Monday, March 24, 2014

CHAPTER 15

Kinematics of Rigid Bodies

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Chapter Outline

1. **Introduction**
2. **Translation**
3. **Rotation About a Fixed Axis**
4. **General Plane Motion**
   - ✓ Absolute and Relative Velocity
   - ✓ Instantaneous Center of Rotation
   - ✓ Absolute and Relative Acceleration
   - ✓ Analysis of Plane Motion in Terms of a Parameter
5. **Rate of Change wrt a Rotating Frame**
6. **Coriolis Acceleration**

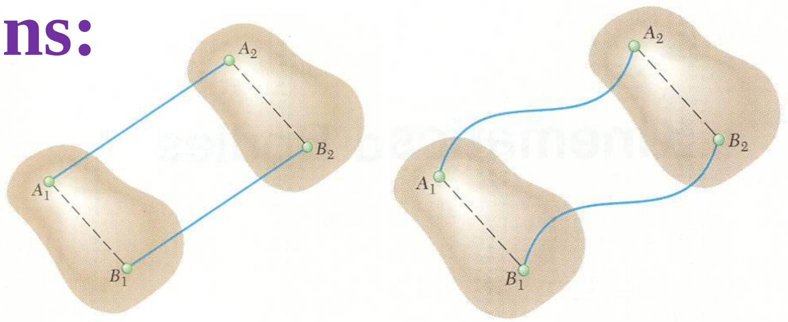
Introduction

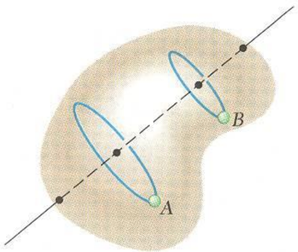
Kinematics of rigid bodies (RB): relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.

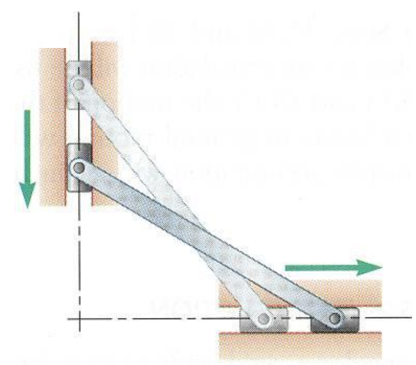
Introduction

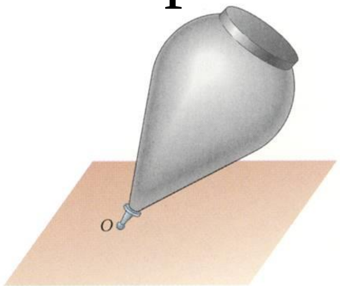
Classification of RB motions:

- Translation:
  - rectilinear translation
  - curvilinear translation
- Rotation about a fixed axis
- General plane motion
- Motion about a fixed point
- General motion

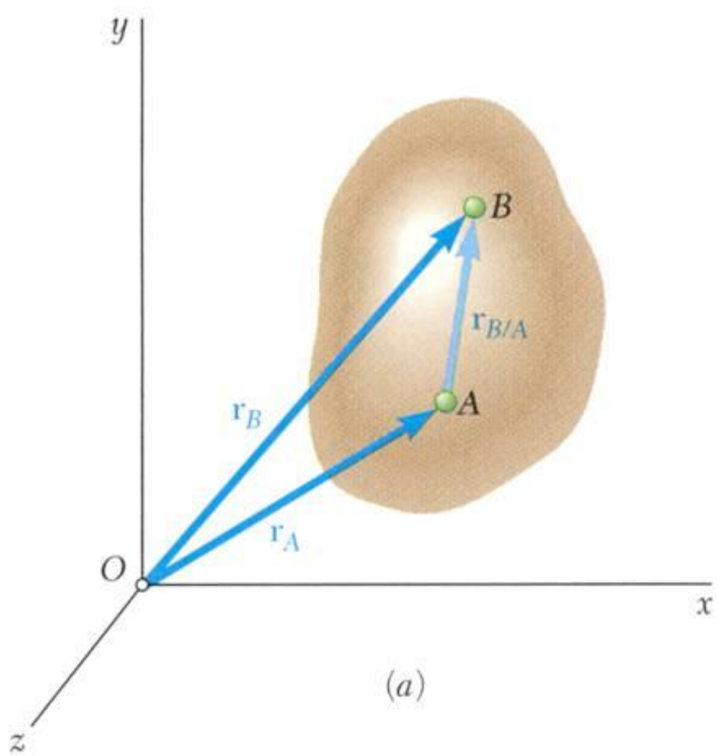








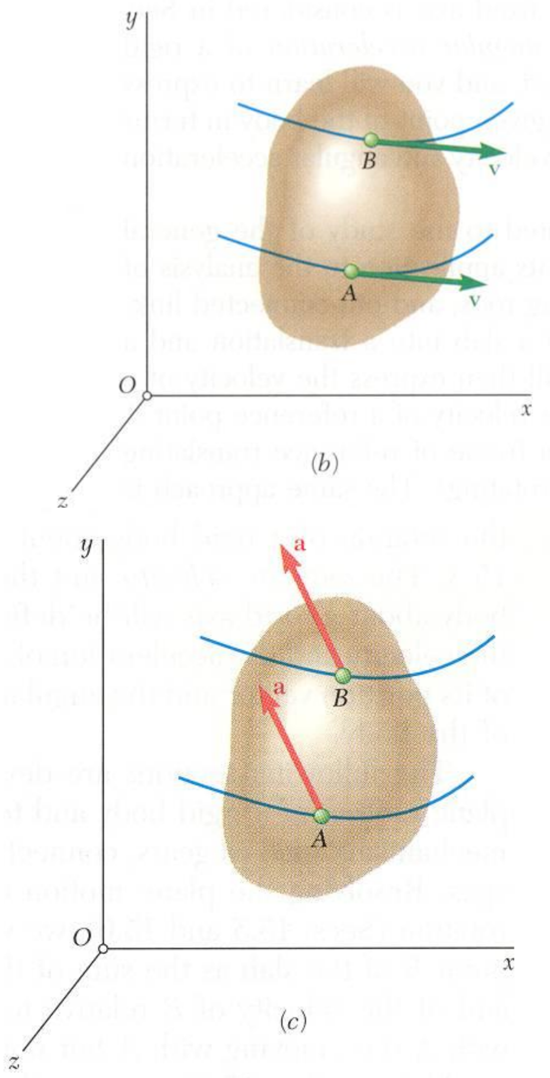
Translation



- **RB in translation:**
  - Direction of any straight line inside body is constant
  - All particles forming RB move in parallel lines.
- For any two particles in the body,

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Translation



- Differentiating wrt time,

$$\dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} = \dot{\vec{r}}_A$$

$$\vec{v}_B = \vec{v}_A$$

All particles have same velocity.

- Differentiating wrt time again,

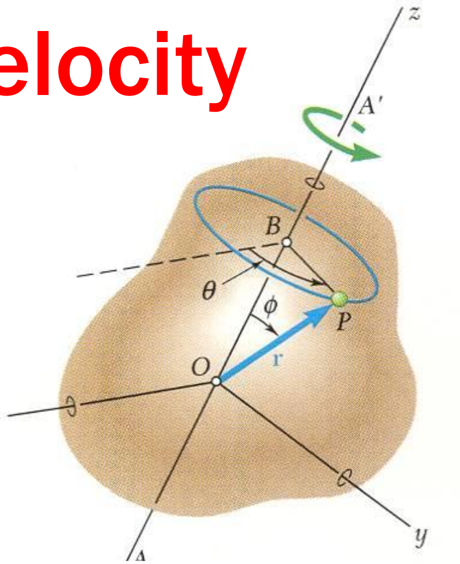
$$\ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A} = \ddot{\vec{r}}_A$$

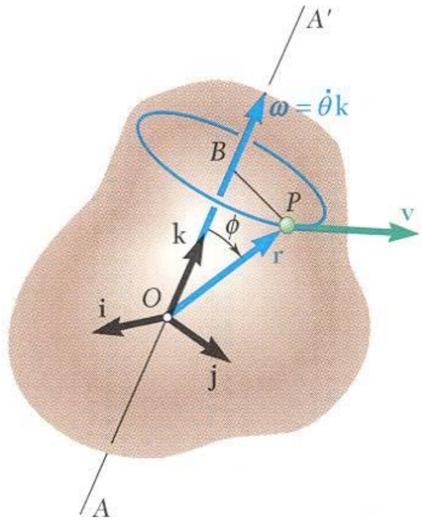
$$\vec{a}_B = \vec{a}_A$$

All particles have same acceleration.

Rotation About a Fixed Axis

Velocity





- Velocity vector $\vec{v} = d\vec{r}/dt$ of particle P is tangent to the path with magnitude $v = ds/dt$

$$\Delta s = (BP)\Delta\theta = (r\sin\phi)\Delta\theta$$

$$v = \frac{ds}{dt} = \lim_{\Delta t \to 0}(r\sin\phi)\frac{\Delta\theta}{\Delta t} = r\dot{\theta}\sin\phi$$

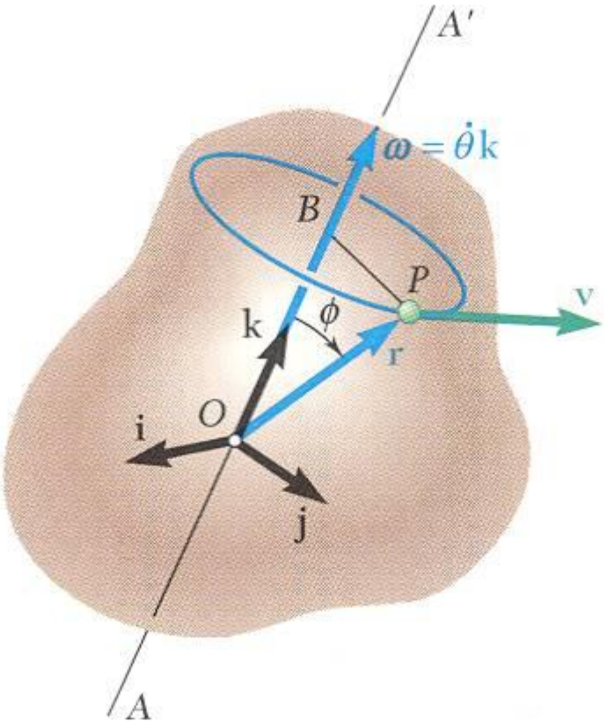
$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega}\times\vec{r}$$

$$\vec{\omega} = \omega\vec{k} = \dot{\theta}\vec{k} = angular\ velocity$$

Rotation About a Fixed Axis

Acceleration

- Differentiating to get,



$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\vec{\omega}\times\vec{r}\right) = \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}$$

$$= \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\vec{v}$$

$$\frac{d\vec{\omega}}{dt} = \vec{\alpha} = angular\ acceleration$$

$$= \alpha\vec{k} = \dot{\omega}\vec{k} = \ddot{\theta}\vec{k}$$

$$\vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{\omega}\times\vec{r}$$

$$\vec{\alpha}\times\vec{r} = \text{tangential acceleration component}$$

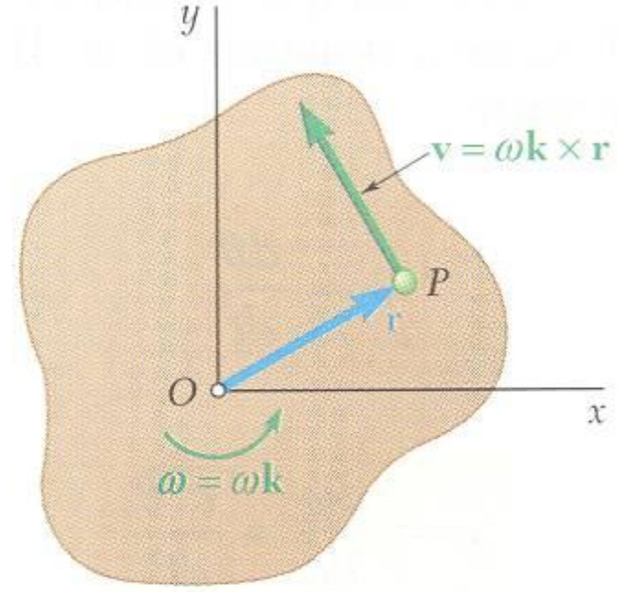
$$\vec{\omega}\times\vec{\omega}\times\vec{r} = \text{radial acceleration component}$$

Rotation About a Fixed Axis

Representative Slab

- Velocity of any point *P* of slab,

$$\vec{v} = \vec{\omega} \times \vec{r} = \omega \vec{k} \times \vec{r} v = r\omega$$



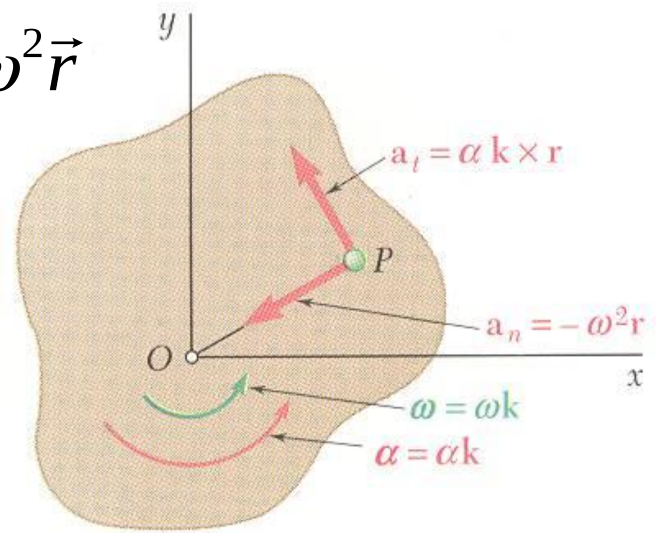
- Acceleration of point *P* of slab,

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r} = \alpha \vec{k} \times \vec{r} - \omega^2 \vec{r}$$

- Tangential & normal components of acceleration ,

$$\vec{a}_t = \alpha \vec{k} \times \vec{r} \qquad a_t = r\alpha$$

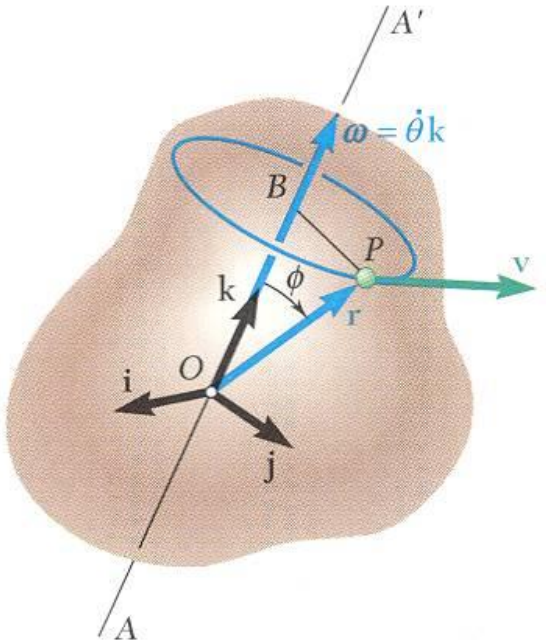
$$\vec{a}_n = -\omega^2 \vec{r} \qquad a_n = r\omega^2$$



Equations Defining Rotation of a RB About a Fixed Axis

$$\omega = \frac{d\theta}{dt} \quad \text{or} \quad dt = \frac{d\theta}{\omega}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta}$$



- *Uniform Rotation,* $\alpha = 0$:

$$\theta = \theta_0 + \omega t$$

- *Uniformly Accelerated Rotation,* α = constant:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

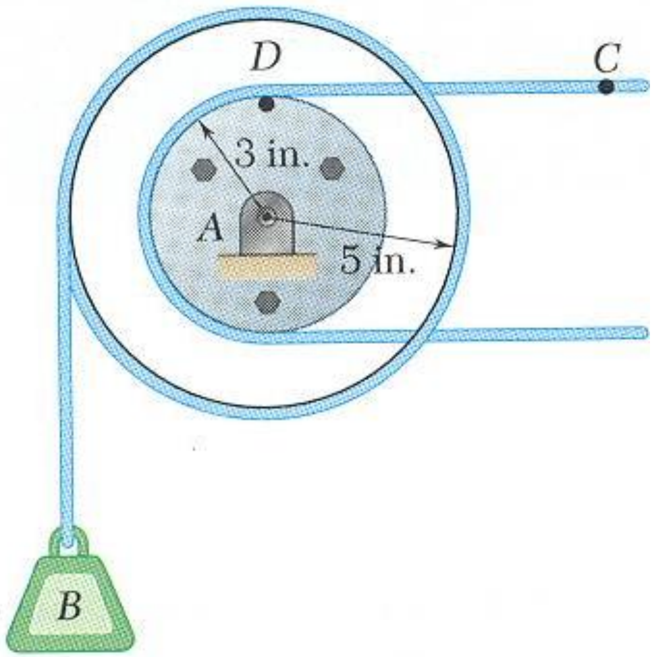
Sample Problem 5.1

Cable *C* has a constant acceleration of 9 in/s^2 and an initial velocity of 12 in/s, both directed to the right. Determine:

a. number of revolutions of pulley in 2 s,

b. velocity & change in position of load *B* after 2 s

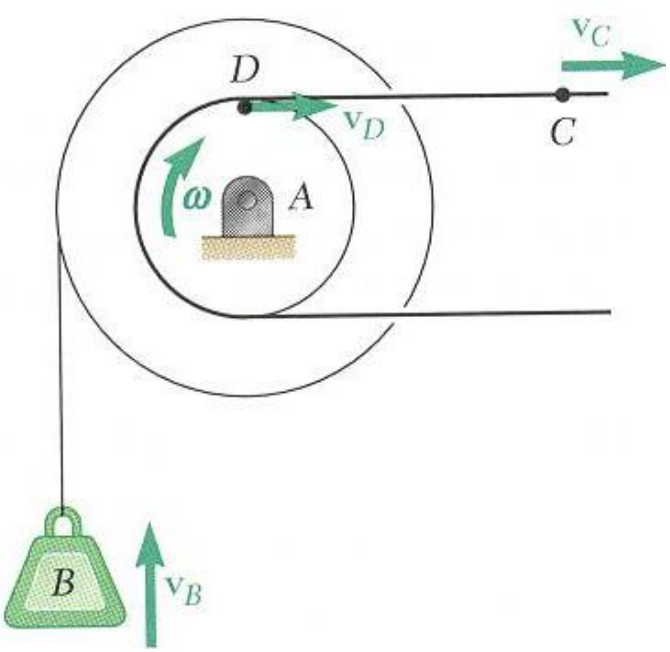
c. acceleration of point *D* on the rim of the inner pulley at $t = 0$.



