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KINEMATIC AND DYNAMIC ANALYSIS OF A 2DOF SERIAL MANIPULATOR BY USING MATLAB

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ABSTRACT

In this article, at first, a 2 Degrees of Freedom (DOF) Manipulator is modelled by V-rep, then calculate the mathematical model of kinematic equations of the Manipulator. The Robot kinematic equations depend on the position of links which are connected with the revolute joints and the required target position of end-effector reached by position and orientation of manipulator links. The simulations of the Manipulator are performed in V-rep (Virtual Robot Experiment Platform, Student Edition). Finally, simulations are performed for the manipulator to reach the end effector to the target point at the best possible.

KEYWORDS: Manipulator, Kinematic, Dynamics, MATLAB & V-rep

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Original Article

INTRODUCTION

In last three decades, engineering science of Robotics has made a lot of attention, because of exploring in the complex environment, space, rescue operation, and accomplish task without human effort etc. Knowledge in multidisciplinary subject required to understand the applications and complexity of robots [1],[2]. Nowadays, most of the robot applications are operated by industrial manipulator for mass production. Similarly, robot manipulator can be used as leg of quadruped robot with changing the position and orientation [3].

The manipulator which are working with an open chain mechanism connected by joints and rigid links. To analyse the rigid body motions in space, efficient tools are required to define a well-researched problem. Kinematically redundant manipulators have many advantages over non-redundant manipulator [4][5]. Redundant manipulator can be defined as a given task, infinite number of solutions in joint variables. Robotics requires systematic ways to analyse the kinematics and dynamics of manipulators. To define the various function of joint values like position, orientation and motion of the end effector, it is necessary to create a database for robot joints. Rotary joints provide the path of an industrial robot's TCP and its workspace. The problem of forward kinematics analysis has been investigated as found in many literatures. Various approaches have been used for the analysis. Milicevic et al. (2007) [6] presented the application of a PC by solving the kinematics and dynamics of manipulators. Patel et al. (2013) [7] analysed the singular configurations of 2DOF robot with forward and inverse kinematics.

The literatures conclude that planar robot manipulators direct kinematics and dynamics mechanism to refine the performance and control of two link planar manipulators. It is desirable to compute kinematics and dynamics analysis with MATLAB. The objectives of the present work are to find the mathematical formulation of the two-link planar robot manipulator forward kinematics and dynamics with the help of D-H convention and Newton-Euler method respectively. Initially, CAD model is developed in V-rep software and then Simulate that mathematical formulation using the MATLAB for the forward kinematic and dynamic analysis of two link planar robot manipulator [8]. Then, forward and inverse kinematics experiments are tested in realistic 2 DOF manipulator.

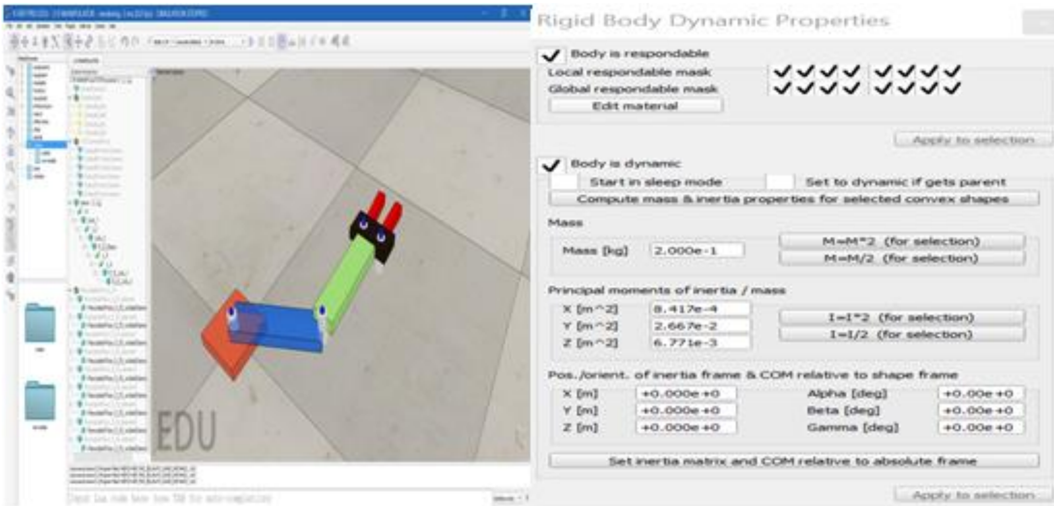

Figure 1: (a) 2 R Robot using V-rep (b) Modify the Rigid Body Dynamics Properties.

