

KINETICS OF A PARTICLE: FORCE MASS AND ACCELERATION

Introduction:

Kinetics is the study of the relations between unbalance forces and the resulting changes in motion. In this chapter we will study the kinetics of particles. this topic requires that we combine our knowledge of the properties of forces, and the kinematics of particle motion previously covered in chapter 2. With the aid of Newton's second law, we can combine these two topics and solve engineering problems involving force, mass, and motion.

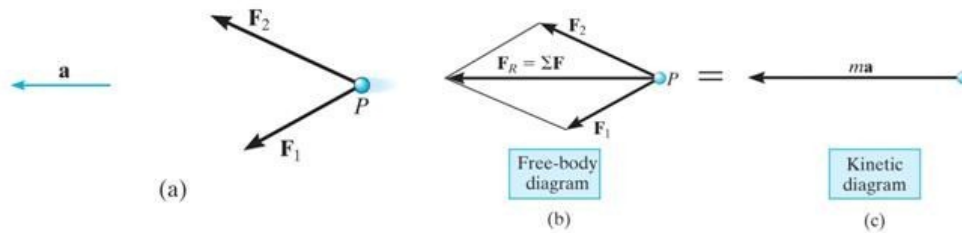
The three general approaches to the solution of kinetic problems are:(A) direct application of Newton's second law which will be discussed in this chapter, (B) use of work energy principles, and (C) solution by impulse-momentum methods. each approach has its special characteristics and advantages the last two will be discussed in the following chapters.

I. Newton's laws of motion:

- **Newton's second law of motion** states that the *unbalanced force* on a particle causes it to accelerate. If the mass of the particle is m and its velocity is \vec{V} , then the second law can be written as

$$\vec{F}_R = \sum F = \frac{d}{dt}(m\vec{V}) = m\vec{a}$$

- This equation is referred to as the *equation of motion* and is one of the most important formulations in mechanics. Its validity is based solely on experimental evidence
- The magnitude and direction of each force acting on the particle are identified using a free body diagram. A kinetic diagram identifies the magnitude and direction of the vector $m\vec{a}$.



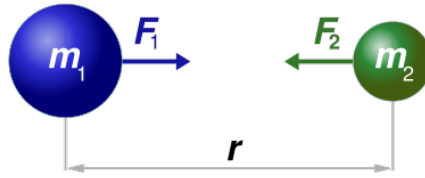
- Newton's first and second laws can appear to be invalid to certain observers. Suppose, for instance that you are a passenger riding in a friend's car. While the car moves at constant speed along a straight line, you don't feel the seat pushing against your back to any unusual extent. This experience is consistent with the first law, which indicates that in the absence of a net force you should move with constant velocity. Suddenly the driver floors the gas pedal. Immediately you feel the seat pressing against your back as the car accelerates. Therefore, you sense that a force is being applied to you. The first law leads you to believe that your motion should change, and, relative to the ground outside, your motion does change. But *relative to the car*, you can see that your motion does not change, because you remain stationary with respect to the car. Clearly Newton's first and second laws do not hold for observers who use the accelerating car as a frame of reference. As a result, such a reference frame is said to be

noninertial. All accelerating references are non inertial. In contrast, observers for whom the law of inertia is valid are said to be using **inertial frame of reference** for their observation.

A Newtonian or inertial frame of reference does not rotate and is either fixed or translates in a given direction with a constant velocity (zero acceleration). This definition ensures that particle's acceleration measured by observers in two different inertial frames of reference will always be the same.

- **Newton's law of gravitational attraction:** The mutual attraction between any two particles is given by:

$$F_1 = F_2 = G \frac{m_1 m_2}{r^2}$$



where $G = 6.673 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$ is the universal constant of gravitation. m_1, m_2 is the mass of each of the particles and r is the distance between the centers of the two particles.

- **Mass and weight.**

- Mass is a property of matter that provides a quantitative measure of its resistance to a change in the velocity. Mass is an absolute quantity since the measurement of mass can be made at any location.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface. we can use the previous expression of gravitational attraction to develop a general expression for finding the weight W of a particle having a mass $m = m_1$. Let m_2 be the mass of the earth and r the distance between the earth's center and the particle. Then, if $g = Gm_2/r^2$ we have:

$$W = mg$$

- In the SI system of units, the mass is specified in kilograms (kg) and the weight in Newtons (N) i.e.,