Sample Problem 5.1

Solution:

- Tangential velocity & acceleration of D are equal to velocity & acceleration of C.



$$(\vec{v}_D)_0 = (\vec{v}_C)_0 = 12\,\text{in./s} \rightarrow$$

$$(v_D)_0 = r\omega_0$$

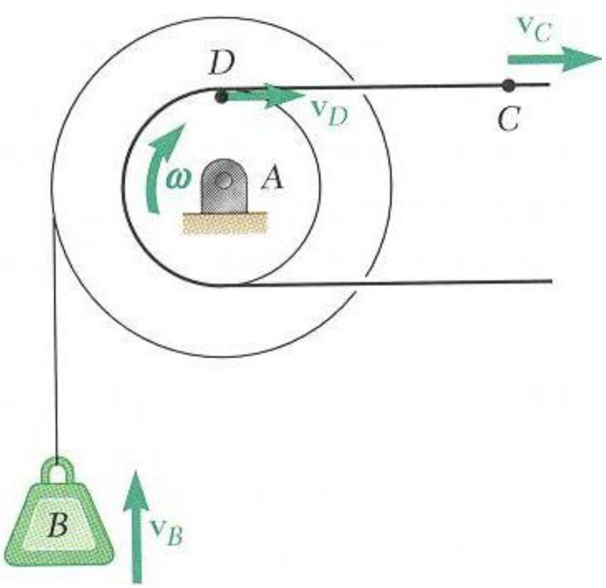
$$\omega_0 = \frac{(v_D)_0}{r} = \frac{12}{3} = 4\,\text{rad/s}$$

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(a_D)_t = r\alpha$$

$$\alpha = \frac{(a_D)_t}{r} = \frac{9}{3} = 3\,\text{rad/s}^2$$

Sample Problem 5.1



- Uniformly accelerated rotation: determine velocity & angular position of pulley after 2 s.

$$\omega = \omega_0 + \alpha t = 4\,\text{rad/s} + \left(3\,\text{rad/s}^2\right)(2\,\text{s}) = 10\,\text{rad/s}$$

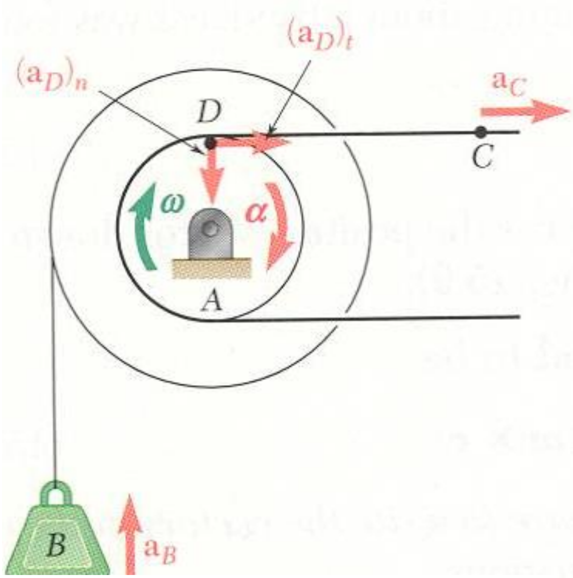
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$= (4\,\text{rad/s})(2\,\text{s}) + \tfrac{1}{2}\left(3\,\text{rad/s}^2\right)(2\,\text{s})^2$$
$$= 14\,\text{rad}$$

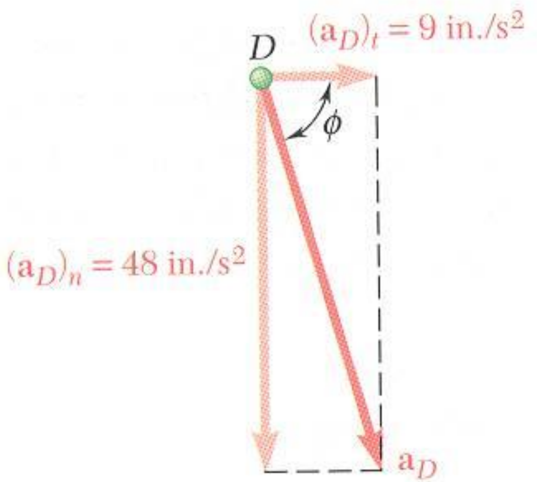
$$N = (14\,\text{rad})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) = \text{number of revs} = 2.23\text{rev}$$

$$v_B = r\omega = (5\,\text{in.})(10\,\text{rad/s}) = 50\ in/s\ (\text{upward})$$

$$\Delta y_B = r\theta = (5\,\text{in.})(14\,\text{rad}) = 70\ in$$

Sample Problem 15.1





- Initial tangential & normal acceleration components of *D*.

$$(\vec{a}_D)_t = \vec{a}_C = 9\,\text{in./s} \rightarrow$$

$$(a_D)_n = r_D\omega_0^2 = (3\,\text{in.})(4\,\text{rad/s})^2 = 48\,\text{in/s}^2 \text{ (downward)}$$

Magnitude & direction of total acceleration,

$$a_D = \sqrt{(a_D)_t^2 + (a_D)_n^2}$$

$$= \sqrt{9^2 + 48^2} = 48.8\,in/s2$$

$$\tan\phi = \frac{(a_D)_n}{(a_D)_t} = \frac{48}{9} \qquad \phi = 79.4°$$

Home Work Assignment # 15.1

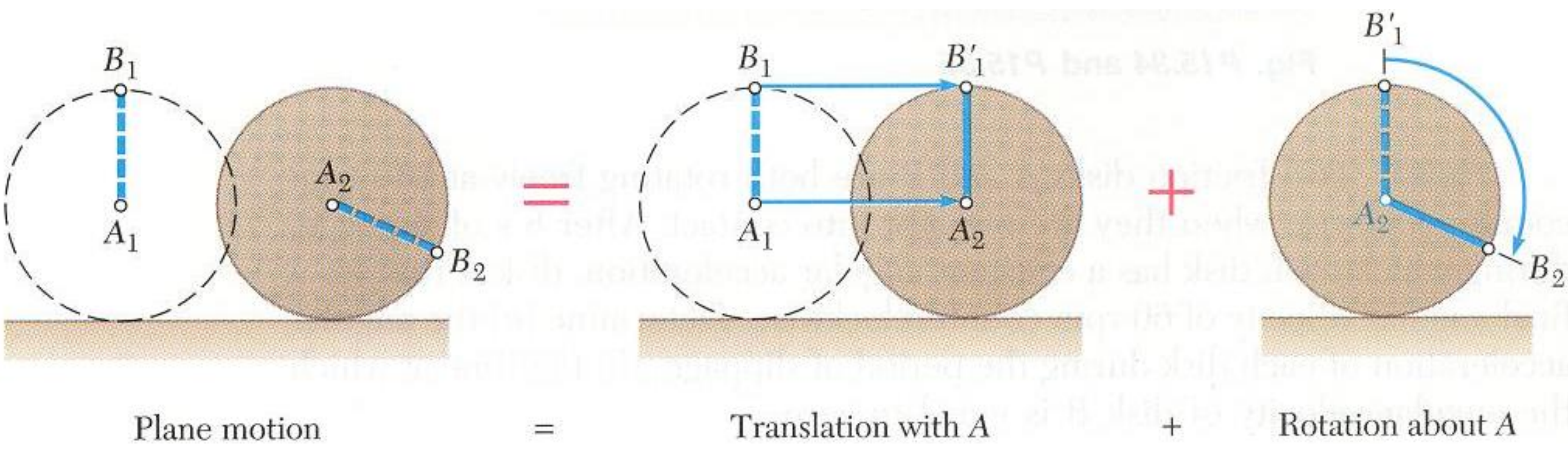
1, 8, 14, 21, 28, 35

Due

Saturday 29/3/2014 (Civil)

Sunday 30/3/2014 (Mechanical)

General Plane Motion (GPM)

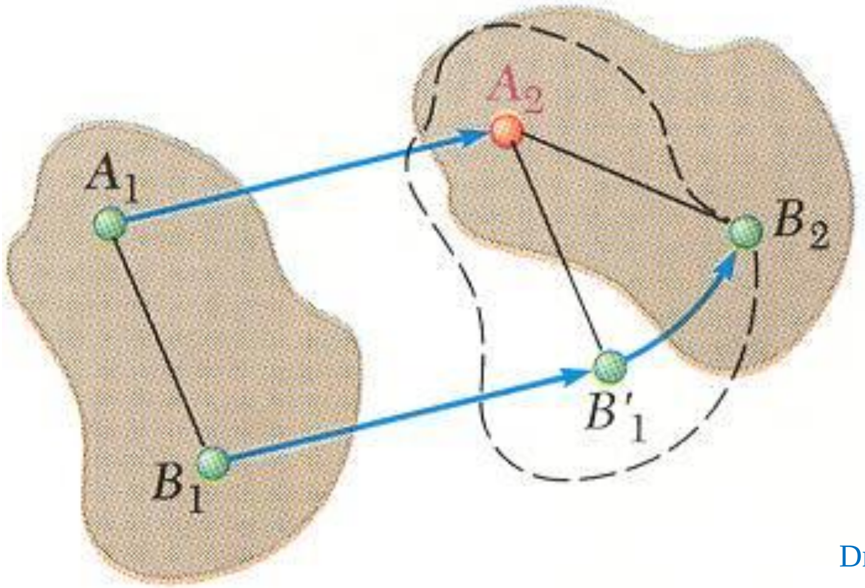


- *GPM is n*either a translation nor a rotation.
- GPM = translation + rotation

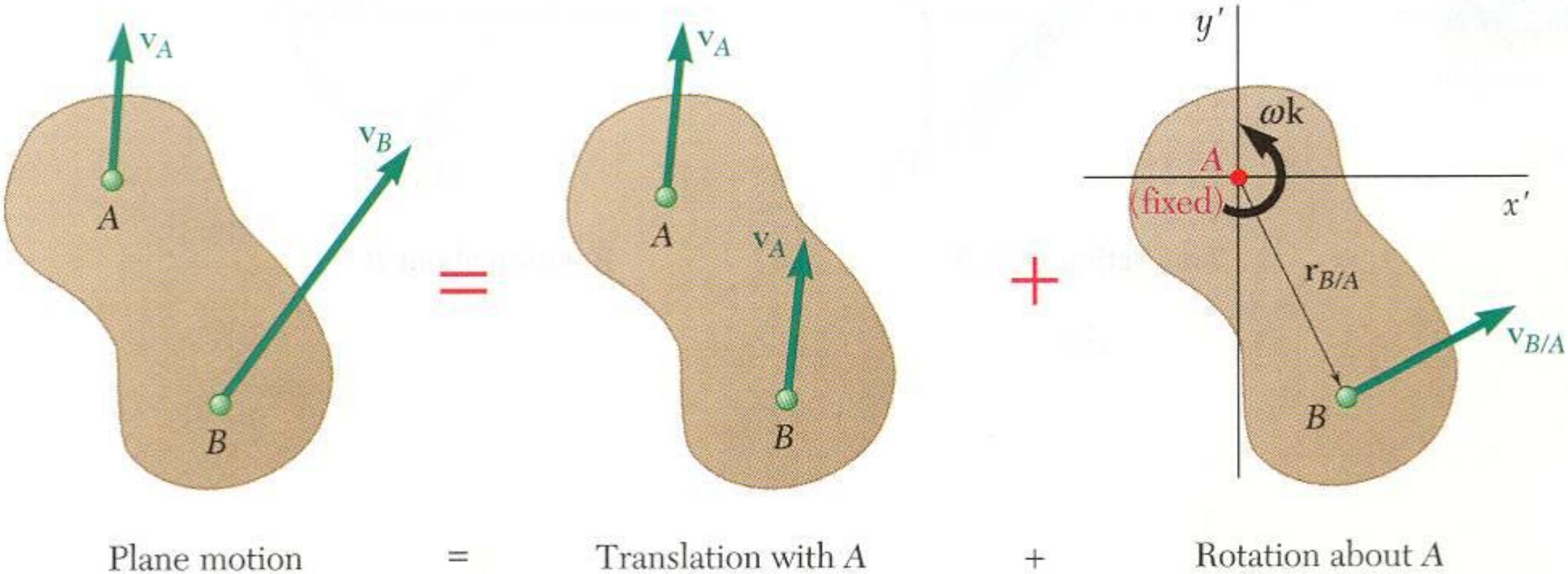
General Plane Motion (GPM)

Displacement of particles A_1 & B_1 to A_2 & B_2 can be divided into two parts:

1. translation to A_2
2. rotation of B_1’ about A_2 to B_2



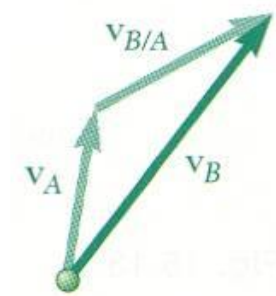
Absolute and Relative Velocity in Plane Motion



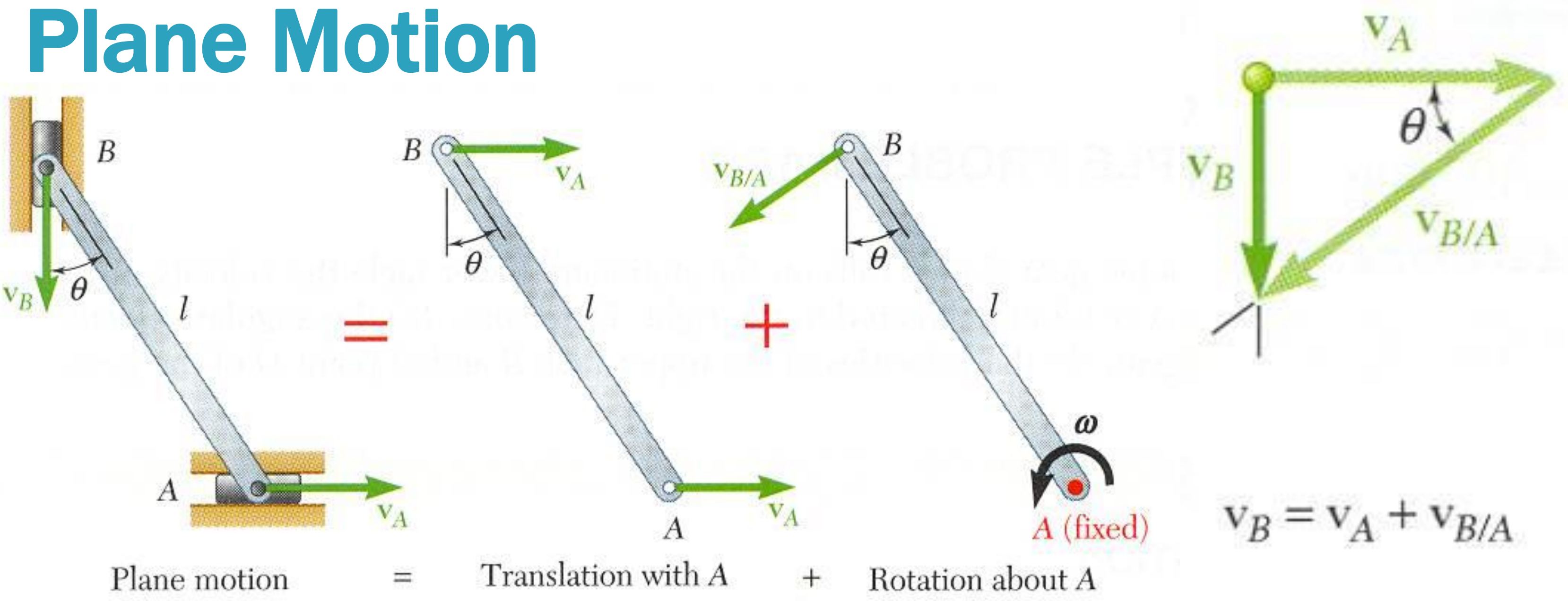
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \omega \vec{k} \times \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \omega \vec{k} \times \vec{r}_{B/A}$$

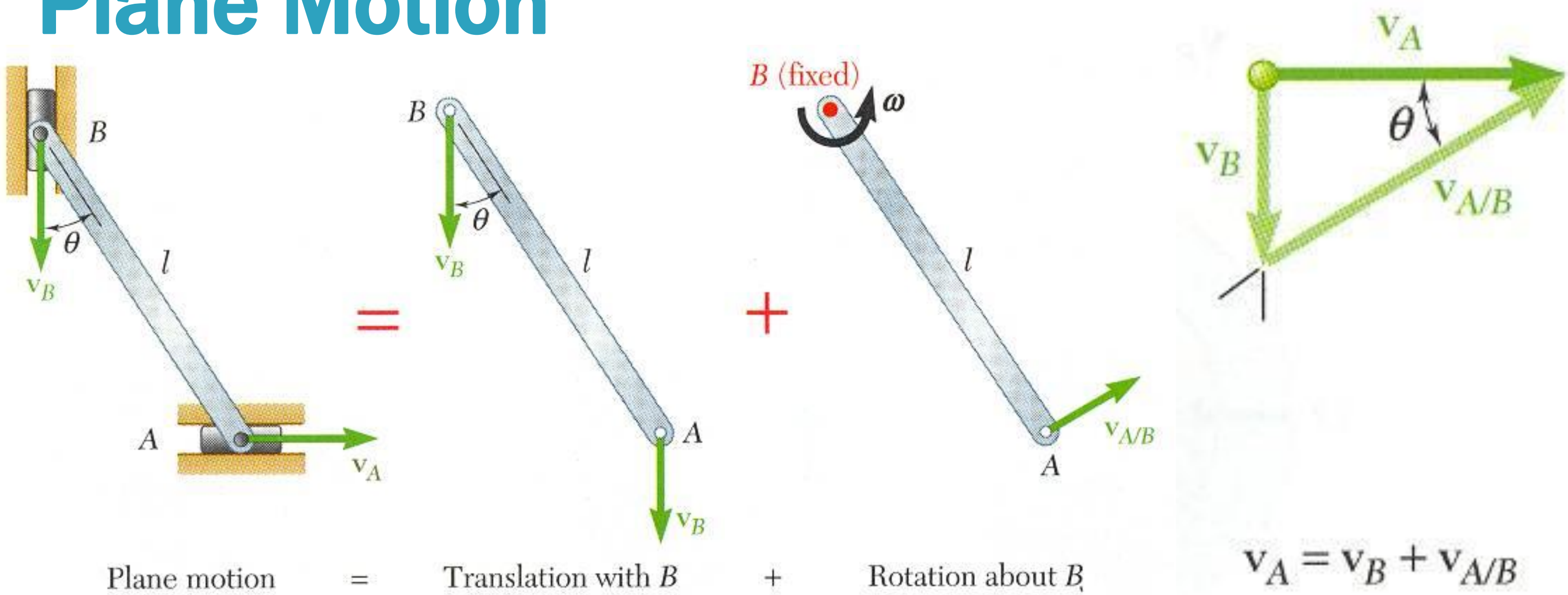


Absolute and Relative Velocity in Plane Motion



Assuming that velocity v_A of end A is known, determine velocity v_B of end B and angular velocity ω in terms of v_A, l, and θ.

Absolute and Relative Velocity in Plane Motion



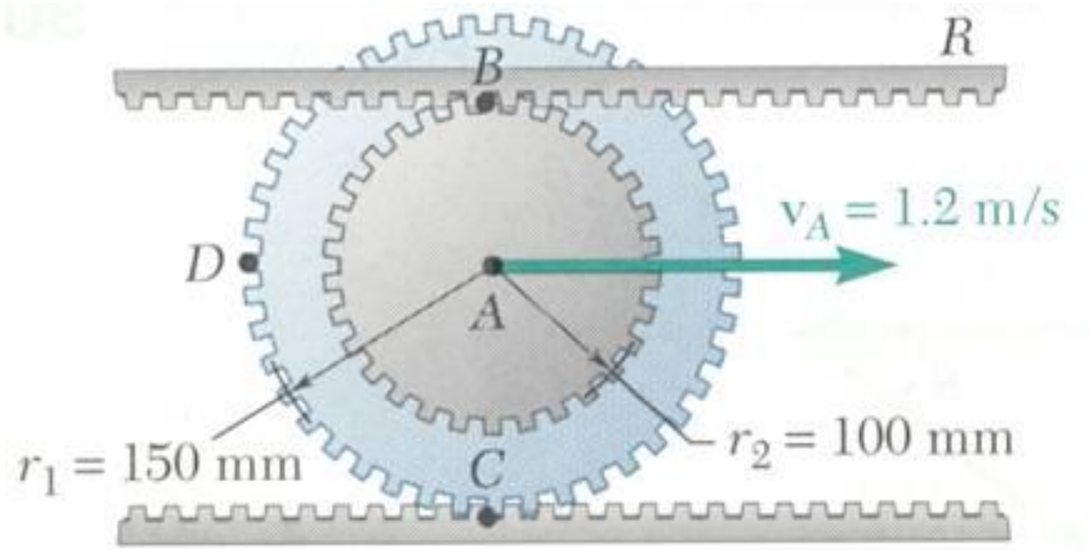
Selecting point B as the reference point and solving for velocity v_A of end A and angular velocity ω leads to an equivalent velocity triangle.

Sample Problem 15.2

The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s. Determine:

a. angular velocity of gear

b. velocities of upper rack *R* and point *D* of gear.



Sample Problem 15.2

Solution:

Displacement of gear center in one revolution = outer circumference

$$\frac{x_A}{2\pi r} = -\frac{\theta}{2\pi} \qquad x_A = -r_1\theta$$

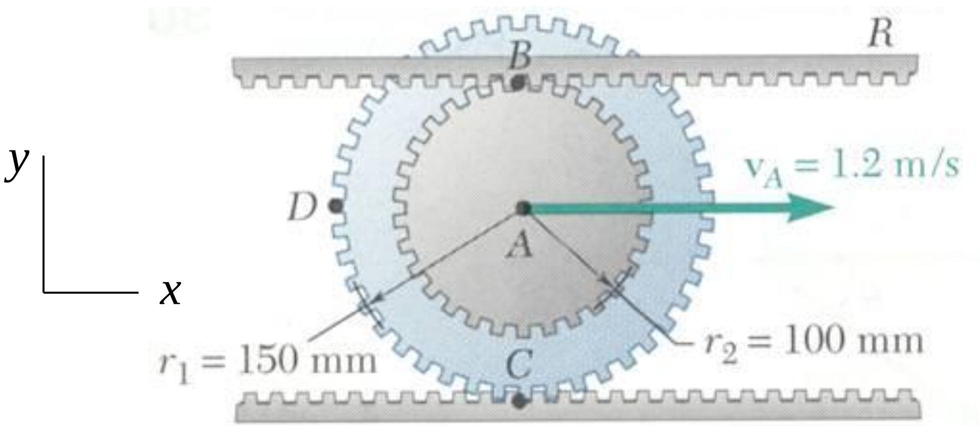
For $x_A > 0$ (to right), $\omega < 0$ (rotates cw),

Differentiate to relate translational & angular velocities.

$$v_A = -r_1\omega$$

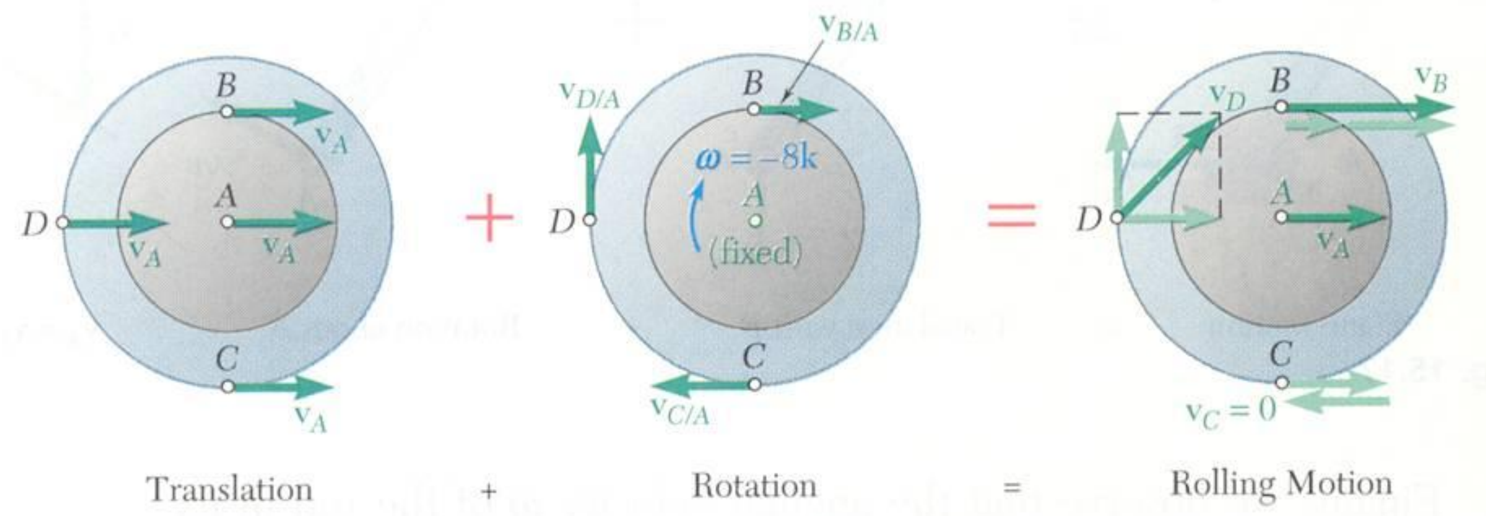
$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\mathrm{m/s}}{0.150\,\mathrm{m}}$$

$$\vec{\omega} = \omega\vec{k} = -(8\,\mathrm{rad/s})\vec{k}$$



Sample Problem 15.2

- For any point P on the gear, $\vec{v}_P = \vec{v}_A + \vec{v}_{P/A} = \vec{v}_A + \omega\vec{k} \times \vec{r}_{P/A}$

