Kepler and Brahe

Laws of planets motion

Tycho Brahe

Born: **14 Dec 1546 in Knutstorp, Skane, Denmark (now Svalöv, Sweden**
Died: **24 Oct 1601 in Prague, Bohemia (now Czech Republic)**

Jorgen Brahe and his wife Inger Oxe had no children of their own, and they acted as foster parents to Tycho until Jorgen's death. Jorgen Brahe was a leading Danish noble while Inger Oxe was the sister of Peder Oxe who was a member of the Rigsraads, the governing council consisting of 20 advisors to the King.

In 1559 Tycho began his studies at the University of Copenhagen. There, following the wishes of his uncle, he studied law but also studied a variety of other subjects and became interested in astronomy.

In 1562 he set off to go to the University of Leipzig. Astronomy was not officially part of his studies, these were classical languages and culture, but he had bought his astronomy books with him together with Dürer's constellation maps. He began making observations and by August 1563.

Had an argument with another Danish student and in the resulting duel Tycho had part of his nose cut off.

Alchemy was a major interest for Tycho

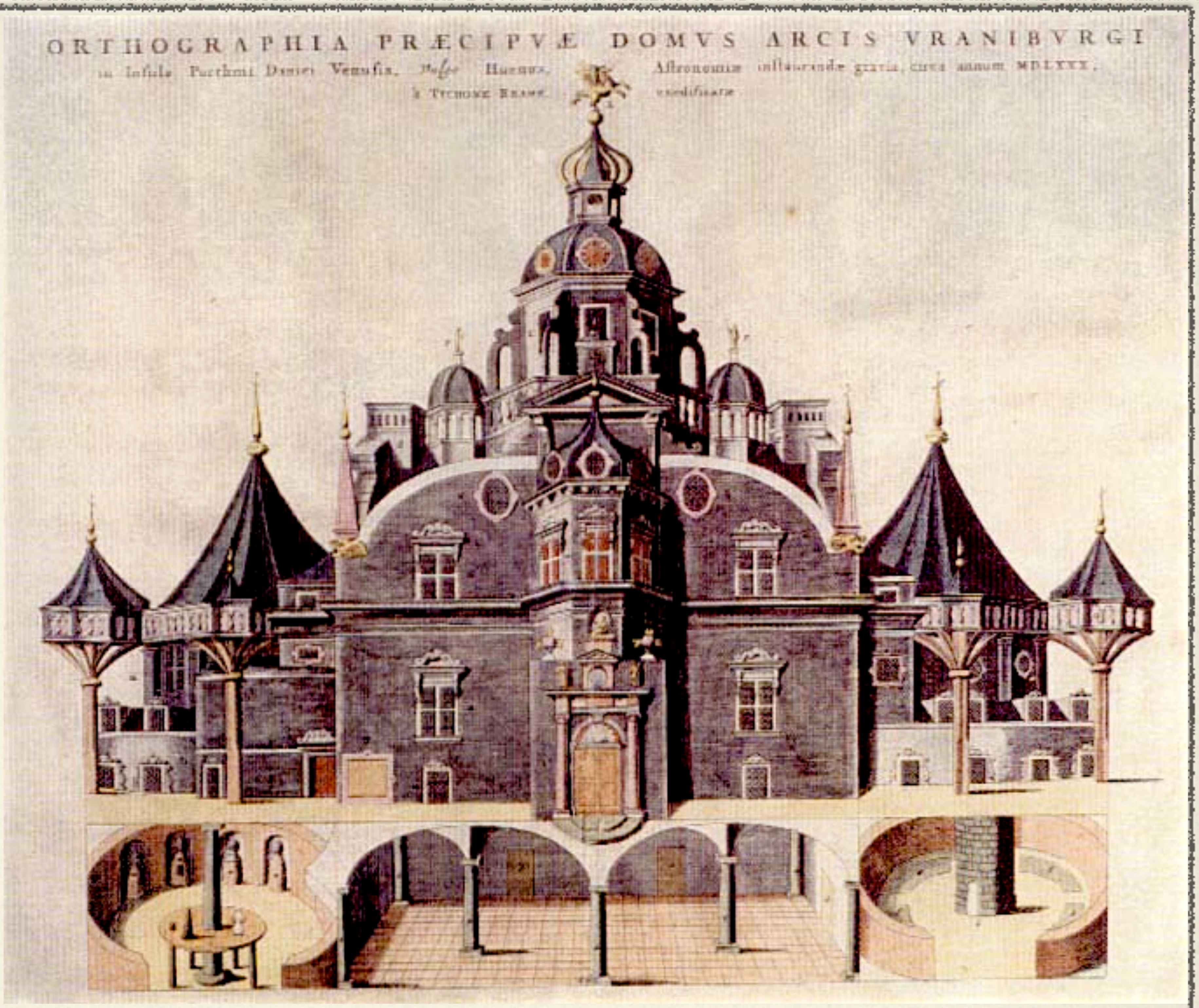
In 1571 with the help of his uncle Steen Bille, Tycho began constructing an observatory in Herrevad Abbey. They also built an alchemy laboratory there.

Tycho Brahe

On 11 *November* 1572, *he emerged into the dark of the early evening, after a long stint of alchemical experimentation, and his first glance at the sky showed him an extra star in the constellation of Cassiopeia, almost directly overhead. He instantly summoned his chemical assistant to confirm that the star really was there. He was not the first to see the new star* (*a supernova*) *but his observations of it* **(published in 1574) did much to prove beyond reasonable doubt that the star really belonged to the firmament and was not merely a local phenomenon** *in the sublunary world* (*as comets were generally believed to be*)*. The star is now usually known as 'Tycho's supernova'. It turned Tycho's interest back to astronomy.*