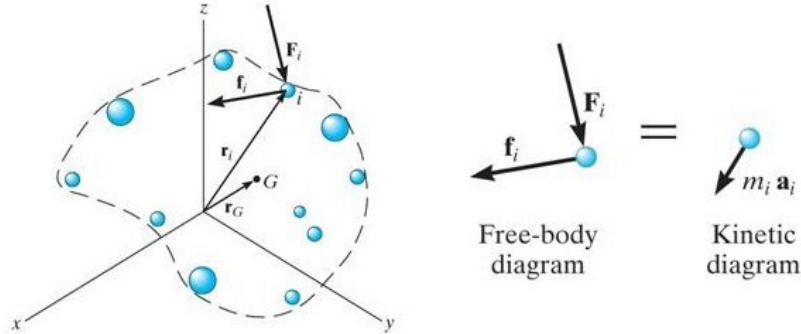
$$W = mg \text{ (N)} \quad (g = 9.81 \text{ m/s}^2)$$

- In the US Customary system of units, the weight in pounds (lb) and the mass is specified in slugs ($slug$) and i.e.,

$$m = \frac{W}{g} \text{ (slug)} \quad (g = 32.2 \text{ ft/s}^2)$$

II. Equation of motion for a system of particles:

The following analysis applies equally well to the motion of a solid, liquid, or gas system. Consider a system made of n particles each of mass m_i . An arbitrary particle i at the instant considered is subject to a system of internal forces and a resultant external force. The resultant internal force $\vec{f}_i = \sum_j \vec{f}_{ji}$ where f_{ji} is the interaction between particles j and i . Note that $\vec{f}_{ij} = -\vec{f}_{ji}$. The resultant external force \vec{F}_i represents for example the effect of gravitational, electrical, magnetic, or contact forces between the i^{th} particle and adjacent bodies or particles not included within the system.



Assuming the coordinates system to be an inertial frame of reference.

$$\vec{f}_i + \vec{F}_i = m_i \vec{a}_i$$

By summing over all the particles we get

$$\sum \vec{f}_i + \sum \vec{F}_i = \sum m_i \vec{a}_i$$

The summation $\sum \vec{f}_i = 0$ since the internal forces between particles all occur in equal but opposite directions. The above equation reduces to:

$$\sum \vec{F}_i = \sum m_i \vec{a}_i$$

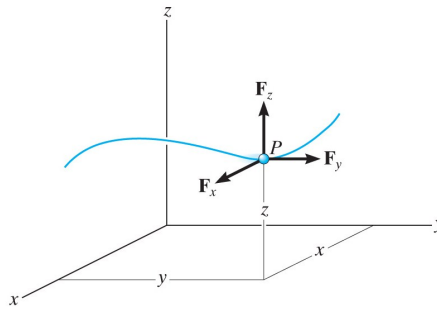
If \vec{r}_G is the position of the center of mass of the system of particles and \vec{a}_G its acceleration then $(\sum m_i) \vec{r}_G = \sum m_i \vec{r}_i$ and $(\sum m_i) \vec{a}_G = \sum m_i \vec{a}_i$. We finally have

$$\sum \vec{F}_i = (\sum m_i) \vec{a}_G$$

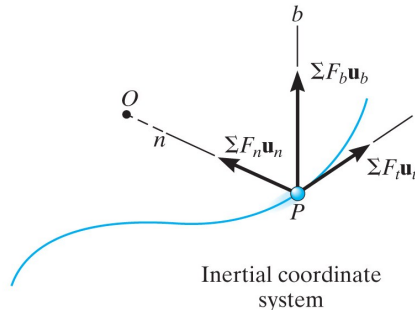
III. Equation of motion and solution of problems:

- The equation of motion as introduced above connect the kinetics of the motion (what causes the motion) and the kinematics of the motion (what describes the motion). Most problems encountered in dynamics will require the used of this equation to identify and solve for one of these variables. In some cases the forces acting on the particle are specified and the question is to determine the resulting motion, in others the acceleration of he particle is either specified or can be determined directly from known kinematic condition and the questions is to solve for the forces that caused that acceleration.
- The vector form of the equation of motion as developed in the previous paragraph can be expanded into a set of scalar equation with an appropriate choice of system of coordinate. The choice of a system of coordinate is usually indicated by the number and geometry of the constraints. For example

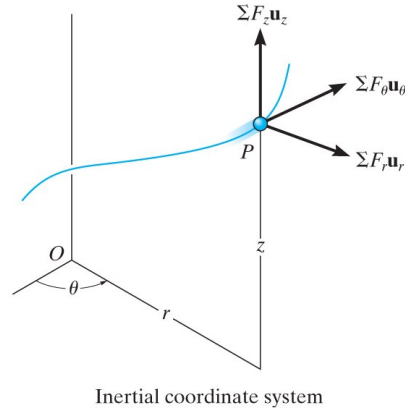
- If a particle is free of mechanical guides and follows a path determined by its initial motion and by the forces which are applied to it from external sources, this type of motion is known as *unconstrained motion* and the particle is said to have *three degrees of freedom* since three independent coordinates are required to specify the position of the particle at any instant in time. In this case all three of the scalar components of the equation of motion would have to be integrated to obtain the space coordinates as a function of time.
- The particle may be constrained to move in a plane, in this case some of the forces acting on the particle may be applied from outside sources, others may be reactions on the particle from the constraining guides. All these forces must be accounted for in the equation of motion and the particle is said to have *two degrees of freedom* and only two coordinates are needed to specify its position.
- The particle may be in some cases constrained to move along a straight line. In this case the particle has only one degree of freedom and only one coordinate is needed to specify its position
- Various types of inertial coordinate systems can be used to apply $\sum \vec{F} = m \vec{a}$ in the component form
 - * Rectangular x, y, z axes are used to describe rectilinear motion along each of the axes