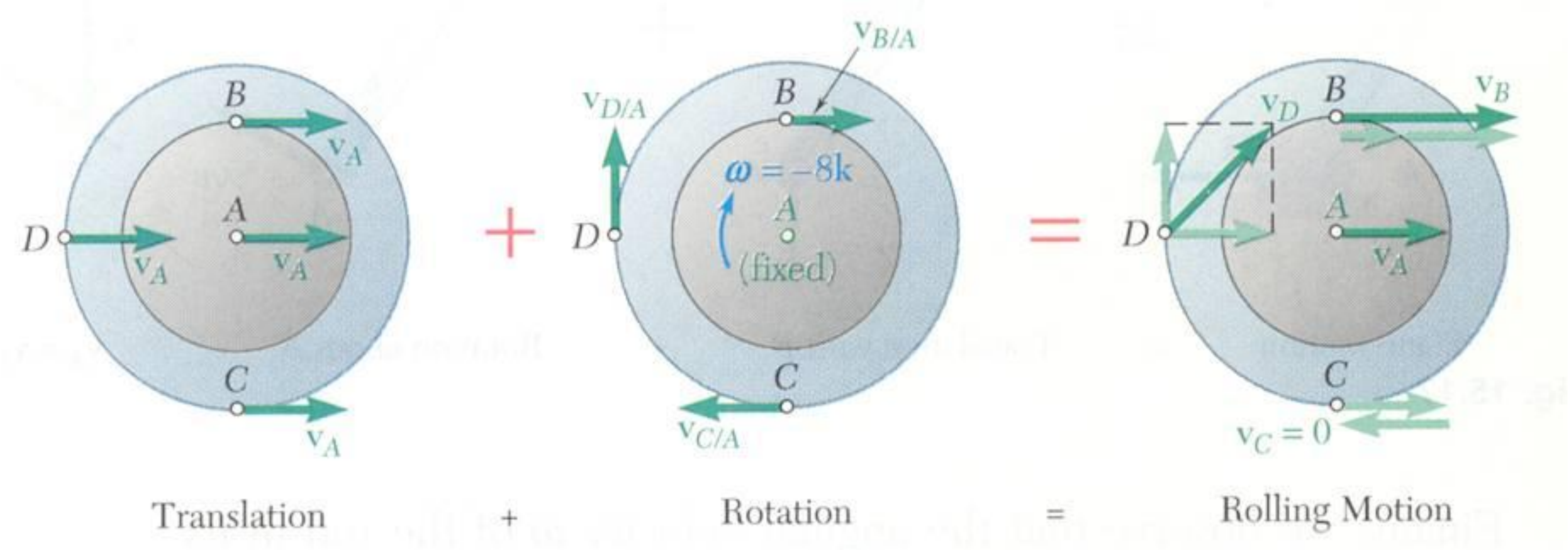


Velocity of upper rack is equal to velocity of point B:

$$\vec{v}_R = \vec{v}_B = \vec{v}_A + \omega\vec{k} \times \vec{r}_{B/A} = (1.2\,\mathrm{m/s})\vec{i} + (8\,\mathrm{rad/s})\vec{k} \times (0.10\,\mathrm{m})\vec{j}$$

$$= (1.2\,\mathrm{m/s})\vec{i} + (0.8\,\mathrm{m/s})\vec{i} \qquad \vec{v}_R = (2\,\mathrm{m/s})\vec{i}$$

Sample Problem 15.2



Velocity of the point *D*:

$$\vec{v}_D = \vec{v}_A + \omega\vec{k} \times \vec{r}_{D/A} = (1.2\,\text{m/s})\vec{i} + (8\,\text{rad/s})\vec{k} \times (-0.150\,\text{m})\vec{i}$$

$$\vec{v}_D = (1.2\,\text{m/s})\vec{i} + (1.2\,\text{m/s})\vec{j}$$

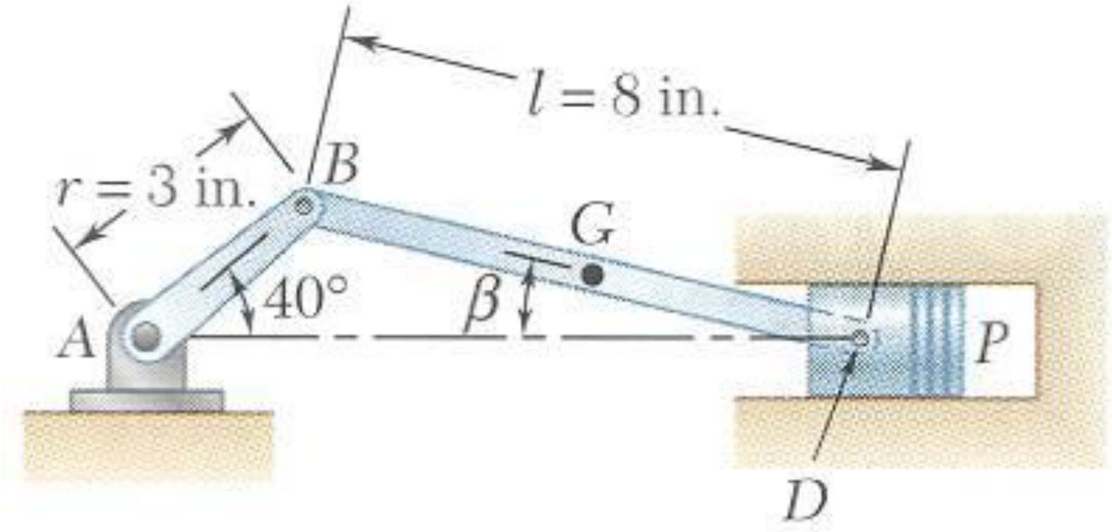
$$v_D = 1.697\,\text{m/s}$$

Sample Problem 15.3

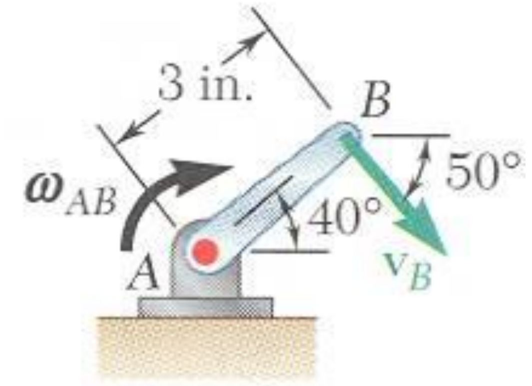
The crank *AB* has a constant cw angular velocity of 2000 rpm. For the crank position indicated, determine:

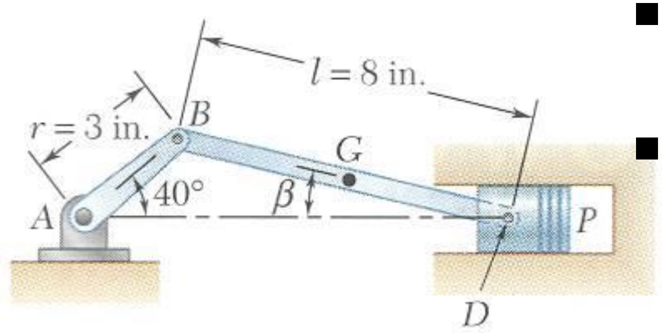
a. angular velocity of connecting rod *BD*

b. velocity of piston *P*



Sample Problem 15.3





Solution: $\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$

$$\omega_{AB} = \left(2000\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{\text{min}}{60\,\text{s}}\right)\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)$$

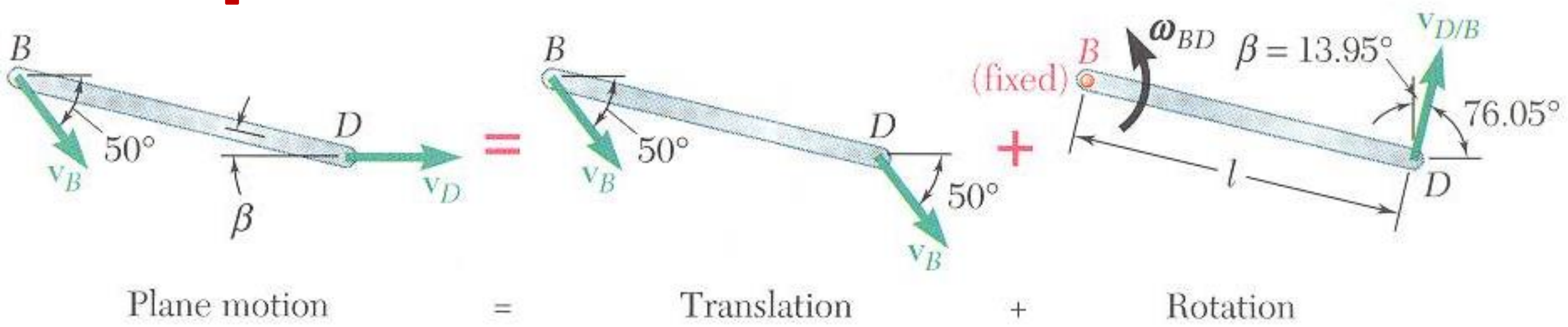
$$= 209.4\ \text{rad/s}$$

$$v_B = (AB)\omega_{AB} = (3\,\text{in.})(209.4\ \text{rad/s})$$

- Direction of $\vec{v}_D$ is horizontal.
- Direction of relative velocity $\vec{v}_{D/B}$ is perpendicular to BD..

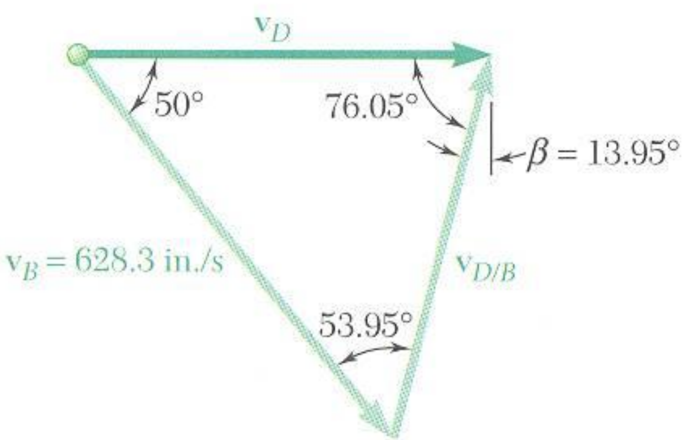
$$\frac{\sin 40^\circ}{8\,\text{in.}} = \frac{\sin\beta}{3\,\text{in.}} \qquad \beta = 13.95^\circ$$

Sample Problem 15.3



$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$\frac{v_D}{\sin 53.95°} = \frac{v_{D/B}}{\sin 50°} = \frac{628.3\,\text{in.}/\text{s}}{\sin 76.05°}$$



$$v_D = v_P = 523.4\,\text{in.}/\text{s} = 43.6\,\text{ft}/\text{s}$$

$$v_{D/B} = 495.9\,\text{in.}/\text{s}$$

$$v_{D/B} = l\omega_{BD}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} = \frac{495.9\,\text{in.}/\text{s}}{8\,\text{in.}} = 62.0\,\text{rad}/\text{s}$$

Home Work Assignment # 15.2

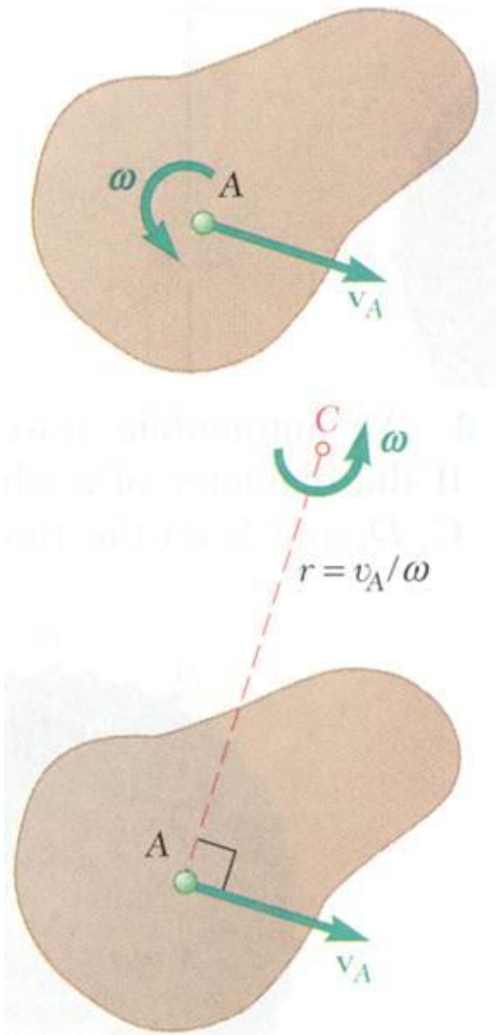
38, 46, 53, 59, 66, 72

Due

Wednesday 2/4/2014 (Civil)

Tuesday 1/4/2014 (Mechanical)

Instantaneous Center of Rotation (ICR) in Plane Motion

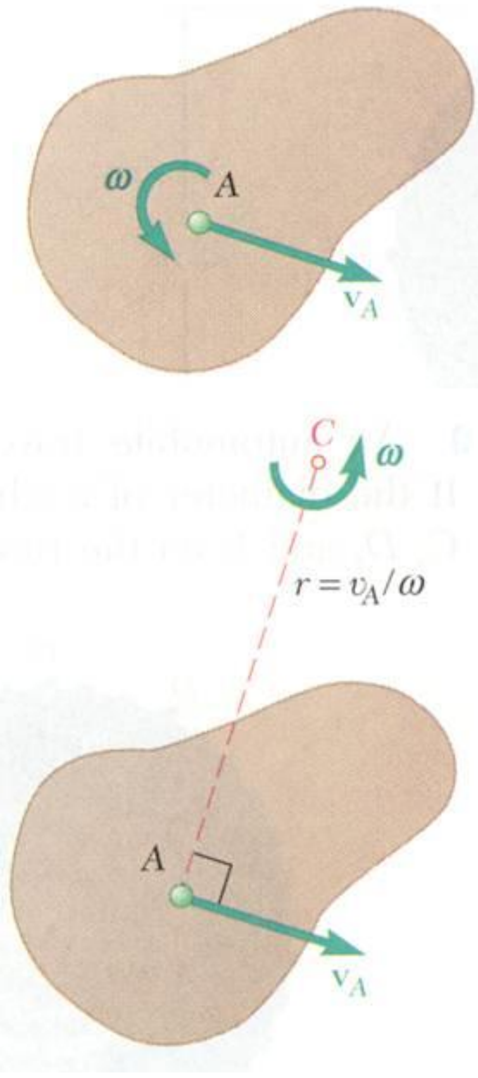


- Plane motion of all particles in a slab can be replaced by:
    1. translation of an arbitrary point *A*
    2. rotation about *A* with an angular velocity that is independent of choice of *A*

Instantaneous Center of Rotation in Plane Motion

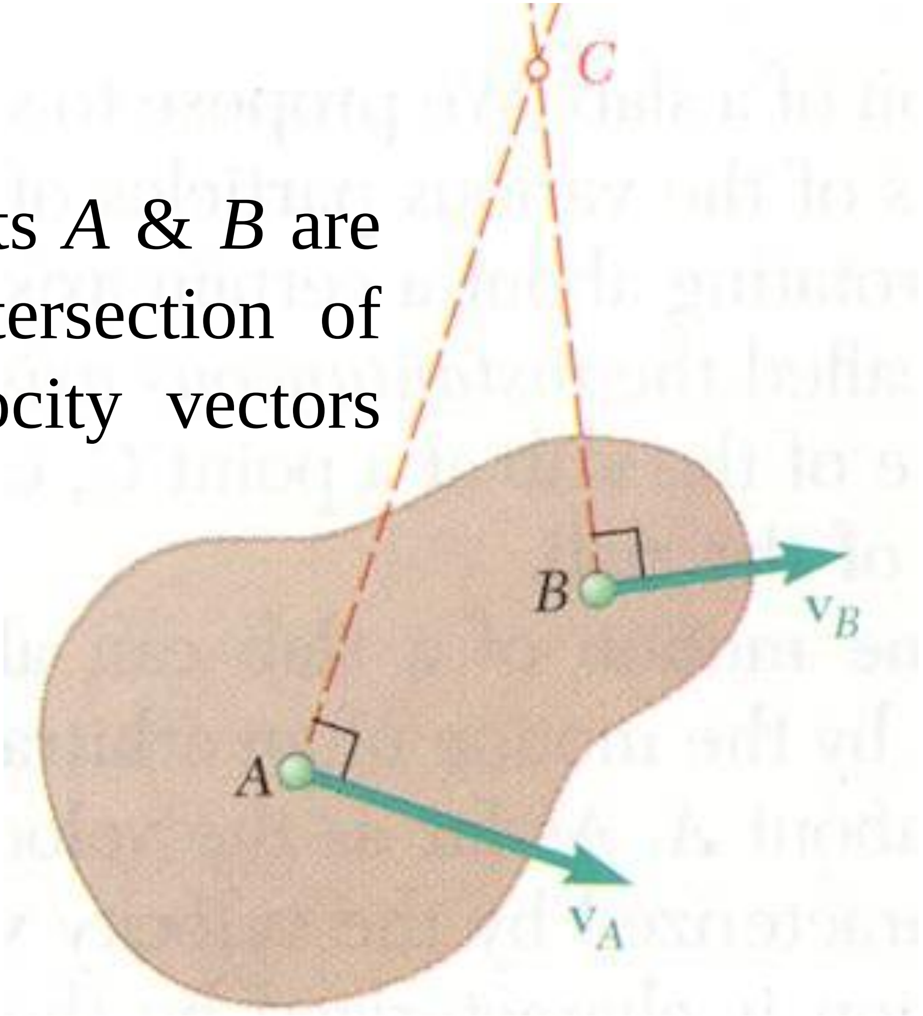
- Same translational & rotational velocities at *A* are obtained by allowing slab to rotate with same angular velocity about some point *C*.

- The slab seems to rotate about the *instantaneous center of rotation C*.



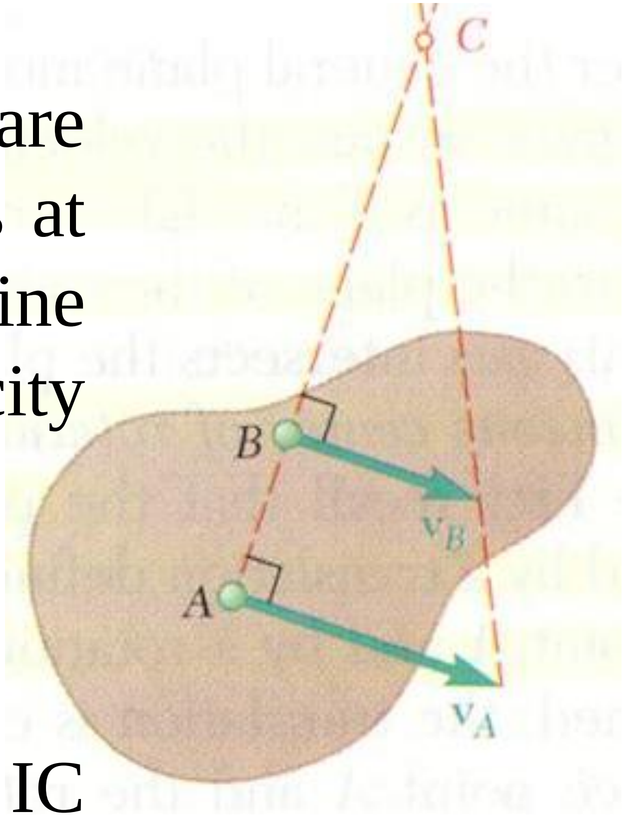
Instantaneous Center of Rotation in Plane Motion

- If velocity at two points *A* & *B* are known, IC lies at intersection of perpendiculars to velocity vectors through *A* & *B*



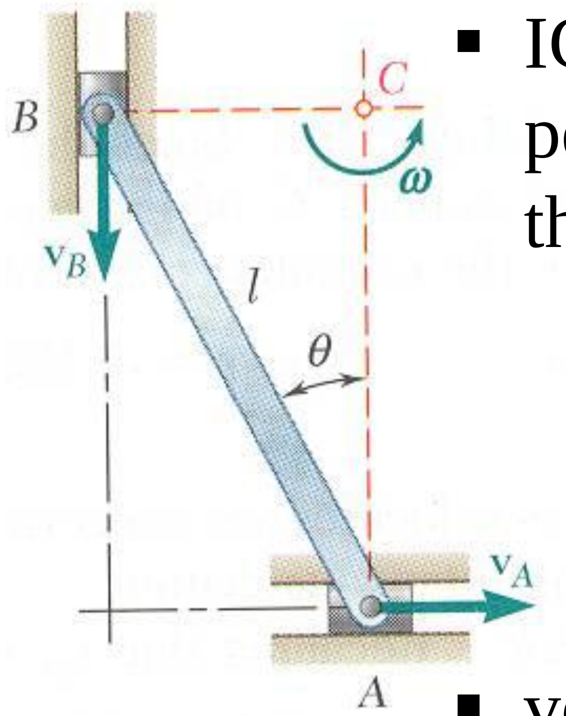
Instantaneous Center of Rotation in Plane Motion

- If velocity vectors at *A* and *B* are perpendicular to line *AB*, IC lies at intersection of line *AB* with line joining extremities of velocity vectors at *A* & *B*.



