NIU Ph.D. candidacy examination Spring 2014 (9/20/2014)

Electricity and Magnetism

Solve 3 out of 4 problems. (40 points each. Total of 120 points)

1. [40 points] ***A hole in a large conducting slab***

   Consider a conducting slab of material of uniform conductivity σ, extending infinitely in the x and y directions and from -b < z < b. A cylindrical hole is made through the slab with radius a and axis along the z axis. Far from the hole, the slab carries a uniform current density $\mathbf{J} = J_0\hat{\mathbf{x}}$. In the following, assume a << b and consider only the region of small |z| << b, so that the z dependence can be neglected.

   (a) Determine the electric potential distributions in the slab and in the hole, respectively. [10 x 2 = 20 points]
   (b) Find the electric field in the hole. [10 points]
   (c) The space of the hole is filled with a material of conductivity σ'. Determine the current density in this space. [10 points]

2. [40 points] ***A linear dielectric sphere***

   A linear dielectric sphere of radius a and dielectric constant κ carries a uniform charge density ρ, surrounded by vacuum.

   (a) Find ***E*** and ***D*** inside and outside the sphere. [15 +10 points]
   (b) Find the energy W of the system. [15 points]

3. [40 points] ***The magnetic field by an infinitely long cylinder with a "frozen-in" magnetization***

   An infinitely long cylinder (radius R) carries a "frozen-in" magnetization parallel to the axis, $\mathbf{M} = ks\hat{\mathbf{z}}$, where k is a constant and s is the distance from the axis; there is no free current anywhere. Find the magnetic field inside and outside of the cylinder by two different methods (a) and (b):

   (a) Locate all the bound currents, and calculate the field **B** they produce. [20 points]
   (b) Use Ampere's law to find **H**, and then get **B**. [20 points]

   Note: Of course, you should get the same final answers obtained by the above two methods. Therefore, expressing the derivations of the answers are the most important point for this problem.

4. [40 points] ***Charged cylindrical tube rotating with increasing angular velocity***
   A long thin-walled cylindrical tube of radius R carries a uniform surface charge density σ, and is rotating about its axis of symmetry with a constant angular acceleration α, so that its angular velocity at time t is ω = αt.
   (a) Find all of the components of the electric and magnetic fields both inside and outside of the cylinder. [26 points]
   (b) Consider a length ℓ of the tube. Find the total flux of Poynting's vector into the tube through its outside surface of radius slightly larger than R, and show that it equals the time rate of change of the stored electromagnetic energy within the volume. [Hint: the result should be non-zero.] [14 points]

NIU Ph.D. candidacy examination Spring 2014 (2/15/2014)

Electricity and Magnetism

Solve 3 out of 4 problems. (40 points each. Total of 120 points)

1. [40 points] ***A conducting sphere and a point charge***

The field of a conducting sphere in the presence of a point charge can be described by

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{|\mathbf{r}+\mathbf{d}|} + \frac{q'}{|\mathbf{r}+\mathbf{d}'|}\right)$$

where q is the point charge and q' is an image charge inside the sphere representing the surface charge of the conducting sphere.

(a) Determine q' and d' if the sphere is grounded assuming that the charge and image charge (located **d** and **d**', respectively) are on the z-axis. [10 points]

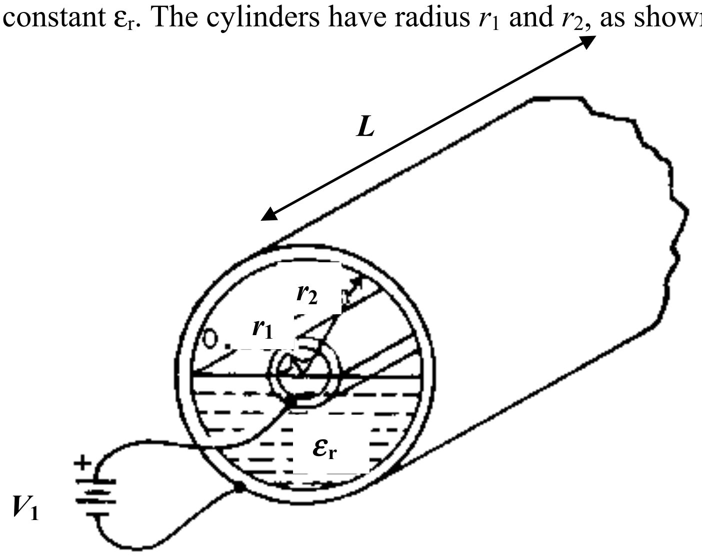
(b) We now want to describe the conducting grounded sphere in a constant electric field. We can use the results of from (a) by considering that two equal and opposite charges at $\pm d\hat{z}$ produce a constant electric field in the z direction $\vec{\mathbf{E}} = E_z\hat{z}$ if the distance between the charges $2d$ is significantly larger than the radius of the sphere R. What is the potential in terms of the real and image charges. [10 points]

