} - R \int \frac{dP}{P}$

The above expressions give the following results:

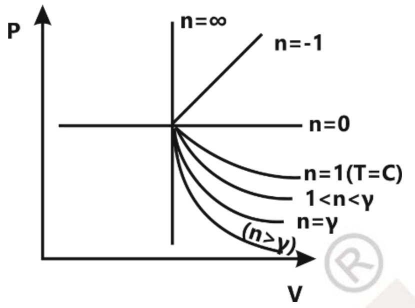
$$s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$$

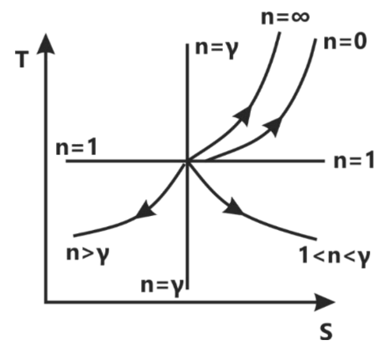
$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$s_2 - s_1 = c_p \ln\frac{V_2}{V_1} + c_v \ln\frac{P_2}{P_1}$$

(g) Two reversible adiabatic processes cannot intersect each other or through one point, only one adiabatic path passes.

Different processes on P-V and T-S diagrams





Note:
On T-S diagram, slope of constant volume process is greater than slope of constant pressure

$$\left(\frac{\partial T}{\partial S}\right)_v = \frac{T}{c_v} \;\&\; \left(\frac{\partial T}{\partial S}\right) = \frac{T}{c_p}$$

$\because c_v < c_p$

Heat Transfer

Fins

Fins are the projections protruding from a hot surface and they are meant for increasing the heat transfer rate by increasing the surface area of heat transfer.

$\text{Area} = zt$

$\text{Perimeter} = 2z + 2t = p$

$q_{x=0} = q_{x+dx} + hP\,dx(T - T_0)$

$q_x = q_x + \frac{\partial(q_x)}{\partial x}dx + q_{convected}$

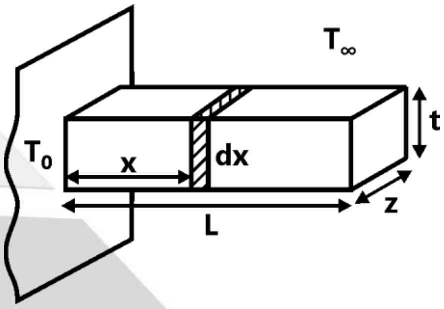
$\frac{\partial^2 T}{\partial x^2} = \frac{hP}{KA}(T - T_\infty)$

$T - T_\infty = \theta$

$m^2 = \frac{hP}{KA}$

$\frac{d^2\theta}{dx^2} = m^2\theta$

$\theta = C_1 e^{-mx} + C_2 e^{mx}$



Now from boundary condition

(i) At $x = 0, T = T_o$, $\theta = \theta_0 = T_0 - T_\infty$

(ii) 2nd boundary condition depends upon different cases.

Case I – Fin is infinitely long

$Q_{\text{through fin}} = \left(\sqrt{hP\,KA_c}\right)\theta_0$ Watt

$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$

Case II- Fin is finite in length and its Tip is insulated

$\left(\frac{\partial T}{\partial x}\right)_{x=L} = 0$ as there is no heat transfer from tip

$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(l - x)}{\cosh ml}$

$Q_{\text{through fin}} = \left(\sqrt{hPKA_c}\right)\tanh(mL)\theta_0$ watt

Note: When no case is mentioned, in any problem use case-II

Case III- Fin is finite in length and also loses heat by convection from its tip.

$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh mL_c}$

$L_c =$ Corrected length

$L_c = L + \frac{t}{2}$ (Rectangular fin)

$L_c = L + \frac{D}{4}$ (Circular fin)

$q_{fin} = \sqrt{hPKA_c}\ \theta_0\ \tan h(mL_c)$

Fin efficiency

It is the ratio of actual heat transfer rate to maximum possible heat transfer rate when entire fin is present at base root temperature.

$$\eta = \frac{Q_{actual}}{Q_{maximum\ possible}} = \frac{\sqrt{hPKA_c}\ \theta_0 \tanh(mL)}{h(PL)\theta_0}$$

(When whole fin is at the base temp)

Depends upon the case which one is to be used

Fin efficiency for insulated tip

$$\eta = \frac{\text{Tanh}(mL)}{mL}$$

Fin efficiency for long fin (infinite)

$$\eta = \frac{1}{mL}$$

Effectiveness of fin

Ratio of heat transfer rate with fins to heat transfer rate without fins. It is a measure of how effective the usage of fins is.

Effectiveness $= \frac{Q_{with\ fin}}{Q_{without\ fin}}$

Effectiveness for case 1 (infinitely long fin)

$$\varepsilon_{fin} = \sqrt{\frac{KP}{hA_c}}$$

Effectiveness for case 2 (Fin is finite in length and its Tip is insulated)

$$\varepsilon_{fin} = \sqrt{\frac{KP}{hA_c}}\text{Tanh}(mL)$$

For fins to be effective, $\varepsilon_{fin} > 1$

→ K↑

→ A_c↓ (i.e why thin fins are preferred)

→ Short in length

→ More in number, so closely spaced

Note: Fins are more effective where convective heat transfer coefficient is less

Engineering Maths

Calculus

Important Series Expansion

a. $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$

b. $(1+x)^{-1} = 1+x+x^2+\ldots\ldots\ldots\ldots$

c. $a^x = 1+x \log a + \frac{x^2}{2!}(x\log a)^2 + \frac{x^3}{3!}(x\log a)^3 + \ldots\ldots\ldots\ldots\ldots\ldots$

d. $\sin x = x - x^3/3! + x^5/5! \ldots\ldots\ldots\ldots\ldots\ldots$

e. $\cos x = 1 - x^2/2! + x^4/4! \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

f. $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots\cdots$

g. $\ln(1+x) = x - x^2/2 + x^3/3 + \ldots\ldots\ldots\ldots, \ |x| < 1$

h. $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots\ldots\ldots\ldots |x| < 1$

i. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$

j. $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\cdots$

Important Limits

a. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

b. $\lim_{x \to 0} \frac{\tan x}{x} = 1$

c. $\lim_{x \to 0} (1+nx)^{1/x} = e^n$

d. $\lim_{x \to 0} \cos x = 1$

e. $\lim_{x \to 0} (1+x)^{1/x} = e$

f. $\lim_{x \to \infty} \left(1 + 1/x\right)^x = e$

g. $\lim_{x \to 0} \frac{(1-\cos(mx))}{x^2} = \frac{m^2}{2}$

h. $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

L – Hospital's Rule

If f (x) and g(x) are two functions such that

$\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$

Then, $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

If f'(x) and g'(x) are also zero as $x \to a$, then we can take successive derivatives till this condition is violated.

For continuity, $\lim_{x \to a} f(x) = f(a)$

For differentiability, $\lim_{h \to 0} \left[\frac{f(x_0 + h) - f(x_0)}{h}\right]$ exists and is equal to $f'(x_0)$

If a function is differentiable at some point then it is continuous at that point but converse may not be true.

Mean Value Theorems

- **Rolle's Theorem**
  If there is a function f(x) such that f(x) is continuous in closed interval a ≤ x ≤ b and f'(x) is existing at every point in open interval a < x < b and f(a) = f(b). Then, there exists a point 'c' such that f'(c) = 0 and a < c < b.

- **Lagrange's Mean value Theorem**
  If there is a function f(x) such that, f(x) is continuous in closed interval a ≤ x ≤ b; and f(x) is differentiable in open interval (a, b) i.e., a < x < b,
  Then there exists a point 'c', such that
  $$f'(c) = \frac{f(b) - f(a)}{(b - a)}$$

Differentiation

Properties: (f + g)' = f' + g' ; (f – g)' = f' – g' ; (f g)' = f' g + f g'

Important derivatives

a. $x^n \to n\, x^{n-1}$

b. $\ell n x \to 1/x$

c. $\log_a x \to (\log_a e)(1/x)$

d. $e^x \to e^x$

e. $a^x \to a^x \log_e a$

f. sin x → cos x

g. cos x → –sin x

h. tan x → $\sec^2 x$

i. sec x → sec x tan x
j. cosec x → − cosec x cot x
k. cot x → $-\operatorname{cosec}^2 x$
l. sin h x → cos h x
m. cos h x → sin h x
n. $\sin^{-1} x \rightarrow \frac{1}{\sqrt{1 - x^2}}$
o. $\cos^{-1} x \rightarrow \frac{-1}{\sqrt{1 - x^2}}$
p. $\tan^{-1} x \rightarrow \frac{1}{1 + x^2}$
q. $\operatorname{cosec}^{-1} x \rightarrow \frac{-1}{x\sqrt{x^2 - 1}}$
r. $\sec^{-1} x \rightarrow \frac{1}{x\sqrt{x^2 - 1}}$
s. $\cot^{-1} x \rightarrow \frac{-1}{1 + x^2}$

Increasing & Decreasing Functions

- $f'(x) \geq 0 \ \forall x \in (a, b)$, then f is increasing in [a, b]
- $f'(x) > 0 \ \forall x \in (a, b)$, then f is strictly increasing in [a, b]
- $f'(x) \leq 0 \ \forall x \in (a, b)$, then f is decreasing in [a, b]
- $f'(x) < 0 \ \forall x \in (a, b)$, then f is strictly decreasing in [a, b]

Maxima & Minima
Local maxima or minima
There is a maximum of f(x) at x = a if f'(a) = 0 and f"(a) is negative.
There is a minimum of f (x) at x = a, if f'(a) = 0 and f" (a) is positive.
To calculate maximum or minima, we find the point 'a' such that f'(a) = 0 and then decide if it is maximum or minima by judging the sign of f"(a).

Global maxima & minima
We first find local maxima & minima & then calculate the value of 'f' at boundary points of interval given eg. [a, b], we find f(a) & f(b) & compare it with the values of local maxima & minima. The absolute maxima & minima can be decided then.

Partial Derivative
If a derivative of a function of several independent variables be found with respect to any one of them, keeping the others as constant, it is said to be a partial derivative.