- If velocity magnitudes are equal, IC is at infinity & angular velocity is zero.

Instantaneous Center of Rotation in Plane Motion



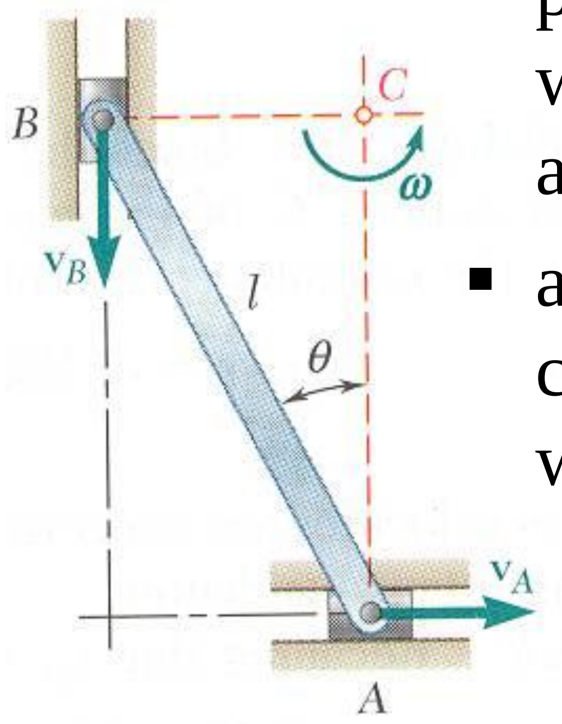
- ICR lies at intersection of perpendiculars to velocity vectors through A and B .

$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta}$$

$$v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta} = v_A\tan\theta$$

- velocities of all particles on rod are as if they were rotated about C.
- particle at ICR has zero velocity.

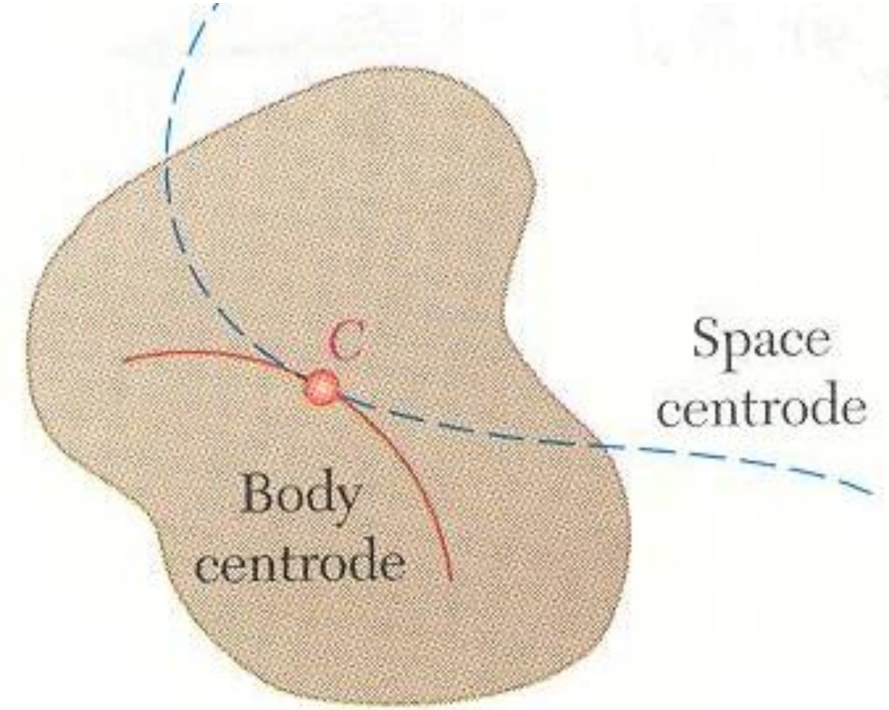
Instantaneous Center of Rotation in Plane Motion



- particle coinciding with ICR changes with time and acceleration of particle at ICR is not zero.
- acceleration of particles in the slab cannot be determined as if the slab were simply rotating about *C*.

Instantaneous Center of Rotation in Plane Motion

- **Body centrode:** trace of locus of ICR on the body
- **space centrode:** trace of locus of ICR in space

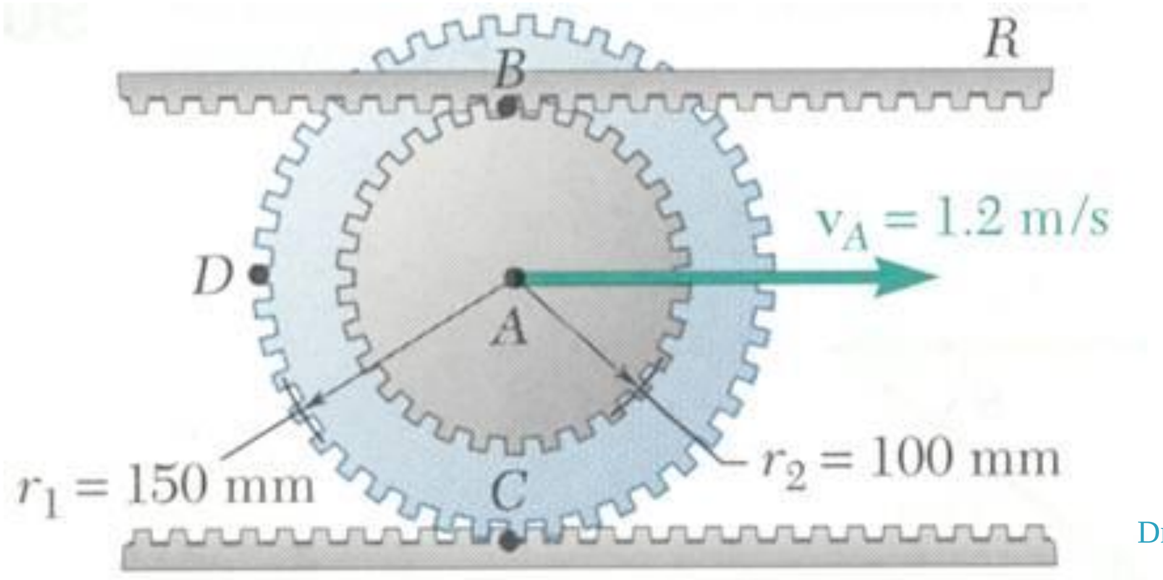


Sample Problem 15.4

The double gear rolls on the stationary lower rack: the velocity of its center is 1.2 m/s. Determine:

a. angular velocity of the gear

b. velocities of upper rack *R* and point *D* of the gear.

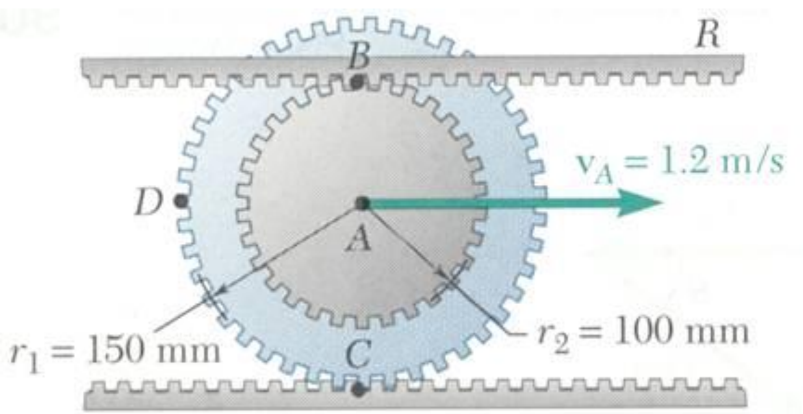


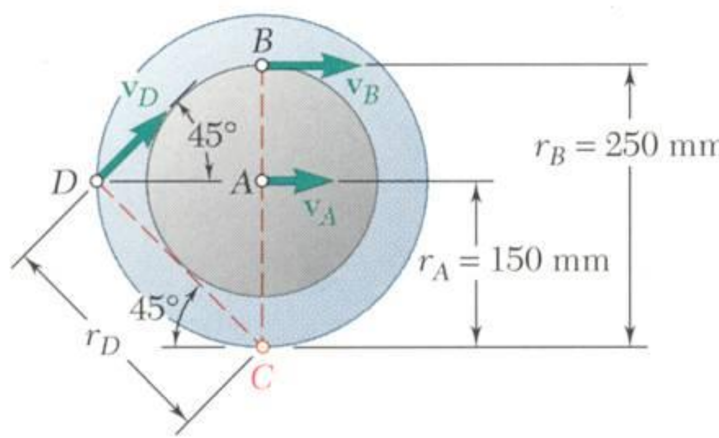
Sample Problem 15.4

Solution:

- point C is in contact with stationary lower rack and, instantaneously, has zero velocity. It must be the location of ICR.
- Determine angular velocity about C based on given velocity at A.

$$v_A = r_A \omega \qquad \omega = \frac{v_A}{r_A} = \frac{1.2\,\mathrm{m/s}}{0.15\,\mathrm{m}} = 8\,\mathrm{rad/s}$$





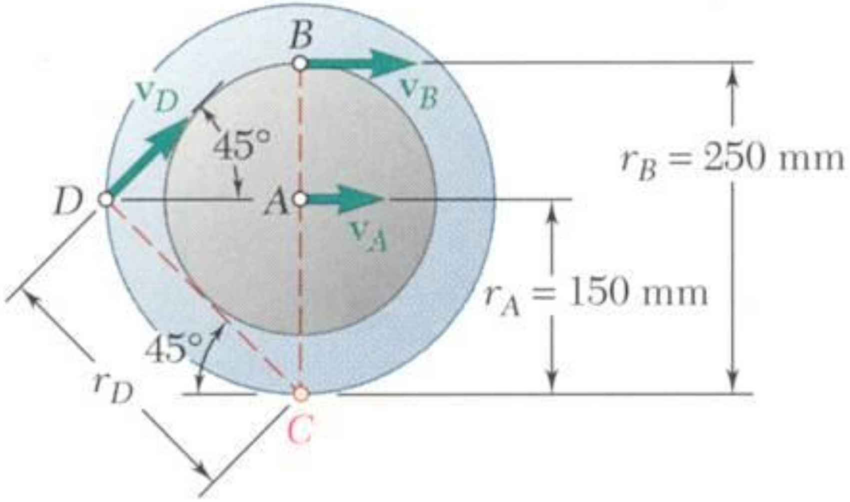
Sample Problem 15.4

- Evaluate velocities at *B* & *D* based on their rotation about *C*.

$$v_R = v_B = r_B\omega = (0.25\,\text{m})(8\,\text{rad/s}) = (2\,\text{m/s})\vec{i}$$

$$r_D = (0.15\,\text{m})\sqrt{2} = 0.2121\,\text{m}$$

$$v_D = r_D\omega = (0.2121\,\text{m})(8\,\text{rad/s})$$



$$v_D = 1.697\,\text{m/s}$$

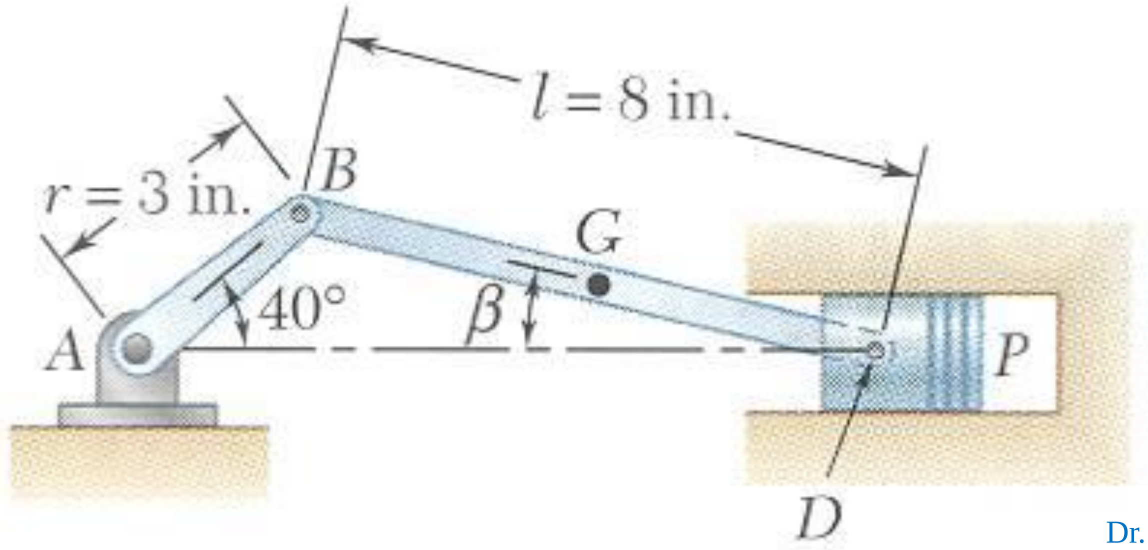
$$\vec{v}_D = (1.2\vec{i} + 1.2\vec{j})(\text{m/s})$$

Sample Problem 15.5

The crank *AB* has a constant cw angular velocity of 2000 rpm. For the crank position indicated, determine:

a. angular velocity of connecting rod *BD*

b. velocity of piston *P*

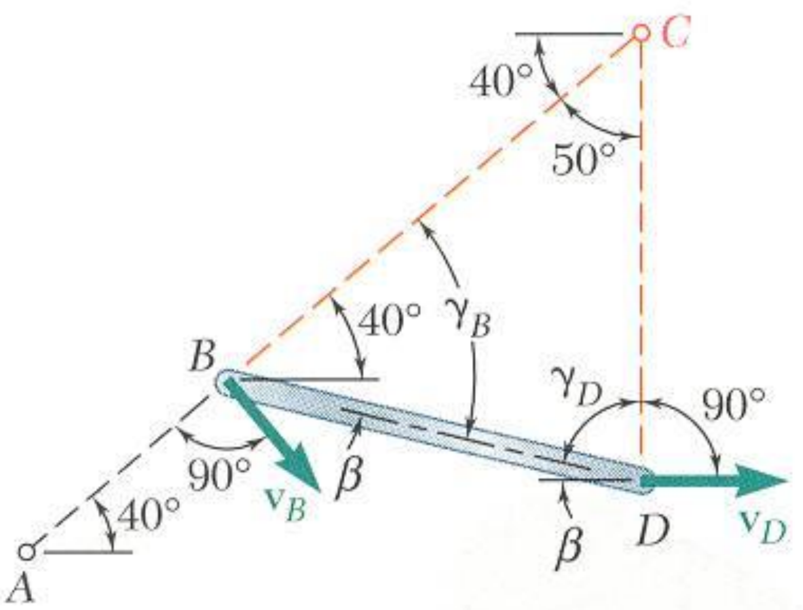


Sample Problem 15.5

- From Sample Problem 15.3,

$$\vec{v}_B = \left(403.9\vec{i} - 481.3\vec{j}\right)(\text{in./s}),\; v_B = 628.3\,\text{in./s},\; \beta = 13.95^\circ$$

- ICR is at intersection of perpendiculars to velocities through *B* & *D*.



$$\gamma_B = 40^\circ + \beta = 53.95^\circ$$

$$\gamma_D = 90^\circ - \beta = 76.05^\circ$$

$$\frac{BC}{\sin 76.05^\circ} = \frac{CD}{\sin 53.95^\circ} = \frac{8\,\text{in.}}{\sin 50^\circ}$$

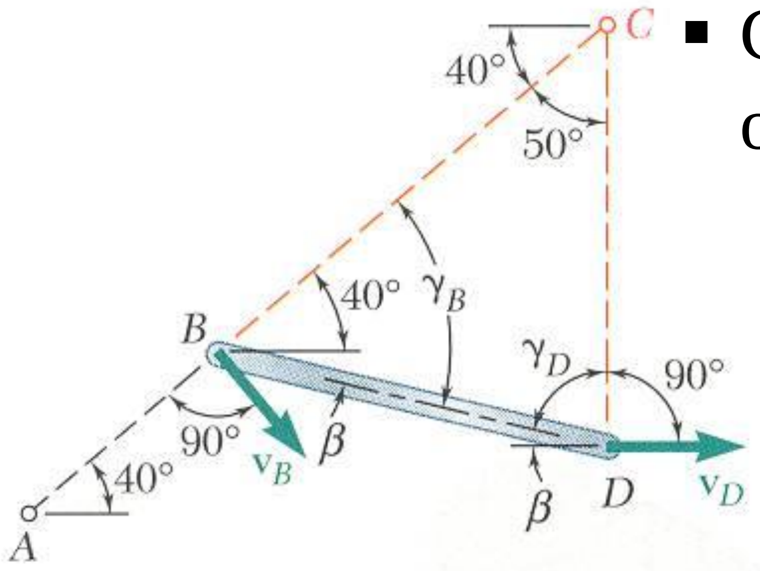
$$BC = 10.14\,\text{in.} \qquad CD = 8.44\,\text{in.}$$

Sample Problem 15.5

- Determine angular velocity about ICR based on velocity at *B*

$$v_B = (BC)\omega_{BD}$$

$$\omega_{BD} = \frac{v_B}{BC} = \frac{628.3\,\text{in./s}}{10.14\,\text{in.}} = 62.0\,rad/s$$



- Calculate velocity at *D* based on its rotation about ICR

$$v_D = (CD)\omega_{BD} = (8.44\,\text{in.})(62.0\,\text{rad/s})$$
$$= 43.6\,ft/s$$

Home Work Assignment # 15.3

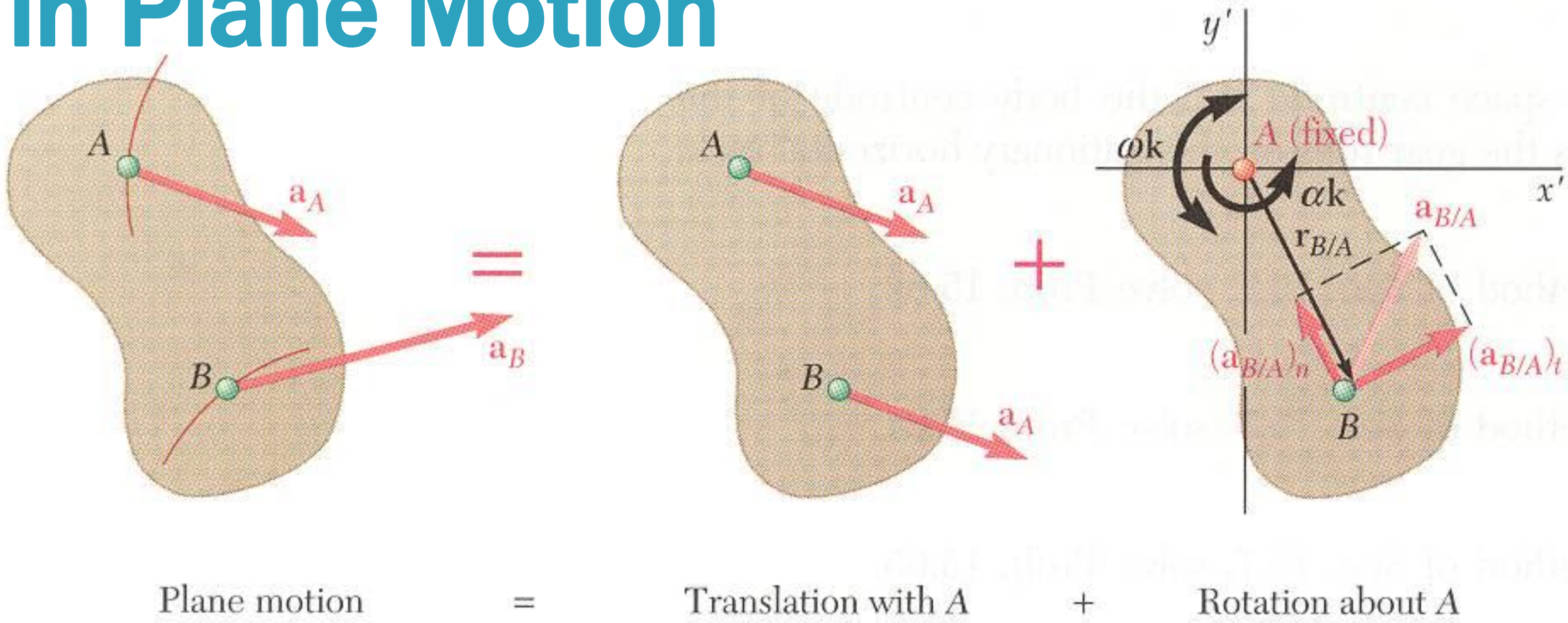
74, 80, 88, 95

Due

Saturday 5/4/2014 (Civil)

Sunday 6/4/2014 (Mechanical)