(c) Expand the potential to lowest order in r/d in the limit that $r/d \ll 1$ and $r/R \ll 1$. Express the potential in terms of the applied electric field E_z. [10 points]

(d) Write the term due to the image charges in terms of the potential of an electric dipole. [10 points]

2. [40 points] ***Partially filled coaxial conducting cylinders***

The space between two coaxial conducting cylinders of length L is half-filled with a dielectric having relative dielectric constant ε_r. The cylinders have radius r_1 and r_2, as shown in fig, and are connected to a V_1 battery.



a) Find the fields $\vec{\mathbf{E}}$ and $\vec{\mathbf{D}}$ in the air and in the dielectric in the space $\mathbf{r_1} < r < r_2$. [20 points]

b) Find the surface charge induced on the inner conductor at points adjacent to the air, and at points adjacent to the dielectric. [10 points]

c) Find the total charge on the inner conductor, and the capacitance. [10 points]

3. [40 points] ***Non-magnetic metal disk under time-dependent external magnetic field***.

   A non-magnetic metal disk of radius a, thickness ℓ, and conductivity σ is located parallel to the xy plane, and centered at the origin. There is a slowly varying but time-dependent external uniform magnetic field $\vec{B} = B_0 \cos(\omega t)\hat{z}$.

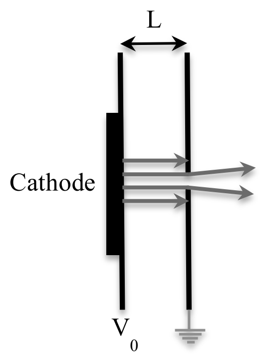
   (a) Find the induced current density $\vec{J}$ in the disk. [20 points]
   (b) A very small wire loop, of radius b, and with resistance R, is located parallel to the disk, and centered above it at the point $(x, y, z) = (0, 0, h)$, with $h \gg a, b, \ell$. Find the current induced in the wire loop due to the magnetic field of the disk. [20 points]

4. [40 points] ***Electron gun*** (*Relativistic electron under electric field*)
   Consider the electron gun in the figure below. The electrons travel from the planar cathode to the planar anode a distance L away, and some of the beam is allowed to pass through the hole of radius $a << L$ into the filed-free region on the right. In the vicinity of the hole, the field has radial components that deflect the electrons away from the axis.

   (a) Use Gauss's law to develop an approximation for the radial components of the field near the axis in terms of the longitudinal field $E_z(z)$ and its derivatives on the axis. [20 points]
   (b) Assume that in the vicinity of the hole, the electrons follow nearly straight-line trajectories at constant velocity.
      i. Compute the change in the radial momentum as the electrons go through the hole to find the deflection of the electrons. Use βc as the final relativistic velocity. [10 points]
      ii. Show that the electrons near the axis are defocused with a focal length

$$f = \frac{2\beta^2\gamma}{\gamma - 1} L \approx 4L$$

      in the nonrelativistic limit, where βc is the final relativistic velocity and γmc^2 the final energy of the electrons, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. As the figure shows, the focal length is for the diverging electrons! Note that the focal length is independent of the electron charge, the electron mass, the radius of the hole and (in the nonrelativistic limit) the electron final energy. [10 points]

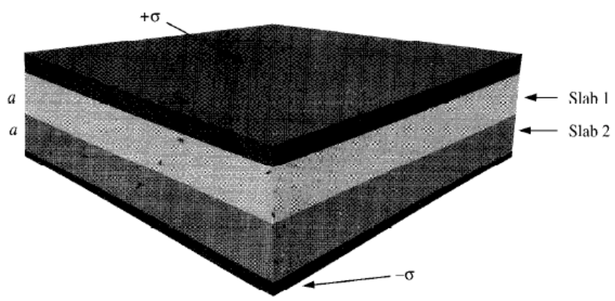


Electron Gun

NIU Ph.D. candidacy examination Fall 2013 (9/21/2013)
Electricity and Magnetism

Solve 3 out of 4 problems. Total of 120 points.