- * Normal and tangential axes are often used when the path of the particle is known. Recall that \vec{a}_n is always directed in the $+\hat{n}$ direction. It indicates the change in the velocity direction. Also recall that $veca_t$ is tangent to the path. It indicates change in the speed

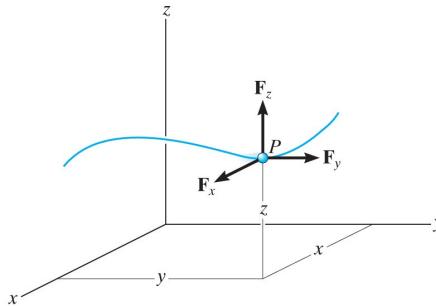


- * Cylindrical coordinates are useful when angular motion of the radial coordinate r is specified or when the path can conventionally be described with these coordinates.



III-1. Equation of motion: Rectangular coordinates:

- When a particle is moving relative to an inertial x, y, z frame of reference, the vector equation of motion is equivalent to the following three scalar equations:



$$\sum F_x = m a_x$$

$$\sum F_y = m a_y$$

$$\sum F_z = m a_z$$

- Only the first two of these equations are used to specify the motion of a particle constrained to move only in the $x - y$ plane.

III-1-1. Procedure for analysis:

- **Free body diagram (FBD)**

- Establish your coordinate system and draw the particles free body diagram showing only external forces. These external forces usually include the weight, normal forces, friction forces, and applied forces. Show the $(m\vec{a})$ vector (sometimes called the inertial force) on a separate diagram.
- Make sure any friction forces act opposite to the direction of motion! If the particle is connected to an elastic spring, a spring force equal to $(F_s = ks)$ should be included on the FBD.

- **Equations of motion**

- If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the scalar form of the equation of motion. In more complex cases (usually 3-D), a Cartesian vector is written for every force and a vector analysis is often best.
- A Cartesian vector formulation of the second law is

$$\vec{F} = m\vec{a} \text{ or}$$

$$F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = m(a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$$

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.

- **Kinematics**

- The second law only provides solutions for forces and accelerations. If velocity or position have to be found, kinematics equations are used once the acceleration is found from the equation of motion.
- Any of the tools learned in the previous chapter may be needed to solve a problem. Make sure you use consistent positive coordinate directions as used in the equation of motion part of the problem!

III-2. Equation of motion: Normal and Tangential coordinates:

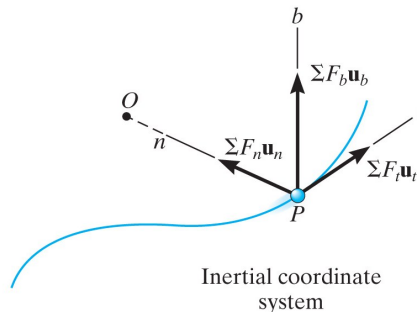
- When a particle moves over a *known curved path*, the equation of motion for the particle may be written in the tangential \hat{u}_t , normal \hat{u}_n and binomial $\hat{u}_b = \hat{u}_t \times \hat{u}_n$ directions giving the three scalar equations of motion:

$$\sum F_t = m a_t$$

$$\sum F_n = m a_n$$

$$\sum F_b = 0$$

(since the particle is constrained to move along the path)



- The tangential acceleration, $a_t = \frac{dv}{dt}$, represents the time rate of change in the magnitude of the velocity. Depending on the direction of F_t , the particles speed will either be increasing or decreasing.
- The normal acceleration, $a_n = \frac{v^2}{r}$, represents the time rate of change in the direction of the velocity vector. Remember, a_n always acts toward the paths center of curvature. Thus, F_n will always be directed toward the center of the path.