Mechanical Design

A model of robot manipulator is designed in V-REP software as shown in Figure 1(a). The robot itself is made up of 2 links namely: base, link_1, link_2 ,where two revolute joints are connected in between links (Fadaei et al, 2017) [9]. The dynamic properties of each link can be found out or modified by selecting each link in model V-rep hierarchy tree. The details of rigid body dynamics properties are shown in fig.1 (b) and Table 1.

Table 1: Material Properties of the Manipulator

Link Descriptions	Unit	Symbol	$Link^1$	$Link^2$
Length	mm	r	200	180
Mass	kg	m	0.021	0.04
Centre of Mass	mm	x_{cm}	19.34	8.33
		y_{cm}	-47.81	-47.81
		z_{cm}	5.00	5.00
Moment of Inertia	$g.mm^2$	I_{xx}	50741.90	42309.72
		I_{yy}	84572	52034.95
		I_{zz}	133860	93132.93

Mathematical Formulation for Analysis

Forward Kinematics

For a serial-chain manipulator, the forward kinematics is essential to find the orientation and position of the end-effectorin cartesian space with the help of all joint angle and link parameters. With the help of all joint angles, forward kinematics gives only one exact solution. The inverse kinematics is the process of finding out the joint space angles with the help of position and orientation of the end-effector. It is a type of serial manipulator which consists of two links and two actuators in 2D plane. The designation “2-R” derives from the fact that the robot has one rotary actuator (i.e., motor) at each of its joints. The details of link parameter for 2-R planer manipulator are as shown in Fig. 2.

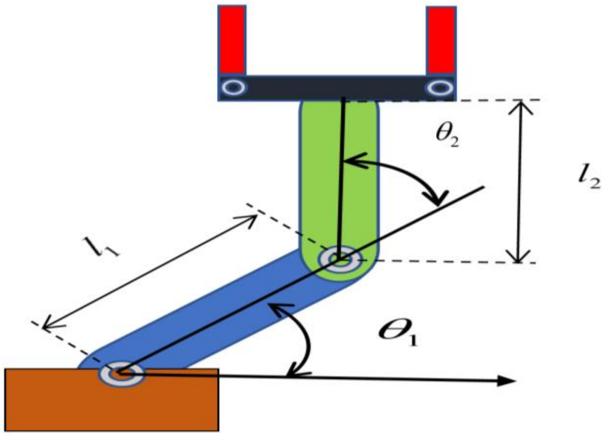

Figure 2: The Planar 2-DOF Manipulator.

D-H Convention

In this D-H convention, transformation related with the joint z_i and x_i. This is the most prominent method for the solving kinematics analysis of robotic system [10]. The four parameters link twist (α_i), link length (r_i), joint angle (θ_i) and link offset (d_i) are associated with link (i) and joint (i) in D-H convention method. Here, z_i is perpendicular to the 2D plane. Where,

Angle α_i is the angle from z_{i-1} to z_i measure about x_i

r_i is the distance from z_{i-1} to z_i measure along x_i

d_i is the offset distance from x_{i-1} to x_i measure along z_{i-1}

Angle θ_i is from x_{i-1} to x_i measure about z_{i-1}

Table 2

Link	r_{i-1} Link length	α_{i-1} Twist Angle	d_i Offset distance	θ_i Joint Angle
1	r_1	0	0	θ_1
2	r_2	0	0	θ_2

The co-ordinate transformation of each link(i) from previous co-ordinate system(i -1) can be obtained as follows in equation (1).

$${}^{i-1}T_i = Rot_x(\alpha_{i-1}).Trans_x(r_{i-1}).Rot_z(\theta_i).Trans_z(d_i)$$

$$= \begin{pmatrix} c\theta_i & -s\theta_i & 0 & r_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & -c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