Homogenous Function

$a_0x^n + a_1x^{n-1}y + a_2x^{n-2}y^2 + \dots\dots\dots + a_ny^n$ is a homogenous function of x & y, of degree 'n'

$$= x^n\left[a_0 + a_1\left(y/x\right) + a_2\left(y/x\right)^2 + \dots\dots\dots + a_n\left(y/x\right)^n\right]$$

Euler's Theorem

If u is a homogenous function of x & y of degree n, then $\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu\right]$

Maxima & minima of multi-variable function

let $r = \left(\frac{\partial^2 f}{\partial x^2}\right)_{\substack{x=a\\y=b}}$; $s = \left(\frac{\partial^2 f}{\partial x \partial y}\right)_{\substack{x=a\\y=b}}$; $t = \left(\frac{\partial^2 f}{\partial y^2}\right)_{\substack{x=a\\y=b}}$

- **Maxima**
  $rt - s^2 > 0$; $r < 0$
- **Minima**
  $rt - s^2 > 0$; $r > 0$
- **Saddle point**
  $rt - s^2 < 0$

Integration

Indefinite integrals are just opposite of derivatives and hence important derivatives must always be remembered.

Some standard integral formula

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int a^x \log a \, dx = a^x + c$$

$$\int e^x dx = a^x + c$$

$$\int uv \, dx = u\int v - \left[u' \cdot \int v\right] + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \tan x \, dx = \ln(\sec x) + c$$

$$\int \operatorname{cosec} x \, dx = \ln(\operatorname{cosec} x - \cot x) + c$$

$$\int \sec x \, dx = \ln(\sec x + \tan x) + c$$

$$\int \cot x \, dx = \ln(\sin x) + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$$

$$\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$$

$$\int \frac{-1}{1+x^2}dx = \cot^{-1}x + c$$

$$\int \frac{-1}{x\sqrt{x^2-1}}dx = \text{cosec}^{-1}x + c$$

$$\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left(\frac{x+a}{x-a}\right) + c$$

$$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right) + c$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right) + c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right) + c$$

Properties of definite integral

a. $\int_a^b f(x)dx = \int_a^b f(t)dt$

b. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

c. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

d. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

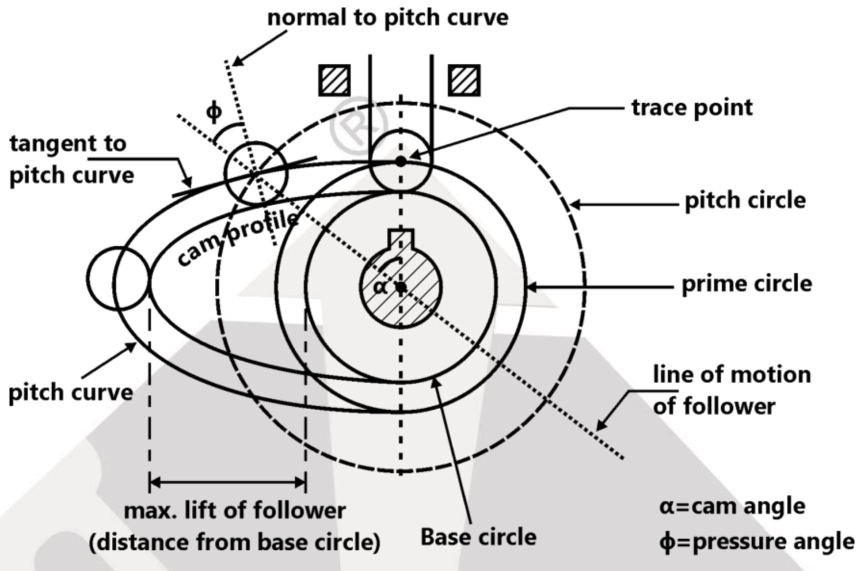
e. $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(2a-x)=f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$

f. $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(x)=f(-x) \\ 0 & \text{if } f(x) = -f(x) \end{cases}$

g. $\frac{d}{dt}\int_{\phi(t)}^{\psi(t)} f(x)dx = f(\psi(t))\psi'(t) - f(\phi(t))\phi'(t)$

Theory of Machine

Terminology of CAMS

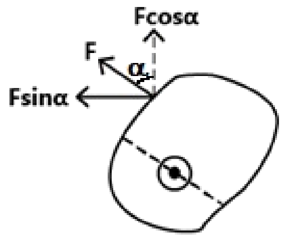


Cam terminologies

Base circle: Smallest circle drawn from the centre of rotation of the cam forming a part of cam profile. Radius of the circle is called the least radius of cam.
Pitch curve: Path of trace point assuming cam is fixed and follower rotates.
Prime circle: Smallest circle that can be drawn from the cam centre and tangent to the pitch curve.
Pitch circle: Circle drawn from the cam centre and passes through the pitch point. Pitch point corresponds to ϕ_{max}
Cam profile: Surface of cam that comes in contact with the follower. While drawing cam profile, we consider that cam is stationary and follower rotates over it.
Dwell: It is zero displacement or absence of rotation of the follower during the motion of the cam.
Angle of Ascent (ϕ_a) **:** It is the angle through which the cam turns during the time follower rise.
Angle of dwell (δ) **:** it is the angle through which cam turns while the follower remains stationary at the highest or lowest position.
Angle of dwell (ϕ_d): It is the angle through which cam turns during the time the follower returns to the initial position.

- For a flat face and knife edge follower, prime circle and the base circle are the same, because in these types, trace point lies on the base circle.\
- Maximum lift of follower = Stroke of follower
- $\phi < 30°$ or $\phi_{max} = 30°$, otherwise a reciprocating type of follower will jam the bearing.

Force exerted by cam



Fcosα: Vertical component which lifts the follower

Fsinα: Exerts lateral pressure on the bearing

$\alpha \uparrow \rightarrow$ Lateral force ↑

α Has to be reduced by making surface more convex and longer

- The minimum value of α cannot be reduced from certain value
- Base circle diameter ↑ $\rightarrow$ Pressure angle ↓

Engineering Mechanics

Impulse, Momentum and Collisions

The **impulse** of a constant force *F* is defined as the product of the force and the time *t* for which it acts.

$$\text{Impulse} = F.t$$

The effect of the impulse on a body can be found using below equations, where *a* is acceleration, u and v are initial and final velocities respectively and t is time.

$$v = u + at \Rightarrow (v - u) = at$$

So

$$I = F \cdot t = ma \cdot t = m(v - u) = \text{change in momentum}$$

So we can say that,
Impulse of a constant force = F.t = change in momentum produced.

Impulse is a vector quantity and has the same units as momentum, Ns or kg m/s
The impulse of a variable force can be defined by the integral

Impulse= $\int_0^t F.dt$, where t is the time for which F acts.

By Newton's 2nd law

$$F = m.a = m(dv/dt)$$

So impulse can also be written

$$\text{Impulse} = \int_u^v m\frac{dv}{dt}dt = \int_u^v mdv = [mv]_u^v$$

which for a constant mass
Impulse =m (v-u)

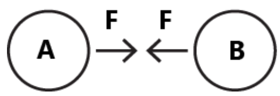
Impulsive force

- Suppose the force *F* is very large and acts for a very short time. During this time the distance moved is very small and under normal analysis would be ignored. Under these conditions, the only effect of the force can be measured is the impulse, or change in momentum which is called an impulsive force.

- In theory this force should be infinitely large and the time of action infinitely small. Some applications where the conditions are approached are collision of snooker balls, a hammer hitting a nail or the impact of a bullet on a target.

Conservation of linear momentum

Consider the direct collision of two spheres A and B.



- When the spheres collide, then by Newton's third law, the force F exerted by A on B is equal and opposite to the force exerted by B on A.
- The time for contact is the same for both. The impulse of A on B is thus equal and opposite to the impulse of B on A. It then follows that the change in momentum of A is equal in magnitude to the change in momentum in B, but it is in the opposite direction. The total change in momentum of the whole system is thus zero.
- This means that the total momentum before and after a collision is equal, or that linear momentum is conserved. This is called the principle of conservation of linear momentum and in summary this may be stated:

The total momentum of a system, in any direction, remains constant unless an external force acts on the system in that direction.
Caution: Take proper sign convention while solving problems.

Impact of inelastic bodies

- When two inelastic bodies collide they remain together. They show no inclination to return to their original shape after the collision.
  An example of this may be two railway carriages that collide and become coupled on impact.
- Problems of this type may be solved by the principle of conservation of linear momentum.

  Momentum before impact = Momentum after impact
  (Take proper sign convention)
- Although momentum is conserved, it is important to realize that energy is always lost in an inelastic collision (it is converted from mechanical energy to some other form such as heat, light or sound.)

Impact of elastic bodies

- In the last section the bodies were assumed to stay together after impact. An elastic body is one which tends to return to its original shape after impact. When two elastic bodies collide, they rebound after collision. An example is the collision of two snooker balls.
- If the bodies are travelling along the same straight line before impact, then the collision is called a direct collision. This is the only type of collision considered here.

Before Collision   After Collision

Consider the two elastic spheres as shown. By the principle of conservation of linear momentum

Momentum before impact = Momentum after impact

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

where the $u_i =$ initial velocity of body *i*.

$v_i =$ final velocity of body *i*.

- When the spheres are inelastic v_1 and v_2 are equal as we saw in the last section. For elastic bodies v_1 and v_2 depend on the elastic properties of the bodies. A measure of the elasticity is the coefficient of restitution, for direct collision this is defined as

$$e = -\frac{(v_1 - v_2)}{(u_1 - u_2)}$$

- The values of 'e' in practice vary between 0 and 1. For completely inelastic collision, $e = 0$ and for completely elastic collision, e=1. In the latter case, no energy is lost in the collision.
- Both the law of restitution & conservation of momentum are applicable along x and y directions in case of oblique collision.

Rolling, torque and angular momentum

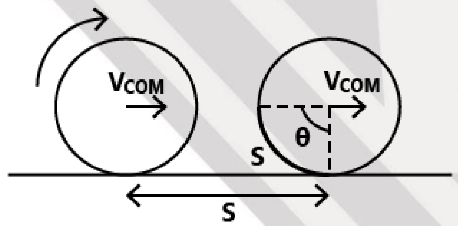
Rolling motion:

Combination of translational motion & rotational motion.
In rolling motion, the centre of the object moves in a line parallel to the surface.

Relation between length and angle of rotation:

When the object rotates through an angle 'θ', a point at a distance R from the rotation axis moves through a distance of S

$S = \theta R$



The arc length S is the same as the distance that the wheel translates.