1. [40 points] **Linear dielectric material**. The space between the plates of a parallel-plate capacitor (Fig.1) is filled with two slabs of linear dielectric material. Each slab has thickness *a,* so the total distance between the plates is *2a.* Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.
   a. Find the electric displacement $\boldsymbol{D}$ in each slab.
   b. Find the electric field $\boldsymbol{E}$ in each slab.
   c. Find the polarization $\boldsymbol{P}$ in each slab.
   d. Find the potential difference between the plates.
   e. Find the location and amount of all bound charge.
   f. From all of the above charge (free and bound), recalculate the field in each slab, and confirm your answer to (b).



2. [40 points] **A magnetic dipole $\vec{\mathbf{m}}$ is imbedded** at the center of a sphere (radius *R*) of linear magnetic mateiral (permeability μ). There is a *bound dipole* at the center. So the *net* dipole moment at the center $\vec{\mathbf{m}}_{center}$ produces a magnetic filed $\vec{\mathbf{B}}_{\substack{center\\dipole}}$. Here magnetic susceptibility is given as χ_m.
   a. Express $\vec{\mathbf{m}}_{center}$ with $\vec{\mathbf{m}}$. [5 points]
   b. Then, show that the magnetic field of a dipole can be written in coordinate-free form
   $$\vec{\mathbf{B}}_{\substack{center\\dipole}}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\frac{1}{r^3}\left[3(\vec{\mathbf{m}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \vec{\mathbf{m}}\right].$$ [10 points]
   c. Assuming $\vec{\mathbf{B}}_{\substack{surface\\current}}$ is proportional to $\vec{\mathbf{m}}$, find the magnetization $\vec{\mathbf{M}}$. [7 points]
   d. Find the bound surface current $\vec{\mathbf{K}}_b$. [8 points]
   e. Find the magnetic field due to the bound surface current $\vec{\mathbf{B}}_{\substack{surface\\current}}$. [5 points]
   f. Finally express the magnetic field inside th sphere $\vec{\mathbf{B}}$ (0 < r ≤ R). [5 points]

3. [40 points] **Electromagnetic fields candidates.** Let E_0, B_0, k, and ω be four parameters and consider the following two vector fields:

   Case (1): $\mathbf{E}(x,t) = E_0 \cos(kx - \omega t)\hat{\mathbf{z}}$ and $\mathbf{B}(x,t) = B_0 \cos(kx - \omega t)\hat{\mathbf{z}}$,

   Case (2): $\mathbf{E}(x,t) = E_0 \cos(kx - \omega t)\hat{\mathbf{z}}$ and $\mathbf{B}(x,t) = B_0 \cos(kx - \omega t)\hat{\mathbf{y}}$.

   The fields are defined over the entire space [coordinate $x = (x, y, z)$] and for arbitrary time *t*. The ^ symbol represents unit vectors.

   a. Can any of these fields represent an electromagnetic field? Consider both cases [Cases (1) and (2)] and for each case detailed your reasoning. Give the physical meaning of the four parameters. [10 points]
   b. Assuming your answer to question (1) is positive for at least one of the cases,
      i. give the necessary condition(s) between the parameters E_0, B_0, k, and ω. [10 points]
      ii. find the corresponding charge distribution and current source, $\rho(x,t)$ and $\mathbf{J}(x,t)$, respectively. [10 points]
   c. Consider the case $\omega = k/\sqrt{\varepsilon_0 \mu_0}$. What are the corresponding charge distribution and current source? For this case find the Poynting vector and its time-averaged value. [10 points]

