

# KETAKSAMAAN DAN NILAI MUTLAK



# URUTAN

## SIFAT – SIFAT URUTAN

- Trikotomi : jika  $x$  dan  $y$  adalah bilangan – bilangan maka pasti satu diantara berikut berlaku :

$$x < y \quad \text{atau} \quad x = y \quad \text{atau} \quad x > y$$

- Transitif :

$$(x < y) \text{ dan } (y < z) \Rightarrow x < z$$

- Penambahan

$$(x < y) \Leftrightarrow x + z < y + z$$

- Perkalian

$$\text{Jika } z > 0 \text{ maka : } x < y \Leftrightarrow x \cdot z < y \cdot z$$

$$\text{Jika } z < 0 \text{ maka : } x < y \Leftrightarrow x \cdot z > y \cdot z$$

# Selang / Interval

- Selang terbuka :  $a < x < b$  atau  $(a, b)$
- Selang tertutup :  $a \leq x \leq b$  atau  $[a, b]$

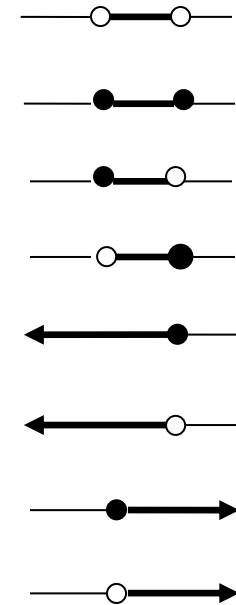
## Penulisan himpunan

- $\{x \in \mathbb{R} \mid a < x < b\}$
- $\{x \in \mathbb{R} \mid a \leq x \leq b\}$
- $\{x \in \mathbb{R} \mid a \leq x < b\}$
- $\{x \in \mathbb{R} \mid a < x \leq b\}$
- $\{x \in \mathbb{R} \mid x \leq b\}$
- $\{x \in \mathbb{R} \mid x < b\}$
- $\{x \in \mathbb{R} \mid x \geq a\}$
- $\{x \in \mathbb{R} \mid x > a\}$

## Selang

- $(a, b)$   
 $[a, b]$   
 $[a, b)$   
 $(a, b]$   
 $(-\infty, b]$   
 $(-\infty, b)$   
 $[a, \infty)$   
 $(a, \infty)$

## Grafik



# KETAKSAMAAN ( $<$ , $\leq$ , $>$ , $\geq$ )

Contoh 1:

Selesaikan ketaksamaan :  $4x - 5 > 2x + 3$

Jawab :  $4x - 5 > 2x + 3$

$$4x - 2x > 3 + 5$$

$$2x > 8$$

$$x > 4 \quad \text{atau : } (4, \infty)$$

$$\text{HP} = \{x \in \mathbb{R} | x > 4\}$$

Maple :

**> Solve (4\*x-5>2\*x+3, {x});**

## Contoh 2:

Selesaikan ketaksamaan :  $x - 2 \geq 4x + 5$

Jawab :  $x - 2 \geq 4x + 5$

$$x - 4x \geq 5 + 2$$

$$- 3x \geq 7$$

$$3x \leq - 7$$

$$x \leq - 7/3 \text{ atau } (- \infty , - 7/3\}$$

Maple :

> **solve (x-2>=4\*x+5, {x}) ;**

### Contoh 3:

Carilah himpunan penyelesaian (HP) dari

$$-3 \leq 2x + 4 < 2$$

Jawab : 
$$-3 \leq 2x + 4 < 2$$

$$-3 - 4 \leq 2x < 2 - 4$$

$$-7 \leq 2x < -2$$

$$-7/2 \leq x < -1$$

$$\text{HP} : \{x \in \mathbb{R} \mid -7/2 \leq x < -1\} \text{ atau } [-7/2, -1)$$

Maple :

```
> solve (2*x+4>=-3 and 2*x+4<2, {x});
```

Contoh 4 : Carilah HP dari

$$6x - 4 < 2x + 4 < 4x + 6$$

Maple :

```
> solve (6*x-4<2*x+4 and  
2*x+4<4*x+6, {x}) ;
```



Contoh 5 : Carilah HP dari

$$2 - 8x \leq 3x + 5 < x - 9$$

Maple :

```
> solve (2-8*x<=3*x+5 and 3*x+5<x-9, {x});
```



# AKAR KUADRAT

Persamaan Kuadrat :  $ax^2+bx+c = 0$

Cara Penyelesaian :

1. Difaktorkan

$$\text{Contoh : } x^2+3x-4=0$$

$$(x+4)(x-1)=0$$

$$x=-4 \quad x=1$$

2. Rumus abc

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Contoh 6 :