The end position coordinate of 2-R manipulator with respect to(O_{io}) can be easily obtained through homogeneous transformation denoted by ${}^{0}T_{2}$. Final homogeneous matrix can be written as

$$ {}^{0}T_{2} = {}^{0}T_{1}.{}^{1}T_{2} = \begin{pmatrix} R_{3\times3} & p_{3\times1} \\ 0 & 1 \end{pmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & r_1c_1 + r_2c_{12} \\ s_{12} & c_{12} & 0 & r_1s_1 + r_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

To find the position of end effector point $P = (X, Y)$ with respect to the origin of base frame, that is

$$ x = r_1 \cos\theta_1 + r_2 \cos(\theta_1 + \theta_2) \tag{3} $$

$$ y = r_1 \sin\theta_1 + r_2 \sin(\theta_1 + \theta_2) \tag{4} $$

Finally, two variables (θ_1, θ_2) can be solved easily by two equations (3)-(4).

Inverse Kinematics

In this paper, details the joint variables are calculated by inverse kinematic equation of 2-R manipulator with the use of end-effectors coordinates (p_x, p_y) w.r.t base global point(O_b).Here, trigonometric method is approached in the problem of inverse kinematics to find θ_1, θ_2, given the co-ordinates of end effector point (p_x, p_y)and other constants such as r_1, r_2.The two joint variables are given directly as follows in Equation (5)-(6).

$$ \theta_2 = \pm\cos^{-1}\left(x^2 + y^2 - r_1^2 - r_2^2 \Big/ 2r_1r_2 \right) \tag{5} $$

$$ \theta_1 = a\tan 2(x, y) - a\tan 2(k_1, k_2) \tag{6} $$

Where $k_1 = r_1 + r_2\cos\theta_2$ and $k_2 = r_2 \sin\theta_2$

Velocity Analysis

When directing a robot arm to move from one place to another, it is not just enough to calculate the joint and end effector coordinates of the target position. It can be only calculated how fast robot end effector moves if the joint velocities are known. Hence, it is required to control the joint velocities to get the desired end effector motion. The Jacobian allows the conversion of velocities of individual joints to differential motions of the end effector. The magnitude of the elements of the Jacobian are function of time as joint angle vary with respect to time.

The forward kinematics of a 2 link, position of end effector manipulator is given as:

$$ x = r_1 \cos\theta_1 + r_2 \cos(\theta_1 + \theta_2) \tag{7} $$

$$y = r_1 \sin\theta_1 + r_2 \sin(\theta_1 + \theta_2) \quad (8)$$

To find end effector velocity ẋ, and the joint velocities $\dot{\theta}$, the solution is written in matrix form as follows

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = J(\theta)\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (9)$$

Where, $J(\theta)$ is Jacobean matrix, can be written as

$$J(\theta) = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} \quad (10)$$

Dynamic Modelling

The fundamental approaches to write equation of motion of a quadrupedal robot mechanism are generally represented in two methods: The Newton-Euler formulation and Langrage formulation [11],[12]. For a rigid body, the spatial equation of motion is used for Newton and Euler's equation. The most common conical form of dynamic motion of robot is the joint space formulation is written in equation (11).

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G_g(\theta) = \tau \quad (11)$$

$M(\theta)$, $C(\theta, \dot{\theta})\dot{\theta}$, $G_g(\theta)$ and τ are inertial matrix, Coriolis and centrifugal, gravitational vector and torque output vector respectively. In this paper, langrage equation is used due to more favourable in complex robotic manipulator configuration. The internal force/reaction forces are neglected in this analysis. The dynamic equation of trotting motion can be developed by using langrage equation,

$$Langrange(L) = T - U(Energies)$$

Where, T and U are the total kinetic and potential energy respectively of the mechanical system. The Lagrange's equation for each generalized co-ordinate of dynamic equation of motion can be written in the form (12)

$$\frac{d}{\partial t}\frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i} = \tau_i \quad (12)$$