The linear (translational) speed V_{COM} of the wheel is $\frac{dS}{dt}$; V_{COM} is the velocity of centre of mass.

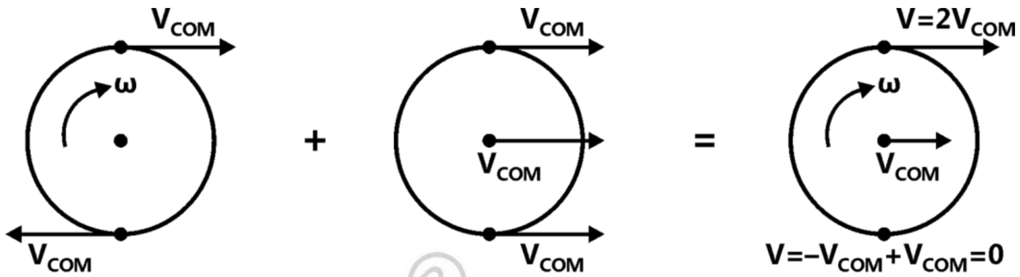
The angular speed of the wheel is $\omega = \frac{d\theta}{dt}$

So, $\frac{dS}{dt} = R\frac{d\theta}{dt}$

$V_{COM} = \omega R$

Rolling motion is the combination of pure rotational motion and pure translational motion.

Pure rotation + pure translation = Rolling motion



The velocity of a point at the top of the rolling wheel is twice that of the centre of the wheel

$V_{top} = \omega(2R) = 2(\omega R) = 2V_{COM}$

Kinetic energy of rolling:

As an object rolls, the point at the very bottom, the contact point with the surface, is instantaneously stationary.

We will call this point P and we can treat rolling about this point.

$K.E = \frac{1}{2} I_P \omega^2$

I_P : Rotational inertia about the point P

Parallel axis theorem says $I_P = I_{COM} + MR^2$

$K.E = \frac{1}{2} I_P \omega^2$

$K.E = \frac{1}{2} I_{COM} \omega^2 + \frac{1}{2} MR^2 \omega^2$

$K.E = \frac{1}{2} I_{COM} \omega^2 + \frac{1}{2} MV_{COM}^2$

Kinetic energy of a rolling object comes from rotational kinetic energy and translational kinetic energy.

Forces in rolling:

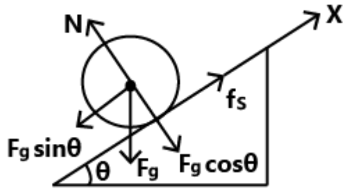
- If a wheel rolls smoothly, there is no sliding at the contact point so there is no friction.
- However if there is an external force that produces an acceleration, there will be an angular acceleration α. The acceleration will make the wheel want to slide at the contact point. Then a frictional force will be on the wheel to oppose the tendency to slide.

Direction of static frictional force:

- If a wheel moving to the right were to accelerate, the bottom of the wheel would want to move to the left compared to the surface. Thus static friction force is to the right.
- If same wheel was to slow down, the direction of the acceleration and angular acceleration would switch and the static friction force will now be pointing towards the left.

Rolling down a ramp:

The direction of the static friction force is the confusing part here. It points up along the ramp. If the wheel were to slide down the ramp, the friction opposing the sliding would be pointing up.



$$f_s - Mg\sin\theta = Ma_{COM}$$
$$\sum \tau = I\alpha$$

Only force on the wheel that produces torque is the friction

$Rf_s = I_{COM}\alpha$

We will need to make use of $a_{COM} = \alpha R$

(a is down the ramp, negative X-direction but the wheel rolls counter-clockwise, α is positive)

$$\alpha = \frac{-a_{COM}}{R}$$

So we can solve for $f_s = -I_{COM}\frac{a_{COM}}{R^2}$

$$a_{COM} = \frac{-g\sin\theta}{1 + \frac{I_{COM}}{MR^2}}$$

Yo-Yo

A Yo-Yo behaves similar to the wheel rolling down a ramp.

1) Instead of rolling down a ramp of angle 'θ', Yo-Yo follows an angle of 90^0 with horizontal.

2) Yo-Yo rolls down a string on a radius R.

3) Instead of friction, the tension shows up in the Yo-Yo.

$$a_{COM} = \frac{-g}{1 + \frac{I_{COM}}{MR_0^2}}$$

Machine Design

Failure Theories

Maximum principal stress theory

- It is also known as Rankine's theory.
- Best for brittle materials.
- For safe design $\sigma_1 \le \frac{S_{yt}}{N}$ or $\frac{S_w}{N}$

  $\sigma_1 =$ Maximum principal stress developed at a critical point

  N = Factor of safety

Maximum shear stress theory (M.S.S.T)

- Also known as Guest and Tresca theory.
- Suitable for ductile materials.
- It gives more safety to component (most safe design).
- Dimension and cost of component is more.
- It is not suitable under hydrostatic state of stress.

For safe design

Absolute $E_{max} \le \frac{S_{Ys}}{N}$ or $\frac{S_{Yt}}{2N}$

$S_{Yt} =$ tensile yield strength

$S_{Ys} =$ Yield shear stress of material

$\left(S_{Ys} = \frac{S_{Yt}}{2}\right)$

For tri-axial state of stress:

$$\tau_{max} = \text{Larger of } \left[\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|\right]$$

For biaxial state of stress:

$$\tau_{max} = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$$

Maximum principal strain theory

Also known as St. Venant's theory

For safe design $\epsilon_1 \le \epsilon_{Y.P}$ or $\frac{S_{Yt}}{N \cdot E}$

$\epsilon_1 =$ Maximum principal strain at critical point

$\epsilon_{Y.P} =$ Strain corresponding to yield point

$N =$ Factor of safety

E= Young modulus of elasticity

$$\in_1 = \frac{1}{E}\left[\sigma_1 - \mu\left(\sigma_2 + \sigma_3\right)\right]$$

Total strain energy theory [T.S.E.T]

- Also known as Haigh's theory
- For safe design, total strain energy per unit volume should be less than that of yield point

  Total $\frac{\text{S.E}}{\text{Volume}} = \; = \frac{1}{2}\sigma_1 \in_1 + \frac{1}{2}\sigma_2 \in_2 + \frac{1}{2}\sigma_3 \in_3 = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]$

  (μ = Poisson's ratio)
- $\left[\frac{(\text{S.E})}{\text{Volume}}\right]_{\text{Yield point}} = \frac{1}{2E}\left(\frac{S_{YL}}{N}\right)^2$

For trivial state of stress condition

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \le \left(\frac{S_{yt}}{N}\right)^2$$

For biaxial state of stress condition

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le \left(\frac{S_{yt}}{N}\right)^2$$

Maximum distortion energy theory (M.D.E.T)

- Also known as Von-Mises theory.
- Best for ductile material.
- Gives economical design (Less cost).
- It is less safer design than those corresponding to maximum shear stress theory.

For safe design

$$\begin{pmatrix}\text{Maximum distortion energy}\\ \text{Per unit volume}\end{pmatrix} \le \begin{pmatrix}\text{Distortion energy per unit volume}\\ \text{At yield point}\end{pmatrix}$$

$$\frac{(\text{D.E})}{\text{Vol}} = \frac{1+\mu}{6E}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]$$

$$\left(\frac{(\text{D.E})}{\text{Vol}}\right)_{\text{Y.P}} = \frac{1+\mu}{3E}\left(\frac{S_{yt}}{N}\right)^2$$

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 \le 2\left(\frac{S_{Yt}}{N}\right)^2$$

For biaxial state of stress condition

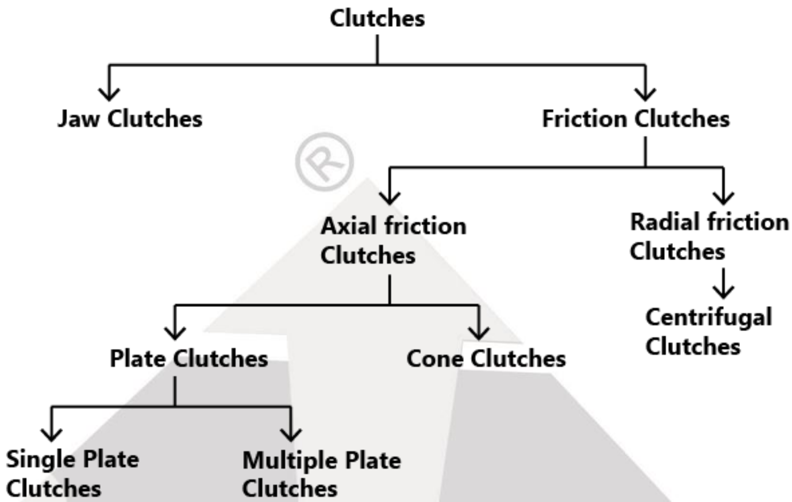
$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \left(\frac{S_{yt}}{N}\right)^2$$

Consolidated table for theories of failure

S.No.	Theories of failure	Design equations	M_e or T_e equations	$(\sigma_t)_{per} = \frac{S_{yt}}{N}$ equations (used when normal stress is acting in only 1 direction)	$\frac{S_{ys}}{S_{yt}}$	Shape of safe boundary i.e. $S_{yc} = -S_{yt}$	Valid for
1.	Max. Principal stress theory	$\sigma_1 \le \left(\frac{S_{yt}}{N} \text{ or } \frac{S_{ut}}{N}\right)$	$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \le \frac{\pi}{32} d^3 (\sigma_t)_{per}$	$= \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$	1	Square	Best for brittle materials. Used for ductile materials when 1.Uniaxial state of stress 2. Biaxial if $\sigma_{1,2}$ are like in nature. 3. Hydrostatic state of stress.
2.	Max. shear stress theory	Larger of $\left[\left\lvert\frac{\sigma_1 - \sigma_2}{2}\right\rvert, \left\lvert\frac{\sigma_2 - \sigma_3}{2}\right\rvert, \left\lvert\frac{\sigma_3 - \sigma_1}{2}\right\rvert\right] \le \frac{S_{yt}}{2 \cdot N}$	$T_e = \sqrt{M^2 + T^2} \le \frac{\pi}{16} d^3 \tau_{per}$	$= \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$	0.5	Hexagon	Used for ductile materials (except hydrostatic state of stress). Gives over safe and uneconomical design.
3.	Max. principal strain theory	$\sigma_1 - \mu(\sigma_2 + \sigma_3) \le \frac{S_{yt}}{N}$	-	-	$\frac{1}{1+\mu}$	Rhombus	-
4.	Total strain energy theory	$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu\left(\sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_3 + \sigma_3 \cdot \sigma_1\right) \le \left(\frac{S_{yt}}{N}\right)^2$	-	-	$\frac{1}{\sqrt{2(1+\mu)}}$	Ellipse Semi-major axis $= \frac{S_{yt}}{\sqrt{1-\mu}}$ Semi-minor axis $= \frac{S_{yt}}{\sqrt{1+\mu}}$	Best for hydrostatic state of stress.
5.	Max. shear strain energy theory/ max. distortion energy theory	$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \le 2\left(\frac{S_{yt}}{N}\right)^2$	$T_e = \sqrt{M^2 + \frac{3}{4}T^2} \le \frac{\pi}{32} d^3 (\sigma_t)_{per}$	$= \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$	$\frac{1}{\sqrt{3}}$	Ellipse Semi-major axis $= \sqrt{2}\, S_{yt}$ Semi-minor axis $= \sqrt{\frac{2}{3}}\, S_{yt}$	Best for ductile materials. Gives safe and economical design.

