

[Assignment View]

Eðlisfræði 2, vor 2007

26. DC Circuits

Assignment is due at 2:00am on Wednesday, February 21, 2007

Credit for problems submitted late will decrease to 0% after the deadline has passed.
 The wrong answer penalty is 2% per part. Multiple choice questions are penalized as described in the online help.
 The unopened hint bonus is 2% per part.
 You are allowed 4 attempts per answer.

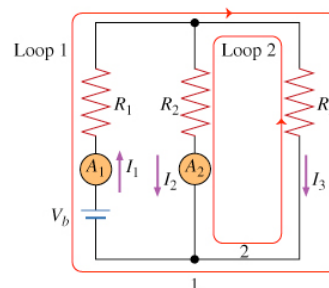
Kirchhoff's Rules and Applying Them

Learning Goal: To understand the origins of both of Kirchhoff's rules and how to use them to solve a circuit problem.

This problem introduces Kirchhoff's two rules for circuits:

- *Kirchhoff's loop rule:* The sum of the voltage changes across the circuit elements forming any closed loop is zero.
- *Kirchhoff's junction rule:* The algebraic sum of the currents into (or out of) any junction in the circuit is zero.

The figure shows a circuit that illustrates the concept of *loops*, which are colored red and labeled loop 1 and loop 2. Loop 1 is the loop around the entire circuit, whereas loop 2 is the smaller loop on the right. To apply the loop rule you would add the voltage changes of all circuit elements around the chosen loop. The figure contains two junctions (where three or more wires meet)--they are at the ends of the resistor labeled R_3 .



The battery supplies a constant voltage V_b , and the resistors are labeled with their resistances. The ammeters are ideal meters that read I_1 and I_2 respectively.

The direction of each loop and the direction of each current arrow that you draw on your own circuits are arbitrary. Just assign voltage drops consistently and sum both voltage drops and currents algebraically and you will get correct equations. If the actual current is in the opposite direction from your current arrow, your answer for that current will be negative. The direction of any loop is even less important: The equation obtained from a counterclockwise loop is the same as that from a clockwise loop except for a negative sign in front of every term (i.e., an inconsequential change in overall sign of the equation because it equals zero).

Part A

The junction rule describes the conservation of which quantity? Note that this rule applies only to circuits that are in steady state.

Hint A.1 At the junction

Hint not displayed

ANSWER: *Answer not displayed*

Part B

Apply the junction rule to the junction labeled with the number 1 (at the bottom of the resistor of resistance R_2).

Hint B.1 Elements in series

Hint not displayed

Answer in terms of given quantities, together with the meter readings I_1 and I_2 and the current I_3 .

ANSWER: $\Sigma I = 0 =$ *Answer not displayed*

Part C

Apply the loop rule to loop 2 (the smaller loop on the right). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow. Remember that the current meter is ideal.

Hint C.1 Elements in series have same current

Hint not displayed

Hint C.2 Sign of voltage across resistors

Hint not displayed

Hint C.3 Voltage drop across ammeter

Hint not displayed

Express the voltage drops in terms of V_b , I_2 , I_3 , the given resistances, and any other given quantities.

ANSWER: $\Sigma(\Delta V) = 0 = \text{Answer not displayed}$

Part D

Now apply the loop rule to loop 1 (the larger loop spanning the entire circuit). Sum the voltage changes across each circuit element around this loop going in the direction of the arrow.

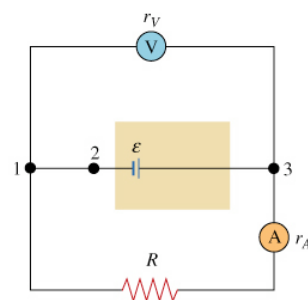
Express the voltage drops in terms of V_b , I_1 , I_3 , the given resistances, and any other given quantities.

ANSWER: $\Sigma(\Delta V) = 0 = \text{Answer not displayed}$

Measurements in Electric Circuits

Learning Goal: To understand the role of the internal resistance of various devices and the use of the ammeter and the voltmeter.

Consider the circuit shown. All wires are considered ideal; that is, they have zero resistance. We will assume for now that all other elements of the circuit are ideal, too: The value of resistance R is a constant, the internal resistances of the battery (r) and the ammeter (r_A) are zero, and the internal resistance of the voltmeter (r_V) is infinitely large.

**Part A**

What is the reading v of the voltmeter?

Express your answer in terms of the EMF ϵ .

ANSWER: $v = \text{Answer not displayed}$

Part B

The voltmeter, as can be seen in the figure, is connected to points 1 and 3. What are the respective voltages between points 1 and 2 and between points 2 and 3?

ANSWER: *Answer not displayed*

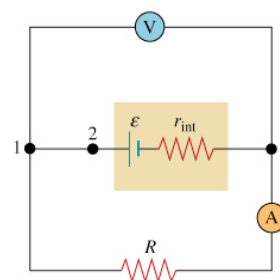
Part C

What is the reading I of the ammeter?

Express your answer in terms of \mathcal{E} and R .

ANSWER: $I = \text{Answer not displayed}$

To make things more interesting, we now assume that the battery has a nonzero internal resistance r (the voltmeter and the ammeter remain ideal).



Part D

What is the reading of the ammeter now?

Express your answer in terms of \mathcal{E} , r , and R .