The dynamic model of a manipulator is useful to calculate the force and torque for the design of joints, links, and actuators. Additionally, dynamic modelling describes the particular dynamic effects (e.g., inertia, Coriolis, centrifugal) to the behaviour of the system. (Matin et al, 2015) For the two -DOF link case, based on Lagrange method, the dynamic equation is derived as follow:

$$\tau_1 = \left\{(m_1 + m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 c_2\right\}\ddot{\theta}_1 + \left\{m_2 r_2^2 + m_2 r_1 r_2 c_2\right\}\ddot{\theta}_2$$
$$-m_2 r_1 r_2 s_2 \dot{\theta}_2^2 - 2m_2 r_1 r_2 s_2 \dot{\theta}_1 \dot{\theta}_2 + (m_1 + m_2) g r_1 s_1 + m_2 g r_2 s_{12} \quad (13)$$

$$\tau_2 = \left\{ m_2 r_2^2 + m_2 r_1 r_2 c_2 \right\} \ddot{\theta}_1 + m_2 r_2^2 \ddot{\theta}_2 + m_2 r_1 r_2 s_2 \dot{\theta}_1^2 + m_2 g r_2 s_{12} \quad (14)$$

Above two equation (13)-(14) can be written in matrix form as

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} (m_1 + m_2) r_1^2 + m_2 r_2^2 + 2 m_2 r_1 r_2 c_2 & m_2 r_2^2 + m_2 r_1 r_2 c_2 \\ m_2 r_2^2 + m_2 r_1 r_2 c_2 & m_2 r_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & -m_2 r_1 r_2 s_2 \\ m_2 r_1 r_2 s_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix}$$
$$+ \begin{bmatrix} -m_2 r_1 r_2 s_2 & -m_2 r_1 r_2 s_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 \\ \dot{\theta}_2 & \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} (m_1 + m_2) g r_1 s_1 + m_2 g r_2 s_{12} \\ m_2 g r_2 s_{12} \end{bmatrix}$$

Where, m_1, m_2 are the mass of the link1, link2 respectively. Also, r_1, r_2 are the lengths of link1,link2 respectively. Where, θ_1, θ_2 are the angles between link1 and link2 with X axis respectively.

ANALYSIS AND RESULT

Experimentation on 2-R Planar Serial Manipulator

In order to validate the forward kinematic equations, a 2R planar robot has been fabricated and controlled using the solutions of equations obtained by solving in MATLAB. The followings apparatus required for making a 2R planar robot are shown in Table 2.

Table 2: Apparatus Required for Making a 2R Planar Robot

Sl. No.	Name	Quantity (in Nos.)	Description
1	Servo motors	2	6V operate voltage
2	Arduino uno	1	7-12V input voltage
3	AAA Pencil battery	6	1.5V each
4	Jumper wires	20	-
5	Aluminium Beams	2	-
6	Bread board	1	-
7	Nuts and bolts	10	-

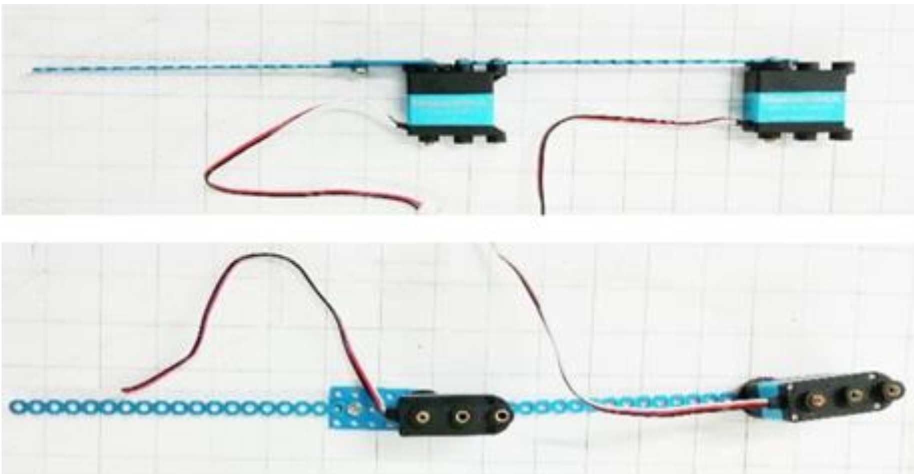

Figure 3: Assembly of 2R Planar Manipulator.