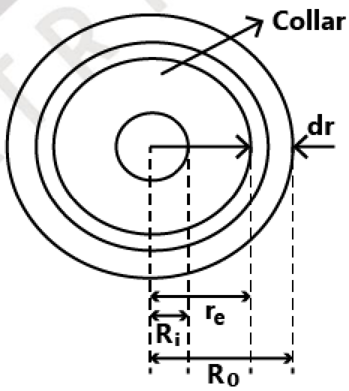
Clutches

It is a mechanical device which is used to engage/disengage the driven shaft to/from driver shaft without stopping the prime mover.



Properties of friction lining material used in clutches:

(i) High coefficient of friction
(ii) High wear resistance
(iii) Higher conductivity
(iv) Lower coefficient of thermal expansion (α)
(v) Good strength



Basic calculation

$R_i =$ Inner radius of clutch

$R_0 =$ Outer radius of clutch

$W =$ Total operating force

$p =$ Pressure intensity

Elemental area $= dA = (2\pi r)dr$

$dW = p \times 2\pi r dr$ $dW =$ elemental force

$dF_f = \mu.dW = \mu.p.2\pi r dr$ $dF_f =$ Frictional force on elemental area

$dT_f = dF_f \times r = \mu.p.2\pi r^2 dr$

$dT_f =$ Torque transmitted by clutch for elemental area

$T_f =$ Total torque transmitted by clutch

Theories of design of clutch:-

(i) Uniform pressure theory (UPT)
(ii) Uniform wear theory (UWT)

(A) Uniform pressure theory

Pressure remains constant over entire friction plate

p= Constant

$$W = \int_{R_i}^{R_0} p.2\pi r.dr = \pi p\left[R_0^{\ 2} - R_i^{\ 2}\right]$$

$$p_{UPT} = \frac{W}{\pi\left[R_0^{\ 2} - R_i^{\ 2}\right]}$$

$$T_f = n\int_{R_i}^{R_0} \mu p \cdot 2\pi r^2 dr = n\int_{R_i}^{R_0} \mu \frac{W}{\pi\left[R_0^{\ 2} - R_i^{\ 2}\right]} \cdot 2\pi r^2 dr = n \cdot \frac{2}{3}\mu W\left[\frac{R_0^{\ 3} - R_i^{\ 3}}{R_0^{\ 2} - R_i^{\ 2}}\right]$$

$$T = n\mu W R_{eff} \text{ where, } R_{eff} = \frac{2}{3}\left[\frac{R_0^{\ 3} - R_i^{\ 3}}{R_0^{\ 2} - R_i^{\ 2}}\right]$$

n = number of frictional surfaces $\Rightarrow$ plates = (n+1)

(B) Uniform wear theory (UWT)

Wear uniformly distributed over entire surface area of clutch

$p \cdot r = \text{constant}$

where, r= radius at any section

$$W_{UWT} = \int_{R_i}^{R_0} p(2\pi r) dr = p \cdot 2\pi r\left[R_0 - R_i\right]$$

$$p_{UWT} = \frac{W}{2\pi r\left[R_0 - R_i\right]}$$

Pressure intensity varies over entire plate

At r = $r_i \Rightarrow p = p_{max}$ [Maximum pressure]

$r = r_0 \Rightarrow p = p_{min}$ [Minimum pressure]

$$T_f = n\int_{R_i}^{R_0} \mu p \cdot 2\pi r^2 dr = n\int_{R_i}^{R_0} \mu \cdot \frac{W}{2\pi r\left(R_0 - R_i\right)} \cdot 2\pi r^2 dr = n\mu W \frac{\left(R_0 + R_i\right)}{2} = n\mu W R_{eff} \quad ; \; R_{eff} = \frac{R_0 + R_i}{2}$$

Note:

- Frictional torque as per uniform pressure theory is more than friction torque by uniform wear theory. That's why for designing clutches, it is better to use uniform wear theory because clutches are used to transmit power by utilising frictional forces. Also pressure is non-uniformly distributed when clutches are in service so UWT is used.
- In case of new clutches, UPT is more appropriate but in case of old clutches UWT is more appropriate. In friction clutches, UWT should be considered.

For multiplate disc clutch, $n = n_1 + n_2 - 1$

where $n_1 =$ discs on driving shaft

$n_2 =$ discs on driven shaft

Industrial Engineering

Sequencing

- Aim of sequencing is to find the order in which different number of jobs are to be proceeded on different machines, so that the idle time can be minimized and utilization may be optimized.
- **Job Flow time:** It is the time from some starting point until that particular job is completed.
- **Make Span time (MST):** It is the time from when processing begins on first job in the set until the last job is completed.
- **Tardiness:** It is the amount of time by which a job is delayed beyond its due date.
- Tardiness = Job Flow time – Due Date
- Average Number of jobs in the system= $\dfrac{\text{Total job flow time}}{\text{Make span time}}$

Sequencing Rule

- **Shortest processing time (SPT)** = Jobs are arranged in increasing order of their processing time.
- **Earliest due date (EDD):** Jobs are arranged in increasing order of due dates.
- **Critical Ratio Rule (CRR):**

  Critical Ratio = $\dfrac{\text{Due date}}{\text{processing time}}$

  Jobs are arranged in increasing order of critical ratio.
- **Slack time remaining (STR):**

  STR = Due date – Processing time

  Jobs are arranged in increasing order of their Slack time remaining.

Sequencing of N-jobs on two machines:

- For sequencing, Johnson's rule is applied.

  Ex:

	Machine – 1	Machine – 2
A	8	4
B	5	9
C	3	7
D	6	7
E	10	5
F	9	2

  Order is

  C → B → D → E → A → F
- Minimum time on machine 1 is sequenced first and minimum time on machine 2 is sequenced last and this continues till all are assigned.

N-jobs on three machines
Ex

	A	B	C
Jobs	A_i	B_i	C_i
1	⋮	⋮	⋮
2	⋮	⋮	⋮
3	⋮	⋮	⋮
⋮	⋮	Max B_i	⋮
⋮	Min A_i	⋮	⋮
⋮	⋮	⋮	Min C_i
N	⋮	⋮	⋮

<u>Conditions to apply Johnson's rule</u>

$\text{Min. } A_i \geq \text{Max. } B_i$

OR

$\text{Min. } C_i \geq \text{Max. } B_i$

$\Rightarrow X_i = A_i + B_i \;\&\; Y_i = B_i + C_i$

Now apply Johnson's rule for machines X & Y

Example: There are S jobs each of which must go through m/c A, B, C in the same order. Find the optimum sequence, make span time and idle time for each m/c.

Jobs	A	B	C
1	8	5	4
2	10	6	9
3	6	2	8
4	7	3	6
5	11	4	5

Solution: Condition is satisfied i.e. $\text{Min. } A_i \geq \text{Max. } B_i$

Jobs	$A + B = X$	$B + C = Y$
1	13	9
2	16	15
3	8	10
4	10	9
5	15	9

So order is

$3 \rightarrow 2 \rightarrow 5 \rightarrow 1 \rightarrow 4$

	m/c A		m/c B		m/c C	
Jobs	In	Out	In	Out	In	Out
3	0	6	6	8	8	16
2	6	16	16	22	22	31
5	16	27	27	31	31	36
1	27	35	35	40	40	44
4	35	42	42	45	45	51=MST

Make span time = 51
Idle time for machine A = 9
Idle time for machine B = 31
Idle time for machine C = 19

Manufacturing Engineering

Material Science

Introduction: Material science is the study of relationship between structure and properties of engineering materials.

Crystal system and bravais lattice

Crystal system	Geometry	Bravais Lattice
Cubic	$a = b = c$ $\alpha = \beta = \gamma = 90°$	SC, BCC, FCC
Tetragonal	$a = b \neq c$ $\alpha = \beta = \gamma = 90°$	ST,BCT
Orthorhombic	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90°$	SO, BCO,FCO, ECO
Rhombohedral	$a = b = c$ $\alpha = \beta = \gamma \neq 90°$	SR
Hexagonal	$a = b \neq c$ $\alpha = \beta = 90°, \gamma = 120°$	SH
Monoclinic	$a \neq b \neq c$ $\alpha = \gamma = 90° \neq \beta$	SM, EC,
Triclinic	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90°$	ST_r

Some important definitions in crystal structure:

Primitive cell: A primitive cell is defined as a simple cubic unit cell having atoms only at the corner.

Crystal lattice: It is a 3D network of lines in space. It is also known as a line lattice.

Space lattice: It is defined as a 3D network of points in space. It is also called as a point lattice.

Lattice parameter: It is the smallest representative group of atoms which when repeated in all the crystallographic directions for infinite number of time result in the development of a crystal lattice.