4. [40 points] **Resonant cylindrical-symmetric cavity for non-relativistic beams:** We consider a resonant cylindrical-symmetric cavity operating on the TM_{010} mode. The axial (accelerating) field on the cavity axis is taken to be $E_z(r = 0, z, t) = E_0 \sin(\omega t)$. [40 points]

   a. We consider a "reference" charged particle with charge *q*, mass *m* and initial energy W_r entering the cavity and crossing its center at time $t = 0$. Assume that the particle velocity does not change within the cavity. What is its final energy? [7 points]
   b. Consider a second identical particle with same initial energy as the reference particle but delayed by a time δt small. What is its final energy W? [7 points]
   c. The two particles are allowed to drift in free space over a length *L* downstream of the accelerating cavity.
      i. Find the final arrival time at the end of the drift associated to each of the particles. [10 points]
      ii. Express the difference in final arrival times as a function of $\delta\gamma = \frac{W - W_r}{mc^2}$, $\gamma = \frac{W + W_r}{2mc^2}$ the mean Lorentz factor of the two-particle system, and δt. [10 points]
      iii. What is the condition between $\frac{\delta\gamma}{\gamma}$ and δt that insures the two particles arrive at the same time (i.e. are bunched)? [6 points]

NIU Ph.D. candidacy examination Spring 2013 (2/16/2013)

Electricity and Magnetism

Solve 3 out of 4 problems. (40 points each. Total of 120 points)

1. [40 points] ***Infinitely long uniform line*** **charge**.

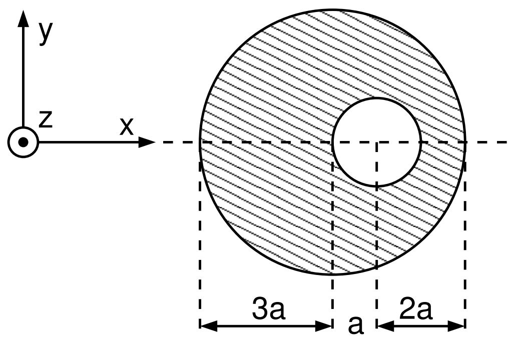
   A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. The wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy-plane.

   a) Find the potential in the region *above* the plane. [20 points]
   b) Find the charge density σ induced on the conducting plane. [20 points]

2. [40 points] ***Wire with*** **hole.**

   The figure shows the cross section of an infinitely long circular cylinder of radius 3a with an infinitely long cylindrical hole of radius a displaced so that the hole's center is at a distance a from the center of the big cylinder. The solid part of the cylinder carries a current I, distributed uniformly over the cross section, and directed out of the plane of the paper.

   a) Find the magnetic field at all points in the plane P containing the axes of the cylinders (in the figure: the xz-plane with y = 0). [20 points]
   b) Determine the magnetic field throughout the hole; it is of a particularly simple character. [20 points]



3. [40 points] ***Rotating solid dielectric cylinder***.

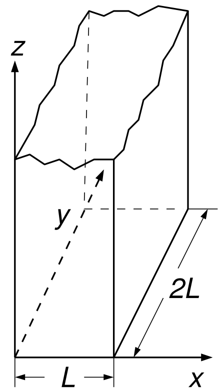
A very long solid dielectric cylinder of radius R with electric susceptibility χ_e is rotating at a constant angular velocity ω about its axis of symmetry, the z-axis. There is also a constant magnetic field in the z direction: $\vec{B} = B\hat{z}$. Find:

a) The electric dipole moment density (per unit volume) as a function of position within the cylinder, [18 points]
b) The volume charge density as a function of position within the cylinder, [12 points]
c) The surface charge density on the cylinder. [10 points]

4. [40 points] ***Waveguide.***

Consider a rectangular waveguide, infinitely long in the z direction, with transverse dimensions $L_x = L$ and $L_y = 2L$ as illustrated in the figure. The walls are a perfect conductor.

a) What are the boundary conditions for the components of B and E at the walls? [8 points]
b) Write the wave equation which describes the E and B fields of the lowest mode. (Hint: The lowest mode has the electric field in the x direction only.) [8 points]
c) For the lowest mode that can propagate, find the phase velocity and the group velocity. [14 points]
d) The possible modes of propagation such waveguides separate naturally into two classes. What are these two classes and how do they differ physically? [10 points]



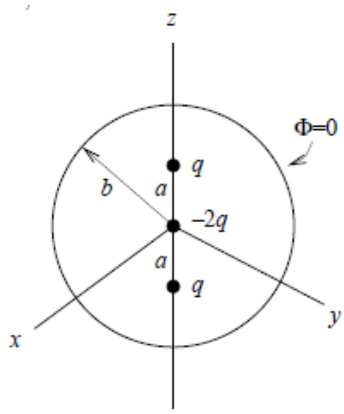
NIU Ph.D. candidacy examination Fall 2012 (9/22/2012)

Electricity and Magnetism

Solve 3 out of 4 problems. (40 points each. Total of 120 points)

1. [40 points] ***Point charges***.