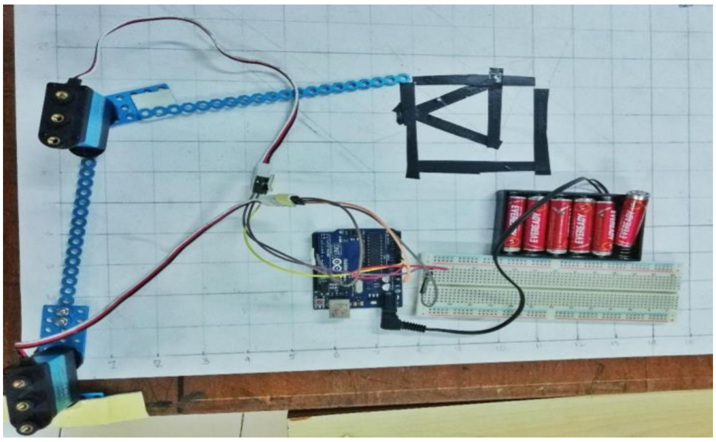

Figure 4: Experimental Setup of 2R Planar Manipulator.

Experimental setup of all the parts 2R planar manipulator are shown in details in fig.3-4. Initially, both the servo motors are set with 0° in such that both servo motor perpendicular to one another. As the experiment is performed on 2D planei.e. X and Y coordinates, servo angles are restricted in between two range.

For servo1 $0^{\circ} \le \theta_1 \le 90^{\circ}$

For servo2 $0^{\circ} \le \theta_2 \le 180^{\circ}$

The workspace of 2-R manipulator will be varied accordingly if angle boundary changes. The workspace of 2-R manipulator with above mentioned boundary conditions is shown in Fig.5For 2R planar manipulator the D-H parameters of different link are shown in Table 3

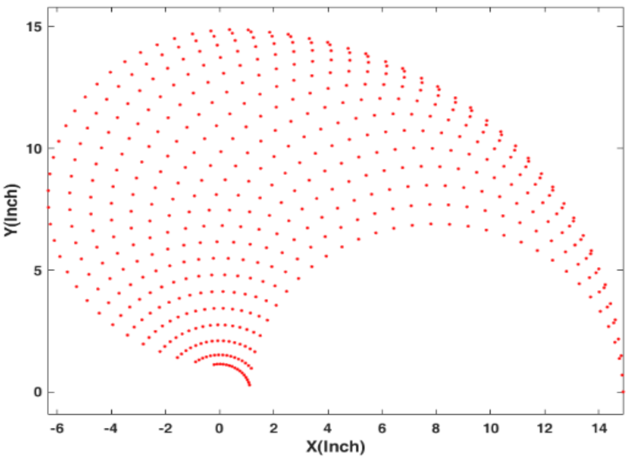

Figure 5: Workspace of 2-R Planar Manipulator.

Table 3: The D-H Parameters of a Planar 2R Manipulator

Link	d		θ_i	r (inch)	α_i
1.	0		25°	8	0
2.	0		45°	6.9	0

Experiment 1: Forward Kinematics

In forward kinematics point P (X, Y) will be traced for given the values of θ_1 and θ_2 and considering all other constants such as r1, r2 fixed.

MATLAB Code

```
clc
clear all
r1 = 8; % length of first arm
r2 = 6.9; % length of second arm
THETA1 = 25*pi/180;
THETA2 = 45*pi/180;
X = r1 * cos(THETA1) + r2 * cos(THETA1 + THETA2); % compute x
coordinates
Y = r1 * sin(THETA1) + r2 * sin(THETA1 + THETA2); % compute y
coordinates
```

To verify the forward kinematics of 2R planar manipulator, firstly have to find co-ordinates of X and Y with the help of above MATLAB code. After the calculations of X and Y coordinates, all those inputs put into Arduino uno, for which a separate code has been written again to conduct the experiment. Then, the codes are burned into Arduino and join all the connections as shown in fig.6.