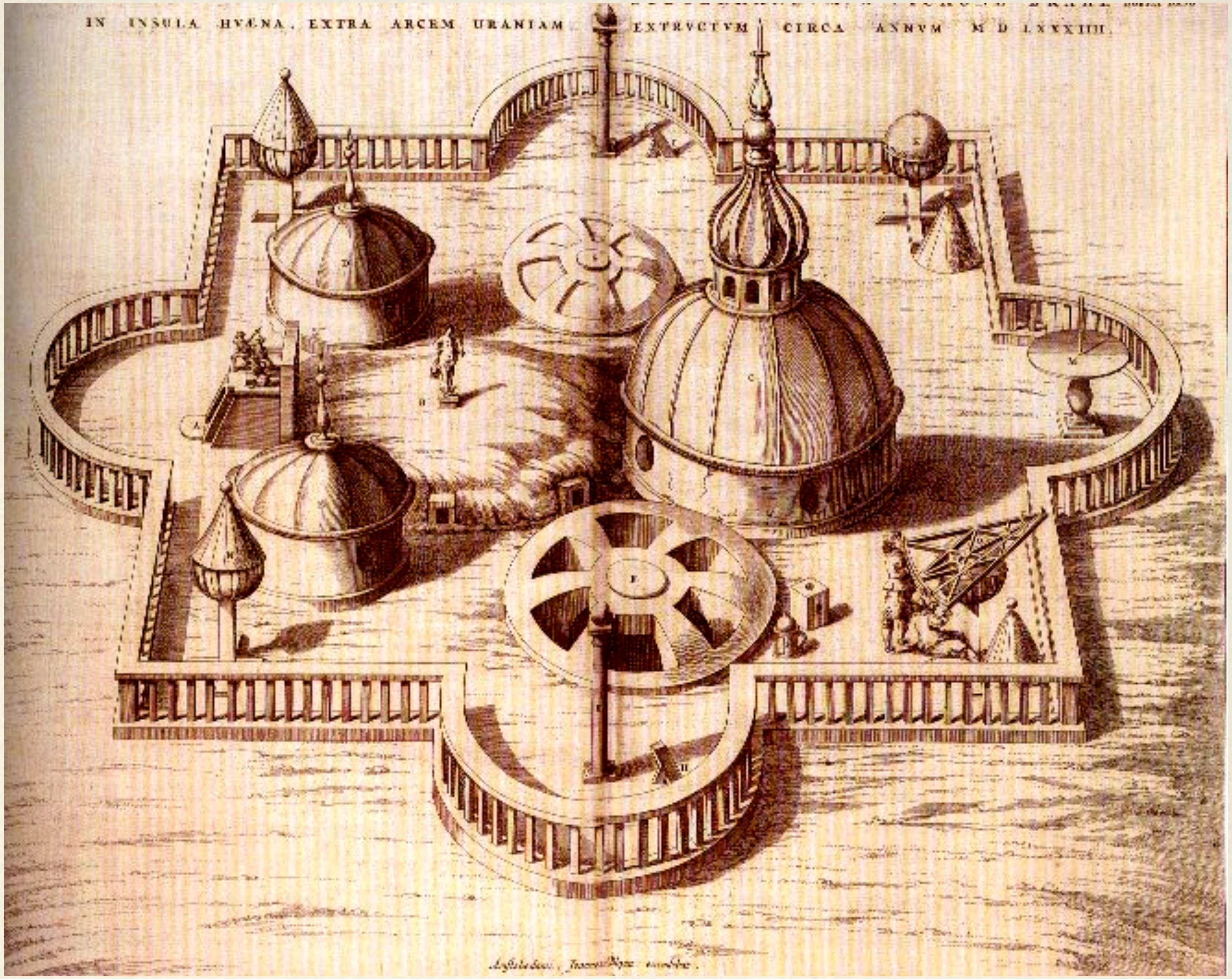
Tycho visited Frankfurt, Basel and finally Venice before returning to Denmark by the end of 1575. By this time he had made a decision to leave Denmark and to settle in Basel, but King Frederick of Denmark was not going to lose his most eminent scientist easily so he made offers to Tycho to entice him to set up an observatory in Denmark. After some offers which Tycho did not find attractive, the King offered Tycho the island of Hven (called today Ven)

Tycho went on to set up a purpose-built observatory, on the island of Hven in Copenhagen Sound. The observatory, called Uraniborg, was equipped with exceptionally large and accurate instruments (and with an alchemical laboratory in its basement). At Uraniborg Tycho made twenty years' worth of astronomical observations.



Here is Tycho's drawing of [the main building at Uraniborg](), taken from his *Astronomiae instauratae mechanica* (1598)

One of the most exciting astronomical events which Tycho observed from Uraniborg was a comet which he first spotted on 13 November 1577. He published his account in *De mundi aetherei recentioribus phaenomenis* (1588) where he draws cosmological conclusions from the fact that his measurements show that the comet is not closer to Earth than the Moon, contradicting Aristotle's model of the cosmos. From his observations Tycho was able to show that the comet was certainly further away than Venus.



The observations have errors falling mostly between about 0.5' and 1.0', that is, about the accuracy of the standard used for comparison. Thus, as was also the case in the earlier study of fixed stars, Kepler's belief that Tycho's observations could be trusted to better than two minutes is amply confirmed.

In his younger days Tycho had been convinced by Copernicus' Sun centered model but his firm belief that theory must be supported by experimental evidence led him away. The problem was, of course, that in the Sun centered model of Copernicus a parallax shift should be observed but despite his attempts to measure such a shift, Tycho could detect none.

The first measurement of the parallax of a star was in 1838 by Bessel who found 0.3" for the parallax of 61 Cygni. Despite the quality of Tycho's measurements, this value in about 100 times smaller that Tycho's observational errors.

Mural quadrant

King Frederick died in April 1588 and, his son Christian (who became King Christian IV) still being a child, a regent was appointed. Support for Tycho continued however

Tycho closed down his observatory on Hven in 1597 (the last recorded observation is on 15 March that year), and moved to Copenhagen. However, things did not go well for him there and he left Denmark with his family and his instruments to seek support and find somewhere to continue his work

In 1599 *he was appointed Imperial Mathematician to the Holy Roman Emperor, Rudolph II, in Prague (then the capital of the Holy Roman Empire).*

Johannes Kepler joined him as an assistant, to help with mathematical calculations.

Tycho intended that this work should prove the truth of his cosmological model, in which the Earth (with the Moon in orbit around it) was at rest in the centre of the Universe and the Sun went round the Earth (all other planets being in orbit about the Sun and thus carried round with it).

Tycho died eleven days after dining at the palace of Peter Vok Ursinus Rozmberk as a result of adhering to the etiquette of the day and refusing to leave the dinner table before his host. Kepler describes his death:

> *Holding his urine longer than was his habit, Brahe remained seated. Although he drank a little overgenerously and experienced pressure on his bladder, he felt less concerned for his state of health than for etiquette. By the time he returned home he could not urinate any more. Finally, with the most excruciating pain, he barely passed some urine, but yet it was blocked. Uninterrupted insomnia followed; intestinal fever; and little by little delirium. ... During his last night, through the delirium in which everything was very pleasant, like a composer creating a song, Brahe these words over and over again: "Let me not seem to have lived in vain."*

When Tycho died, Kepler succeeded him as Imperial Mathematician.