ANSWER: $I = \text{Answer not displayed}$

Part E

What is the reading of the voltmeter now?

Hint E.1 Using Ohm's law

Hint not displayed

Express your answer in terms of \mathcal{E} , r , and R .

ANSWER: $V = \text{Answer not displayed}$

Now assume that the ammeter has nonzero resistance $R_A > 0$. The battery still has nonzero internal resistance.

Part F

Compared to their values when $R_A = 0$, how would the readings of the ammeter and the voltmeter change when $R_A > 0$?

Hint F.1 How to approach this part

Hint not displayed

ANSWER: *Answer not displayed*

Part G

What is the new reading of the ammeter?

Express your answer in terms of \mathcal{E} , R_A , r , and R .

ANSWER: $I = \text{Answer not displayed}$

Now assume that the ammeter again has zero resistance, but the resistance of the voltmeter is less than infinity. The battery still has nonzero internal resistance.

Part H

Compared to their values when $R_V = \infty$, how would the readings of the ammeter and the voltmeter change when R_V is some large but finite value?

Hint H.1 Consider the voltmeter first

Hint not displayed

Hint H.2 The change in the battery current

Hint not displayed

Hint H.3 The reading of the ammeter

Hint not displayed

ANSWER: *Answer not displayed*

Suppose now that the piece of ideal wire between points 1 and 2 is removed and replaced by a nonideal wire with a nonzero resistance.

Part I

How would this change affect the readings of the ammeter and the voltmeter?

Hint I.1 How to approach this part

Hint not displayed

ANSWER: *Answer not displayed*

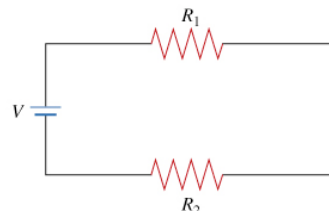
Resistor and Battery Circuits

Ohmic Resistors in Series

Two Resistors in Series

Consider the setup shown of two ideal ohmic resistors.

Resistor 1 has resistance R_1 , and resistor 2 has resistance R_2 . They are connected in series with a constant voltage of magnitude V . When the two resistors are connected in this way, they form a system equivalent to a single resistor of resistance R , as shown in the next diagram.



Part A

What is the effective resistance R of the two-resistor system?

Part A.1 Voltage drop across R_1

Part not displayed

Part A.2 Voltage drop across R

Part not displayed

Hint A.3 How to put it all together

Hint not displayed

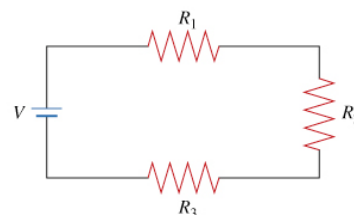
Express the effective resistance in terms of R_1 and R_2 .

ANSWER: $R =$ *Answer not displayed*

Three Resistors in Series

Now consider three resistors set up in series, as shown.

The resistances are R_1 , R_2 , and R_3 , and the applied constant voltage is again V .

**Part B**

Find the effective resistance, R_T , of the three-resistor system.

Hint B.1 Different strategies to use

Hint not displayed

Express the effective resistance in terms of R_1 , R_2 , and R_3 .

ANSWER: $R_T =$ *Answer not displayed*

Two Resistors

Two resistors of resistances R_1 and R_2 , with $R_2 > R_1$, are connected to a voltage source with voltage v_0 . When the resistors are connected in series, the current is I_s . When the resistors are connected in parallel, the current I_p from the source is equal to $10I_s$.

Part A

Let r be the ratio R_1/R_2 . Find r .

Part A.1 Calculate the source voltage

When resistors are connected in series, the total current must travel through both resistors. Therefore, in a series circuit, the current is equal everywhere. The voltage drops by a certain amount through each resistor, so the voltage drop for the series is the sum of the voltage drops through each resistor. Use this information to find the

source voltage V_0 .

Express your answer in terms of I_s and the resistances R_1 and R_2 .

ANSWER: $V_0 = I_s \cdot (R_1 + R_2)$

Part A.2 Find another expression for the source voltage

When resistors are connected in parallel, the current splits, with a portion of it going through each resistor. Therefore, the net current is the sum of the currents through each resistor. The voltage drop across each resistor is the same, and this drop is equal to the source voltage. Use this information to find another expression for the source voltage V_0 .

Hint A.2.a The equivalent resistance of resistors in parallel

Hint not displayed

Express your answer in terms of the resistances R_1 and R_2 and the observed parallel current I_p .

ANSWER: $V_0 = I_p \cdot ((R_1 \cdot R_2) / (R_1 + R_2))$

Hint A.3 Equate the two expressions for voltage

Equate the two expressions for the voltage that you have obtained, making the substitutions $r = R_1/R_2$ and $I_p = 10I_s$. Then solve for r . Keep in mind that $R_2 > R_1$.

Hint A.4 Formula for the roots of a quadratic equation

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Hint A.5 General answer

More generally, if the ratio of currents is i , then r is the smaller of the two roots of the equation $r^2 + (2-i)r + 1 = 0$. That is, $r = \frac{i}{2} - 1 - \sqrt{\left(\frac{i}{2} - 1\right)^2 - 1}$. This would be unity for $i = 4$, which would result if the two resistors were equal.