   Three point charges (q, $-2q$, q) are located in a straight line with separation a and with the middle charge ($-2q$) at the origin of a *grounded* conducting spherical shell of radius b, as indicated in the sketch below:



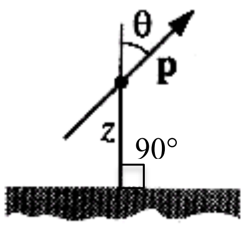
   (a) Write down the potential due to the three charges ***in the absence of*** the grounded sphere. [10 points]

   (b) Find the limiting form of the potential as $a \to 0$, but the product $qa^2 = Q$ remains finite. [15 points]

   (c) Write this latter answer in spherical coordinates. [15 points]

2. [40 points] ***Electric dipole***.

   A perfect dipole $\boldsymbol{p}$ is located at a distance z above an infinite grounded conducting plane as shown below. The dipole makes an angle θ with a perpendicular to the plane.



   a) Find the torque on $\boldsymbol{p}$. [25 points]

   b) If the dipole is free to rotate, in what orientation will it come to rest? [15 points]

3. [40 points] ***Magnetic charges***

We assume the existence of magnetic charge ρ_m ("magnetic monopoles") related to the magnetic field $\vec{B}$ by the local relation, $\nabla \cdot \vec{B} = \mu_0 \rho_m$, analogous to Gauss' law for electric charges.

a) Using the divergence theorem, obtain the magnetic field due to a point magnetic charge located at the origin $\boldsymbol{r} = 0$. [10 points]

b) In the absence of magnetic charge, the curl of the electric field is given by Faraday's law in differential form. Show that this law is incompatible with a magnetic charge density that is a function of time. [10 points]

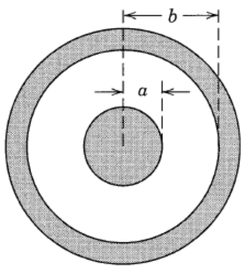
c) Assuming that magnetic charge is conserved, derive the <u>*local*</u> relation between the magnetic charge density ρ_m and the magnetic charge current density $\boldsymbol{j}_m$. Interpret your result. [10 points]

d) Modify Faraday's law to obtain a law consistent with the presence of a magnetic charge density, $\rho_m = \rho_m(\mathbf{r}, t)$

Demonstrate that the modified law is consistent with your solution to part (c). [10 points]

4. [40 points] ***TEM mode propagated along a transmission line***.

A transmission line consisting of two concentric circular cylinders of metal with conductivity σ and skin depth δ, as shown, is filled with a uniform lossless dielectric (μ, ε). A TEM mode is propagated along this line.



a) Show that the time-averaged power flow along this line is

$$P = \sqrt{\frac{\mu}{\varepsilon}} \pi a^2 |H_0|^2 \ln\left(\frac{b}{a}\right)$$

where H_0 is the peak value of the azimuthal magnetic field at the surface of the inner conductor. [10 points]

b) The transmitted power is attenuated along the line in the form of $P(z) = P_0 e^{-2\gamma z}$. Express γ with conductivity σ, skin depth δ, and dielectric constants of (μ, ε). [10 points]

[Hint] The power loss per unit length of the waveguide is expressed as:

$$-\frac{dP}{dz} = \frac{1}{2\sigma\delta} \oint_C \left|\hat{n} \times \vec{H}\right|^2 dl = \frac{1}{2\sigma\delta} |H_0|^2 \oint_C \frac{a^2}{\rho^2} dl \ ,$$

where ρ is the radial cylindrical coordinate and C is a circular contour on the surface of the conductor.

c) The characteristic impedance of the line, Z_0, is defined as the ratio of the voltage between the cylinders to the axial current flowing in one of them at any position z. Express the characteristic impedance Z_0 for this line. [10 points]

d) Express the inductance per unit length of the line, L, where μ_C is the permeability of the conductor. Take into account the correction to the inductance due to the penetration of the flux into the conductors by a distance of order δ. [10 points]

[Hint] First express the energy per unit length stored in the magnetic field, U_{vol} (inside the volume of the waveguide) and U_{walls} (penetrated into the wall of the waveguide). Here the magnetic field penetrated into the conducting walls can be approximated to:

$$H(\zeta) = H_{\parallel} e^{-\zeta/\sigma} e^{i\zeta/\sigma}$$, where ζ is the distance into the conductor.