Tycho's observations of planetary positions, which were made using instruments with open sights (a telescope was not used for astronomy until about 1609), were much more accurate than any made by his predecessors.

They allowed Kepler, who (unlike Tycho) was a convinced follower of Copernicus, to deduce his three laws of planetary motion (1609, 1619) and to construct astronomical tables, the Rudolphine Tables (Ulm, 1627), whose enduring accuracy did much to persuade astronomers of the correctness of the Copernican theory.

However, until at least the mid-seventeenth century, Tycho's model of the planetary system was that favored by most astronomers. It had the advantage of avoiding the problems introduced by ascribing motion to the Earth.

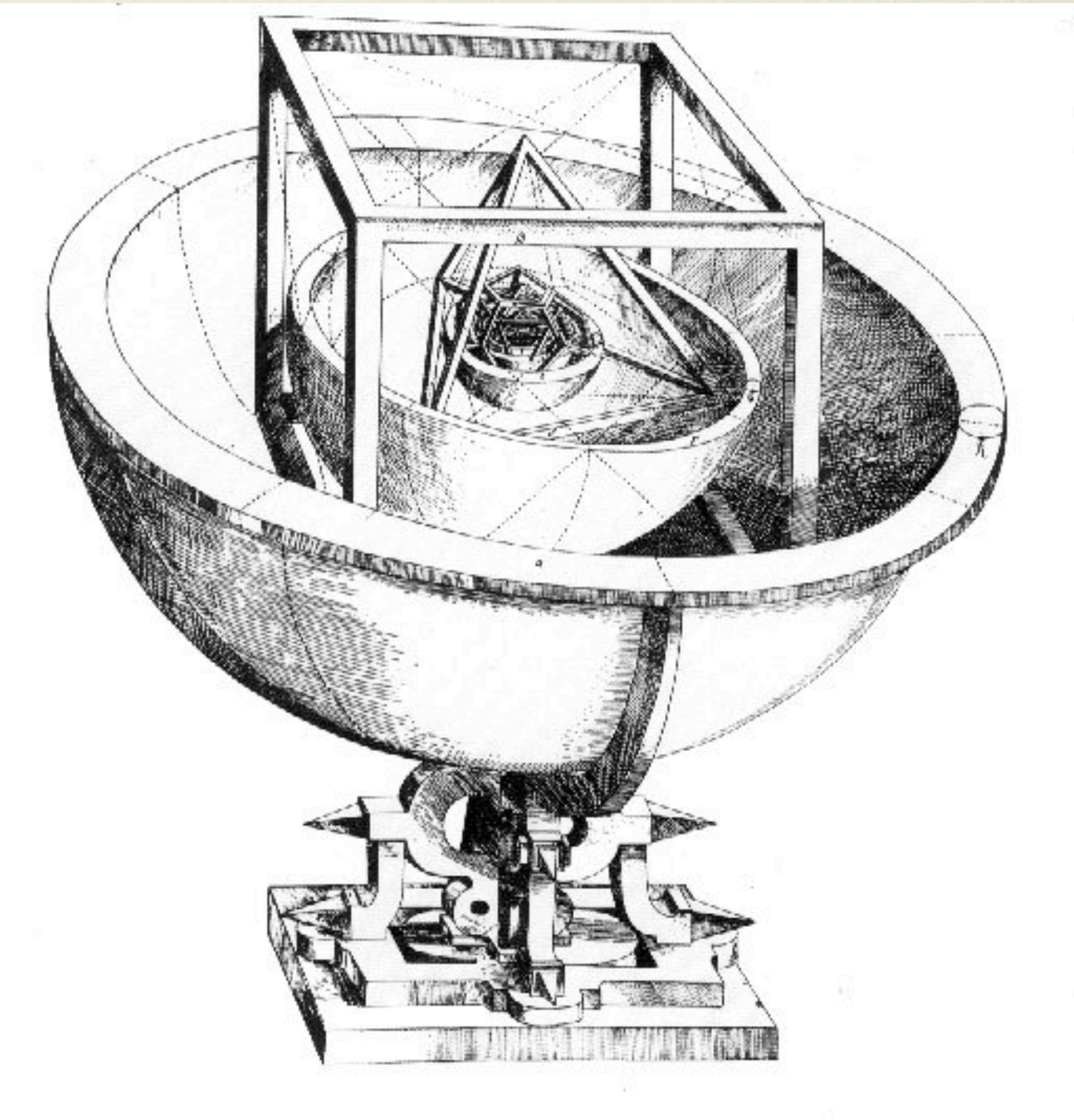
Johannes Kepler (1571-1630)

Johannes Kepler was born in Weil der Stadt in Swabia, in southwest Germany.

In 1584 he entered the Protestant seminary at Adelberg, and in *1589* he began his university education at the Protestant university of Tübingen. Here he studied theology and read widely. He passed the M.A. examination in 1591 and continued his studies as a graduate student.
Kepler's teacher in the mathematical subjects was Michael Maestlin (1580-1635). Maestlin was one of the earliest astronomers to subscribe to Copernicus's heliocentric theory, although in his university lectures he taught only the Ptolemaic system. Only in what we might call graduate seminars did he acquaint his students, among whom was Kepler, with the technical details of the Copernican system. Kepler stated later that at this time he became a Copernican for "physical or, if you prefer, metaphysical reasons."

In 1594 Kepler accepted an appointment as professor of mathematics at the Protestant seminary in Graz (in the Austrian province of Styria). He was also appointed district mathematician and calendar maker. Kepler remained in Graz until 1600, when all Protestants were forced to convert to Catholicism or leave the province, as part of Counter Reformation * measures. For six years, Kepler taught arithmetic, geometry (when there were interested students), Virgil, and rhetoric. In his spare time he pursued his private studies in astronomy and astrology.