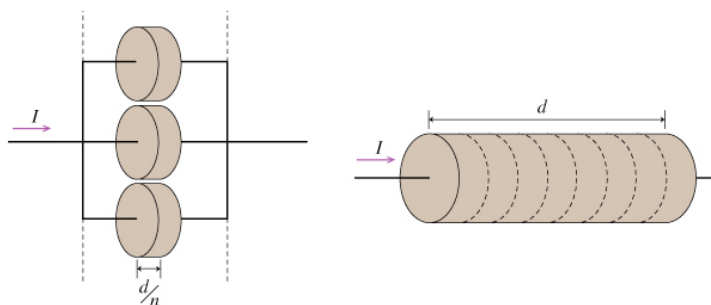
Note that the other root of the quadratic equation is greater than 1 and is equal to $\frac{1}{r} = \frac{R_2}{R_1}$.

Round your answer to the nearest thousandth.

ANSWER: $r = R_1/R_2$ **0.127**

Resistance and Wire Length

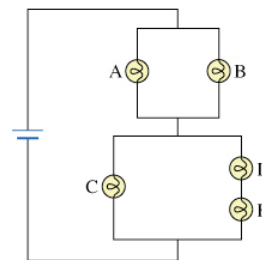
You have been given a long length of wire. You measure the resistance of the wire, and find it to be R_{long} . You then cut the wire into n identical pieces, and connect them in parallel, as shown in the figure.



Part A

Brightness of Light Bulbs

Consider a circuit containing five identical lightbulbs and an ideal battery.



Part A

Rank the brightness of the five bulbs (A through E) from brightest to dimmest. (The more current flowing through a bulb, the brighter it will be.)

Hint A.1 Comparing bulb A to bulb B

Hint not displayed

Hint A.2 Comparing bulb D to bulb E

Hint not displayed

Hint A.3 Comparing bulb C to bulb D or E

Hint not displayed

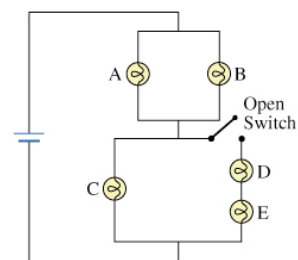
Hint A.4 Comparing bulb C to bulb A or B

Hint not displayed

List the bulbs in order from brightest to dimmest. Between each pair of bulbs, use the symbol $>$ to indicate that the left-hand bulb is brighter than the right-hand bulb, or $=$ to indicate that the bulbs have the same brightness. For example, " $B=C=E>A>D$ " means that bulbs B, C, and E all have the same brightness, and that they are brighter than bulb A, which in turn is brighter than bulb D.

ANSWER: *Answer not displayed*

Now consider what happens when a switch in the circuit is opened.



Part B

What happens to bulb A?

Hint B.1 How to approach this part

Hint not displayed

ANSWER: *Answer not displayed*

Part C

What is the current I_C now flowing in bulb C?

Express your answer in terms of the applied voltage \mathcal{E} and R , the resistance of a single bulb.

ANSWER: $I_C =$ *Answer not displayed*

Part D

What happens to bulb C?

Part D.1 Current in bulb C earlier

Part not displayed

ANSWER: *Answer not displayed*

Comparing brightness of light bulbs

Consider five identical light bulbs (A - E) connected to a battery as shown in the circuit below.

Part A

Rank the brightness of all five bulbs from brightest to dimmest.

Hint A.1 Brightness

The brightness of a light bulb is related to the power P dissipated by that bulb. Recall that power depends upon the square of the current I that passes through the bulb and the resistance R of the bulb. Since each light bulb in this circuit has the same resistance, the lightbulb with the largest current passing through it will be the brightest.

Part A.2 Compare the brightness of bulbs A and B

ANSWER: The brightness of bulb A is the brightness of bulb B.

Part A.3 Compare the brightness of bulbs D and E

ANSWER: The brightness of bulb D is the brightness of bulb E.

Part A.4 Compare the brightness of bulbs C and D**Part A.4.a Compare the current through bulbs C and D**

Part not displayed

ANSWER: The brightness of bulb C is the brightness of bulb D.

Part A.5 Compare the brightness of bulbs C and A**Part A.5.a Compare the current through bulbs C and A**

Part not displayed

ANSWER: The brightness of bulb C is the brightness of bulb A.

Part A.6 Compare the brightness of bulbs D and A**Part A.6.a Compare the current through bulbs C and A***Part not displayed*

ANSWER: The brightness of bulb D is the brightness of bulb A.

Rank the bulbs from brightest to dimmest.

ANSWER:

[View](#)

How does this change effect the circuit?

Suppose bulb E is unscrewed and removed from its socket. (The empty socket remains in the circuit.)

Part B

Does bulb A get brighter, dimmer, or stay the same brightness?

Hint B.1 How to approach the problem

When light bulb E is unscrewed and removed from the circuit, the current I that leaves the battery changes. This will change the amount of current that flows through light bulb A. Keep in mind that the brightness of a light bulb is related to the square of the current that passes through the bulb and the resistance of the bulb.

Part B.2 Determine how the current changes

When light bulb E is unscrewed and removed from the circuit, will the current that leaves the battery increase or decrease?