NIU Ph.D. candidacy examination Spring 2012 (2/18/2012)

Electricity and Magnetism

Solve 3 out of 4 problems.

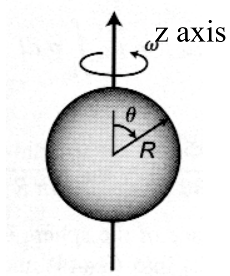
1. [40 points] The electric field of a charged sheet.

   (a) Find the electric field at a height z above the center of a square sheet (side a) carrying a uniform surface charge density σ. [20 points]

   (b) Find the electric field, keeping the leading non-zero term, when $a \to \infty$ (infinite plane). [10 points]

   (c) Find the electric field, keeping the leading non-zero term, when $z >> a$. [10 points]

2. [40 points] Consider a uniform thin shell of charge spinning about the z axis with the angular velocity ω, with the total charge q and the radius of the sphere R.



   (a) Express its magnetic dipole moment, $\vec{m}$. [12 points]

   (b) Express the magnetic scalar potential (as a function of radius r) outside the spinning shell of charge using ω, q, R. [14 points]

   (hint: The magnetic scalar potential can be represented by the expansion

$$\begin{cases} \Phi_{out} = \sum_{m=0}^{\infty} \frac{a_m}{r^{m+1}} P_m(\cos\theta) & \text{(Outside the sphere)} \\ \Phi_{in} = \sum_{m=1}^{\infty} b_m r^m P_m(\cos\theta) & \text{(Inside the sphere)} \end{cases},$$

   where $P_l(\zeta)$ is the Legendre polynomials, a_m and b_m are some coefficients.

   (c) Find the magnetic field outside the spinning shell of charge, in terms of the magnetic dipole moment, $\vec{m}$. [14 points]

3. [40 points] Induction of a toroidal coil.

   (a) Find the self-inductance of a toroidal coil with rectangular cross section (inner radius a, outer radius b, height h), which carries a total of N turns. [20 points]

   (b) Calculate the energy stored in this toroidal coil. [20 points]

4. [40 points] An electromagnetic wave with angular frequency ω moves through a material that obeys Ohm's Law with conductivity σ. The permittivity and permeability of the material are the same as that of vacuum.

   (a) Derive the separate second-order wave equations for the electric and magnetic fields $\vec{E}$ and $\vec{B}$. [12 points]

   (b) Find expressions for the electric and magnetic fields of a wave moving in the $\hat{z}$ direction and polarized in the $\hat{x}$ direction. [16 points]

   (c) Find the distance that the wave travels for which its intensity is decreased by a factor of 10. [12 points]

NIU Ph.D. candidacy examination Fall 2011 (9/17/2011)
Electricity and Magnetism

Solve 3 out of 4 problems.

1. [40 points] The time-averaged potential of a neutral atom is given by

$$\Phi = \frac{q}{4\pi\varepsilon_0}\frac{e^{-\alpha r}}{r}(1+\beta r),$$

where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius.
   (a) Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically. [30 points]
   (b) Find the relationship between α and β such that the charge distribution could describe a neutral atom. [10 points]

2. [40 points] The retarded vector potential in spherical coordinates (r,θ,ϕ) for a general oscillating magnetic dipole $\vec{m} = m_0\cos(\omega t)\hat{z}$ is

$$\vec{A} = \frac{\mu_0\omega m_0}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\left[\omega(t - r/c)\right]\hat{\phi}.$$

Now consider a circular wire loop of radius a that is fed by a source of alternating current $I = I_0\cos(\omega t)$. The loop is centered at the origin and lies within the xy plane. Find the following in terms of a, I_0, ω, constants of nature, and the spherical coordinates:
   (a) Calculate the leading behavior of the electric and magnetic fields in the large distance $(r \gg a, c/\omega)$ limit. [20 points]
   (b) Describe the intensity pattern for the radiation as a function of angle, and find the total time-averaged radiated power. [20 points]