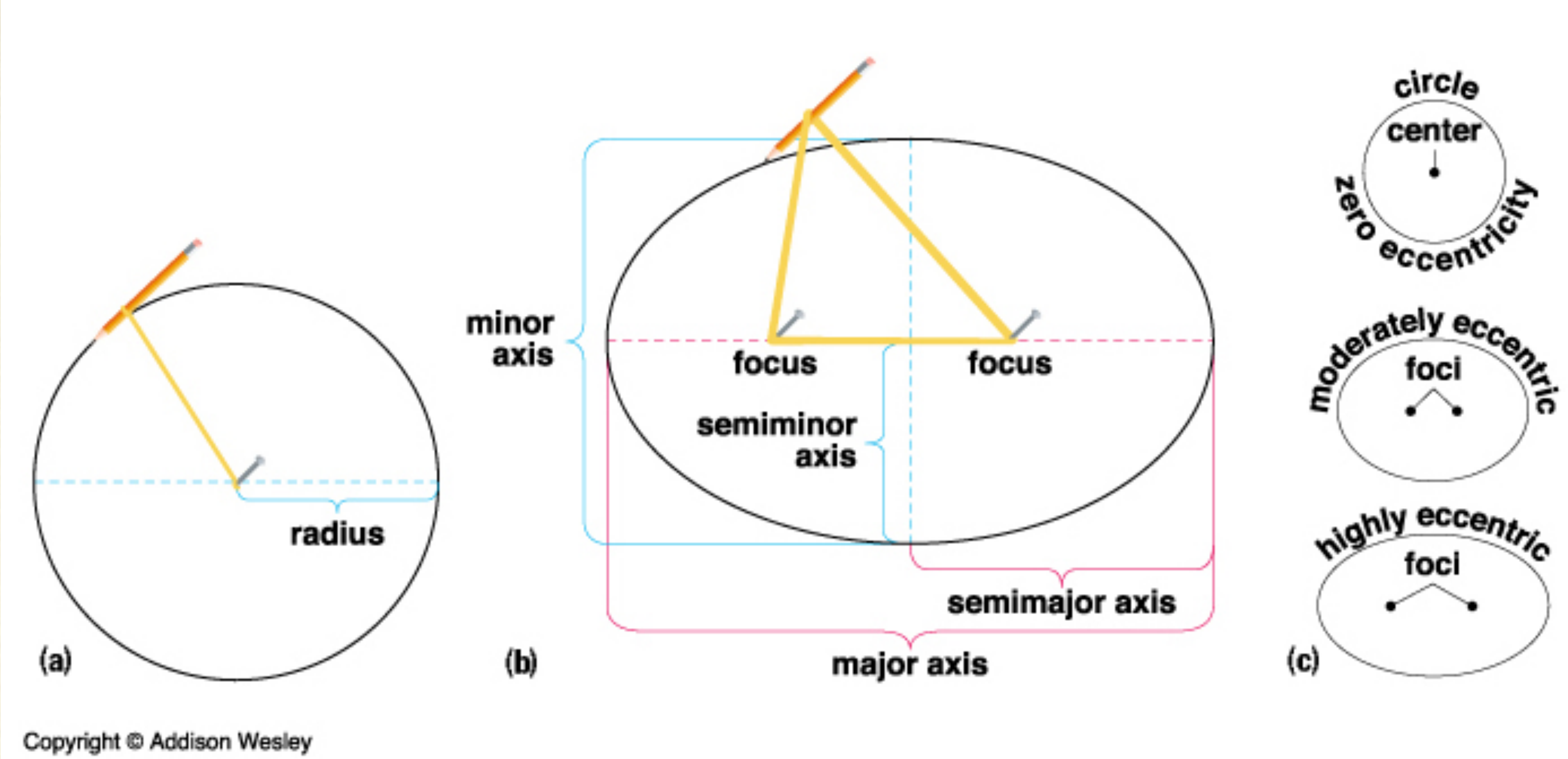
he published his first important work, *The Cosmographic Mystery*, in which he argued that the distances of the planets from the Sun in the Copernican system were determined by the five regular solids, if one supposed that a planet's orbit was circumscribed about one solid and inscribed in another.



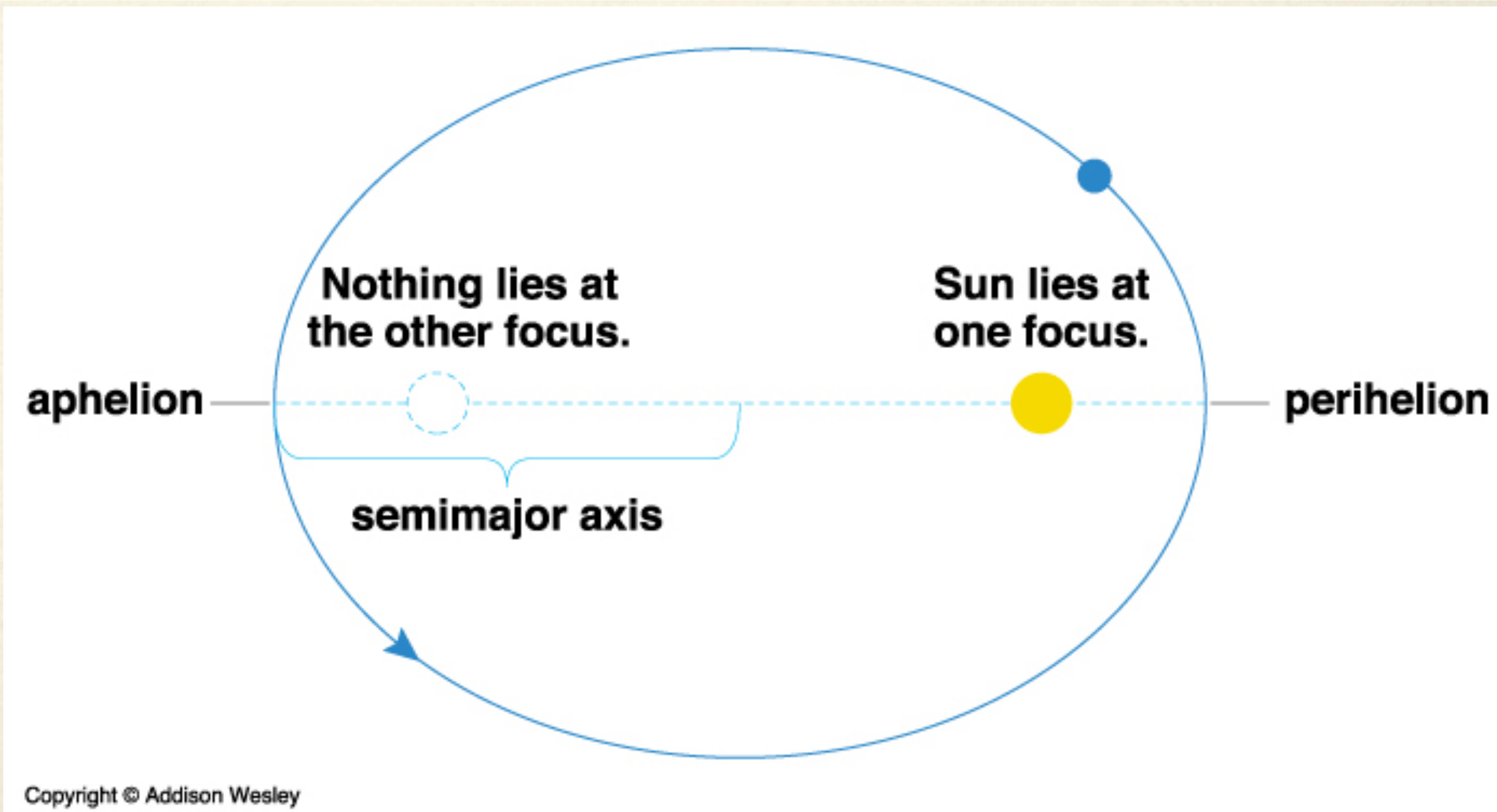
It was a wrong description of planet orbits

Kepler was invited by Tycho Brahe to Prague to become his assistant and calculate new orbits for the planets from Tycho's observations. Kepler moved to Prague in 1600.