Hint B.2.a Relationship between resistance and current*Hint not displayed***Part B.2.b Determine how the total resistance changes***Part not displayed*

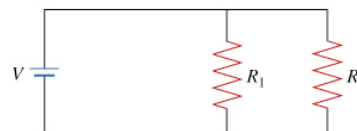
ANSWER: It will increase. It will decrease

ANSWER: Light bulb A gets brighter. Light bulb A gets dimmer. Light bulb A stays the same brightness.

Resistors in Parallel

A circuit contains a source of constant voltage V and two resistors connected in parallel, as shown in the figure.

Resistor 1 has resistance R_1 , and resistor 2 has resistance R_2 .



Part A

Which two of the following statements are true?

1. The voltage source provides a constant voltage, a part of which, v_1 , drops off across resistor 1, and the remainder, v_2 , across resistor 2. Hence $v = v_1 + v_2$.
2. The voltage drops v_1 across resistor 1 and v_2 across resistor 2 are the same, and they are independent of the values of R_1 and R_2 . Both v_1 and v_2 are equal to the voltage provided by the source. Hence $v = v_1 = v_2$.
3. The current I provided by the voltage source splits: A part of it, I_1 , flows through resistor 1, while the remainder, I_2 , flows through resistor 2. Hence $I = I_1 + I_2$.
4. The currents I_1 through resistor 1 and I_2 through resistor 2 are the same, and they are equal to the current I provided by the source. Hence $I = I_1 = I_2$.

ANSWER: *Answer not displayed*

Part B

Part not displayed

Part C

Part not displayed

Series And Parallel Connections

Learning Goal: To learn to calculate the equivalent resistance of the circuits combining series and parallel connections.

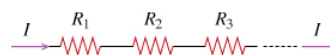
Resistors are often connected to each other in electric circuits. Finding the **equivalent resistance** of combinations of resistors is a common and important task. Equivalent resistance is defined as the single resistance that can replace the given combination of resistors in such a manner that the currents in the rest of the circuit do not change.

Finding the equivalent resistance is relatively straightforward if the circuit contains only series and parallel connections of resistors.

An example of a **series connection** is shown in the diagram:

For such a connection, the current is the same for all individual resistors and the total voltage is the sum of the voltages across the individual resistors.

Using Ohm's law ($R = \frac{V}{I}$), one can show that, for a series connection, the equivalent resistance is the sum of the individual resistances.



Mathematically, these relationships can be written as:

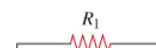
$$I = I_1 = I_2 = I_3 = \dots$$

$$V = V_1 + V_2 + V_3 + \dots$$

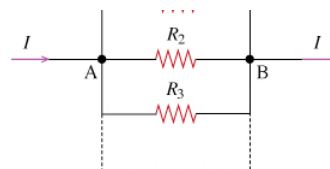
$$R_{\text{eq-series}} = R_1 + R_2 + R_3 + \dots$$

An example of a **parallel connection** is shown in the diagram:

For resistors connected **in parallel** the voltage is the same for all individual resistors because they are all connected to the same two points (A and B on the diagram). The total current is the sum of the currents through the individual



resistors. This should makes sense as the total current "splits" at points A and B.



Using Ohm's law, one can show that, for a parallel connection, the reciprocal of the equivalent resistance is the sum of the reciprocals of the individual resistances.

Mathematically, these relationships can be written as:

$$V = V_1 = V_2 = V_3 = \dots$$

$$I = I_1 + I_2 + I_3 + \dots$$

$$\frac{1}{R_{\text{eq-parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

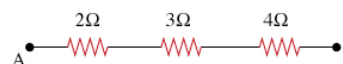
NOTE: If you have already studied capacitors and the rules for finding the equivalent capacitance, you should notice that the rules for the capacitors are similar - but not quite the same as the ones discussed here.

In this problem, you will use the the equivalent resistance formulas to determine R_{eq} for various combinations of resistors.

Part A

For the combination of resistors shown, find the equivalent resistance between points A and B.

Express your answer in Ohms.



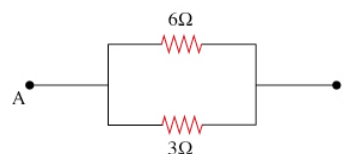
ANSWER: $R_{\text{eq}} = 9 \ \Omega$

These resistors are connected in *series*; the current through each is the same.

Part B

For the set-up shown, find the equivalent resistance between points A and B.

Express your answer in Ohms.

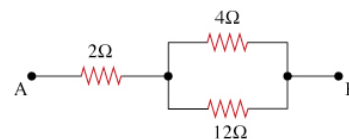


ANSWER: $R_{\text{eq}} = 2 \ \Omega$

This is a *parallel* connection since the voltage across each resistor is the same.

Part C

For the combination of resistors shown, find the equivalent resistance between points A and B.



Hint C.1 How to approach the question

Hint not displayed

Part C.2 What kind of connection is this?

Part not displayed

Express your answer in Ohms.

ANSWER: $R_{\text{eq}} = 5 \ \Omega$

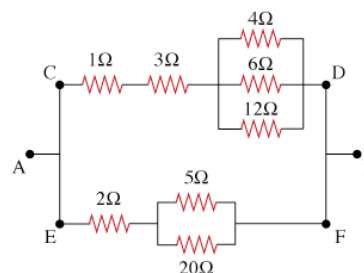
In this case, you cannot say that *all three* resistors are connected either in series or in parallel. You have a combination of a series and a parallel connection.

