KwaZulu-Natal

PINETOWN DISTRICT

This revision guide contains important mathematical definitions, proofs, theorems and formula for

Paper 1 and **Paper 2**.

This content is CAPS compliant and suitable for Grade 11 and Grade 12 learners.

PAPER 1
Arithmetic and geometric series

Proof 1: Prove that the sum of n terms of an **arithmetic sequence** is given by the formula:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_n = a + (a+d) + ...a + (n-1)d$$

$$S_n = [a + (n-1)d] + [a + (n-2)d] + ... + a$$

$$2S_n = 2a + (n-1)d + ...2a + (n-1)d$$

$$2S_n = n[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Prove 1 (alternative):

The general term (last term) of an arithmetic series is $T_n = a + (n-1)d = l$

$S_n = a + [a + d] + [a + 2d] + \dots + [l - d] + l$ … equation 1

In reverse: $S_n = [a + (n-1)d] + [a + (n-2)d] + [a + (n-3)d] + \dots + [a + d] + a$

$S_n = l + [l - d] + [l - 2d] + \dots + [a + d] + a$ … equation 2

Adding equation 1 and equation 2:

$2S_n = [a + l] + [a + l] + \dots + [a + l]$ to n terms

$2S_n = n[a + l]$

$\therefore S_n = \frac{n}{2}[a+l]$

Replacing l with $a + (n-1)d$: $\therefore S_n = \frac{n}{2}[2a+(n-1)d]$

Proof 2: Prove that the sum to n terms of a **geometric sequence** is given by:

$$S_n = \frac{a(r^n-1)}{r-1} \quad ; \quad r \neq 1$$

$$S_n = a + ar + ar^2 + ... + ar^{n-1}$$

$$rS_n = ar + ar^2 + ... + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

Information Sheet – formula for paper 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula. Used to find roots of a quadratic (parabola). Parts of this formula can be used to find *x*-value of TP. Also the discriminant ($\Delta = b^2 - 4ac$) for Nature of Roots.

$$A = P(1 + ni)$$

Simple Interest / growth.

$$A = P(1 - ni)$$

Simple Depreciation / straight line depreciation.

$$A = P(1 - i)^n$$

Reducing / diminished balance depreciation.

$$A = P(1 + i)^n$$

Compound Interest / growth. Also used for inflation.

$$T_n = a + (n-1)d$$

Linear patterns. Arithmetic Progression. Finding the general term of an AP. *Common difference.

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Linear patterns. Arithmetic Series. Finding the sum of an Arithmetic Series. LEARN THE PROOF FOR THIS.

$$T_n = ar^{n-1}$$

Geometric patterns. Geometric Progression. Finding the general term of a GP. *Common ratio.

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1} \; ; \; r \neq 1$$

Geometric patterns. Geometric Series. Finding the sum of a GS. LEARN THE PROOF FOR THIS.

$$S_\infty = \frac{a}{1 - r} \quad ; \; -1 < r < 1$$

Sum to infinity

Condition for convergence.

$$F = \frac{x\left[(1+i)^n - 1\right]}{i}$$

Future Value Annuity. Investments and savings. Used when working with sinking funds.

$$P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

Present Value Annuity. Loans. Also used when living off an inheritance / large amount of money.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Finding the derivative using FIRST PRINCIPLES. *REMEMBER NOTATION.

$$P(A) = \frac{n(A)}{n(S)}$$

Probability definition.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Fundamental Probability Identity.

Information Sheet – formula for paper 2

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Distance between two coordinates.

$$M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$$

Midpoint between two coordinates.

$$m=\tan\theta$$

Angle of inclination.

$$y=mx+c$$

Standard form of straight line with *c* being the y-intercept.

$$y-y_1=m(x-x_1)$$

Gradient / point form of a straight line.

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Gradient between two coordinates.

$$(x-a)^2+(y-b)^2=r^2$$

Standard form of a circle with centre at (a ; b).

In ΔABC: $$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

SINE RULE. Used to find unknown sides or angles in non right angled triangles.

$$a^2=b^2+c^2-2bc.\cos A$$

COSINE RULE. Used to find unknown sides or angles in non right angled triangles when you have SSS or SAS.

$$\text{area}\,\Delta ABC=\frac{1}{2}ab.\sin C$$

AREA RULE. Used to find unknown area in non right angled triangles when you have SAS.

$$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta \qquad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta \qquad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$$

Cosine and Sine Compound Angle Expansion

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases} \qquad \sin 2\alpha=2\sin\alpha.\cos\alpha$$

Cosine and Sine Double Angle Expansion

$$\bar{x}=\frac{\sum fx}{n}$$

Mean for grouped data

$$\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$$

To calculate variance and standard deviation. Calculator work.

$$\hat{y}=a+bx$$

Regression Line / line of least squares regression.

Calculator work.

$$b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

To calculate the correlation coefficient. Calculator work.