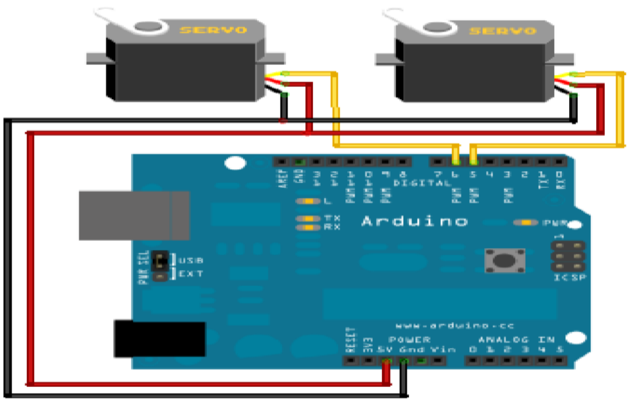

Figure 6: Servo and Arduino Connections.

Experiment 2: Inverse Kinematics

In this experiment will involve finding the values of θ_1 and θ_2,given the co-ordinates of point P=(X,Y) and other constants such as r_1 and r_2 .For smooth conduct of inverse kinematics, three different cases like tracing a line, tracing a triangle, tracing a square are considered to perform the experiment.

- Tracing a line
- Tracing a triangle
- Tracing a square

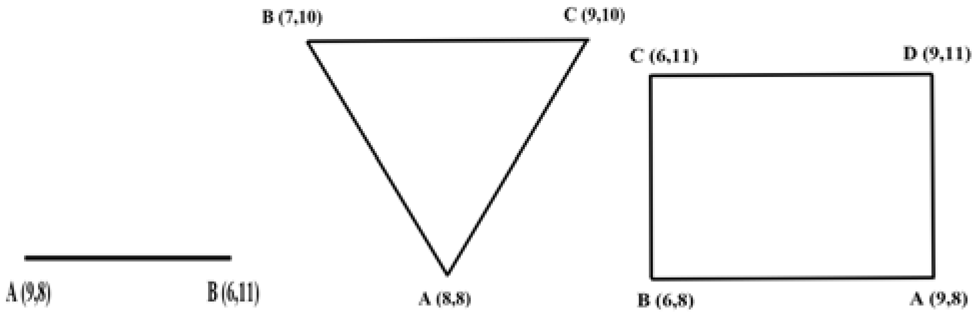

Figure 7

To trace a line, at least two coordinates of a point in a plane are required. The co-ordinates of each point are mentioned in details. Then, the degrees of rotation for both servo1 and servo 2 are to be determined with the help of MATLAB. The details angle variation of each tracing is shown in fig. (7)-(9).

MATLAB Code

```
clc
clear all
r1= 8;
r2= 6.9;
X= linspace(9,6,10);
Y= linspace(8,11,10);
c2 = (X.^2 + Y.^2 - r1^2 - r2^2)/(2*r1*r2);
s2 = sqrt(1 - c2^2);
THETA2D = atan2(s2,c2); % theta2 is deduced
k1 = r1 + r2.*cos(THETA2D);
k2 = r2*sin(THETA2D);
THETA1D = atan2(Y,X) - atan2(k2,k1); % theta1 is deduced
```

Arduino Code

```
#include <Servo.h>
Servo myservo1;
Servo myservo2;
int theta1[10]={ 8.5284,10.6643,12.9109,15.2613,17.7092,20.2491, 22.8765, 25.5881,28.3820,31.2582};
int   theta2[10]={72.3954,72.6373,72.6373,72.3954,71.9108,71.1813,70.2037,68.9731 ,67.4828,65.7235};
inti;
```

```
void setup()
{
myservo1.attach(5);
myservo2.attach(6);   }
void loop()
{
myservo1.write(0);
myservo2.write(0);
delay(3000);
for(i=0; i<=9; i++)
  {
myservo1.write(theta1[i]);
myservo2.write(theta2[i]);
delay(1500);
```

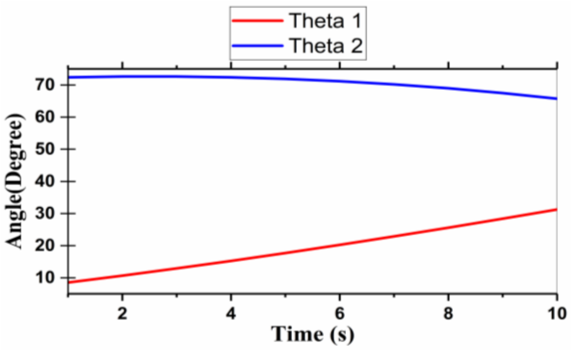

Figure 7: The Rotating Angle θ_1 and θ_2 during Tracing a Line.