Some circuits may contain a large number of resistors connected in various ways. To determine the equivalent resistance of such circuits, you have to take several steps, carefully selecting the "sub-combinations" of resistors connected in relatively obvious ways. Good record-keeping is essential here.

The next question helps you practice this skill.

Part D

For the combination of resistors shown, find the equivalent resistance between points A and B.



Hint D.1 How to approach the question

Hint not displayed

Part D.2 Find R_{eq} for the "4-6-12" combination

Part not displayed

Part D.3 Find R_{eq} for the top branch

Part not displayed

Part D.4 Find R_{eq} for the bottom branch

Part not displayed

Express your answer in Ohms.

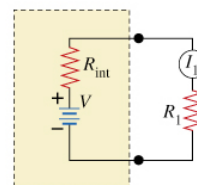
ANSWER: $R_{\text{int}} = 3 \ \Omega$

The next level of analyzing a circuit is to determine the voltages across and the currents through the various branches of the circuit. You will practice that skill in the future.

Of course, there are circuits that *cannot* possibly be represented as combinations of series and parallel connections. However, there are ways to analyze those, too.

Finding Current by Changing Resistors

A battery provides a voltage V and has unknown internal resistance R_{int} . When the battery is connected across a resistor of resistance R_1 , the current in the circuit is I_1 .



Part A

If the external resistance is then changed to R_2 (i.e., by replacing the resistor), what will be the value I_2 of the new current in the circuit?

Hint A.1 How to approach the problem

Find two expressions for the internal resistance, one involving the resistance R_1 and one involving R_2 . Eliminate the internal resistance from the resulting system of equations, and solve for I_2 .

Hint A.2 Internal resistance explained

All sources of EMF have an attribute called internal resistance. For example, the chemical reactions that provide the EMF in a DC battery occur in a medium that has its own resistance to the flow of electrons as current. We can model this as an extra resistor with resistance R_{int} connected in series with the EMF source. (In reality, this internal resistance is an inherent part of the EMF source and is not something we can remove.)

Part A.3 Find the internal resistance in terms of R_1

Find the internal resistance R_{int} of the EMF source.

Hint A.3.a Kirchhoff's rules

Remember that according to Kirchhoff's rules, the sum of the voltages around a closed loop must equal zero.

Express your answer in terms of V , I_1 , and R_1 .

ANSWER: $R_{\text{int}} = (V - I_1 R_1) / I_1$

Part A.4 Find the internal resistance in terms of R_2

Find the internal resistance R_{int} of the EMF source.

Hint A.4.a Kirchhoff's rules

Hint not displayed

Express your answer in terms of V , I_2 , and R_2 .

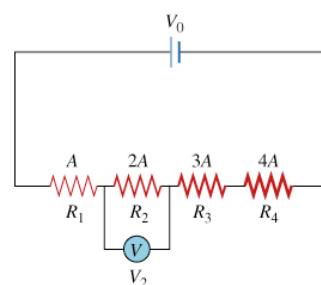
More Challenging Resistor and Battery Circuits

Series Resistors with Different Areas

Four wires are made of the same highly resistive material, cut to the same length, and connected in series.

1. Wire 1 has resistance R_1 and cross-sectional area A .
2. Wire 2 has resistance R_2 and cross-sectional area $2A$.
3. Wire 3 has resistance R_3 and cross-sectional area $3A$.
4. Wire 4 has resistance R_4 and cross-sectional area $4A$.

A voltage V_0 is applied across the series, as shown in the figure.

**Part A**

Find the voltage V_2 across wire 2.

Part A.1 Resistance of wires

Part not displayed

Part A.2 Find the current through wire 2

Part not displayed

Hint A.3 The voltage across wire 2

Hint not displayed

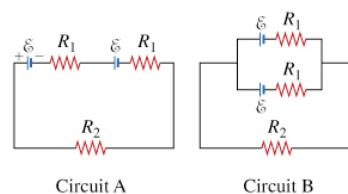
Give your answer in terms of V_0 , the voltage of the battery.

ANSWER: $V_2 =$ *Answer not displayed*

Batteries in Series or Parallel

You are given two circuits with two batteries of emf \mathcal{E} and internal resistance R_1 each. Circuit A has the batteries connected in series with a resistor of resistance R_2 , and circuit B has the batteries connected in parallel to an equivalent resistor.

Note that the symbol \mathcal{E} should be entered in your answers as EMF.



Part A

In which direction does the current in circuit A flow?

Hint A.1 Conventions

Remember that the conventional current flows from a positive to a negative terminal.

ANSWER: clockwise counterclockwise

Part B

What is the current through the resistor of resistance R_2 in circuit A?

Hint B.1 Which formula to use

You are given the voltage and the resistance; hence Ohm's law is the relevant one here.

Hint B.2 Total resistance of the circuit

One can do this by inspection or by applying the Kirchhoff's loop rule. Kirchhoff's loop rule states that

$$\sum_{i=0}^n \Delta V_i = 0.$$

For circuit A, ΔV_i is the change in total potential difference due to the presence of electrical components in the circuit. Remember that each resistor contributes voltage equal to IR .

Express the current in terms of \mathcal{E} , R_1 , and R_2 .