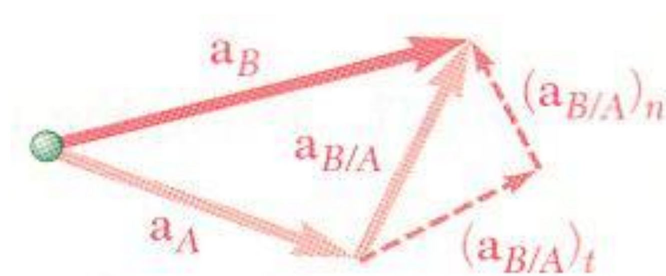
Absolute and Relative Acceleration in Plane Motion



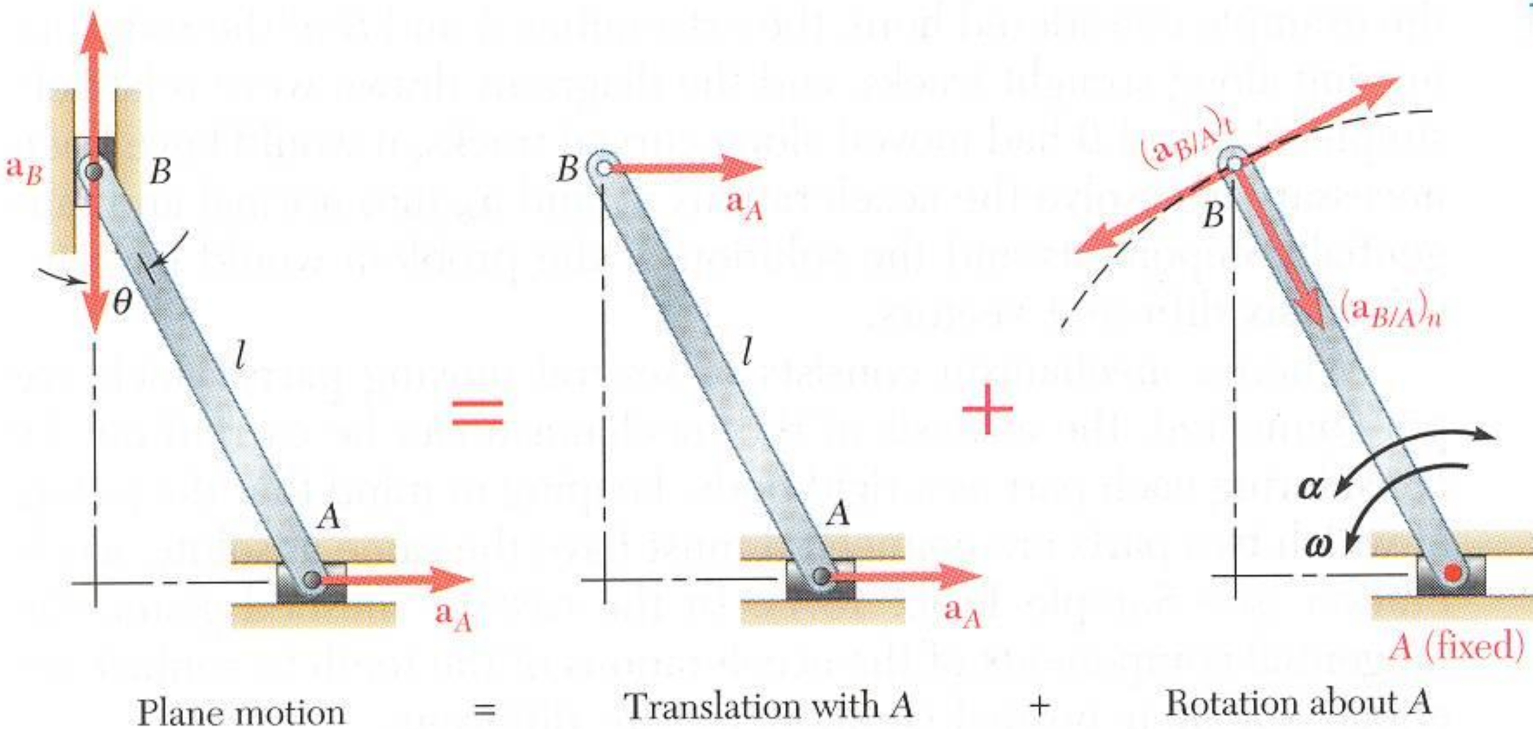
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\left(\vec{a}_{B/A}\right)_t = \alpha\vec{k} \times \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_t = r\alpha$$

$$\left(\vec{a}_{B/A}\right)_n = -\omega^2 \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_n = r\omega^2$$



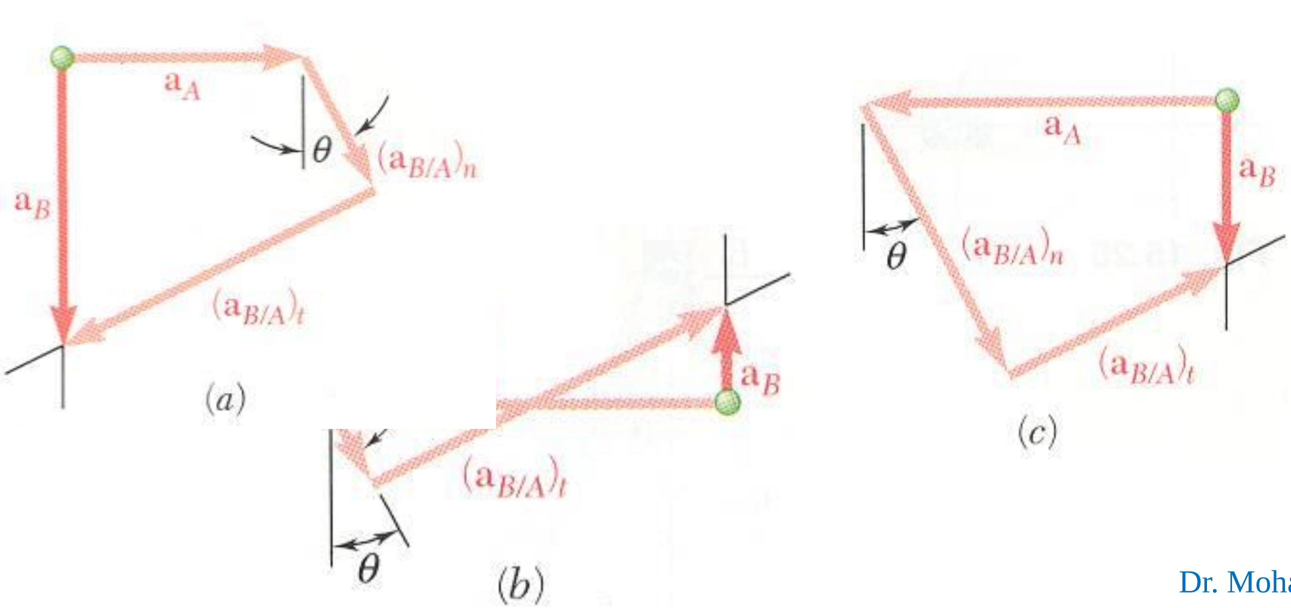
Absolute and Relative Acceleration in Plane Motion

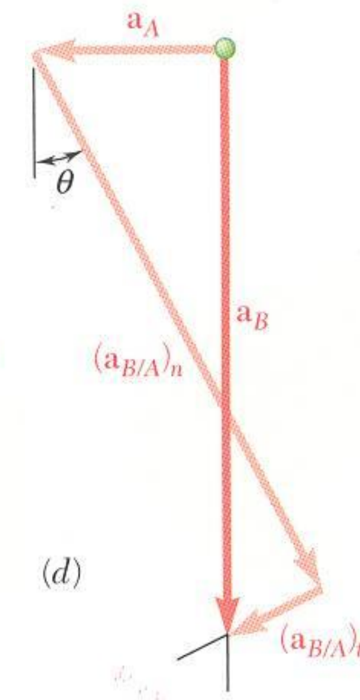


$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{a}_A + \left(\vec{a}_{B/A}\right)_n + \left(\vec{a}_{B/A}\right)_t$$

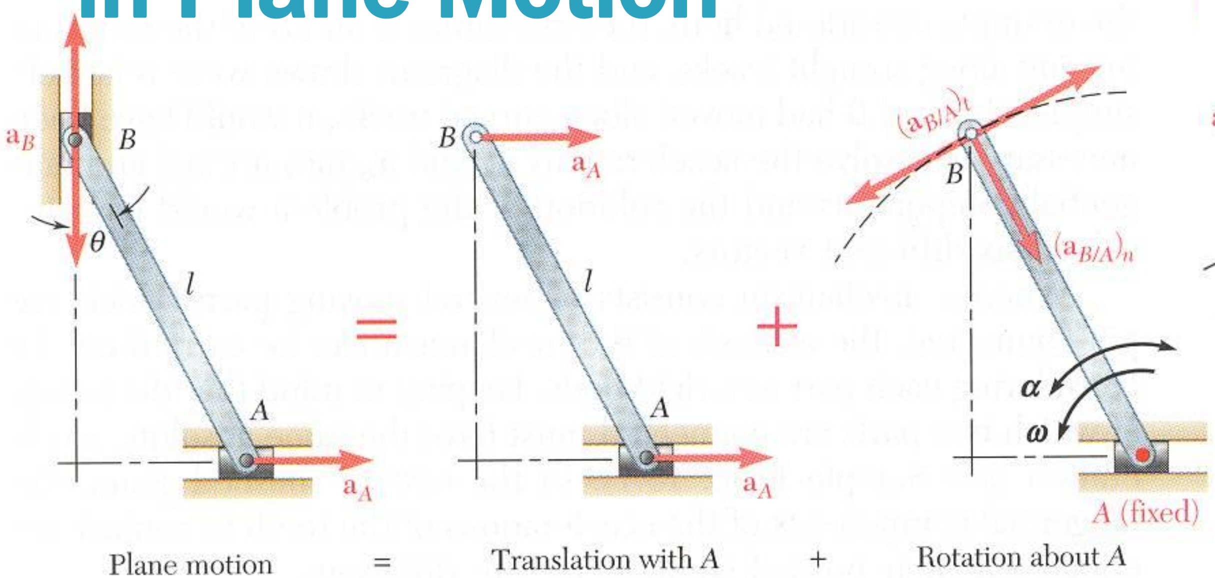
Absolute and Relative Acceleration in Plane Motion

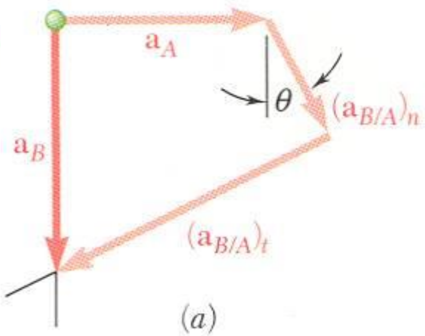
- Vector result depends on sense of and the relative magnitudes of a_A and $\left(a_{B/A}\right)_n$
- Must also know angular velocity ω.





Absolute and Relative Acceleration in Plane Motion



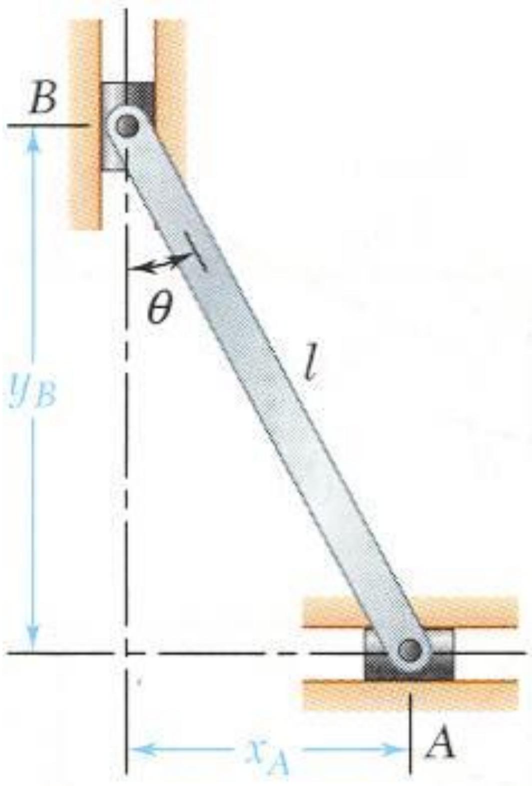


$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$\rightarrow$ $+x$ components: $0 = a_A + l\omega^2 \sin\theta - l\alpha\cos\theta$

$+\uparrow$ y components: $-a_B = -l\omega^2\cos\theta - l\alpha\sin\theta$

Analysis of Plane Motion in Terms of a Parameter



$$x_A = l\sin\theta \qquad y_B = l\cos\theta$$

$$v_A = \dot{x}_A = l\dot{\theta}\cos\theta = l\omega\cos\theta$$

$$v_B = \dot{y}_B = -l\dot{\theta}\sin\theta = -l\omega\sin\theta$$

$$a_A = \ddot{x}_A = -l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta$$

$$= -l\omega^2\sin\theta + l\alpha\cos\theta$$

$$a_B = \ddot{y}_B = -l\dot{\theta}^2\cos\theta - l\ddot{\theta}\sin\theta$$

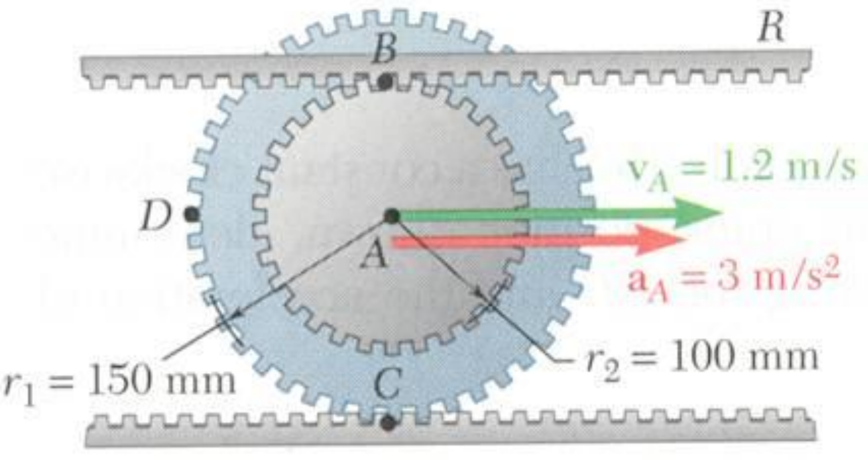
$$= -l\omega^2\cos\theta - l\alpha\sin\theta$$

Sample Problem 15.6

The center of the double gear has a velocity and acceleration to the right of 1.2 m/s and 3 m/s^2, respectively. The lower rack is stationary. Determine:

a. angular acceleration of the gear

b. acceleration of points *B*, *C*, *D*



Sample Problem 15.6

Solution:

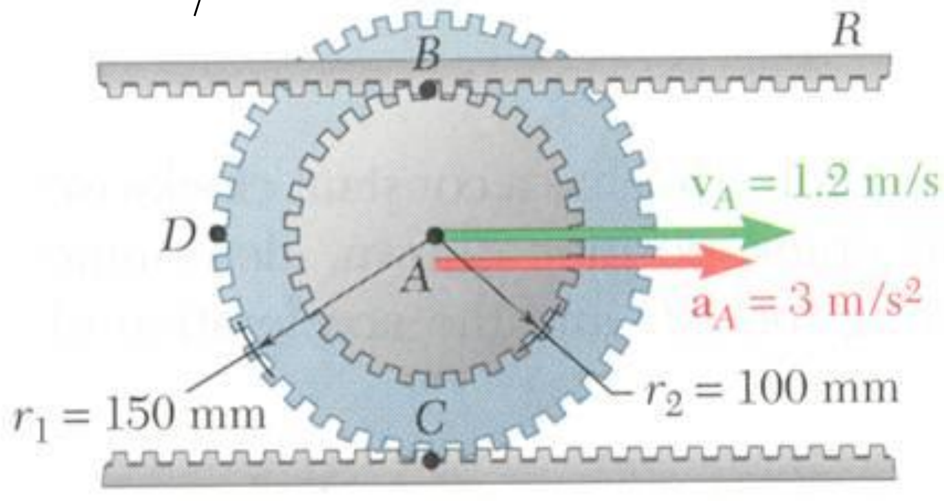
$$x_A = -r_1\theta, \quad v_A = -r_1\dot{\theta} = -r_1\omega$$

$$\omega = -\frac{v_A}{r_1} = -\frac{1.2\,\mathrm{m/s}}{0.150\,\mathrm{m}} = -8\,\mathrm{rad/s}$$

$$a_A = -r_1\ddot{\theta} = -r_1\alpha$$

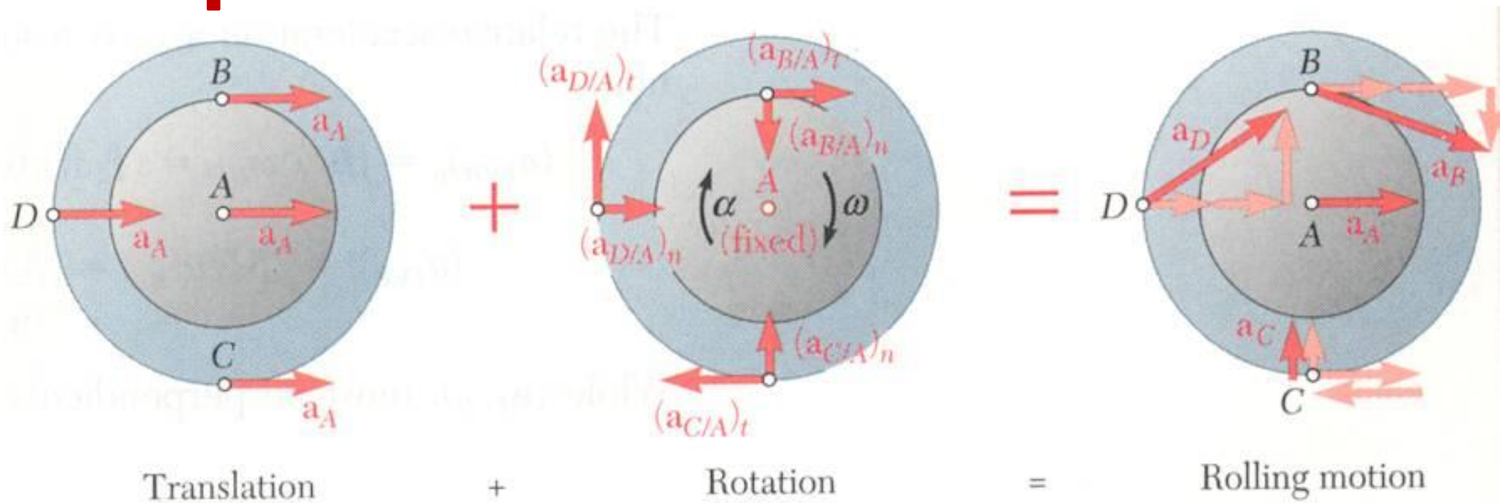
$$\alpha = -\frac{a_A}{r_1} = -\frac{3\,\mathrm{m/s^2}}{0.150\,\mathrm{m}}$$

$$\vec{\alpha} = \alpha\vec{k} = -\left(20\,\mathrm{rad/s^2}\right)\vec{k}$$

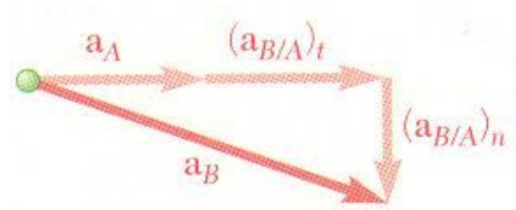


Sample Problem 15.6

Dr. Mohammad Suliman Abuhaiba, PE



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \left(\vec{a}_{B/A}\right)_t + \left(\vec{a}_{B/A}\right)_n$$



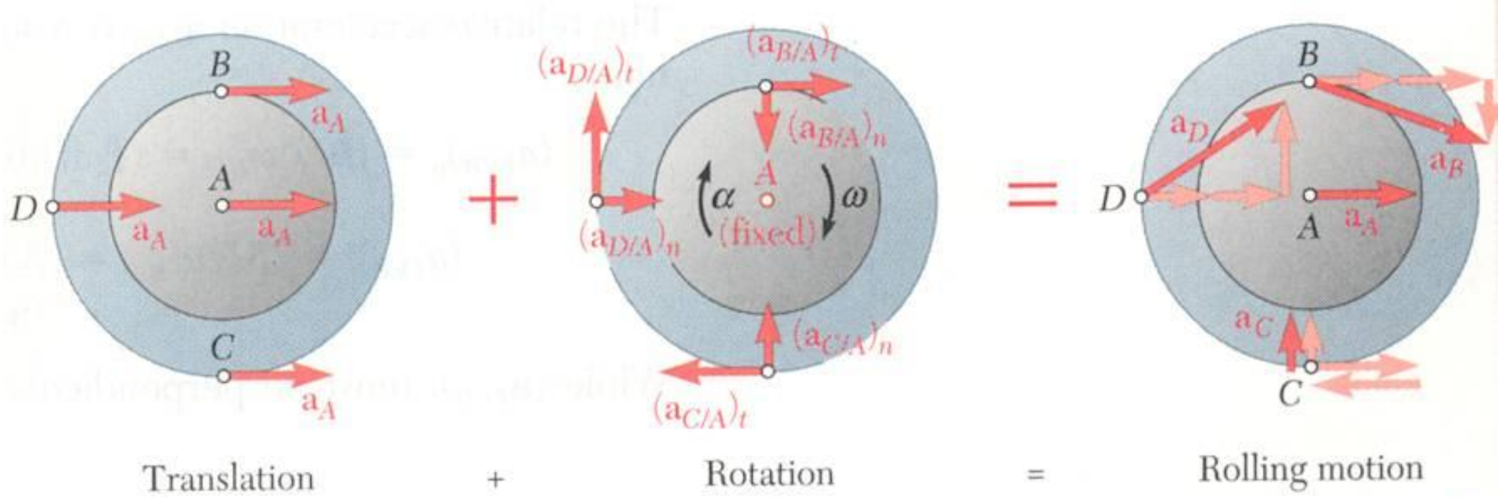
$$= \vec{a}_A + \alpha\vec{k} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