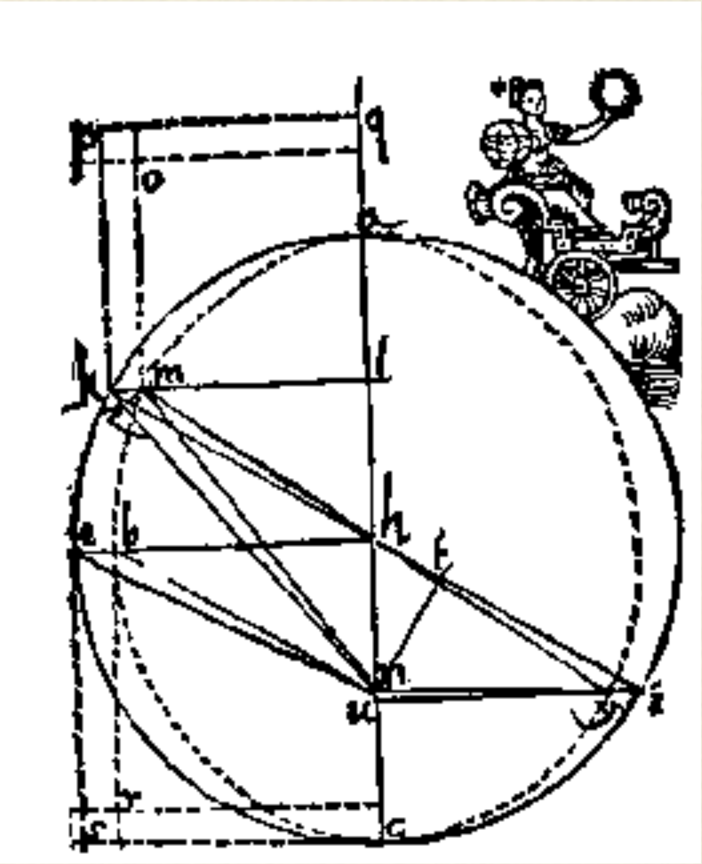
Kepler served as Tycho Brahe's assistant until the latter's death in 1601 and was then appointed Tycho's successor as Imperial Mathematician, the most prestigious appointment in mathematics in Europe. He occupied this post until, in 1612, Emperor Rudolph II was deposed.



in 1609 his *Astronomia Nova* ("New Astronomy") appeared, which contained his first two laws (planets move in elliptical orbits with the sun as one of the foci, and a planet sweeps out equal areas in equal times). Whereas other astronomers still followed the ancient precept that the study of the planets is a problem only in kinematics, Kepler took an openly dynamic approach, introducing physics into the heavens.

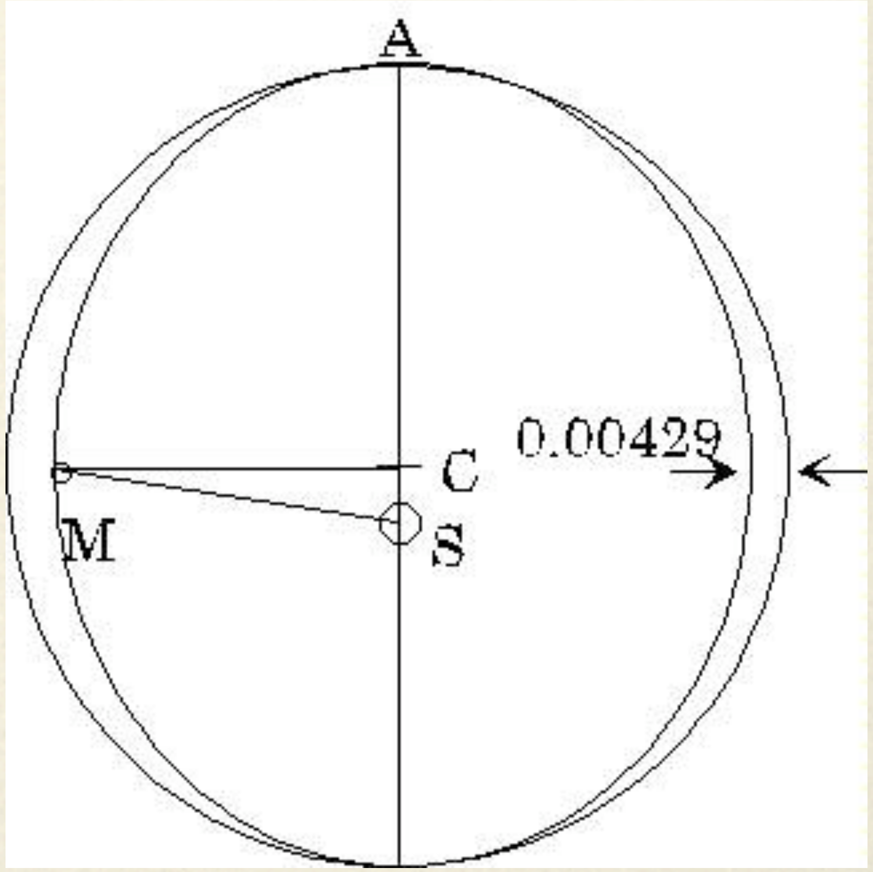


In an effort to find a better description of planetary motions, Kepler re-done Ptolemy's calculations. He found an error in placing of the Sun. He improves the geocentric model. It is now much more accurate than it used to be. Yet, it makes an error in prediction of position of Mars - 8 arcminutes. Accuracy of Brahe's measurements is 0.5 arcmin. Thus, the geocentric model still fails.