Crystal structure characteristics

Let a = lattice parameter
r = atomic radius

Characteristic	BCC	FCC	HCP
a to r relation	$a=\frac{4r}{\sqrt{3}}$	$a=\frac{4r}{\sqrt{2}}$	$a=2r$
Average no. of atoms (N_{avg})	$\frac{8}{8}+\frac{0}{2}+\frac{1}{1}=2$	$\frac{8}{8}+\frac{6}{2}+\frac{0}{1}=4$	$\frac{12}{6}+\frac{2}{2}+\frac{3}{1}=6$
Co-ordination numbers	8	12	12
Atomic packing factor	0.68	0.74	0.74

Example of BCC elements are: Fe (Except in 910–1400°C), W, Cr, V, Mo etc.

- BCC elements are generally hard and brittle.
- BCC elements are mixed up with other material then the hardness and brittleness of combination being increased (wear resistance also increased).
- All BCC elements are carbide formation elements and gives fine grains structure.

Example of FCC elements are: Fe (in 910–1400°C), Al, Cu, Ni, Au, Ag, Pt etc.

- FCC elements are generally strong and ductile.
- Toughness depends upon both strength and ductility.

Example of HCP: Ti, Mg, Zn, Zr, Co, Cd, etc.

- HCP elements are relatively less ductile compared to FCC elements.
- HCP elements are best suited for soild lubricant.

Allotropy: Allotropy is defined as the tendency of an element to exist in the different crystalline structure at different temperature and pressure.
The crystal structure which have higher value of atomic factor are called as closed packed structure. Ex: FCC and HCP

Miller indices for plane: These are defined as rationalized reciprocals of fractional intercepts taken along the three crystallographic directions and written under parenthesis without a repeating comma between them.

- They are denoted by (h k *l*)
- Always expressed as smallest integers

Characteristics of miller indices of plane:

- When a plane is parallel to an axis its miller indices on the axis is zero.
- Two parallel planes will have quantatively the same miller indices.
- Two planes $(h_1 k_1 \ell_1) \& (h_2 k_2 \ell_2)$ will be perpendicular if $h_1h_2 + k_1k_2 + \ell_1\ell_2 = 0$.
- The angle θ between two intersecting planes $(h_1 k_1 \ell_1) \& (h_2 k_2 \ell_2)$ is given by

$$\cos\theta = \frac{h_1h_2 + k_1k_2 + \ell_1\ell_2}{\sqrt{h_1^2 + k_1^2 + \ell_1^2} \cdot \sqrt{h_2^2 + k_2^2 + \ell_2^2}}$$

- Planes having low indices are far away from the origin than those having high indices.

- Interplanar distance denotes the distance between two planes, one of which is passing through the origin.

  Let d= interplanar distance, then $d = \frac{a}{\sqrt{h^2 + k^2 + \ell^2}}$ where $a =$ lattice parameter,

  $(h\,k\,\ell) =$ Miller indices

Miller indices of direction: These are defined as rationlised components of a given direction vector taken along the three crystallographic directions and written inside square brackets without a separating comma between them.

- Denoted by [u v w]
- Always written as smallest integers.

Characteristics of miller indices of direction:

- When a direction is perpendicular to an axis its miller indices on that direction is zero.
- Parallel direction will have quantatively the same miller indices.
- Two directions $[u_1\,v_1\,w_1]$ & $[u_2\,v_2\,w_2]$ will be perpendicular if $u_1u_2 + v_1v_2 + w_1w_2 = 0$
- The angle θ between two intersecting directions $[u_1v_1w_1]$ & $[u_2v_2w_2]$ is given by

$$\cos\theta = \frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \cdot \sqrt{u_2^2 + v_2^2 + w_2^2}}$$

- A plane and a direction having the same miller indices will be perpendicular to each other.

Density calculation in crystal structures:

(1). Volume density $(\rho_v) = \frac{\text{weight of } N_{avg}}{\text{Volume of unit cell}}$

(2) Planar density $(\rho_\ell) = \frac{\text{number of atoms}}{\text{area of plane}}$

(3) Linear density $(\rho_\ell) = \frac{\text{number of atoms}}{\text{length of direction vector}}$

Structural imperfection

- **Point defect (zero dimension)**
  1. Vacancies
  2. Self interstitials
  3. Impurities
     - Interstitial
     - Substitutional
  4. Frenkel defect
  5. Schottky defect
- **Line defect (1 Dimensional)**
  1. Dislocation
     - Edge dislocation
     - Screw dislocation
- **Surface defect (2-dimensional)**
  - Grain boundary
  - Twin boundaries
  - Low angle boundaries
  - Stacking faults
- **Volume defect (3-dimensional)**
  1. Casting defects
  2. Welding defects
  3. Forming defects

Kuestions

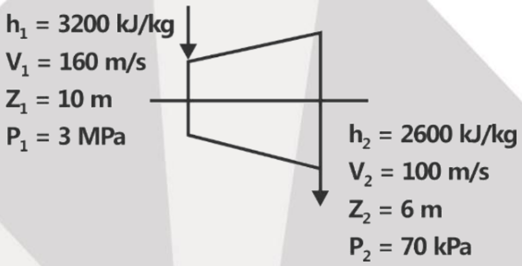
Thermodynamics

Heat & Work for open system

Sample Problem

Linked Answer Questions

The inlet and the outlet conditions of steam for an adiabatic steam turbine are as indicated in the notations are as usually followed.



a. If mass flow rate of steam through the turbine is 20 kg/s, the power output of the turbine (in MW) is

(a) 12.157 (b) 12.941

(b) 168.001 (d) 168.785

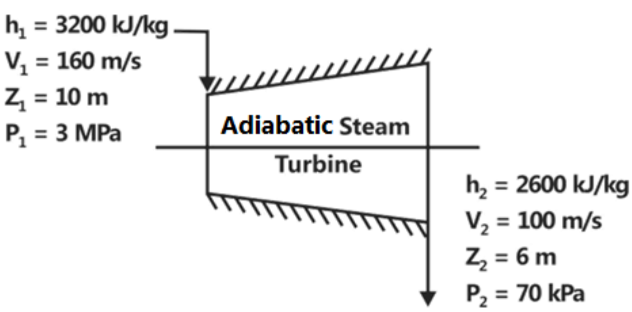
b. Assume the above turbine to be part of a simple Ranking cycle. The density of water at the inlet to the pump is 1000 kg/m^3. Ignoring kinetic and potential energy effects, the specific work (in kJ/kg) supplied to the pump is

(a) 0.293 (b) 0.351

(c) 2.930 (d) 3.510

Solution:

a. (a)



Applying Steady Flow Energy Equation (SFEE)

$$h_1 + \frac{V_1^2}{2} + gZ_1 + q = h_2 + \frac{V_2^2}{2} + gZ_2 + w$$

q = 0; because it is an adiabatic steam turbine.

$$3200\times10^3 + \frac{(160)^2}{2} + 9.81\times10 + 0 = 2600\times10^3 + \frac{(100)^2}{2} + 9.81\times6 + w$$

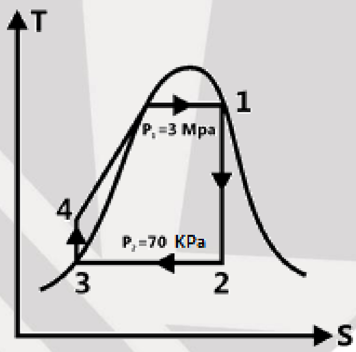
(Note that all work and energy values are to be taken in J)

w = 607839.24 J/kg = 607.84 kJ/kg

Mass flow rate of steam through turbine is 20 kg/s.

The power output of the turbine is = 20 × 607.84 = 12156.78 kW = 12.157 MW

Solution:
b. (c)



Neglecting Kinetic and potential energy effects

SFEE:

$h_1 + q = h_2 + w_T$ (For Turbine)

3200 = 2600 + w_T

w_T = 600 kJ/kg (q=0 for adiabatic steam turbine)

Specific work supplied to the pump is

$$w_P = -\int vdP = v(P_1 - P_2) = \frac{P_1 - P_2}{\rho} = \frac{(3000-70)\ kN/m^2}{1000\ kg/m^3} = 2.930\ kJ/kg$$

Problems

01. 0.5 kg of air is compressed in a piston-cylinder device. At an instant of time when T = 400K, the rate at which work is being done on the air is 8.165kW and heat is being removed at a rate of 1.0 kW. The rate of temperature rise (in k/s) will be

(a) 10 (b) 30

(c) 25 (d) 20

01. Ans: (d)
Solution:

$$\dot{Q} - \dot{W} = \frac{dE}{dt} = \frac{dU}{dt} = \frac{d}{dt}(mu) = (mc_v T) = mc_v \frac{dT}{dt}$$

$$-1 + 8.165 = 0.5 \times 0.718 \times \frac{dT}{dt}$$

$$\frac{dT}{dt} = 19.95 \ K/s \approx 20 \ K/s$$

02. An insulated bottle is fitted with valve through which air from atmosphere at 1.013 bar and 25°C is allowed to flow slowly to fill the bottle. If bottle's initial temperature is 25°C and pressure is 0.5 bar then what will be the final temperature of the bottle (in °C) when the pressure in the bottle is 1.013bar.

(a) 75.4
(b) 348.4
(c) 278.5
(d) 982.2

02. Ans: (a)
Solution:

$$\frac{dm}{dt} = \dot{m}_i - \cancel{\dot{m}_e}^{\,0} \qquad (h_{inlet} = h_i = h_0)$$

$$\frac{du}{dt} = m_i h_i - \cancel{m_e h_e}^{\,0}$$

$$\frac{du}{dt} = \frac{dm}{dt} \times h_i$$

$$du = dm \times h_i$$

$$(U_2 - U_1) = (m_2 - m_1) h_i$$

$$(m_2 u_2 - m_1 u_1) = (m_2 - m_1) h_0$$

$$\left(\frac{P_2 V}{RT_2} c_V T_2 - \frac{P_1 V}{RT_1} c_V T_1\right) = \left(\frac{P_2 V}{RT_2} - \frac{P_1 V}{RT_1}\right) c_P T_0$$

$$\frac{c_V V}{R}\left[P_2 - P_1\right] = \frac{c_P T_0 V}{R}\left[\frac{P_2}{T_2} - \frac{P_1}{T_1}\right]$$

$$(P_2 - P_1) = \gamma T_0 \left(\frac{P_2}{T_2} - \frac{P_1}{T_1}\right)$$

$$(1.013 - 0.5) \times 10^5 = 1.4 \times 298 \left(\frac{1.013}{T_2} - \frac{0.5}{298}\right) \times 10^5$$

$$T_2 = 347.92\,K$$

$$t_2 = 74.92°C$$

03. During steady flow compression process of a gas with mass flow rate of 2 $^{kg/s}$, an increase in specific enthalpy is 15 kJ/kg and decrease in kinetic energy is 2 kJ/kg. The rate of heat rejection to the environment is 3 kW. The power needed to drive the compressor is

(a) 23 kW (b) 26 kW
(c) 29 kW (d) 37 kW

03. Ans: (c)
Solution:
Compression process

$\dot{m} = 2\text{kg/s}$

$h_2 - h_1 = 15\text{kJ/kg}$

$\frac{V_2^2 - V_1^2}{2000} = -2\text{kJ/kg}$

$\dot{Q} = -3\text{kW}$

$$\left(h_1 + \frac{V_1^2}{2000}\right)\dot{m} + \dot{Q} = \left(h_2 + \frac{V_2^2}{2000}\right)\dot{m} + \dot{W}$$

$$\dot{Q} = \dot{m}\left[(h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2000}\right)\right] + \dot{W}$$

$$-3 = 2\times[15-2] + \dot{W}$$

$\dot{W} = -29$ kW

-ve sign indicates work is done on the system.