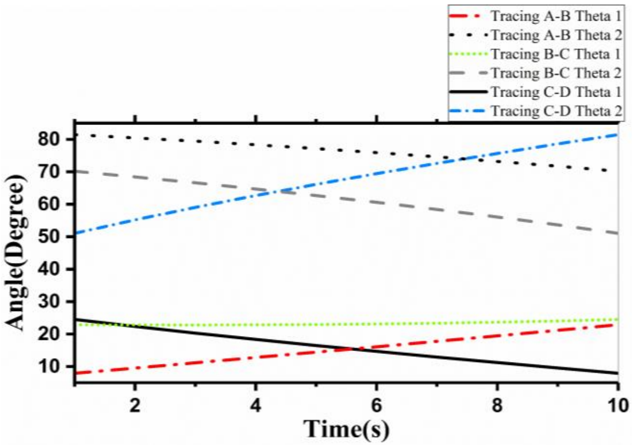

Figure 8: The Rotating Angle θ_1 and θ_2 during Tracing a Triangle

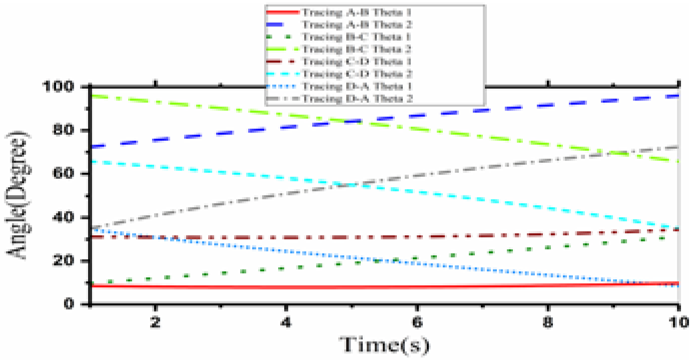

Figure 9: The Rotating Angle θ_1 and θ_2 during Tracing a Square.

Robot Dynamics Analysis

Equations are evaluated in numeric or recursive manner. The problem formulation for equations of motion of direct dynamics mechanical system of two link planar robot manipulator has been carried out using Newton-Euler formulation which is based on the force-moment balance where the sum of forces which are acting on the links is equal to rate of change of linear momentum. The variation of joint angle, angular velocity, torques at each joint are obtained by two approaches MATLAB and V-rep. The simulation analyses of both the approaches are shown in Fig.10-12. In fig.12, It is observed that joint 1 produced more torque compared to joint 2 during the tracing. The maximum torque in joint 1 and 2 are observed 105Nm and 60 Nm respectively. The simulation results of both approaches are quite similar during the line tracing.

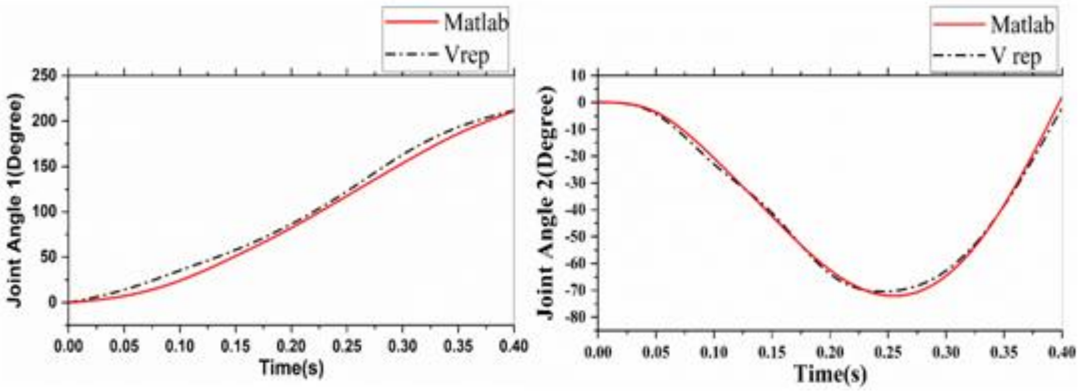

Figure 10: Joint Angle Simulation in Matlab and V-rep.