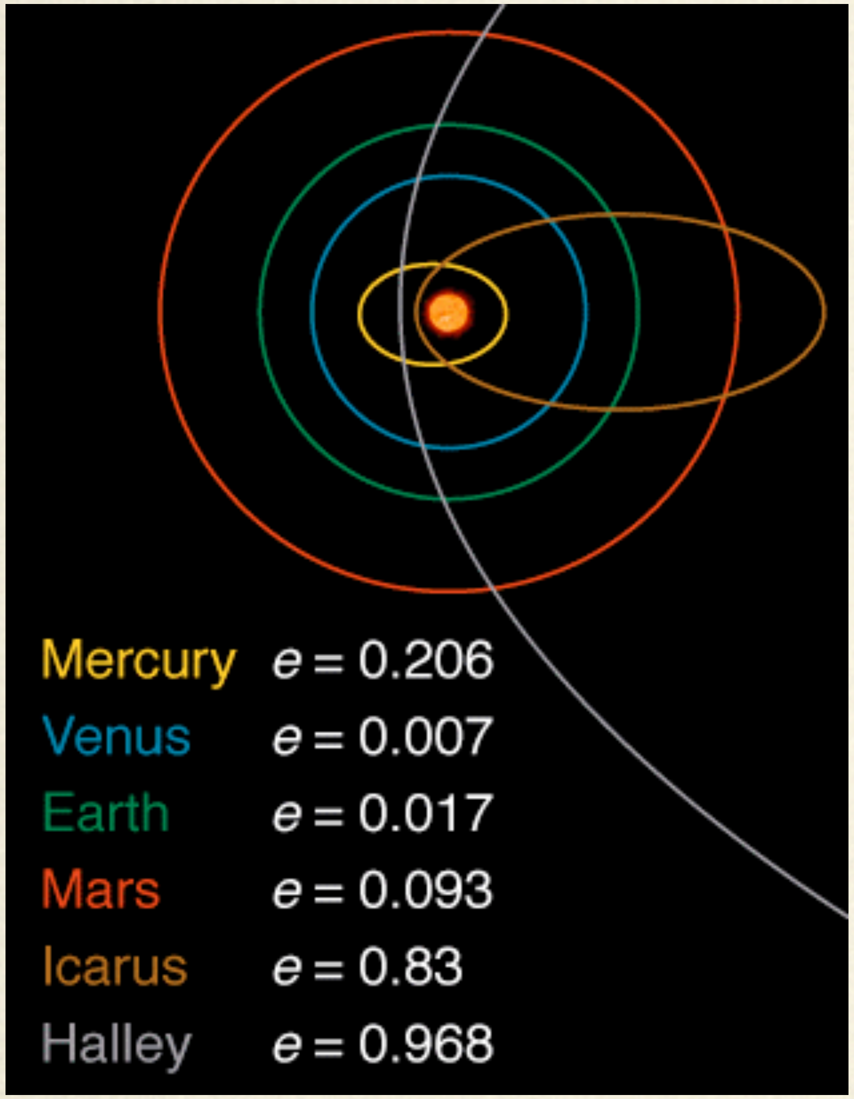


You can see a diagram from *Astronomia Nova* (1609) showing Kepler's elliptical orbit for Mars..

It took 6 years and thousands of pages of calculations to derive the law of elliptical motion

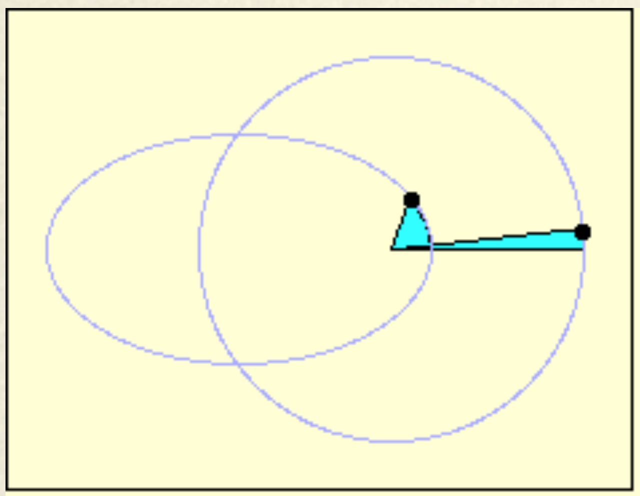


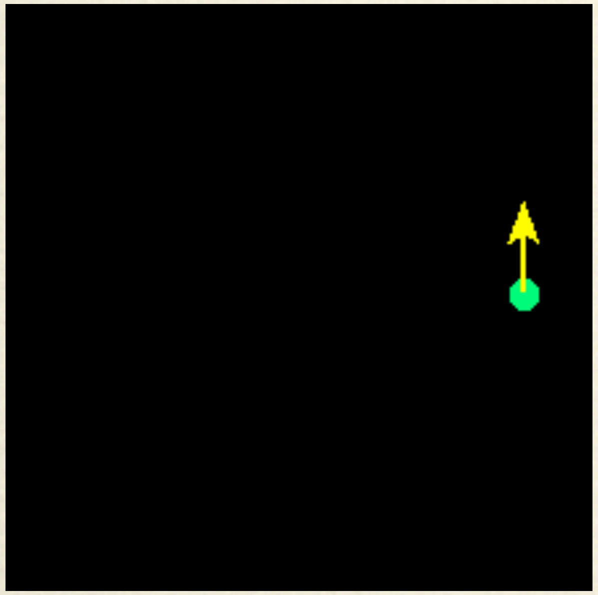
Relative deviation of the Mars orbit from a circle is actually very small - 0.00429. In the plot it is exaggerated