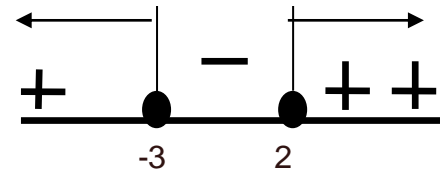
Carilah Hp dari pertidaksamaan :  $x^2 + x \geq 6$

Jawab :

$$x^2 + x \geq 6$$
$$x^2 + x - 6 \geq 0$$
$$(x + 3)(x - 2) \geq 0$$

$\swarrow$                        $\searrow$

$$x + 3 = 0 \quad x - 2 = 0$$
$$x = -3 \quad x = 2$$



Hp :  $\{ x \in \mathbb{R} \mid x \leq -3 \text{ atau } x \geq 2 \}$

Maple : `> solve(x^2+x>=6, {x}) ;`

Contoh 7: Selesaikan  $x^2 - 2x - 4 > 0$

Penyelesaian :

$$x_{1,2} = \frac{-(-2) \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{2 \pm \sqrt{4 + 16}}{2}$$

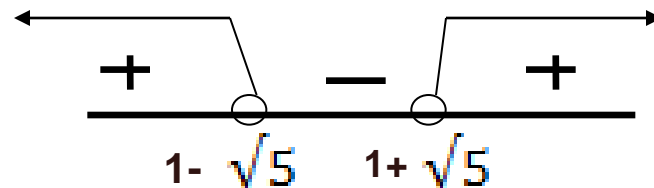
$$= \frac{2 \pm \sqrt{20}}{2}$$

$$= \frac{2 \pm 2\sqrt{5}}{2}$$

$$= 1 \pm \sqrt{5}$$

$$x_1 = 1 - \sqrt{5}$$

$$x_2 = 1 + \sqrt{5}$$



HP :  $\{ x \in \mathbb{R} \mid x < 1 - \sqrt{5} \text{ atau } x > 1 + \sqrt{5} \}$

> `solve(x^2 - 2*x - 4 > 0, {x}) ;`

Selesaikan ketaksamaan :

$$8. x^3 + 2x^2 - 5x - 6 \leq 0$$

$$9. (x+4)(2x-1)^2(x-3) \leq 0$$



Contoh 10:

Selesaikan :  $\frac{2x-5}{x-2} \leq 1$

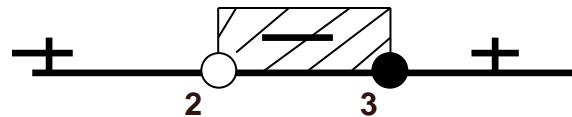
Jawab :  $\frac{2x-5}{x-2} - 1 \leq 0$

$$\frac{2x-5}{x-2} - \frac{x-2}{x-2} \leq 0$$

$$\frac{2x-5-x+2}{x-2} \leq 0$$

$$\frac{x-3}{x-2} \leq 0$$

Hp :  $\{ x \in \mathbb{R} \mid 2 < x \leq 3 \}$



# NILAI MUTLAK

■ Definisi :

$$|x| = x \text{ jika } x \geq 0$$

$$|x| = -x \text{ jika } x < 0$$

■ Contoh :

$$|5| = 5$$

$$|0| = 0$$

$$|-3| = -(-3) = 3$$

## Sifat-sifat nilai mutlak

$$\blacksquare |a \cdot b| = |a| \cdot |b|$$

$$\blacksquare \left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

$$\blacksquare |a + b| \leq |a| + |b|$$

$$\blacksquare |a - b| \geq |a| - |b|$$

## Ketaksamaan nilai mutlak

$$|x| \leq a \Rightarrow -a \leq x \leq a$$

$$|x| \geq a \Rightarrow x \leq -a \text{ atau } x \geq a$$

## Contoh 1 :

Carilah HP dari  $|x - 4| < 1$

Jawab :

$$-1 < x - 4 < 1$$

$$-1 + 4 < x < 1 + 4$$

$$3 < x < 5$$

Hp  $\{x \mid 3 < x < 5\}$  atau  $(3, 5)$

Maple :  **$\> \text{solve}(\text{abs}(x-4)<1,\{x\});$**



Contoh 2:

Selesaikan Ketaksamaan :  $| 3x - 5 | \geq 4$

Jawab :

$$3x - 5 \leq -4 \text{ atau } 3x - 5 \geq 4$$

$$3x \leq 1$$

$$3x \geq 9$$

$$x \leq 1/3$$

$$x \geq 3$$

Hp :  $\{ x \mid x \leq 1/3 \text{ atau } x \geq 3 \}$

atau :  $(-\infty, 1/3] \cup [3, \infty)$

Maple := **solve(abs(3\*x-5)>=4,{x});**

# KUADRAT

$$|x|^2 = x^2$$

$$|x| < |y| \Leftrightarrow x^2 < y^2$$

Contoh 1:

Carilah HP dari  $|x + 1| > |x - 3|$

Jawab :

$$(x + 1)^2 > (x - 3)^2$$

$$x^2 + 2x + 1 > x^2 - 6x + 9$$

$$8x > 8 \rightarrow x > 1$$

HP :  $\{x | x > 1\}$  atau :  $(1, \infty)$

Maple :  **$> \text{solve}(\text{abs}(x+1) > \text{abs}(x-3), \{x\});$**

## Contoh 2:

Carilah HP dari :  $|2x - 3| \leq |x + 3|$

Jawab :

$$(2x - 3)^2 \leq (x + 3)^2$$

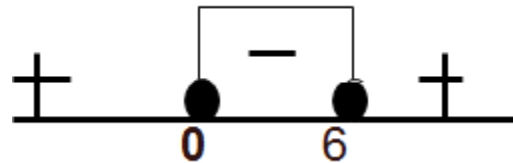
$$4x^2 - 12x + 9 \leq x^2 + 6x + 9$$

$$3x^2 - 18x \leq 0$$

$$x^2 - 6x \leq 0$$

$$x(x - 6) \leq 0$$

$$x = 0 \quad x = 6$$



HP :  $\{ x \mid 0 \leq x \leq 6 \}$  atau  $[0,6]$

Maple : `> solve(abs(2*x-3)<=abs(x+3),{x});`

# Contoh 3

$$|x - 2| > 2 \cdot |x - 1|$$



**PR:** Tentukan HP

1.  $2x + 6 \leq 5x - 3 \leq 2x - 9$

2.  $x^4 - 5x^2 + 4 \geq 0$

3.  $\frac{2x}{x-5} \geq -x$

4.  $2x^2 + 4x - 5 \geq 0$

5.  $\frac{2x+6}{x+1} < x+3$

# PR: Tentukan HP

1.  $2x + 6 \leq 5x - 3 \leq 2x - 9$

2.  $x^4 - 5x^2 + 4 \geq 0$

3.  $\frac{2x}{x-5} \geq -x$

4.  $2x^2 + 4x - 5 \geq 0$

5.  $\frac{2x+6}{x+1} < x+3$

6.  $\left| \frac{1}{2} - 3x \right| < 7$

7.  $\left| \frac{2}{x} \right| > 3$

8.  $\left| \frac{2x+1}{x-1} \right| \leq 2$

9.  $|x - 2| < 3|x + 7|$

10.  $|2x - 5| < |x + 4|$



# Soal-Soal Ketaksamaan

1).  $4x - 7 < 3x + 5$

2).  $7x - 1 \leq 10x + 4$

3).  $3x + 5 > 7x + 15$

4).  $-6 < 2x + 3 \leq -1$

5).  $-3 < 4x - 9 < 11$

6).  $2 + 3x \leq 5x + 1 < 16$

7).  $2x - 4 \leq 6 - 7x \leq 3x + 6$

8).  $x^2 + x - 12 \leq 0$

9).  $x^2 - 5x + 6 > 0$

10).  $3x^2 - 11x - 4 \geq 0$

11).  $2x^2 + 7x - 15 \geq 0$

12)  $\frac{x+5}{2x-1} \leq 0$

13)  $\frac{2x-3}{x+1} > 1$

14)  $\frac{1}{x} < 5$

15)  $\frac{1}{3x-2} \geq 4$

# SOAL - SOAL NILAI MUTLAK:

1.  $|x + 1| < 4$

2.  $|3x + 4| < 8$

3.  $|\frac{x}{3} - 2| \geq 6$

4.  $|2x - 7| > 3$

5.  $|\frac{3x}{5} + 1| \leq 4$

6.  $|\frac{x}{2} + 7| \geq 2$

7.  $2x^2 - 5x - 4 \leq 4$

8.  $3x^2 + x - 1 > 0$

9.  $4x^2 + x - 2 > 0$

10.  $x^2 + 2x - 5 < 0$

11)  $|x - 2| < 3|x + 7|$

12)  $|2x - 5| < |x + 4|$

13)  $2|2x - 3| < |x + 10|$

14)  $|3x - 11| < 2|x + 6|$



Contoh 6 : Carilah HP dari

$$2x + 6 \leq 5x - 3 \leq 2x - 9$$

Maple :

```
> solve (2*x+6<=5*x-3 and 5*x-3<=2*x-9, {x}) ;
```



## Contoh 13:

Selesaikan :  $\frac{2x+6}{x+1} < x+3$

$$\text{HP} = \{ x \in \mathbb{R} \mid -3 < x < -1 \text{ atau } x > 1$$