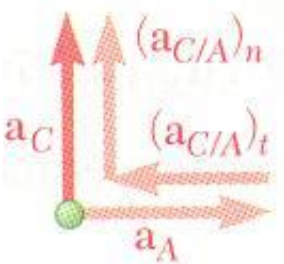
$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k} \times (0.100\,\mathrm{m})\vec{j} - (8\,\mathrm{rad/s})^2(-0.100\,\mathrm{m})\vec{j}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(2\,\mathrm{m/s^2}\right)\vec{i} - \left(6.40\,\mathrm{m/s^2}\right)\vec{j}$$

$$\vec{a}_B = \left(5\,\mathrm{m/s^2}\right)\vec{i} - \left(6.40\,\mathrm{m/s^2}\right)\vec{j} \qquad a_B = 8.12\,\mathrm{m/s^2}$$

Sample Problem 15.6





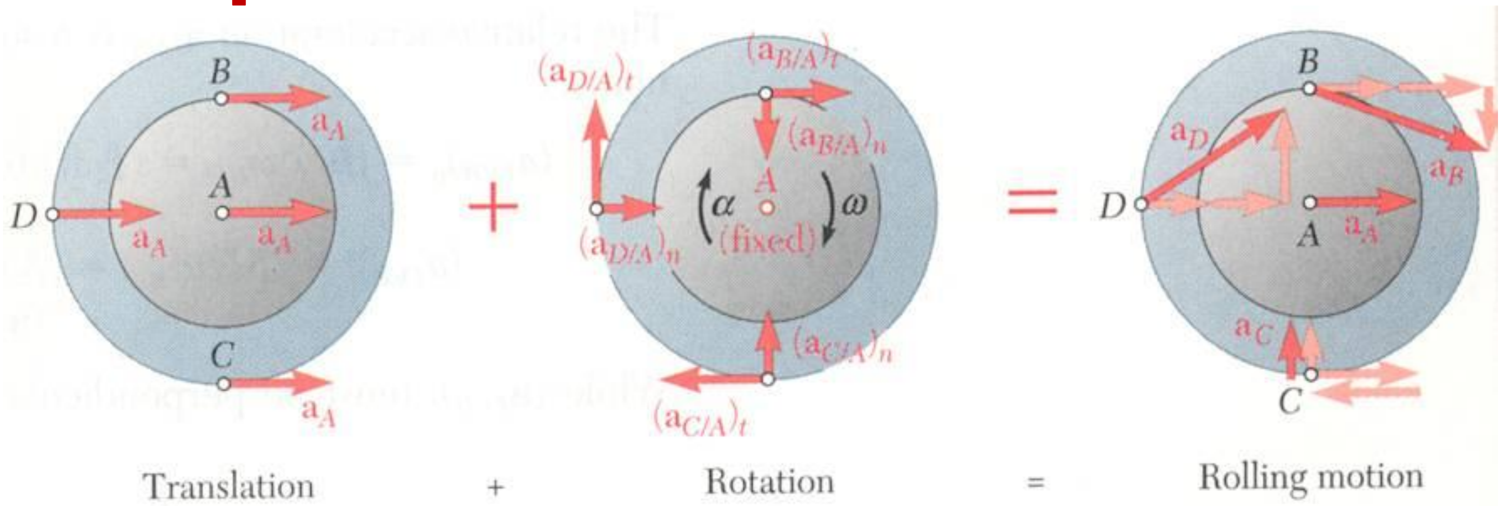
$$\vec{a}_C = \vec{a}_A + \vec{a}_{C/A} = \vec{a}_A + \alpha\vec{k} \times \vec{r}_{C/A} - \omega^2 \vec{r}_{C/A}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(20\,\mathrm{rad/s^2}\right)\vec{k} \times (-0.150\,\mathrm{m})\vec{j} - (8\,\mathrm{rad/s})^2(-0.150\,\mathrm{m})\vec{j}$$

$$= \left(3\,\mathrm{m/s^2}\right)\vec{i} - \left(3\,\mathrm{m/s^2}\right)\vec{i} + \left(9.60\,\mathrm{m/s^2}\right)\vec{j}$$

$$\vec{a}_c = \left(9.60\,\mathrm{m/s^2}\right)\vec{j}$$

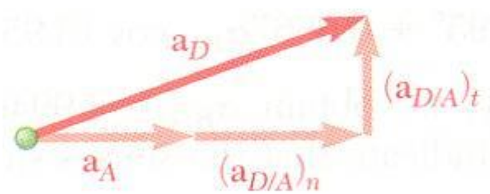
Sample Problem 15.6



$$\vec{a}_D = \vec{a}_A + \vec{a}_{D/A} = \vec{a}_A + \alpha\vec{k} \times \vec{r}_{D/A} - \omega^2 \vec{r}_{D/A}$$

$$= \left(3\,\text{m/s}^2\right)\vec{i} - \left(20\,\text{rad/s}^2\right)\vec{k} \times (-0.150\,\text{m})\vec{i} - (8\,\text{rad/s})^2(-0.150\,\text{m})\vec{i}$$

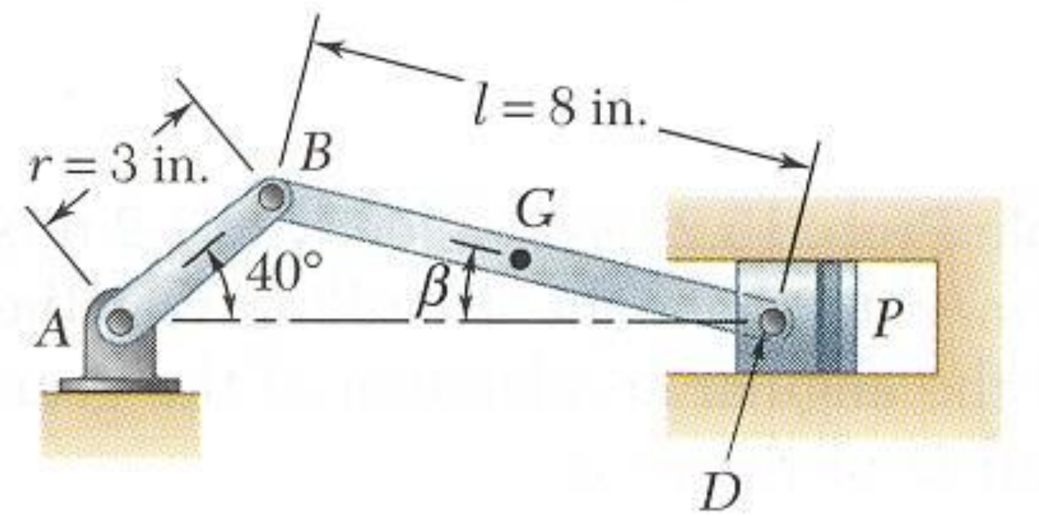
$$= \left(3\,\text{m/s}^2\right)\vec{i} + \left(3\,\text{m/s}^2\right)\vec{j} + \left(9.60\,\text{m/s}^2\right)\vec{i}$$



$$\vec{a}_D = \left(12.6\,\text{m}/s^2\right)\vec{i} + \left(3\,\text{m/s}^2\right)\vec{j} \qquad a_D = 12.95\,\text{m/s}^2$$

Sample Problem 15.7

Crank *AB* of the engine system has a constant cw angular velocity of 2000 rpm. For the crank position shown, determine angular acceleration of the connecting rod *BD* and the acceleration of point *D*.

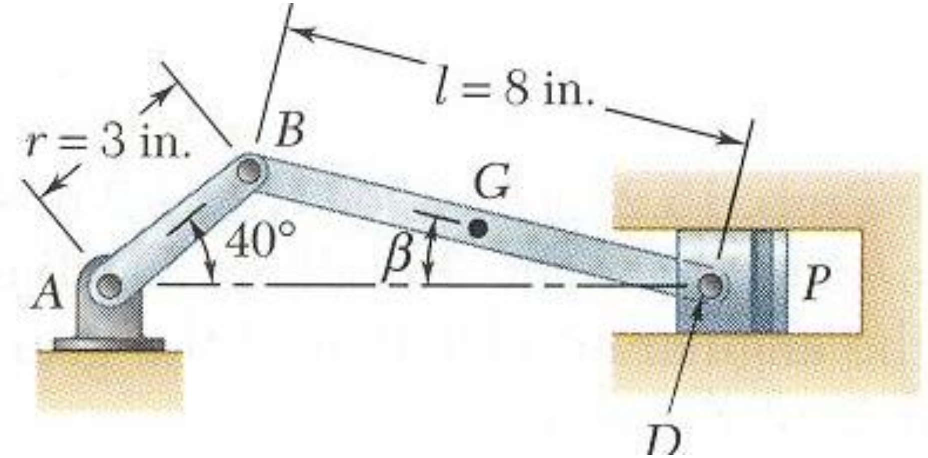


Sample Problem 15.7

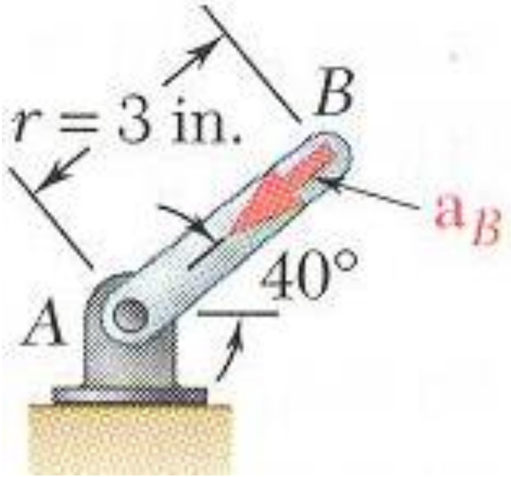
Solution:

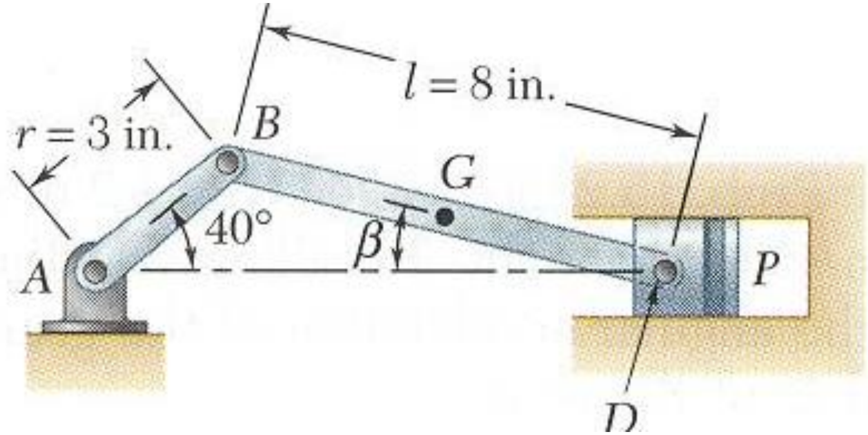
- Angular acceleration of connecting rod *BD* and acceleration of point *D* will be determined from

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$



Sample Problem 15.7





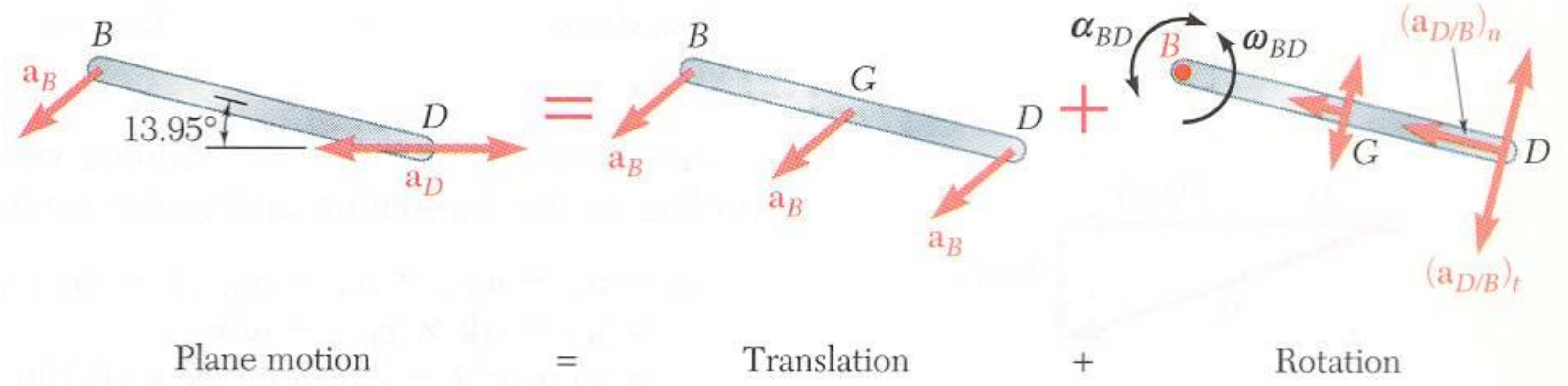
$$\omega_{AB} = 2000\,\text{rpm} = 209.4\,\text{rad/s} = \text{constant}$$

$$\alpha_{AB} = 0$$

$$a_B = r\omega_{AB}^2 = \left(\tfrac{3}{12}\,\text{ft}\right)\left(209.4\,\text{rad/s}\right)^2 = 10{,}962\,\text{ft/s}^2$$

$$\vec{a}_B = \left(10{,}962\,\text{ft/s}^2\right)\left(-\cos 40°\,\vec{i} - \sin 40°\,\vec{j}\right)$$

Sample Problem 15.7



$$\vec{a}_D = \mp a_D \vec{i}$$

Sample Problem 15.3: $\omega_{BD} = 62.0$ rad/s, $\beta = 13.95^o$.

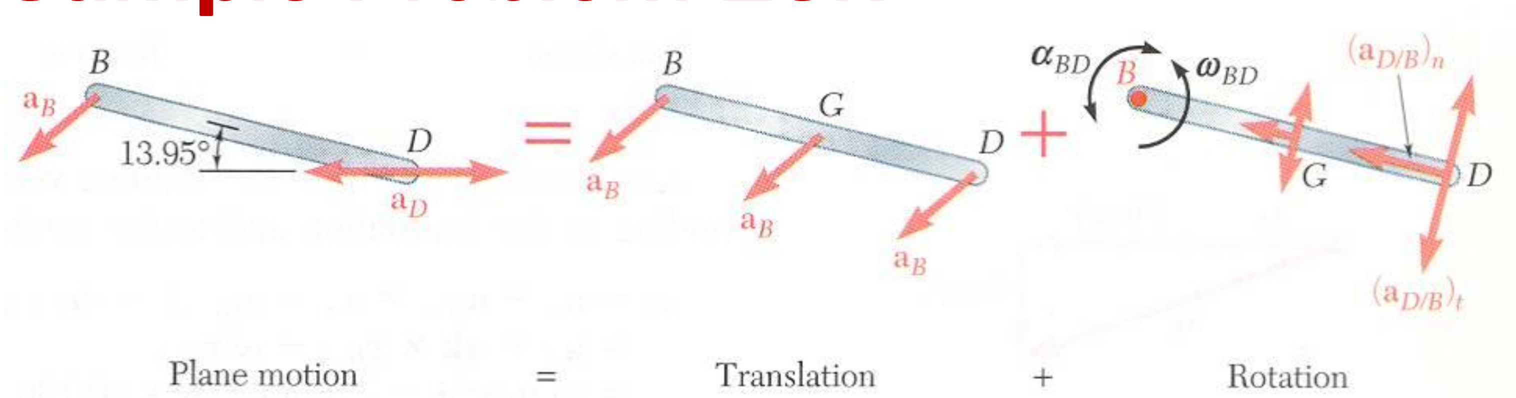
$$\left(a_{D/B}\right)_n = (BD)\omega_{BD}^2 = \left(\tfrac{8}{12}\text{ ft}\right)(62.0\,\text{rad/s})^2 = 2563\,\text{ft/s}^2$$

$$\left(\vec{a}_{D/B}\right)_n = \left(2563\,\text{ft/s}^2\right)\left(-\cos 13.95°\vec{i} + \sin 13.95°\vec{j}\right)$$

$$\left(a_{D/B}\right)_t = (BD)\alpha_{BD} = \left(\tfrac{8}{12}\text{ ft}\right)\alpha_{BD} = 0.667\alpha_{BD}$$

$$\left(\vec{a}_{D/B}\right)_t = \left(0.667\alpha_{BD}\right)\left(\pm \sin 76.05°\vec{i} \pm \cos 76.05°\vec{j}\right)$$

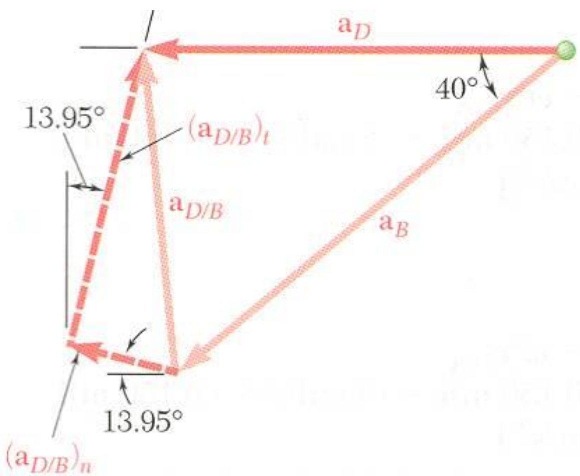
Sample Problem 15.7



$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B} = \vec{a}_B + \left(\vec{a}_{D/B}\right)_t + \left(\vec{a}_{D/B}\right)_n$$

***x* components:** $-a_D = -10{,}962\cos 40° - 2563\cos 13.95° + 0.667\alpha_{BD}\sin 13.95°$

***y* components:** $0 = -10{,}962\sin 40° + 2563\sin 13.95° + 0.667\alpha_{BD}\cos 13.95°$

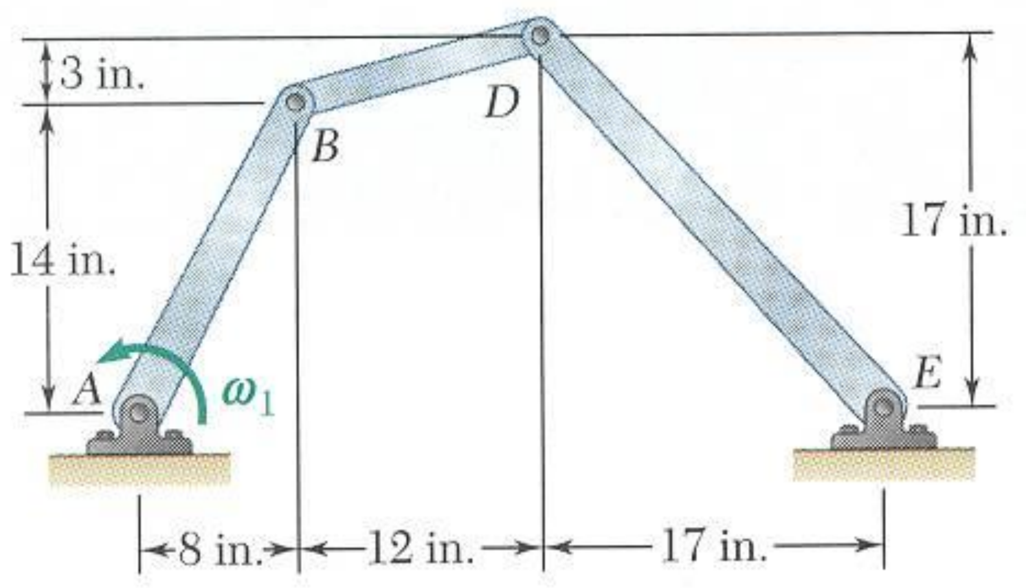


$$\vec{\alpha}_{BD} = \left(9940\,\mathrm{rad/s^2}\right)\vec{k}$$

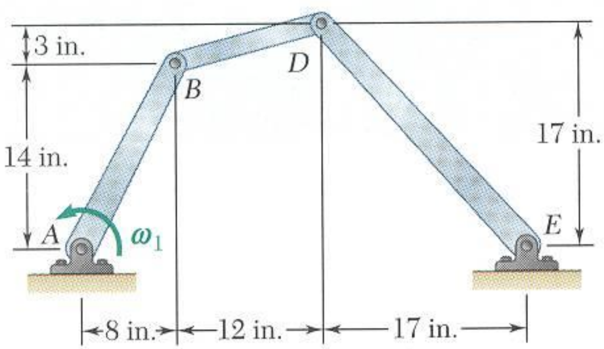
$$\vec{a}_D = -\left(9290\,\mathrm{ft/s^2}\right)\vec{i}$$

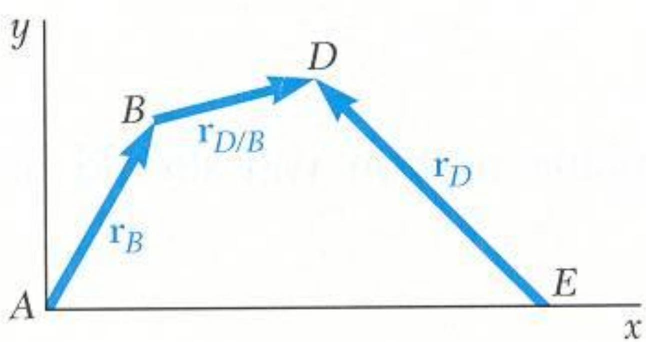
Sample Problem 15.8

In the position shown, crank *AB* has a constant angular velocity ω_1 = 20 rad/s ccw. Determine angular velocities and angular accelerations of connecting rod *BD* and crank *DE*.