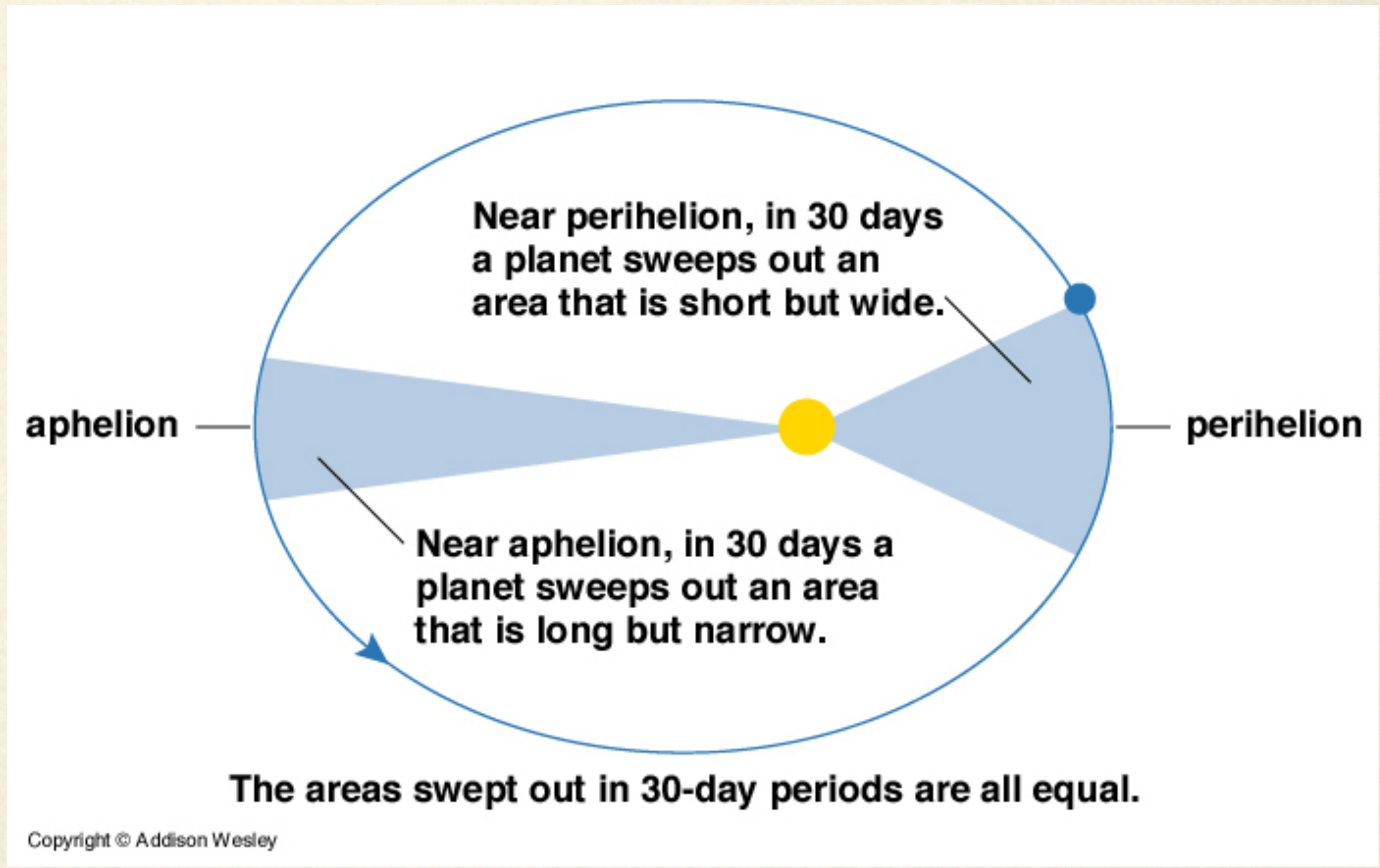

Mercury e = 0.206
Venus e = 0.007
Earth e = 0.017
Mars e = 0.093
Icarus e = 0.83
Halley e = 0.968

a planet sweeps out equal areas in equal times

II. The line joining the planet to the Sun sweeps out equal areas in equal times as the planet travels around the ellipse.







III. The ratio of the squares of the revolutionary periods for two planets is equal to the ratio of the cubes of their semimajor axes

$$P^2 = A^3$$

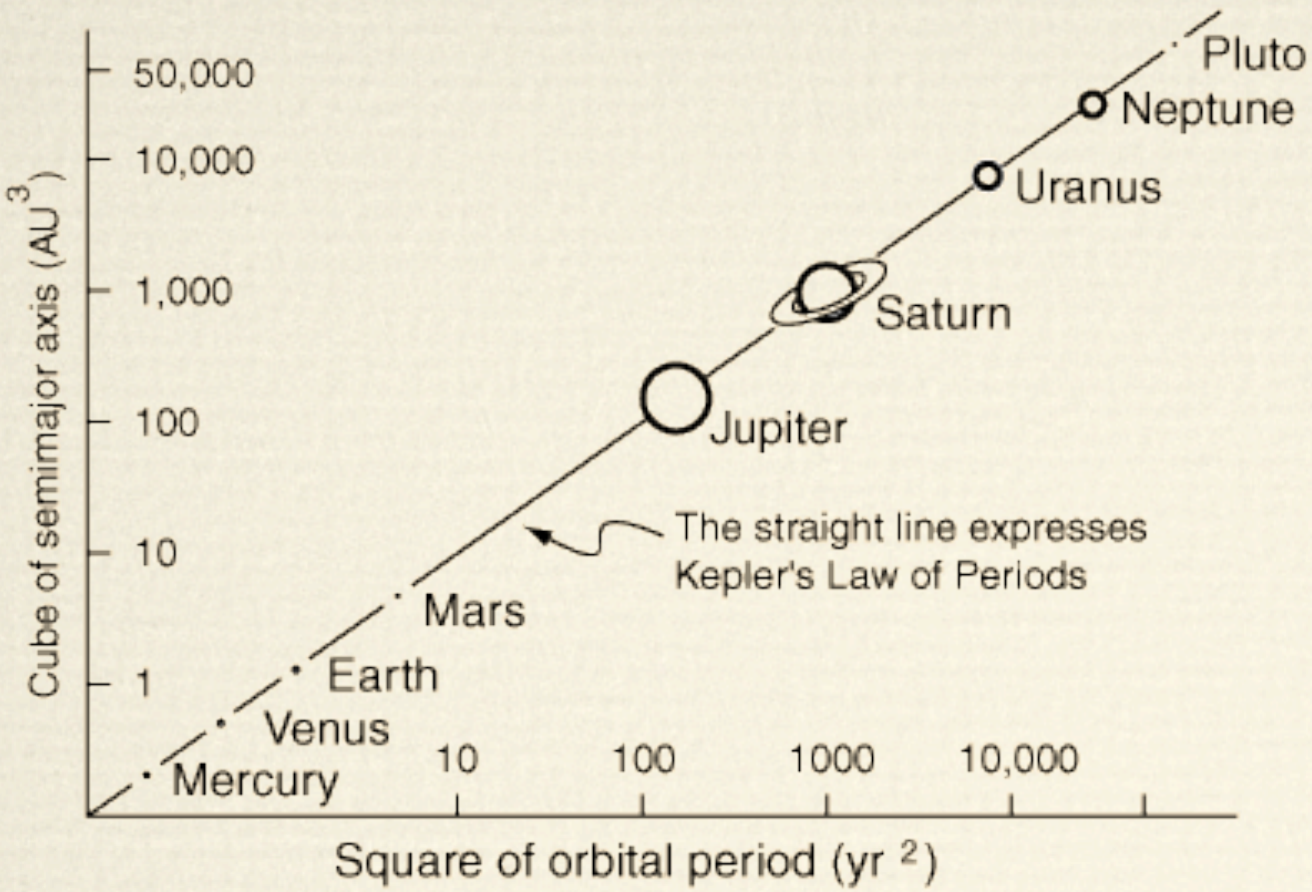
In 1612 Kepler accepted the position of district mathematician in the city of Linz, a position he occupied until 1626.