3. [40 points] A very long coaxial cylinder of length l is formed from an inner conductor of radius a and an outer conductor of radius b. A solid dielectric plastic tube of permittivity ε just fills the space between the conductors. Suppose that the charges on the inner and outer conducting cylinders are held constant at +Q and –Q, respectively.
   (a) Find the capacitance. [20 points]
   (b) If the plastic tube is withdrawn by a distance x from one end, find the magnitude and direction of the force on it. (Assume that the space left by the partially withdrawn plastic tube is air, with permittivity ε_0.) [20 points]

4. [40 points] Consider the resonant cavity produced by closing off the two ends of a rectangular wave guide (cross sectional size a and b), at z = 0 and at z = d, making a perfectly conducting empty box.
   (a) Find the resonant frequencies for TE mode. [20 points]
   (b) Find the associated electric field. [10 points]
   (c) Find the associated magnetic fields. [10 points]

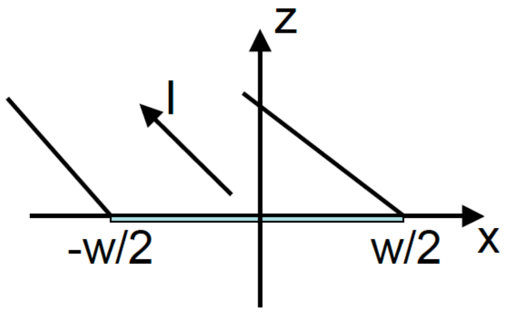
NIU Ph.D. candidacy examination Spring 2011 (2/19/2011)
Electricity and Magnetism
Solve 3 out of 4 problems.

<u>Problem 1</u>: [40 points]

An infinitely long strip (oriented along the y-axis) has width w and negligible thickness, see the figure. It carries a total current I uniformly distributed over the width.

a) Find expressions for the magnetic induction produced by this strip. [25 points]

b) Make sketches of B_z and B_x as function of x along traces at height z_0 above the strip for $z_0/w << 1$ and for $z_0/w >>1$ and discuss the results. [15 points]



Hints:

$$\int \frac{x'}{(x')^2 + z_0^2} dx' = \frac{1}{2} \ln\left((x')^2 + z_0^2\right)$$

$$\int \frac{z_0}{(x')^2 + z_0^2} dx' = \arctan\left(\frac{x'}{z_0}\right)$$

<u>Problem 2</u>: [40 points]

A surface charge density $\sigma(x,y) = \sigma_0 \sin^2(\alpha x)$ is located on the xy-plane.

a) Find the electrostatic potential $\phi(x,y,z)$ produced by this charge distribution and discuss the behavior at large values of z. [20 points]

b) Verify that your solution fulfills Poisson equation for the given charge distribution. [20 points]

Problem 3: [40 points]

An infinite flat sheet of charge density per unit area σ, located in the xy plane, is forced to oscillate along the x-axis. The velocity of charges at time *t* is given by $\mathbf{v} = \hat{x} v_0 \cos(\omega t)$, resulting in electromagnetic radiation.

(a) Solve for all components of the electromagnetic fields everywhere. [20 points]
(b) How much energy per unit area is radiated away in a time T? (You may assume T >> 1/ω.) [20 points]

Problem 4: [40 points]

A collimated beam of protons has the form of a very long cylinder of radius *R*. The speed of the protons is *v* along the cylinder's axis direction, and the number of protons per unit volume inside the cylinder is a constant, *n*. Suppose that the beam now leaves the region where it has been collimated by external forces, and enters a region of empty space.

(a) Find the electric and magnetic fields inside and outside of the beam. [16 points]
(b) What is the total force (magnitude and direction) on a proton within the beam at a distance *r* from the beam axis? [12 points]
(c) Now suppose that a similarly collimated cylindrical antiproton beam of the same radius and number density is moving through the proton beam, with the same beam axis, but in the opposite direction with speed *v*'. What is the total force (magnitude and direction) on a proton at a distance *r* from the beam axis? [12 points]

(Proton have charge |*e*|, antiprotons have charge -|*e*|.)