- Recall, if the path of motion is defined as $y = f(x)$, the radius of curvature at any point can be obtained from

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|}$$

III-2-1. Procedure for analysis:

- Use $n - t$ coordinates when a particle is moving along a known, curved path.
- Establish the $n - t$ coordinate system on the particle.
- Draw free-body and kinetic diagrams of the particle. The normal acceleration (a_n) always acts inward (the positive n -direction). The tangential acceleration (a_t) may act in either the positive or negative t direction.
- Apply the equations of motion in scalar form and solve.
- It may be necessary to employ the kinematic relations:

$$a_t = \frac{dv}{dt} = v \frac{dv}{ds} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

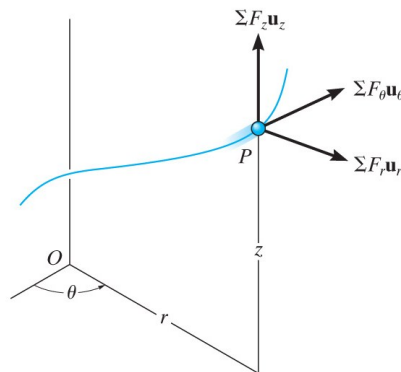
III-3. Equation of motion: Cylindrical coordinates:

- When all the forces acting on the particle are resolved into cylindrical components, i.e., along the unit-vector directions $\hat{u}_r, \hat{u}_\theta, \hat{u}_z$ the equations of motion for the particle may be written in the (cylindrical) r, θ, z directions giving three scalar equations of motion:

$$\sum F_r = m a_r$$

$$\sum F_\theta = m a_\theta$$

$$\sum F_z = m a_z$$

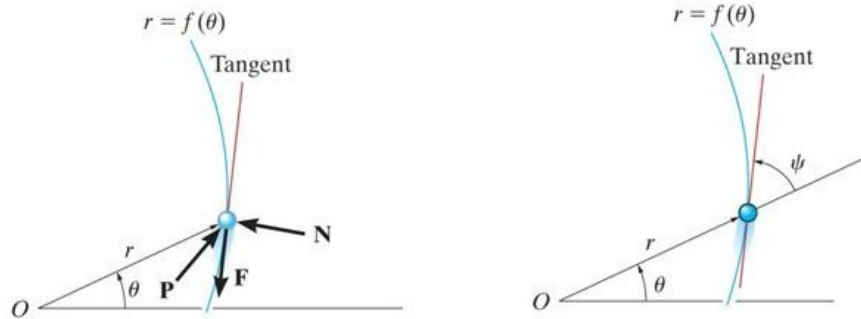


Inertial coordinate system

- Note that if the particle is constrained to move only in the $r - \theta$ plane, then only the first two equations are used to specify the motion

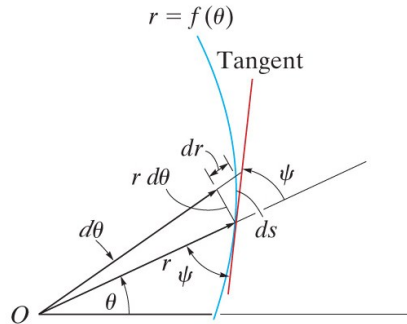
– The most straight forward type of problems involving cylindrical coordinates requires the determination of the resultant force components $\sum F_r$, $\sum F_\theta$, $\sum F_z$ which cause a particle to move with a known acceleration. If, however, the particle's acceleration is not completely specified at the given instant, then some information regarding the directions or magnitudes of the forces acting on the particle must be known or computed.

* If a force \vec{P} causes the particle to move along a path defined by $r = f(\theta)$, the normal force \vec{N} exerted by the path on the particle is always perpendicular to the path's tangent. The frictional force \vec{F} always acts along the tangent in the opposite direction of motion. The directions of \vec{N} and \vec{F} can be specified relative to the radial coordinate by using angle ψ .



* The angle ψ , defined as the angle between the extended radial line and the tangent to the curve, can be required to solve some problems. It can be determined from the following relationship.

$$\tan(\psi) = \frac{r \, d\theta}{dr} = \frac{r}{dr/d\theta}$$



III-3-1. Procedure for analysis:

Cylindrical or polar coordinates are a suitable choice for the analysis of a problem for which data regarding the angular motion of the radial line r are given, or in cases where the path can be conventionally expressed in terms of these coordinates. Once these coordinates have been established, the equations of motion can be applied in order to relate the forces acting on the particle to its acceleration components. The specific procedure is as follows:

- Establish the inertial r, θ, z coordinate system and draw the particle's free body diagram
- Assume that \vec{a}_r , \vec{a}_θ , \vec{a}_z , act in the positive directions of r, θ, z if they are unknown

- Identify the unknowns in the problem
- Apply the equations of motion as prescribed in the previous paragraph
- Determine r and the time derivatives \dot{r} , \ddot{r} , $\dot{\theta}$, $\ddot{\theta}$, \dot{z} , and evaluate the acceleration components $a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$, $a_z = \ddot{z}$
- If any of the acceleration components is computed as a negative quantity, it indicates that it acts in its negative coordinate direction
- Use the chain rule of calculus to calculate the time derivative of $r = f(\theta)$