ANSWER: $I_A = (2 \cdot (EMF)) / (2R_1 + R_2)$

Part C

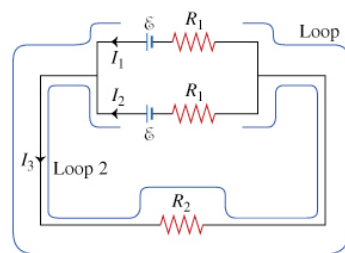
Calculate the current I_B through the resistor of resistance R_2 for circuit B.

Hint C.1 Which rule to use

Apply Kirchhoff's loop rule to the circuit. You should divide the circuit into two loops, then apply Kirchhoff's loop rule to each loop to obtain two equations. To obtain the equation for I_B , you will also need to apply Kirchhoff's junction rule (current rule).

Part C.2 What is the emf for loop 1?

The diagram shows the circuit divided into two loops: I_1 is the current in the topmost branch, I_2 is the current in the branch below it, while $I_3 = I_B$ is the current in the lowest branch, which contains R_2 . Find an expression for the emf using the voltage drops across the two resistors in loop 1.



Express your answer in terms of I_1 , I_2 , R_1 , and R_2 .

ANSWER: $\mathcal{E} = I_1 \cdot R_1 + I_2 \cdot R_2$

Part C.3 What is the emf for loop 2?

Now look at loop 2. Find an expression for the emf using the voltage drops across the two resistors in the loop.

Express your answer in terms of I_2 , I_3 , R_1 , and R_2 .

ANSWER: $\mathcal{E} = I_2 R_1 + I_3 R_2$

Part C.4 Application of Kirchhoff's junction rule (current rule)

You should now have two equations involving all the variables in the circuit diagram. To solve for I_3 , you need a relationship between I_1 and I_2 . Choose the correct relation by applying Kirchhoff's junction rule to one of the junctions. Recall that Kirchhoff's junction rule states that the algebraic sum of all the currents into a junction is zero:

$$\sum_{i=0}^n I_i = 0.$$

ANSWER: $I_1 - I_2 = I_3$
 $I_2 - I_1 = I_3$
 $I_1 + I_2 = I_3$

Now solve the three equations you have obtained for the currents in each branch to obtain an expression for I_3 ($= I_B$). To do this, you could either add the two equations other than the one above, or substitute for I_1 and I_2 from the other equations into this one.

Express your answer in terms of \mathcal{E} , R_1 , and R_2 .

ANSWER: $I_B = 2(EMF)/(R_1 + 2R_2)$

Part D

What is the power dissipated by the resistor of resistance R_2 for circuit A, given that $\mathcal{E} = 10$ V, $R_1 = 300$ ohms, and $R_2 = 5000$ ohms?

Hint D.1 What formula to use

There are two formulas for working out the power dissipated by a resistor:

$$P = V^2/R$$

and

$$P = I^2 R,$$

where R is the resistance of a resistor and V is the voltage across the resistor. You have worked out I from a previous part of the question, so the latter equation looks more useful.

Calculate the power to two significant figures.

ANSWER: $P = 0.064$ W

Part E

For what ratio of R_1 and R_2 would power dissipated by the resistor of resistance R_2 be the same for circuit A and circuit B?

Hint E.1 Getting started

Hint not displayed

Hint E.2 Finding R_1/R_2

Hint not displayed

ANSWER: $R_1/R_2 = 1$

Part F

Under which of the following conditions would power dissipated by the resistance R_2 in circuit A be bigger than that of circuit B?

Hint F.1 How to think about the problem

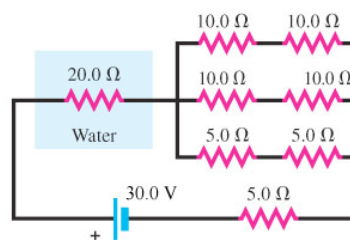
Hint not displayed

Some answer choices overlap; choose the most restrictive answer.

ANSWER: $R_2 < R_1$
 $R_2 > R_1$
 $R_2 < 2R_1$
 $R_2 < 0.5R_1$

Heating a Water Bath

In the circuit in the figure, a 20-ohm resistor sits inside 101 g of pure water that is surrounded by insulating Styrofoam.



Part A

If the water is initially at temperature 10.4°C, how long will it take for its temperature to rise to 57.0°C?

Hint A.1 How to approach the problem

First reduce the system of resistors to a single equivalent resistor; then use this simplified circuit to calculate the current flowing through the battery. Determine the current flowing through the resistor in the water and calculate its power output. Finally, use the calculated power output to calculate the time needed to heat the water bath.

Part A.2 Calculate the resistance of the circuit

Calculate the total resistance R_{total} of the network of resistors shown in the figure.

Hint A.2.a Reducing a network of resistors to an equivalent resistor

Hint not displayed

Part A.2.b Combining the resistors in the middle section

Part not displayed

Hint A.2.c Combining the rest of the resistors

Hint not displayed

Express your answer in ohms using three significant figures.

ANSWER: $R_{\text{total}} = 30.0 \ \Omega$

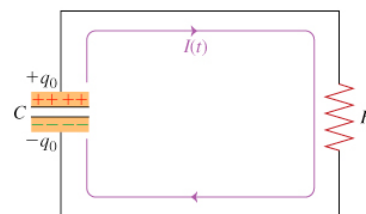
Part A.3 Calculate the current in the equivalent resistor

Circuits with Capacitors and Varying Current

An R-C Circuit

Learning Goal: To understand the behavior of the current and voltage in a simple R-C circuit

A capacitor with capacitance C is initially charged with charge q_0 . At time $t=0$ a resistor with resistance R is connected across the capacitor.