Sample Problem 15.8





$$r_B = 8i + 14j$$
$$r_D = -17i + 17j$$
$$r_{D/B} = 12i + 3j$$

Solution:

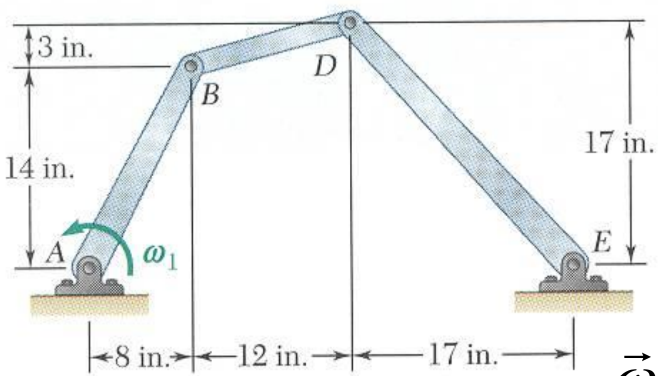
$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

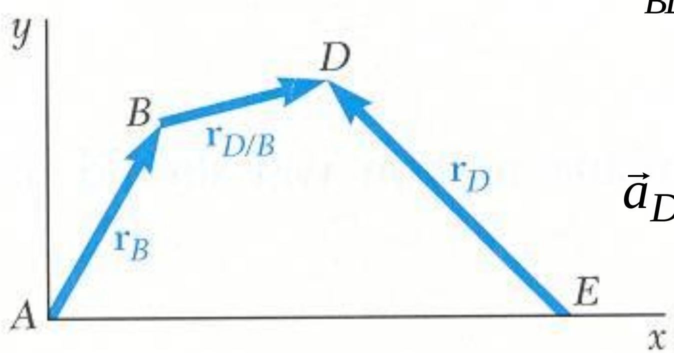
$$\vec{v}_D = \vec{\omega}_{DE} \times \vec{r}_D = \omega_{DE}\vec{k} \times \left(-17\vec{i} + 17\vec{j}\right)$$
$$= -17\omega_{DE}\vec{i} - 17\omega_{DE}\vec{j}$$

$$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_B = 20\vec{k} \times \left(8\vec{i} + 14\vec{j}\right)$$
$$= -280\vec{i} + 160\vec{j}$$

$$\vec{v}_{D/B} = \vec{\omega}_{BD} \times \vec{r}_{D/B} = \omega_{BD}\vec{k} \times \left(12\vec{i} + 3\vec{j}\right)$$
$$= -3\omega_{BD}\vec{i} + 12\omega_{BD}\vec{j}$$

Sample Problem 15.8





$r_B = 8i + 14j$

$r_D = -17i + 17j$

$r_{D/B} = 12i + 3j$

x components:

$$-17\omega_{DE} = -280 - 3\omega_{BD}$$

y components:

$$-17\omega_{DE} = +160 + 12\omega_{BD}$$

$$\vec{\omega}_{BD} = -(29.33\,\text{rad/s})\vec{k},\quad \vec{\omega}_{DE} = (11.29\,\text{rad/s})\vec{k}$$

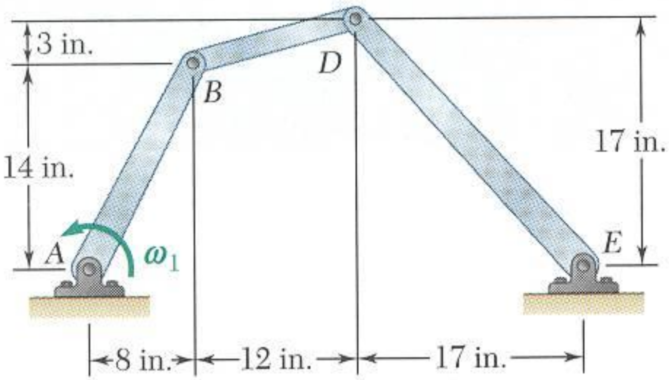
$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$$

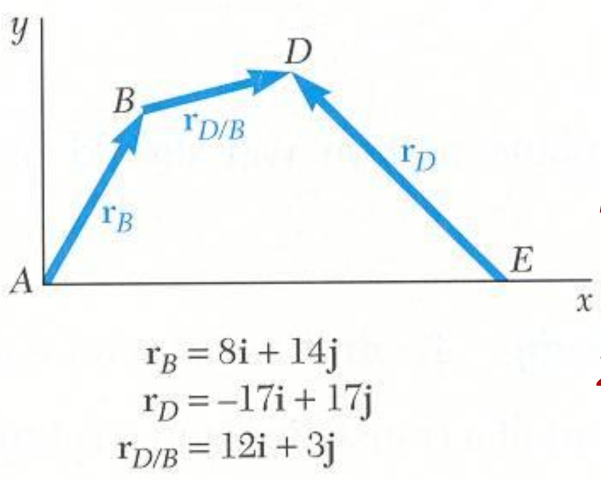
$$\vec{a}_D = \vec{\alpha}_{DE} \times \vec{r}_D - \omega_{DE}^2 \vec{r}_D$$

$$= \alpha_{DE}\vec{k} \times \left(-17\vec{i} + 17\vec{j}\right) - (11.29)^2\left(-17\vec{i} + 17\vec{j}\right)$$

$$= -17\alpha_{DE}\vec{i} - 17\alpha_{DE}\vec{j} + 2170\vec{i} - 2170\vec{j}$$

Sample Problem 15.8





$$\vec{a}_B = \vec{\alpha}_{AB} \times \vec{r}_B - \omega_{AB}^2 \vec{r}_B = 0 - (20)^2 \left(8\vec{i} + 14\vec{j}\right)$$
$$= -3200\vec{i} + 5600\vec{j}$$

$$\vec{a}_{D/B} = \vec{\alpha}_{BD} \times \vec{r}_{B/D} - \omega_{BD}^2 \vec{r}_{B/D}$$
$$= \alpha_{B/D}\vec{k} \times \left(12\vec{i} + 3\vec{j}\right) - (29.33)^2 \left(12\vec{i} + 3\vec{j}\right)$$
$$= -3\alpha_{B/D}\vec{i} + 12\alpha_{B/D}\vec{j} - 10{,}320\vec{i} - 2580\vec{j}$$

***y* components:**

$$-17\alpha_{DE} - 12\alpha_{BD} = -6010$$

***x* components:**

$$-17\alpha_{DE} + 3\alpha_{BD} = -15{,}690$$

$$\vec{\alpha}_{BD} = -\left(645\,\mathrm{rad/s^2}\right)\vec{k} \qquad \vec{\alpha}_{DE} = \left(809\,\mathrm{rad/s^2}\right)\vec{k}$$

Home Work Assignment # 15.4

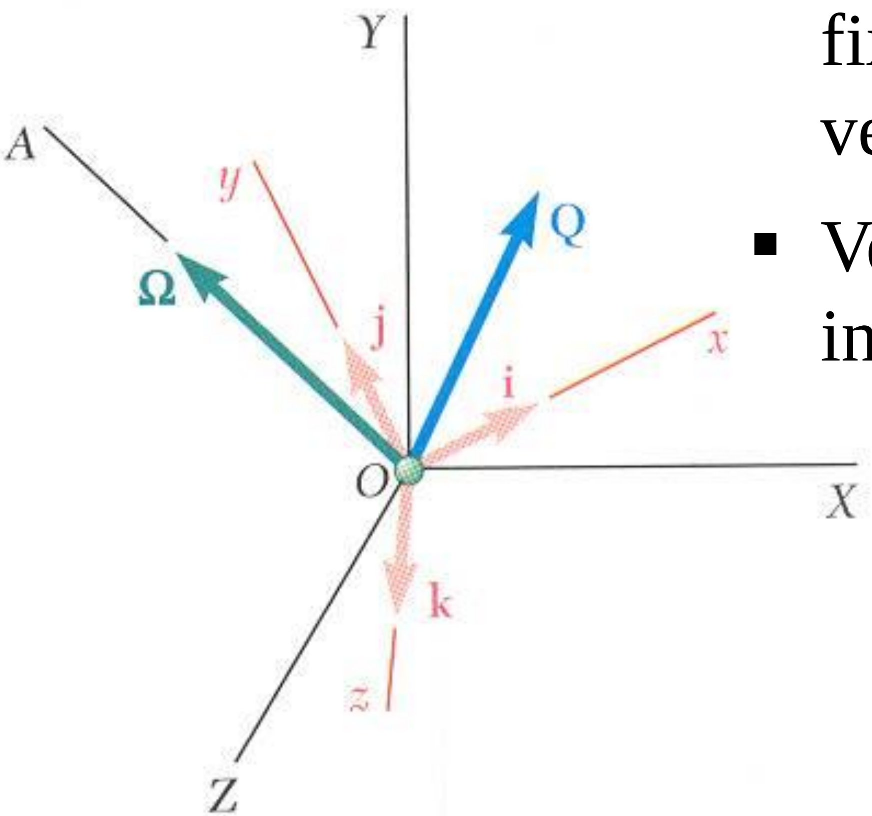
105, 112, 119, 126, 133, 141

Due

Wednesday 9/4/2014 (Civil)

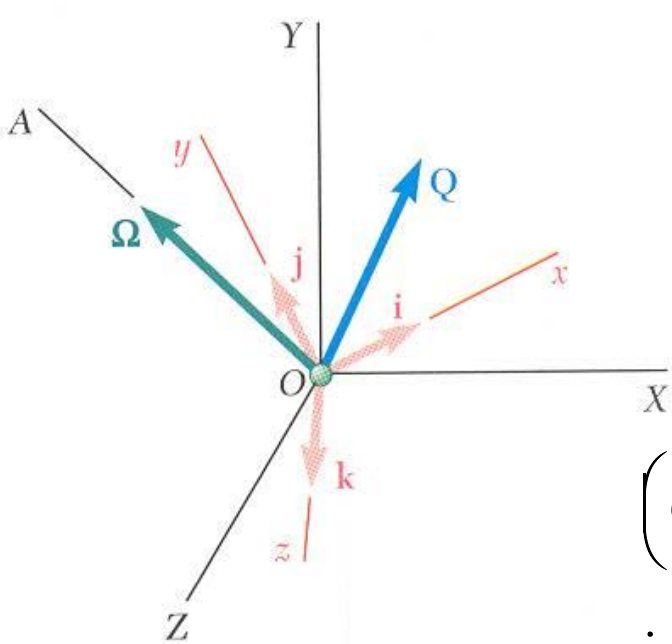
Tuesday 8/4/2014 (Mechanical)

Rate of Change wrt a Rotating Frame



- Frame *OXYZ* is fixed.
- Frame *Oxyz* rotates about fixed axis *OA* with angular velocity $\vec{\Omega}$
- Vector function $\vec{Q}(t)$ varies in direction and magnitude.

Rate of Change wrt a Rotating Frame



- Wrt the rotating *Oxyz* frame,

$$\vec{Q} = Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}$$

$$\left(\dot{\vec{Q}}\right)_{Oxyz} = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k}$$

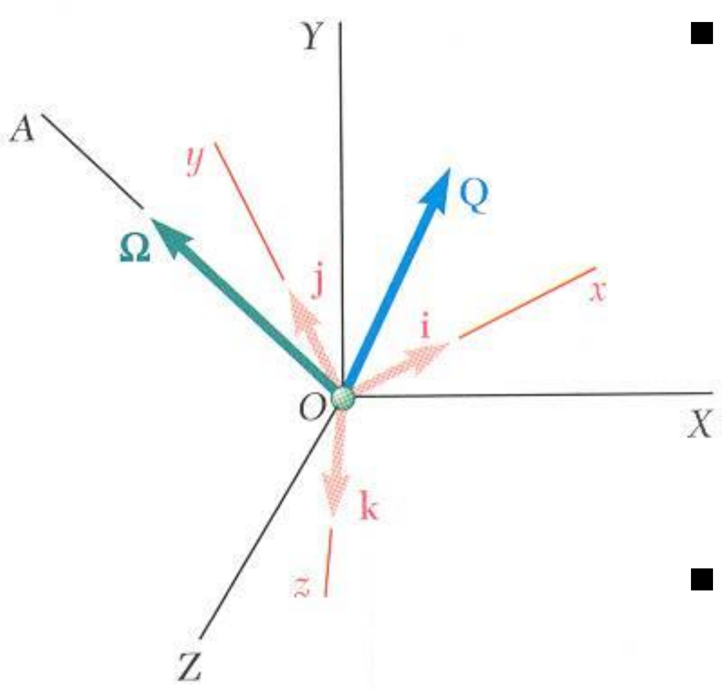
- Wrt the fixed *OXYZ* frame,

$$\left(\dot{\vec{Q}}\right)_{OXYZ} = \dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k} + Q_x\dot{\vec{i}} + Q_y\dot{\vec{j}} + Q_z\dot{\vec{k}}$$

$$\dot{Q}_x\vec{i} + \dot{Q}_y\vec{j} + \dot{Q}_z\vec{k} = \left(\dot{\vec{Q}}\right)_{Oxyz}$$

$$= \text{rate of change wrt rotating frame}$$

Rate of Change wrt a Rotating Frame



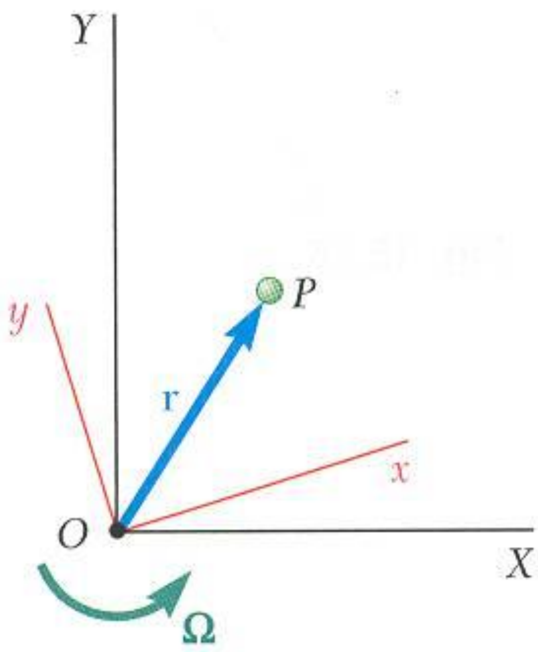
- If $\vec{Q}$ were fixed within *Oxyz* then $\left(\dot{\vec{Q}}\right)_{OXYZ}$ is equivalent to velocity of a point in a rigid body attached to *Oxyz* and

$$Q_x\dot{\vec{i}}+Q_y\dot{\vec{j}}+Q_z\dot{\vec{k}}=\vec{\Omega}\times\vec{Q}$$

- wrt to the fixed *OXYZ* frame,

$$\left(\dot{\vec{Q}}\right)_{OXYZ}=\left(\dot{\vec{Q}}\right)_{Oxyz}+\vec{\Omega}\times\vec{Q}$$

Coriolis Acceleration



- Frame *OXY* is fixed
- frame *Oxy* rotates with angular velocity $\vec{\Omega}$.
- Position vector $\vec{r}_P$ for particle *P* is the same in both frames
- Rate of change depends on choice of frame
- Absolute velocity of particle *P* is

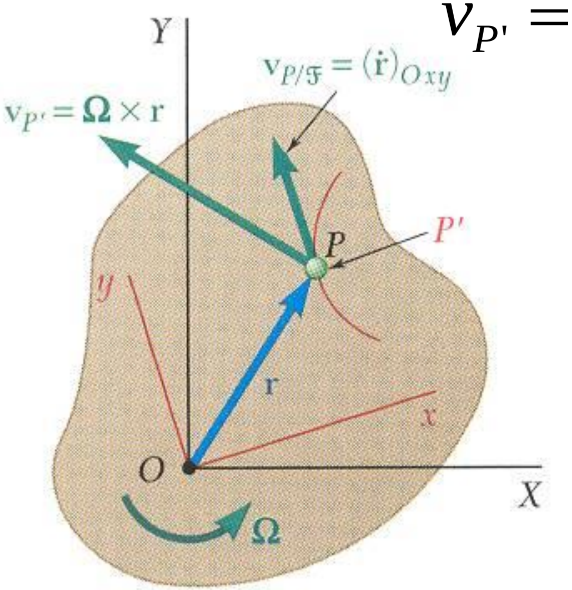
$$\vec{v}_P = \left(\dot{\vec{r}}\right)_{OXY} = \vec{\Omega}\times\vec{r} + \left(\dot{\vec{r}}\right)_{Oxy}$$

Coriolis Acceleration

- A rigid slab attached to rotating frame *Oxy* ($\mathfrak{F}$).
- *P'* = point on slab which corresponds instantaneously to position of particle *P*

$$\vec{v}_{P/\mathbf{F}} = \left(\dot{\vec{r}}\right)_{Oxy} = \text{velocity of P along its path on the slab}$$

$$\vec{v}_{P'} = \text{absolute velocity of point P' on the slab}$$



- Absolute velocity for particle P may be written as

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/\mathfrak{F}}$$

$$\vec{v}_P = \vec{\Omega} \times \vec{r} + \left(\dot{r}\right)_{Oxy}$$

$$= \vec{v}_{P'} + \vec{v}_{P/\mathfrak{F}}$$

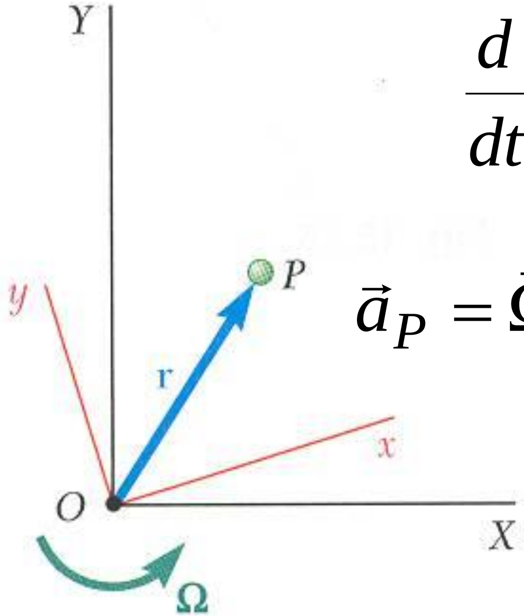
Coriolis Acceleration

- Absolute acceleration for particle *P* is

$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\dot{\vec{r}}\right)_{OXY} + \frac{d}{dt}\left[\left(\dot{\vec{r}}\right)_{Oxy}\right]$$

$$\left(\dot{\vec{r}}\right)_{OXY} = \vec{\Omega} \times \vec{r} + \left(\dot{\vec{r}}\right)_{Oxy}$$

$$\frac{d}{dt}\left[\left(\dot{\vec{r}}\right)_{Oxy}\right] = \left(\ddot{\vec{r}}\right)_{Oxy} + \vec{\Omega} \times \left(\dot{\vec{r}}\right)_{Oxy}$$



$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) + 2\vec{\Omega} \times \left(\dot{\vec{r}}\right)_{Oxy} + \left(\ddot{\vec{r}}\right)_{Oxy}$$

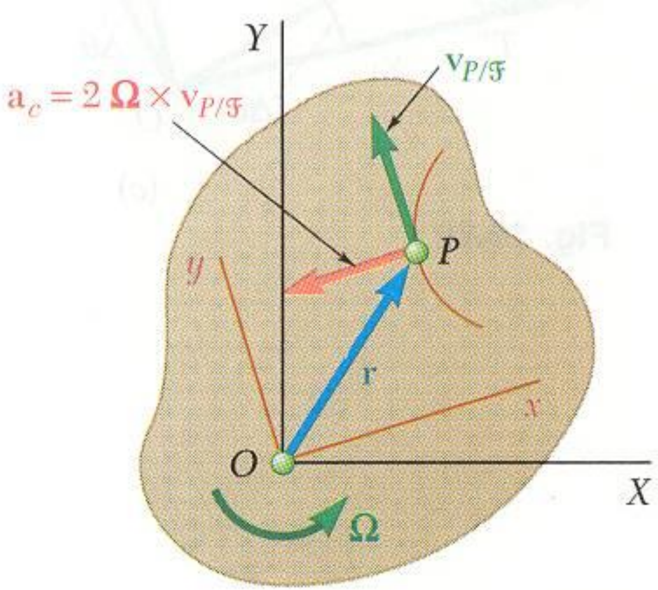
Coriolis Acceleration

- Utilizing the conceptual point *P'* on the slab,

$$\vec{a}_{P'} = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right)$$

$$\vec{a}_{P/\mathfrak{F}} = \left(\ddot{\vec{r}}\right)_{Oxy}$$

- Absolute acceleration for the particle *P* becomes



$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/\mathbf{F}} + 2\vec{\Omega} \times \left(\dot{\vec{r}}\right)_{Oxy}$$

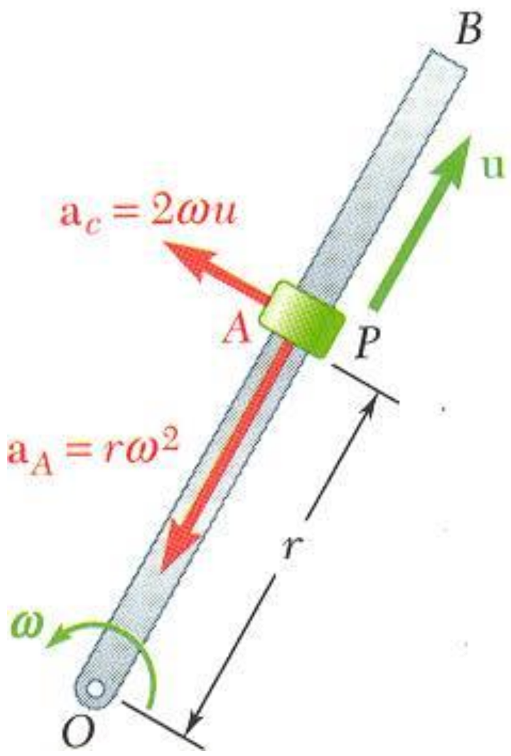
$$= \vec{a}_{P'} + \vec{a}_{P/\mathbf{F}} + \vec{a}_c$$

$$\vec{a}_c = 2\vec{\Omega} \times \left(\dot{\vec{r}}\right)_{Oxy} = 2\vec{\Omega} \times \vec{v}_{P/\mathbf{F}} =$$

$$= \text{Coriolis acceleration}$$

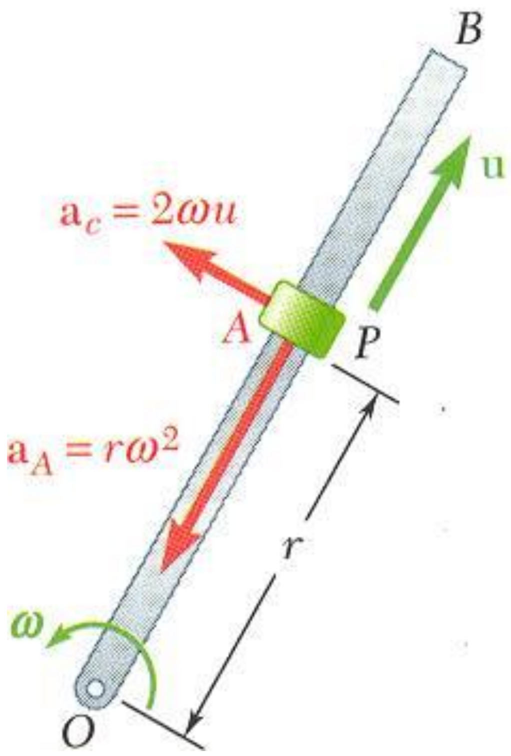
Dr. Mohammad Suliman Abuhaiba, PE

Coriolis Acceleration



- Collar P is sliding at constant relative velocity u along rod OB.
- The rod is rotating at a constant angular velocity w.
- Point A on the rod corresponds to instantaneous position of P.