04. In an isentropic flow through nozzle, air flows at the rate of 600 kg/hr. At inlet to the nozzle, pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.5 MPa. Initial air velocity is 300m/s. $(c_p = 1.003\text{kJ/Kg}-\text{K})(R = 287\text{J/kg}-\text{K})$. The exit velocity (m/s) of air is

04. Ans: 576–612
Solution:

$T_1 = 127 + 273 = 400\text{K}$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$[\gamma = 1.4 \text{ for air}]$

$$T_2 = 400\left(\frac{5}{20}\right)^{\frac{0.4}{1.4}} = \frac{400}{(4)^{0.286}} = 269\text{ K}$$

Applying energy equation to the nozzle, we can write.

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

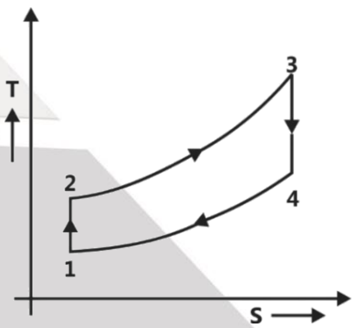
$$V_2 = \sqrt{V_1^2 + 2c_P(T_1 - T_2)} = \sqrt{300^2 + 2\times1.003(400-269)\times10^3} = 593.95\text{ m/s}$$

Brayton cycle

Sample Problem

In an ideal Brayton cycle, atmospheric air (ratio of specific heats, c_p/c_v = 1.4, specific heat at constant pressure = 1.005 kJ/kg.K) at 1 bar and 300 K is compressed to 8 bar. The maximum temperature in the cycle is limited to 1280 K. If the heat is supplied at the rate of 80 MW, the mass flow rate (in kg/s) of air required in the cycle is ___________.

Solution: 108.071 kg/s



Given, $\frac{c_p}{c_v} = \gamma = 1.4$

c_p = 1.005 kJ/kgK
P_1 = 1 bar
T_1 = 300 K
P_2 = 8 bar
$T_3 = T_{max}$ = 1280 K
$Q_s = mc_p(T_3 - T_2)$ = 80 MW
For process 1 – 2,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 8^{\frac{0.4}{1.4}} = 1.8114$$

$$\therefore \quad T_2 = 300 \times 1.8114 = 543.43\ \text{K}$$

$$m = \frac{80 \times 10^6}{1.005 \times 10^3 (1280 - 543.43)} = 108.071\ \text{kg/s}$$

Problems

01. A gas turbine plant operates on Brayton cycle between 300 K and 1073 K. The maximum cycle efficiency (in %) and maximum work done per kg of air (in kJ), respectively are:
(a) 60 and 300
(b) 40 and 250
(c) 52 and 254
(d) 47 and 240

01. Ans: (d)
Solution:

$$\eta_{cycle} = 1 - \sqrt{\frac{T_{min}}{T_{max}}} = 1 - \sqrt{\frac{30}{1073}} = 47.12\%$$

$$w_{max} = c_p\left[\sqrt{T_{max}} - \sqrt{T_{min}}\right]^2 = 1.005\left[\sqrt{1073} - \sqrt{300}\right]^2 = 239.47\ \text{kJ/kg}$$

02. A gas turbine power plant working on the Brayton cycle with regeneration of 80% effectiveness. The air at the inlet to the compressor is at 1 bar and 300 K the pressure ratio is 6 and maximum cycle temperature is 1200 K. The turbine and compressor each have an efficiency of 80%. What will be the efficiency of the cycle with the regeneration?

(a) 58.6
(b) 30.6
(c) 20.5
(d) 40.2

02. Ans: (b)

Solution:

Given:

$\eta_{Regeneration} = 0.8$

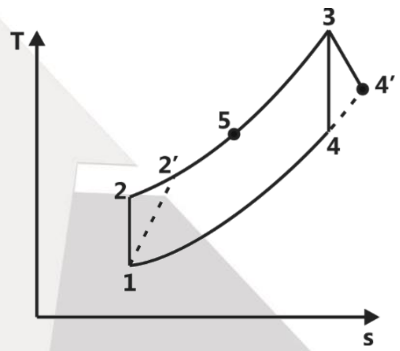
$\eta_T = 0.8 = \eta_C$

$P_1 = 1$ bar

$T_1 = 300$ K

$T_3 = 1200$ K

$r_P = \frac{P_2}{P_1} = 6$



$P_2 = 6 \times 1 = 6$ bar

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{0.4}{1.4}}$$

$T_2 = 500.55$ K

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = 1200 \times \left(\frac{1}{6}\right)^{\frac{0.4}{1.4}} = 719.2 \text{ K}$$

$$\eta_C = \frac{T_2 - T_1}{T_2' - T_1}$$

$$0.8 = \frac{500.6 - 300}{T_2' - 300}$$

$T_2' = 550.68$ K

$$\eta_T = \frac{T_4' - T_3}{T_4 - T_3}$$

$$0.8 = \frac{T_4' - 1200}{719.2 - 1200}$$

$T_4' = 815.6$ K

$$\eta_{reg} = \frac{T_5 - T_2'}{T_4' - T_2'}$$

$$0.8 = \frac{T_5 - 550.68}{815.36 - 550.68}$$

$T_5 = 762.42$ K

$$\eta = \frac{W_T - W_C}{Q_S} = \frac{c_P\left(T_3 - T_4'\right) - c_P\left(T_2' - T_1\right)}{c_P\left(T_3 - T_5\right)} = \frac{(1200 - 815.36) - (550.68 - 300)}{(1200 - 762.72)} = 30.6\%$$

03. In a gas turbine operating on Brayton cycle, with intercooling.
i. Efficiency of the cycle increases
ii. Work ratio increases
iii. Compressor work decreases

Which of the above statements are true?

(a) I and III
(b) I and II
(c) II and III
(d) I, II and III

03. Ans: (c)
Solution:
Efficiency of Brayton cycle decreases with use of intercooling.

Heat Transfer

Heat Transfer Coefficient and Nusselt number

Sample Problem

Linked Answer Questions

An uninsulated air conditioning duct of rectangular cross section 1 m × 0.5 m, carrying air at 20°C with a velocity of 10 m/s, is exposed to an ambient temperature of 30°C. Neglect the effect of duct construction material. For air in the range of 20-30°C, data is as follows: thermal conductivity = 0.025 W/m. K; viscosity = $18\ \mu Pa.s$; Prandtl number = 0.73; density = 1.2 kg/m^3. For laminar flow Nusselt number is equal to 3.4 for constant wall temperature conditions and, for turbulent flow, Nu = 0.023 $Re^{0.8}$ $Pr^{0.33}$.

a. The Reynolds number for the flow is

(a) 444
(b) 890
(c) 4.44×10^5
(d) 5.33×10^5

b. The heat transfer per meter length of the duct, in watt, is

(a) 3.8
(b) 5.3
(c) 89
(d) 769

Solution:

a. (c)

Equivalent length of duct,

$$L = \frac{4\ A}{P} = \frac{4\ ab}{2(a+b)} = \frac{2ab}{a+b} = \frac{2\times1\times0.5}{1+0.5} = 0.667\ m$$

We know that Reynolds number

$$Re = \frac{\rho VL}{\mu} = \frac{1.2\times10\times0.667}{18\times10^{-6}}$$

Re = 4.44×10^5

b. (d)

For pipes, since Re > 2000, flow is turbulent. Hence using turbulent flow correlation, we get

Nu = 0.023 $R^{0.8}Pr^{0.33}$

$$\frac{hL_c}{k} = 0.023\times(4.44\times10^5)^{0.8}\times(0.73)^{0.33} = 683.715$$

$$\frac{h\times0.667}{0.025} = 683.715$$

$$h = \frac{683.715\times0.025}{0.667}$$

h = 25.63 W/m^2-k

Now, total area of duct

A = 2 × 1 × L + 2 × 0.5 × L = 3L

(where L is the length of duct)
Heat transfer rate
$q = hA\,(T_1 - T_2) = 25.63 \times 3 \times L \times (30 - 20)$

$$\therefore \quad \frac{q}{L} = 768.8\ \text{W/m}$$

$$\therefore \quad \frac{q}{L} \approx 769\ \text{W/m}$$

Problems

01. In an experiment, the local heat transfer over a flat plate was correlated in the form of local Nusselt number as expressed by the following correlation.

$$Nu_x = 0.035\, Re_x^{0.8}\, Pr^{\frac{1}{3}}$$

The ratio of the average convection heat transfer coefficient $\left(\bar{h}\right)$ over the entire plate length to the local convection heat transfer coefficient $\left(h_x\right)$ at x = L is ________________.