**Part A**

Use the Kirchhoff loop rule and Ohm's law to express the voltage across the capacitor $v(t)$ in terms of the current $I(t)$ flowing through the circuit.

Express your answer in terms of $I(t)$ and R .

ANSWER: $v(t) =$ *Answer not displayed*

Part B

Part not displayed

Part C

Part not displayed

Part D

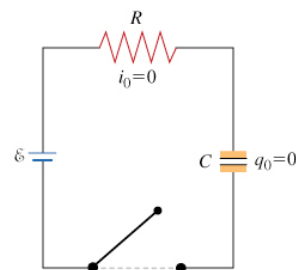
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Charging and Discharging a Capacitor in an R-C Circuit

Learning Goal: To understand the dynamics of a series R-C circuit.

Consider a series circuit containing a resistor of resistance R and a capacitor of capacitance C connected to a source of EMF \mathcal{E} with negligible internal resistance. The wires are also assumed to have zero resistance. Initially, the switch is open and the capacitor discharged.

Let us try to understand the processes that take place after the switch is closed. The charge of the capacitor, the current in the circuit, and, correspondingly, the voltages across the resistor and the capacitor, will be changing. Note that at any moment in time during the life of our circuit, Kirchhoff's loop rule holds and indeed, it is helpful: $\mathcal{E} - v_R - v_C = 0$, where v_R is the voltage across the resistor, and v_C is the voltage across the capacitor.

**Part A**

Part B

Immediately after the switch is closed, what is the voltage across the resistor?

- ANSWER:**
- \mathcal{E}
 - $\frac{1}{2}\mathcal{E}$
 - zero

Part C

Immediately after the switch is closed, what is the direction of the current in the circuit?

- ANSWER:**
- clockwise
 - counterclockwise
 - There is no current because the capacitor does not allow the current to pass through.

While no charge can physically pass through the gap between the capacitor plates, it can flow in the rest of the circuit. The current in the capacitor can be thought of as a different sort of current, not involved with the flow of charge, but with an electric field that is increasing with time. This current is called the *displacement current*. You will learn more about this later. Of course, when the charge of the capacitor is not changing, then there is no current.

Part D

After the switch is closed, which plate of the capacitor eventually becomes positively charged?

- ANSWER:**
- the top plate
 - the bottom plate
 - both plates
 - neither plate because electrons are negatively charged

Part E

Eventually, the process approaches a steady state. In that steady state, the charge of the capacitor is not changing. What is the current in the circuit in the steady state?

Hint E.1 Charge and current

Hint not displayed

- ANSWER:**
- $\frac{\mathcal{E}}{R}$
 - $\frac{\mathcal{E}}{R+C}$
 - $\frac{\mathcal{E}}{R-C}$
 - $\frac{\mathcal{E}}{C-R}$
 - zero

Part F

In the steady state, what is the charge q of the capacitor?

Hint F.1 Voltage in the steady state

Hint not displayed

Express your answer in terms of any or all of \mathcal{E} , R , and C .

- ANSWER:** $q = C\mathcal{E}$

Part G

How much work w is done by the voltage source by the time the steady state is reached?

Hint G.1 Charge and EMF

By definition, the EMF of the source is defined as the ratio of the work done by the source "pushing" a charge q through the circuit and the magnitude of that charge: $\mathcal{E} = \frac{W}{q}$.

Express your answer in terms any or all of \mathcal{E} , R , and C .

ANSWER: $W = \mathcal{E}^2 C$

Note that the work done depends on C , but not on R ! This is because it is the capacitor that determines the amount of charge flow through the circuit. Charge flow stops when $qC = \mathcal{E}$. The resistance does however affect the *rate* of charge flow i.e. the current. You will calculate this effect in the parts that follow.

Also note that of the total work done by the battery, half is stored as electrostatic energy in the capacitor, while half is dissipated as losses in the resistor. So such a simple charging circuit has a high loss percentage, independent of the value of the resistance of the circuit.

Now that we have a feel for the state of the circuit in its steady state, let us obtain expressions for the charge of the capacitor and the current in the resistor as functions of time. We start with the loop rule: $\mathcal{E} - V_R - V_C = 0$. Note that $V_R(t) = I(t)R$, $V_C(t) = \frac{q(t)}{C}$, and $I(t) = \frac{dq(t)}{dt}$. Using these equations, we obtain $\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$, and then, $\frac{dq(t)}{q(t) - C\mathcal{E}} = -\frac{dt}{RC}$.

Part H

Integrate both sides of the equation $\frac{dq(t)}{q(t) - C\mathcal{E}} = -\frac{dt}{RC}$ to obtain an expression for $q(t)$.

Hint H.1 Constant of integration

To obtain the constant of integration, use the initial condition that at time $t = 0$, the charge on the capacitor is zero.

Express your answer in terms of any or all of \mathcal{E} , R , t , and C . Enter **exp(x)** for e^x .