Coriolis Acceleration



- Absolute acceleration of collar is

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P/\mathfrak{F}} + \vec{a}_c$$

$$\vec{a}_A = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r} \right) \qquad a_A = r\omega^2$$

$$\vec{a}_{P/\mathfrak{F}} = \left(\ddot{\vec{r}} \right)_{Oxy} = 0$$

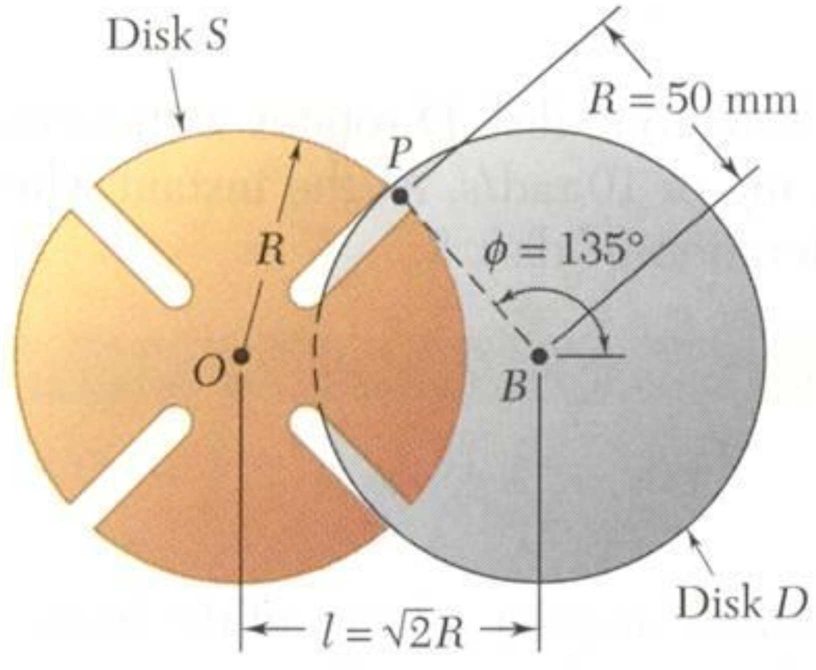
$$\vec{a}_c = 2\vec{\Omega} \times \vec{v}_{P/\mathfrak{F}} \qquad a_c = 2\omega u$$

Sample Problem 15.9

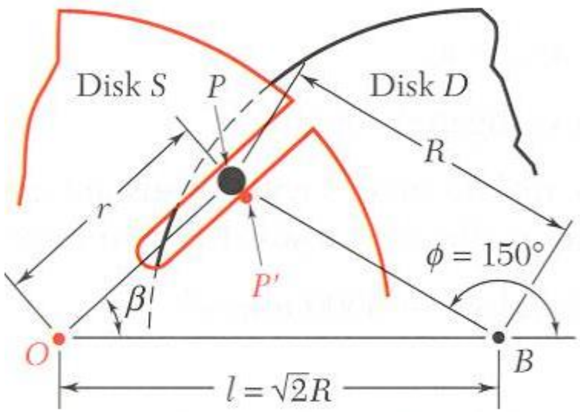
Disk D of the Geneva mechanism rotates with constant ccw angular velocity $\omega_D = 10$ rad/s. At the instant when $\phi = 150^o$, determine:

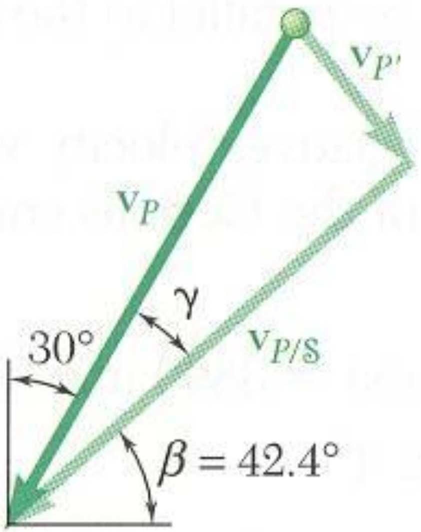
a. angular velocity of disk *S*

b. velocity of pin *P* relative to disk *S*



Sample Problem 15.9





Solution:

- Absolute velocity of point P:

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/s}$$

- Magnitude & direction of absolute velocity of pin P:

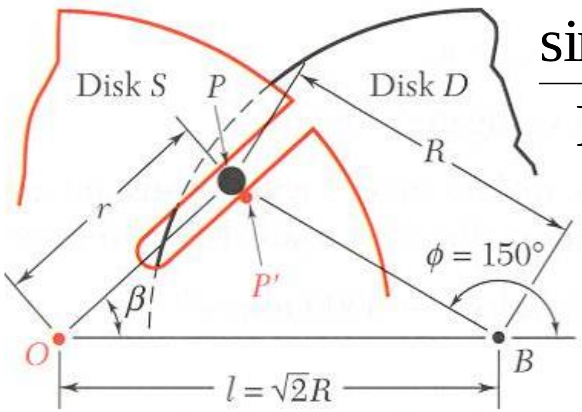
$$v_P = R\omega_D = (50\,\text{mm})(10\,\text{rad/s}) = 500\,\text{mm/s}$$

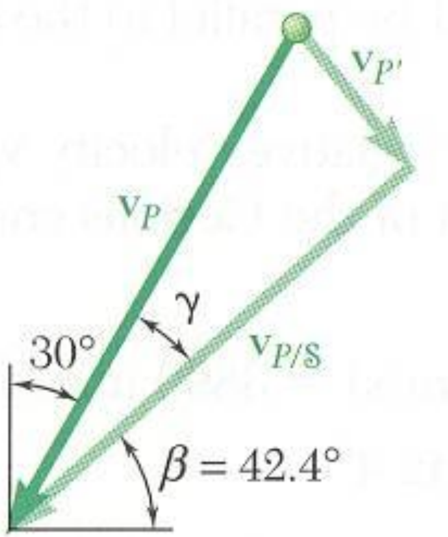
- Direction of velocity of P wrt S is parallel to slot. From the law of cosines,

$$r^2 = R^2 + l^2 - 2Rl\cos 30° = 0.551R^2$$

$$r = 37.1\,\text{mm}$$

Sample Problem 15.9





$$\frac{\sin\beta}{R} = \frac{\sin 30°}{r} \qquad \sin\beta = \frac{\sin 30°}{0.742} \qquad \beta = 42.4°$$

$$\gamma = 90° - 42.4° - 30° = 17.6°$$

$$v_{P'} = v_P \sin\gamma = (500\,\text{mm/s})\sin 17.6° = 151.2\,\text{mm/s}$$

$$= r\omega_s \qquad \omega_s = \frac{151.2\,\text{mm/s}}{37.1\,\text{mm}}$$

$$\vec{\omega}_s = (-4.08\,\text{rad/s})\vec{k}$$

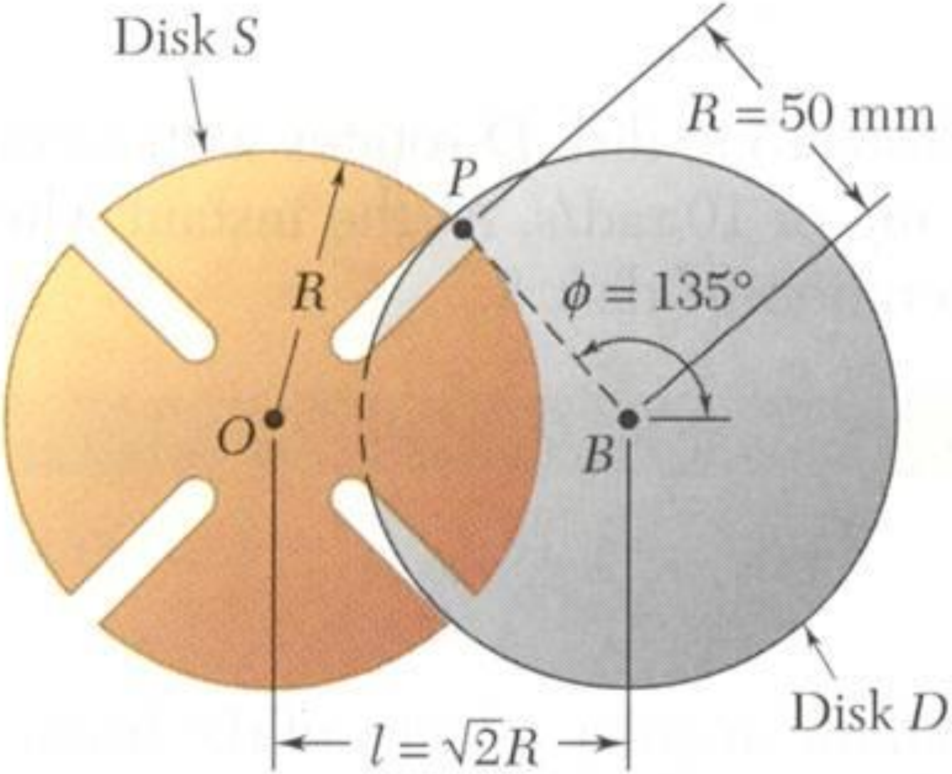
$$v_{P/s} = v_P \cos\gamma = (500\,\text{m/s})\cos 17.6°$$

$$\vec{v}_{P/s} = (477\,\text{m/s})\left(-\cos 42.4°\vec{i} - \sin 42.4°\vec{j}\right)$$

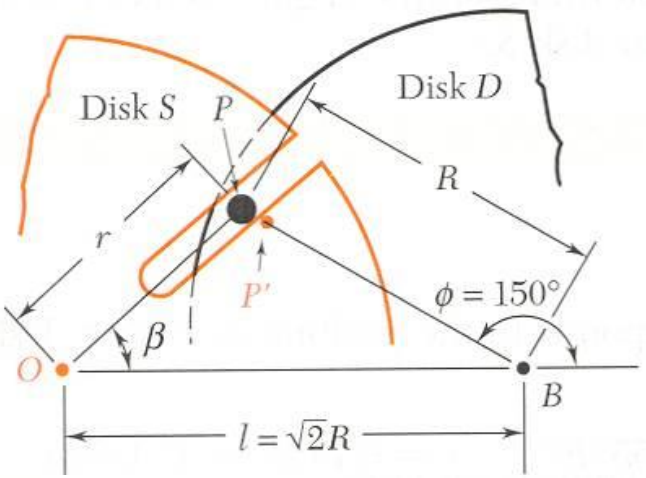
$$v_P = 500\,\text{mm/s}$$

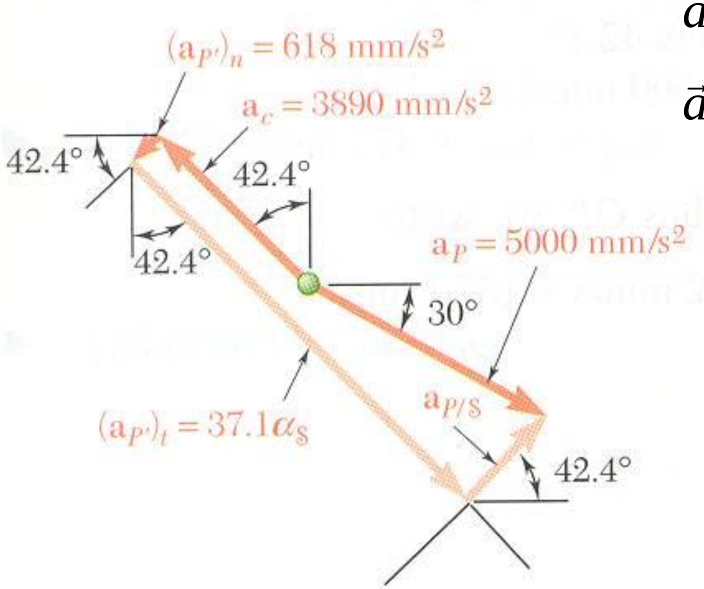
Sample Problem 15.10

In the Geneva mechanism, disk *D* rotates with a constant ccw angular velocity of 10 rad/s. At the instant when $\varphi = 150^o$, determine angular acceleration of disk *S*.



Sample Problem 15.10





Solution:

$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/s} + \vec{a}_c$$

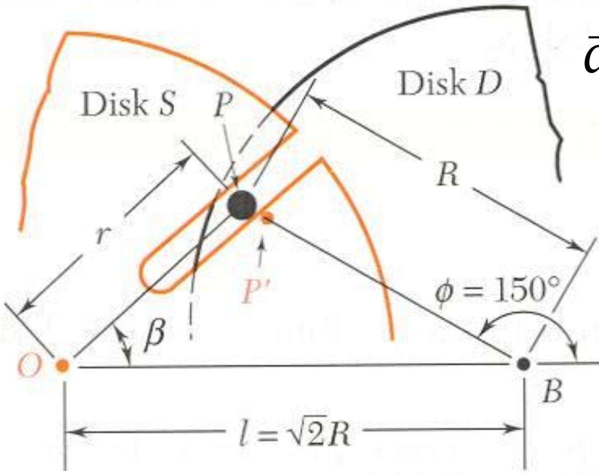
$$\beta = 42.4^\circ \qquad \vec{\omega}_S = (-4.08\,\text{rad/s})\vec{k}$$

$$\vec{v}_{P/s} = (477\,\text{mm/s})\left(-\cos 42.4^\circ \vec{i} - \sin 42.4^\circ \vec{j}\right)$$

$$a_P = R\omega_D^2 = (500\text{mm})(10\,\text{rad/s})^2 = 5000\,\text{mm/s}^2$$

$$\vec{a}_P = \left(5000\,\text{mm/s}^2\right)\left(\cos 30^\circ \vec{i} - \sin 30^\circ \vec{j}\right)$$

Sample Problem 15.10

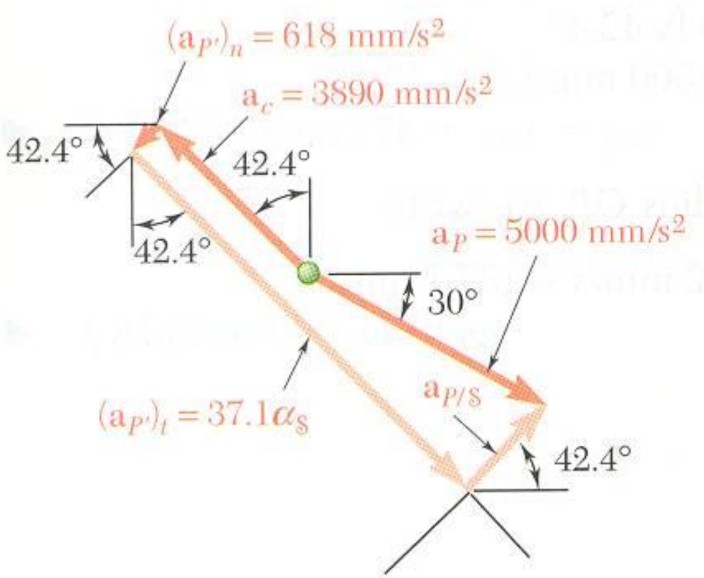


$$\vec{a}_{P'} = (\vec{a}_{P'})_n + (\vec{a}_{P'})_t$$

$$(\vec{a}_{P'})_n = (r\omega_S^2)(-\cos 42.4°\vec{i} - \sin 42.4°\vec{j})$$

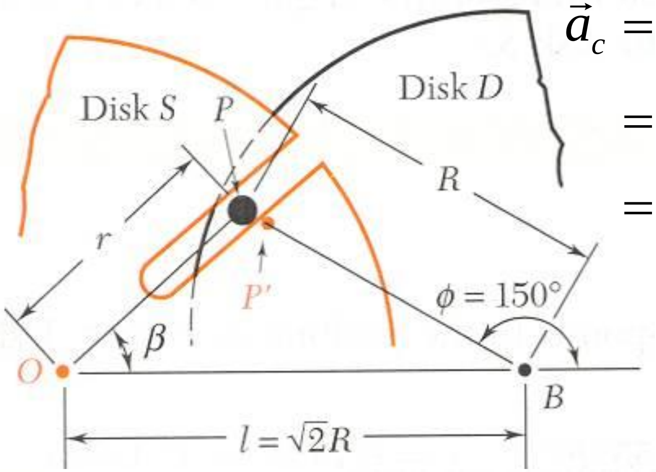
$$(\vec{a}_{P'})_t = (r\alpha_S)(-\sin 42.4°\vec{i} + \cos 42.4°\vec{j})$$

$$(\vec{a}_{P'})_t = (\alpha_S)(37.1\text{mm})(-\sin 42.4°\vec{i} + \cos 42.4°\vec{j})$$



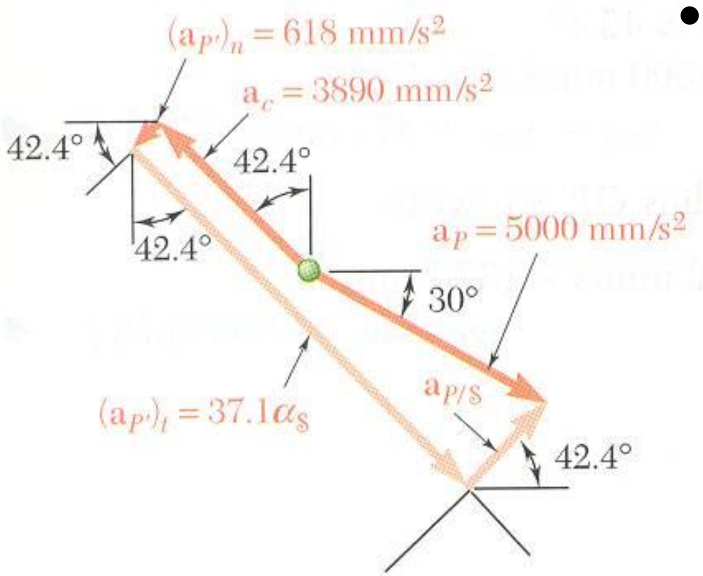
- Direction of Coriolis acceleration is obtained by rotating direction of relative velocity $\vec{v}_{P/s}$ by 90º in the sense of ω_S.

Sample Problem 15.10



$$\vec{a}_c = \left(2\omega_S v_{P/s}\right)\left(-\sin 42.4°\vec{i} + \cos 42.4\vec{j}\right)$$
$$= 2(4.08\,\text{rad/s})(477\,\text{mm/s})\left(-\sin 42.4°\vec{i} + \cos 42.4\vec{j}\right)$$
$$= \left(3890\,\text{mm/s}^2\right)\left(-\sin 42.4°\vec{i} + \cos 42.4\vec{j}\right)$$

- Relative acceleration $\vec{a}_{P/s}$ must be parallel to the slot.



- Equating components perpendicular to slot,

$$37.1\alpha_S + 3890 - 5000\cos 17.7° = 0$$
$$\alpha_S = -233\,\text{rad/s}$$
$$\vec{\alpha}_S = (-233\,\text{rad/s})\vec{k}$$

Home Work Assignment # 15.5

150, 158, 166, 172, 179

Due

Saturday 12/4/2014 (Civil)

Sunday 13/4/2014 (Mechanical)