In 1619 he published *Harmonice Mundi* ("Harmony of the World"), in which he derived the heliocentric distances of the planets and their periods from considerations of musical harmony. In this work we find his third law, relating the periods of the planets to their mean orbital radii.

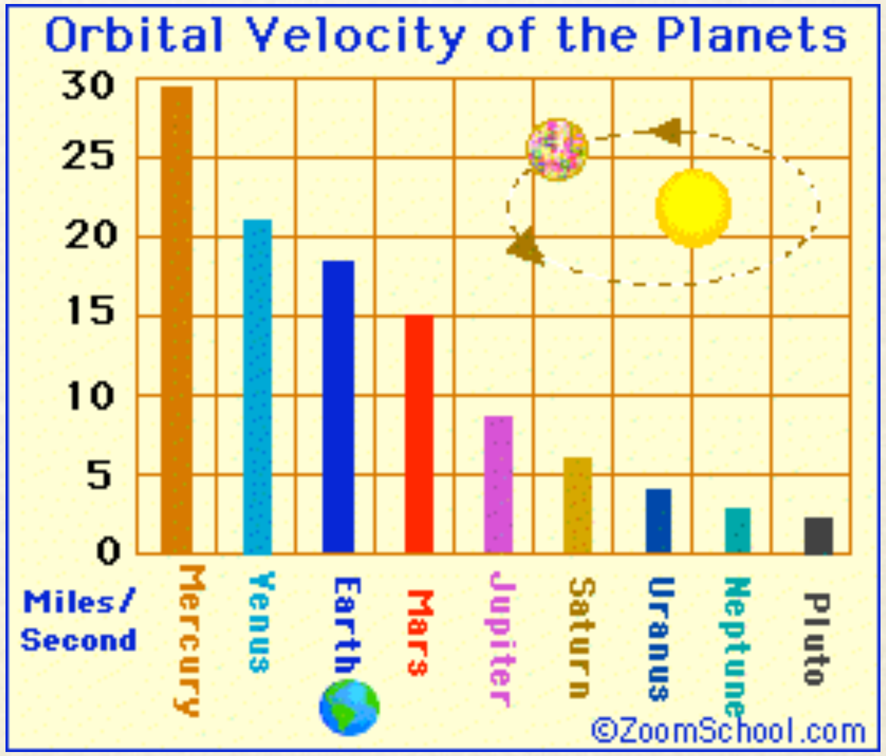
In 1615-16 there was a witch hunt in Kepler's native region, and his own mother was accused of being a witch. It was not until late in 1620 that the proceedings against her ended with her being set free. At her trial, her defense was conducted by her son Johannes.

1618 marked the beginning of the Thirty Years War, a war that devastated the German and Austrian region. Kepler's position in Linz now became progressively worse, as Counter Reformation measures put pressure on Protestants in the Upper Austria province of which Linz was the capital. Because he was a court official, Kepler was exempted from a decree that banished all Protestants from the province, but he nevertheless suffered persecution.

He and his family left Linz in 1626. Kepler now had no position and no salary. He tried to obtain appointments from various courts and returned to Prague in an effort to pry salary that was owed him from his years as Imperial Mathematician from the imperial treasury. He died in Regensburg in 1630.



Cube of semimajor axis (AU³)
50,000
10,000
1,000
100
10
1
Pluto
Neptune
Uranus
Saturn
Jupiter
The straight line expresses
Kepler's Law of Periods
Mars
Earth
Venus
Mercury
10
100
1000
10,000
Square of orbital period (yr ²)



Orbital Velocity of the Planets
30
25
20
15
10
5
0
Miles/
Second
Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
Pluto
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However scientists certainly did not accept Kepler's first two laws with enthusiasm. The first was given a cool reception and was certainly thought to require further work to confirm it. The second of Kepler's laws suffered an even worse fate in being essentially ignored by scientists for around 80 years.

Kepler's third law, that the squares of the periods of planets are proportional to the cubes of the mean radii of their orbits, appeared in *Harmonice mundi* (1619) and, perhaps surprisingly in view of the above comments, was widely accepted right from the time of its publication.

The reason why the laws where not easily accepted was in the lack of explanations why the orbits should be elliptical. That explanation will come much later

In 1679 Hooke wrote a letter to Newton. In the letter he explained how he considered planetary motion to be the result of a central force continuously diverting the planet from its path in a straight line. Newton did not answer this directly but explained his own idea that the rotation of the Earth could be proved from the fact that an object dropped from the top of a tower should have a greater tangential velocity than one dropped near the foot of the tower.

Newton provided a sketch of the path that the particle would follow, quite incorrectly showing it spiral towards the centre of the Earth. Hooke replied that his theory of planetary motion would lead to the path of the particle being an ellipse so that the particle, were it not for the fact that the Earth was in the way, would return to its original position after traversing the ellipse.

- Kepler devised the following Laws of Planetary Motion:
  - *Law of Ellipses* - The orbit of a planet is an ellipse with the Sun at one focus. (Eccentricity of an ellipse is the ratio of the distance between the focii divided by the length of the major axis.)
  - *Law of Equal Areas* - A hypothetical line drawn between a planet and the Sun sweeps out equal areas in equal intervals of time. In simpler terms, this means that a planet travels faster when it is closer to the Sun.
  - *Harmonic Law* - If P is a planet's orbital period around the Sun (measured in Earth years) and A is the average distance between the planet and the Sun (measured in AU), then
  -
  -
  -
  -

$$P^2 = A^3.$$

  - For example, if a planet had an orbital period of 8 yrs, then ; so A = 4 AU. So, by measuring the orbital period of a planet through careful observations of the planet with respect to the stars, one can determine its distance from the Sun.
  - Kepler's laws are a correct description of planetary motion. However, they lack any explanation as to why planets move in this fashion.

Tycho Brahe and Johannes Kepler

- *Tycho Brahe's (1546-1601) precise observations of Mars set the stage for Johannes Kepler (1571-1630) to make major modifications and improvements on the heliocentric model.*