ANSWER: $q(t) = C \cdot \mathcal{E} \cdot (1 - \exp(-t/(RC)))$

Part I

Now differentiate $q(t)$ to obtain an expression for the current $I(t)$.

Express your answer in terms of any or all of \mathcal{E} , R , t , and C . Enter **exp(x)** for e^x .

ANSWER: $I(t) = (\mathcal{E}/R) \cdot \exp(-t/(RC))$

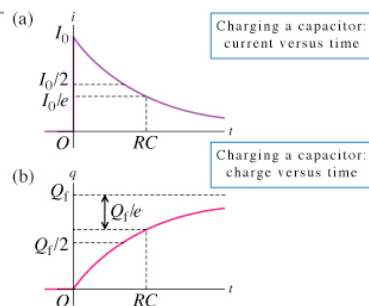
Theoretically, the steady state is never reached: The exponential functions approach their limits as $t \rightarrow \infty$ asymptotically. However, it does not take very long for the values of $q(t)$ and $I(t)$ to get very close to their limiting values. The next few questions illustrate this point. Note that the quantity RC has dimensions of time and is called the *time constant*, or the *relaxation time*. It is often denoted by τ . Using τ , one can rewrite the expressions for charge and current as follows:

$$q(t) = C\mathcal{E} \left(1 - \exp\left(-\frac{t}{RC}\right) \right) \rightarrow q(t) = C\mathcal{E} \left(1 - \exp\left(-\frac{t}{\tau}\right) \right)$$

and

$$I(t) = \frac{\mathcal{E}}{R} \exp\left(-\frac{t}{RC}\right) \rightarrow I(t) = \frac{\mathcal{E}}{R} \exp\left(-\frac{t}{\tau}\right).$$

Graphs of these functions are shown in the figure.



Part J

Find the time t_2 that it would take the charge of the capacitor to reach 99.99% of its maximum value given that $R = 12.0 \Omega$ and $C = 500 \mu\text{F}$.

Part J.1 Find an expression for the time

Find the time t_1 that it takes the charge of the capacitor to reach 99.99% of its maximum value.

Hint J.1.a How to approach this part

What is the maximum charge on the capacitor? Set $q(t)$ to be 99.99% of this value and solve for the corresponding time.

Express your answer in terms of τ . Use three significant figures for any numerical terms.

ANSWER: $t_1 = 9.210\tau$

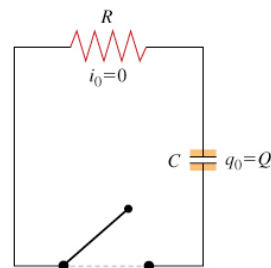
It would take the same amount of time for the current to drop to 0.01% of its initial (maximum) value; compare the expressions for $q(t)$ and $I(t)$ to see this for yourself.

Express your answer numerically in seconds. Use three significant figures in your answer.

ANSWER: $t_2 = 5.53 \times 10^{-2}$

Notice how quickly the circuit approaches steady state for these typical values of resistance and capacitance!

Let us now consider a different R - C circuit. This time, the capacitor is initially charged ($q(0) = q_0$), and there is no source of EMF in the circuit. We will assume that the top plate of the capacitor initially holds positive charge. For this circuit, Kirchhoff's loop rule gives $IR + \frac{q}{C} = 0$, or equivalently, $IR = -\frac{q}{C}$.



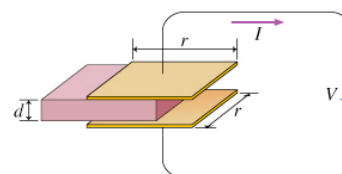
Part K

Find the current $I(t)$ as a function of time for this circuit.

Part K.1 Find the charge $q(t)$ on the capacitor

Changing Capacitance Yields a Current

Each plate of a parallel-plate capacitor is a square with side length r , and the plates are separated by a distance d . The capacitor is connected to a source of voltage v . A plastic slab of thickness d and dielectric constant K is inserted slowly between the plates over the time period Δt until the slab is squarely between the plates. While the slab is being inserted, a current runs through the battery/capacitor circuit.



Part A

Assuming that the dielectric is inserted at a constant rate, find the current I as the slab is inserted.

Part A.1 What is the effect of the dielectric on capacitance?

Part not displayed

Part A.2 What is the current in the circuit?

Part not displayed

Part A.3 What is the initial capacitance?

Part not displayed

Part A.4 What is the change in capacitance?

Part not displayed

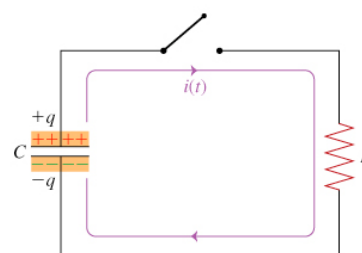
Express your answer in terms of any or all of the given variables V , K , r , d , Δt , and ϵ_0 , the permittivity of free space.

ANSWER: $I = \text{Answer not displayed}$

Charged Capacitor and Resistor

Learning Goal: To study the behavior of a circuit containing a resistor and a charged capacitor when the capacitor begins to discharge.

A capacitor with capacitance C is initially charged with charge q . At time $t=0$, a switch is thrown to close the circuit connecting the capacitor in series with a resistor of resistance R .



Part A

Part B

Part not displayed

Summary	7 of 17 problems complete (36.15% avg. score) 30.73 of 35 points
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