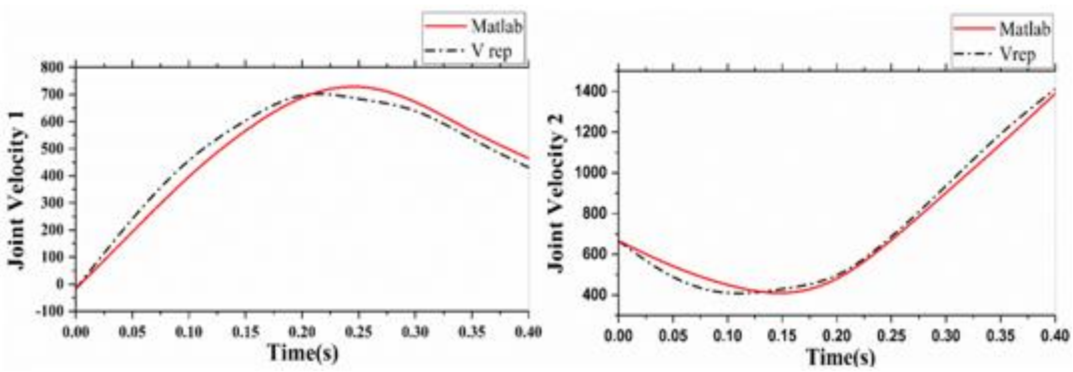

Figure 11: Joint Velocity Analysis in Matlab and V-rep.

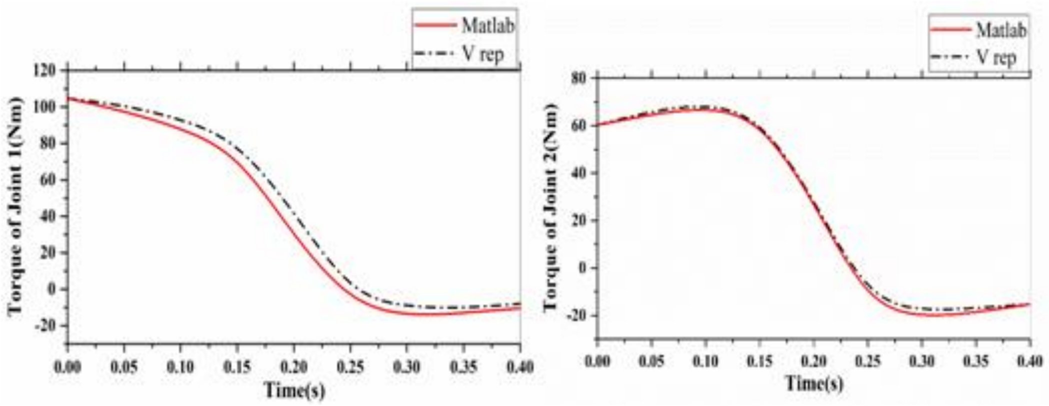

Figure 12: Joint Torque Simulation in Matlab and V-rep.

CONCLUSIONS

In this paper, the complete mathematical formulation for forward kinematics and dynamics modelling of two link planar robot manipulator having two degree of freedom are derived. By using the D-H parameters and homogeneous transformation matrices, the mathematical equations for the position, velocity and acceleration of end effectorare deduced. The Denavit Hartenberg parameters are used to determine the homogeneous coordinate transformation matrices through their different orientations and position of link. Similarly, for direct dynamics modelling, mathematical equations of motion for two link planar robot manipulator system by using Newton-Euler formulation which is based on the force-moment balance. For the computational analysis of mathematical formulation of complete forward kinematics and dynamics of the system MATLAB code are developed in the form of several M-files. The various simulations analysis has been performed and corresponding results are also plotted in details.

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