01. Ans: 1.1–1.3
Solution:

$$Nu_x = \frac{h_x x}{k} = 0.035\left(\frac{\rho V x}{\mu}\right)^{0.8} Pr^{\frac{1}{3}}$$

$$h_x \propto x^{-0.2}$$

and, $\bar{h} = \frac{1}{L}\int_{x=0}^{L} h_x dx$

We know, $h_x \propto x^{-0.2}$

$$h_x = cx^{-0.2}$$

$$h_{x=L} = cL^{-0.2}$$

$$c = \frac{h_{x=L}}{L^{-0.2}} = L^{0.2} h_{x=L}$$

$$\bar{h} = \frac{1}{L}\int_0^L cx^{-0.2} dx = \frac{c}{L}\int_0^L x^{-0.2} dx = \frac{L^{0.2} \times h_{x=L}}{L}\left[\frac{x^{-0.2+1}}{(-0.2+1)}\right]$$

$$\bar{h} = \frac{h_{x=L}}{L^{0.8}}\left[\frac{L^{0.8}}{0.8}\right]$$

$$\frac{\bar{h}}{h_{x=L}} = 1.25$$

02. Air at 25°C approaches a 0.9 m long 0.6 m wide plate with an approach velocity of 4.5 m/sec. The plate is heated to a surface temperate of 135°C.
[$\nu = 21.09 \times 10^{-6}$ m^2/sec, K = 0.0304 W/mK, Pr = 0.692]
Average heat transfer coefficient of the plate is

(a) 32.5 W/m^2K (b) 5.83 W/m^2K
(c) 15.25 W/m^2K (d) 8.7 W/m^2K

02. Ans: (d)
Solution:

$$Re_L = \frac{4.5\times 0.9}{21.09\times 10^{-6}} = 192034.13 < 5\times 10^5 \Rightarrow \text{laminar}$$

$$(Nu)_L = \frac{\bar{h}L}{k} = 0.664\times (Re_L)^{0.5}(Pr)^{0.33}$$

$$\frac{\bar{h}\times 0.9}{0.0304} = 0.664\times (192034.13)^{0.5}(0.692)^{0.33}$$

$$\bar{h} = 8.7\ W/m^2K$$

03. Air flows over a heated flat plate at a velocity of 100 m/s. The local skin friction coefficient at a point on the plate is 0.008.
Estimate the local heat transfer coefficient at this point. Density = 0.88 kg/m^3.
Viscosity = 2.286×10^{-5} kg/ms. c_P = 1.001 kJ/kgK, K = 0.035 W/mK.
Use $St.\ Pr^{\frac{2}{3}} = \frac{C_f}{2}$

(a) 21.71 W/m^2K (b) 46.779 W/m^2K
(c) 583.93 W/m^2K (d) 467.79 W/m^2K

03. Ans: (d)
Solution:

$$Pr = \frac{\mu c_p}{k} = \frac{2.286\times 10^{-5}\times 1.001\times 10^3}{0.035} = 0.6537$$

$$St_x = \frac{h_x}{\rho c_p V} = \frac{h_x}{0.88\times 1.001\times 10^3\times 100} = \frac{h_x}{88088}$$

$$St_x.Pr^{\frac{2}{3}} = \frac{C_f}{2}$$

$$\frac{h_x(0.6537)^{\frac{2}{3}}}{88088} = \frac{0.008}{2}$$

$$h_x = 467.79\ W/m^2K$$

Engineering Mechanics

Rectilinear Motion

Sample Problem

The initial velocity of an object is 40 m/s. The acceleration 'a' of the object is given by the following expression.
a = -0.1 v, Where v is the instantaneous velocity of the object. The velocity of the object after 3 seconds will be ____________

Solution: 29.63 m/s

$U = 40\ m/s$

$a = -0.1\ V$

$V = ?$

$T = 3\ s$

Using expression for acceleration

$$a = \frac{dv}{dt} = -0.1\ v$$

Rearranging and integrating both sides

$$\int_{40}^{v} \frac{dv}{v} = \int_{0}^{3} -0.1 dt$$

$$\left[\ln v\right]_{40}^{v} = -0.1\ [t]$$

$$\ln v - \ln 40 = -0.1\ \left[3.0\right] = -0.3$$

$$\ln v = \ln 40 - 0.3$$

$$\ln v = 3.38887$$

$$v = e^{3.38887} = 29.6327\ m/s$$

Problems

01. A stone is thrown upto a slope of inclination θ at a speed of V and an angle (beta) β to slope. The stone reaches its greatest distance from the slope after a time of

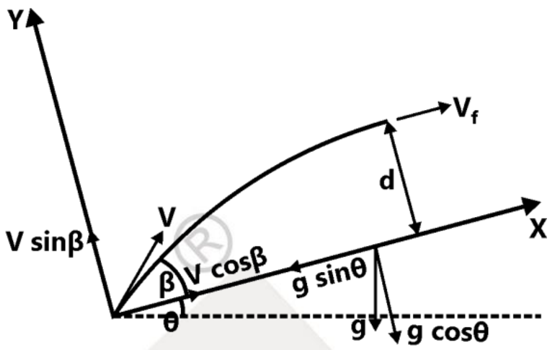
(a) $\frac{V\sin\theta}{g\cos\beta}$

(b) $\frac{V\cos\beta}{g\sin\theta}$

(c) $\frac{V\sin\beta}{g\cos\theta}$

(d) $\frac{V\cos\theta}{g\sin\beta}$

01. Ans: (c)
Solution:



Take the reference planes as shown
This '$g\cos\theta$' will affect the vertical component of the projectile on this reference plane.

$(V_i)_x = V\cos\beta$; $(V_i)_Y = V\sin\beta$

$(V_f)_x = V_f$; $(V_f)_Y = 0$

Along Y-direction

$(V_f)_Y = (V_i)_Y + at$

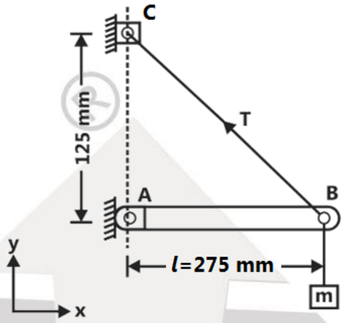
$0 = V\sin\beta - g\cos\theta \cdot t$ (Taking -ive sign with $g\cos\theta$ $\because$ it is acting in downward direction)

$t = \dfrac{V\sin\beta}{g\cos\theta}$

Free body diagram

Problems

A mass 35 kg is suspended from a weightless bar AB which is supported by a cable CB and a pin at A as shown in figure. The pin reactions at A on the bar AB are



(a) R_x = 343.4 N, R_y =755.4 N
(b) R_x = 343.4 N, R_y = 0
(c) R_x = 755.4 N, R_y = 343.4 N
(d) R_x = 755.4 N, R_y = 0

Solution: (d) is correct option

$T \sin\theta + R_y = mg$

$T \cos\theta = R_x$

Now, $\tan\theta = \frac{125}{275}$

$\theta = 24.44°$

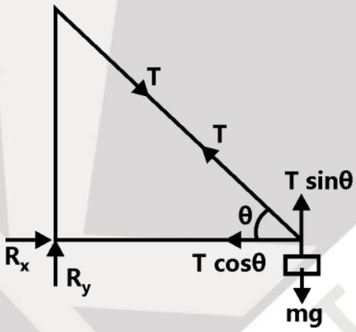
Taking moment about A

$l \times T\sin\theta = l \times mg$

$T = \frac{35\times 9.81}{\sin 24.44} = 829.74$

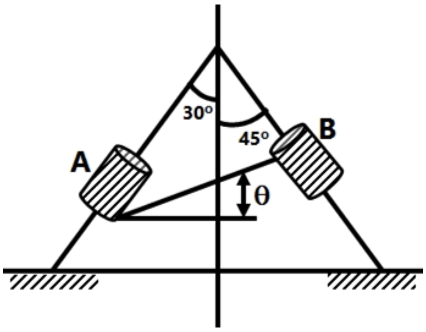
R_x = 755.4

R_y = 0

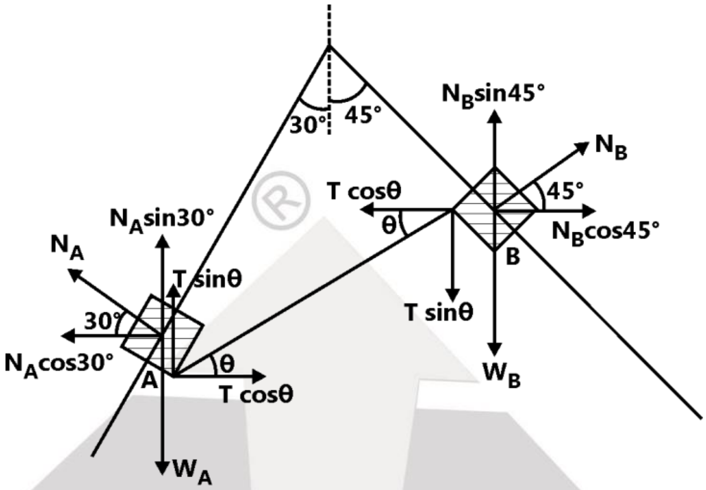


Problems

01. The collars A and B hang on vertical frame composed of two smooth rods. If the mass of collar A is 10 kg and the mass of collar B is 5 kg, determine the equilibrium angle θ (in degree)



01. Ans: 24-26
Solution:
Construct free body diagram.



For equilibrium of A

$T\cos30° - N_A\cos30° = 0$ ....(1)

$T\sin30° + N_A\sin30° - W_A = 0$ ....(2)

For equilibrium of B

$-T\cos\theta + N_B\cos45° = 0$ ....(3)

$-T\sin\theta + N_B\sin45° - W_B = 0$ ....(4)

From (1) and (3), $N_A = 0.816N_B$

From (2) and (4)

$N_A\sin30° + N_B\sin45° = W_A + W_B = g(M_A + M_B) = 147.15$

After putting value of N_A

$(0.816\times\sin30° + \sin45°)N_B = 147.15$

$N_B = 131.96$ N

$N_A = 107.679$ N

From (1) $T\sin\theta = 107.679\times\cos30° = 93.2533$

From (2) $T\sin\theta = W_A - N_A\sin30° = 10\times9.81 - 107.679\times\sin30° = 44.2601$

$\tan\theta = \frac{44.2601}{93.2533}$

$\theta = 25.39°$

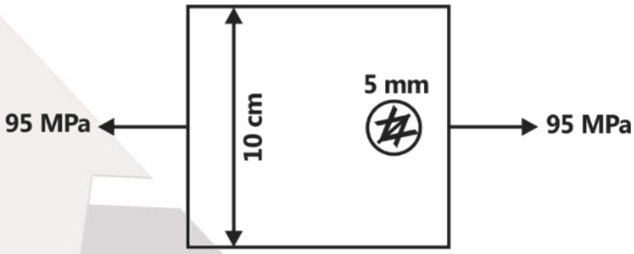
Machine Design

Stress concentration and Theories of Failure

Sample Problem

A large uniform plate containing a rivet hole subjected to uniform uniaxial tension of 95 MPa. The maximum stress in the plate is

(a) 100 MPa
(b) 285 MPa
(c) 190 MPa
(d) Indeterminate



Solution: (b) is correct option

$k_t = \left(1 + \frac{2b}{a}\right)$; a = b for circular hole

$k_t = 3$

Max. stress $= 3 \times \sigma_{aw} = 285$ MPa due to stress concentration.

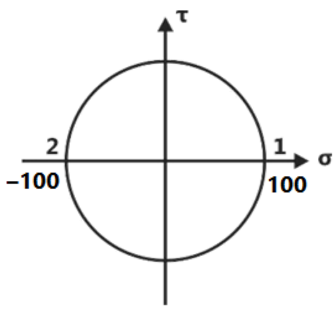
Sample Problem

A shaft is subjected to pure torsional moment. The maximum shear stress developed in the shaft is 100 MPa. The yield and ultimate strengths of the shaft material in tension are 300 MPa and 500 MPa, respectively. The factor of safety using maximum distortion energy (von-Mises) theory is ____________.

Solution: 1.732

$\sigma_1 = 100$ MPa & $\sigma_2 = -100$ MPa

As per Von-Mises yield theory:



$$\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right] = 2\left(\frac{\sigma_y}{N}\right)^2$$

In this case, $\sigma_3 = 0$

$$2\left[\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2\right] = 2\left(\frac{\sigma_y}{N}\right)^2$$

$$N^2\left[(100)^2 + (100)^2 + (100)^2\right] = (300)^2$$

$$N^2 = \frac{9}{3}$$

$$N = \sqrt{3}$$

$\therefore\ N = 1.732$

Problems

01. At a critical section in a shaft, bending stress of 60 MPa and torsional shear stress of 40 MPa are induced. If yield stress is 300 MPa and Poisson's ratio is 0.3, then factor of safety according to maximum principal strain theory is ____________.

01. Ans: 3.3 – 3.6

Solution:

Given:

$\sigma_t = 60\text{MPa}$

$\tau = 40\text{MPa}$

$\sigma_y = 300\text{MPa}$

$\mu = 0.3$

Maximum and Minimum principal stresses.

$$\sigma_{1,2} = \frac{1}{2}\left[\sigma_t \pm \sqrt{\sigma_t^2 + 4\tau^2}\right] = \frac{1}{2}\left[60 \pm \sqrt{60^2 + 4\times 40^2}\right]$$

$\sigma_1 = 80\text{MPa}$

$\sigma_2 = -20\text{MPa}$

According to maximum principal strain theory

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \frac{\sigma_y}{(\text{F.S})}$$

$$\text{Factor of safety, F.S.} = \frac{\sigma_y}{\sigma_1 - \mu\sigma_2} = \frac{300}{80 - 0.3(-20)} = 3.5$$

02. According to the maximum shear stress theory of failure, permissible twisting in a circular shaft is T. The permissible twisting moment in the same shaft as per the maximum principal stress theory of failure will be

(a) $\frac{T}{2}$

(b) T

(c) $\sqrt{2}\,T$

(d) 2T

02. Ans: (d)

Solution:

As per MSST, $T_{per} = T$

As per MPST, let $T_{per} = T_1$

$$T = Z_p(\tau_{per}) = \frac{\pi d^3}{16}(\tau_{per}) = \frac{\pi}{16}d^3\left(\frac{S_{ys}}{N}\right)$$

For a given 'd' and 'N', $T \propto S_{ys}$

$$\therefore \frac{(T_{per})_{MPST}}{(T_{per})_{MSST}} = \frac{(S_{ys})_{MPST}}{(S_{ys})_{MSST}} \Rightarrow \frac{T_1}{T} = \frac{S_{yt}}{S_{yt}/2} = 2$$

$T_1 = 2T$

Industrial Engineering

Linear Programming (Graphical & Simplex Method)

Sample Problem

For the linear programming problem:

Maximize $Z = 3x_1 + 2x_2$

Subject to

$-2x_1 + 3x_2 \le 9$

$x_1 - 5x_2 \ge -20$

$x_1, x_2 \ge 0$

The above problem has

(a) unbounded solution
(b) infeasible solution
(c) alternative optimum solution
(d) degenerate solution

Solution: (a) is correct option

Objective function to be maximized $Z = 3x_1 + 2x_2$

Constraints

$-2x_1 + 3x_2 \le 9$

$x_1 - 5x_2 \ge -20$

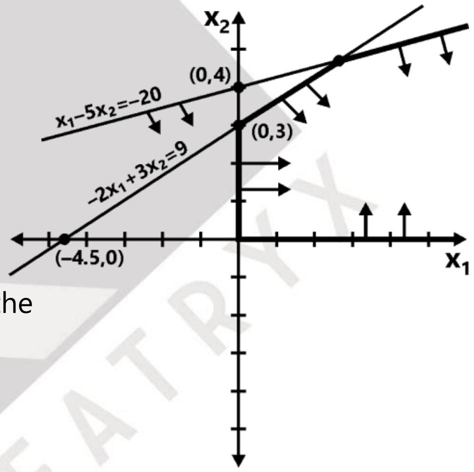
$x_1, x_2 \le 0$

$\therefore$ Changing the inequalities to equalities and plotting the same on graph

$-2x_1 + 3x_2 = 9$

$x_1 - 5x_2 = -20$

$\therefore$ The solution is unbounded.



Problems

02. Maximize $Z = 3x_1 + 4x_2$

Subject to

$x_1 + x_2 \le 1$

$-3x_1 + x_2 \ge 3$

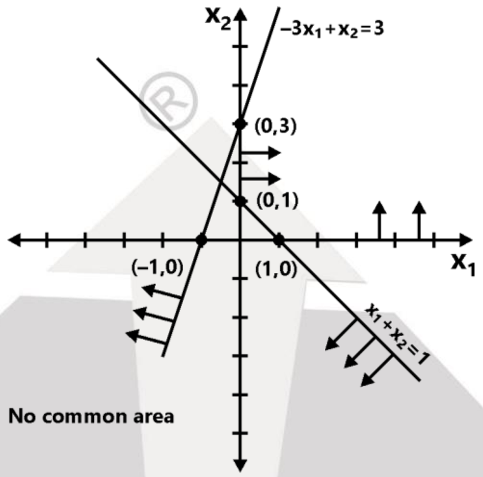
$x_1 \ge 0, x_2 \ge 0$

(a) The LPP has a unique optimal solution
(b) The LPP is unbounded
(c) The LPP has multiple optimal solution
(d) The LPP is infeasible

02. Ans: (d)

Solution:

The problem is depicted graphically as shown in figure, there is no point (x_1, x_2) which can lie in both the regions (satisfy both the constraints), there exists no solution to the given problem. Hence, there is infeasible solution.



Manufacturing Engineering

Solidification and Cooling

Sample Problem

A cube and a sphere made of cast iron (each of volume 1000 cm^3) were cast under identical conditions. The time taken for solidifying the cube was 4 s. The solidification time (in s) for the sphere is ___________.

Solution: 6.157 s

Time of solidification (T) $= k\left[\dfrac{\text{volume}}{\text{surface area}}\right]^2$

$$\left[\frac{T_{sphere}}{T_{cube}}\right] = \left[\frac{A_{cube}}{A_{sphere}}\right]^2 = \left[\frac{6a^2}{4/3\,\pi r^3}\right]^2 - (1)$$

$\because\ [V_{sphere} = V_{cube}]$

V_{cube} = a^3 = 1000 cm^3

a = 10 cm

Similary,

$$V_{sphere} = \frac{4}{3}\pi r^3 = 1000$$

r = 6.2035 cm

Putting in (1), we get

$$\frac{T_{sphere}}{T_{cube}} = \left[\frac{6\times 10^2}{4\pi\times(6.2035)^2}\right]^2$$

$T_{sphere} = 6.157$ s

Problems

01. A hollow casting is produced using a cylindrical core of height = diameter = 100 mm

Density of liquid metal $= 2700\ kg/m^3$.

Density of core $= 1600\ kg/m^3$

Calculate the net buoyancy force on core.

01. Ans: 8-9

Solution:

Net buoyancy force $= (\rho_m - \rho_c)gV = 8.4709$ N

02. A casting of size 150 mm × 100 mm × 10 mm is required. Assume volume shrinkage of casting as 8%. If the height of the riser is 40 mm and the riser volume desired is three times the shrinkage in casting, the approximate riser diameter will be __________ mm.

02. Ans: 32.5-35.0
Solution:
Volume of riser= $3\times(\%)$ of shrinkage volume

$$\frac{\pi}{4}\times d^2\times 40 = 3\times\frac{8}{100}\times(150\times100\times10) \qquad \left(\because\ \text{\% of shrinkage volume} = \frac{8}{100}\times150\times100\times10\right)$$

$d = 33.85\text{mm}$

Validating,

$$\left(V/A\right)_{riser} = \frac{\frac{\pi}{4}\times d^2\times h}{2\times\frac{\pi}{4}d^2+\pi dh} = 5.946$$

$$\left(V/A\right)_{casting} = \frac{L\times B\times H}{2(LB+BH+HL)} = \frac{150\times100\times10}{2\times(150\times100+100\times10+10\times150)} = 4.285$$

$$\left(V/A\right)_{riser} > \left(V/A\right)_{casting}$$

$\Rightarrow$ solidification time for riser is more than that of casting which is always required.
$d_{riser} = 33.85\text{ mm}$ is valid.

03. A spherical drop of molten metal of radius 2mm was found to solidify in 10 seconds. A similar drop of radius 4mm would solidify in
(a) 14.14 seconds
(b) 20 seconds
(c) 18.30 seconds
(d) 40 seconds

03. Ans: (d)
Solution:

$$t_{solidification} = k\left(V/A_s\right)^2$$

$$t \propto \left(V/A_s\right)^2$$

$$t \propto \left(d/6\right)^2 \rightarrow \text{For sphere}$$

$$t \propto \left(r/3\right)^2$$

$$\frac{t_1}{t_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{10}{t_2} = \left(\frac{2}{4}\right)^2 \Rightarrow t_2 = 40\,\text{sec}$$