Grade 11 / 12 Trigonometry "Cheat Sheet"

COMPOUND ANGLES (G12)

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

DOUBLE ANGLES (G12)

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta-\sin^2\theta$

$= 2\cos^2-1$

$= 1-2\sin^2\theta$

CO-FUNCTIONS

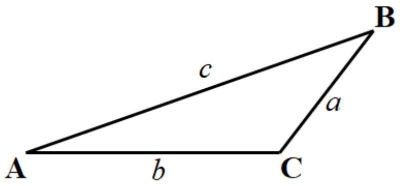
$\sin(90°+\theta)= \cos\theta$

$\sin(90°-\theta)= \cos\theta$

$\cos(90°+\theta)= -\sin\theta$

$\cos(90°-\theta)= \sin\theta$

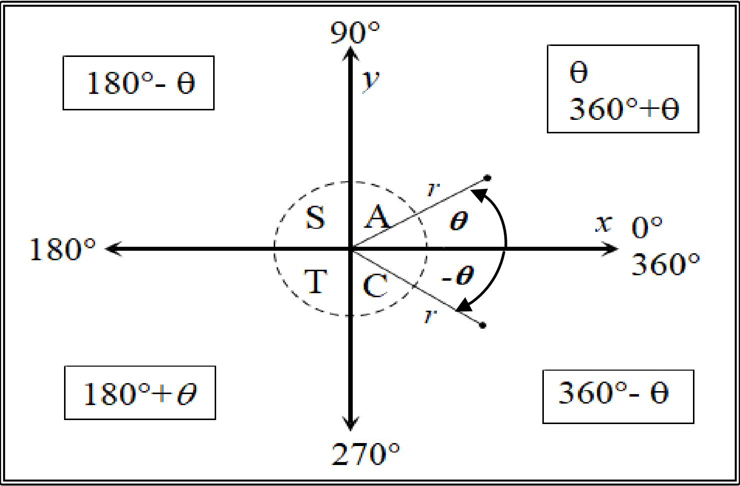
TRIANGLE SOLUTIONS

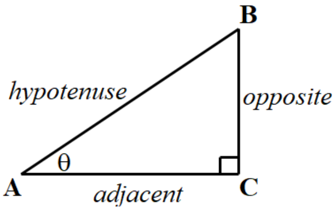


Sine Rule: $\left\{\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right\}$ $\left\{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}\right\}$

Cosine Rule: $\left\{\begin{array}{l} a^2=b^2+c^2-2bc\cos A \\ b^2=a^2+c^2-2ac\cos B \\ c^2=a^2+b^2-2ab\cos C \end{array}\right\}$

Area Rule: $\left\{\begin{array}{l} A=\frac{1}{2}ab.\sin C \\ A=\frac{1}{2}ac.\sin B \\ A=\frac{1}{2}bc.\sin A \end{array}\right\}$





RIGHT TRIANGLE DEFINITION

$\sin\theta=\frac{O}{H}$; $\cos\theta=\frac{A}{H}$; $\tan\theta=\frac{O}{A}$

CARTESIAN PLANE TRIG

$\sin\theta=\frac{y}{r}$; $\cos\theta=\frac{x}{r}$; $\tan\theta=\frac{y}{x}$

TAN IDENTITY

$\tan\theta=\frac{\sin\theta}{\cos\theta}$

NEGATIVE ANGLES

$\sin(-\theta)=-\sin\theta$

$\cos(-\theta)=\cos\theta$

$\tan(-\theta)=-\tan\theta$

GENERAL SOLUTIONS

$\sin\theta$: $\left\{\begin{array}{l} \theta=180°-ref\angle+k.360°;\ k\in\mathbb{Z} \\ \theta=ref\angle+k.360°;\ k\in\mathbb{Z} \end{array}\right\}$

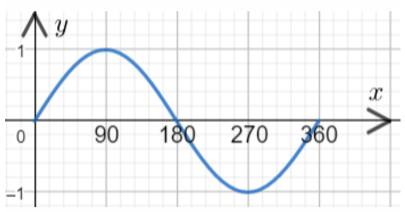
$\cos\theta$: $\left\{\begin{array}{l} \theta=ref\angle+k.360°;\ k\in\mathbb{Z} \\ \theta=-ref\angle+k.360°;\ k\in\mathbb{Z} \end{array}\right\}$

$\tan\theta$: $\{\theta=ref\angle+k.180°;\ k\in\mathbb{Z}\}$

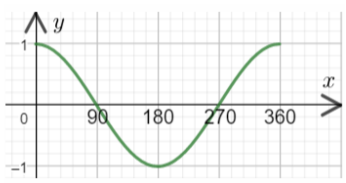
PYTHAGOREAN IDENTITY

$\sin^2\theta+\cos^2\theta=1 \rightarrow \sin^2\theta=1-\cos^2\theta \rightarrow \cos^2\theta=1-\sin^2\theta$

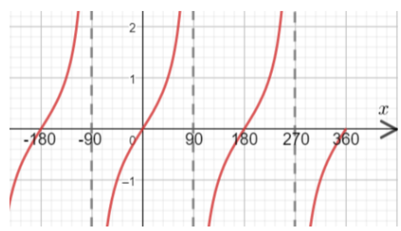
f(x)= sinx



f(x)= cosx



f(x)= tanx



Grade 11 – Circle Geometry Theorems

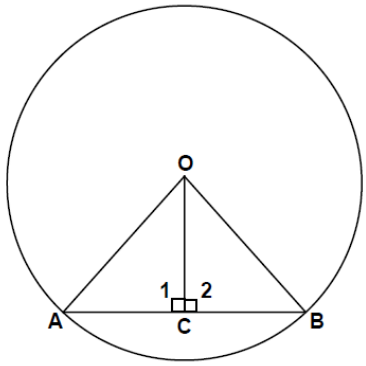
1. The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

Given : Any circle O with OC ⊥ chord AB .

Required : Prove that AC = BC .

Construction : Draw OA and OB .

Proof : In Δ AOC and Δ BOC :



1) AO = OB (radii)

2) OC is common

3) $\hat{C}_1 = \hat{C}_2 = 90°$ (given)

$\therefore \ \Delta AOC \equiv \Delta BOC$ (RHS)

∴ AC = BC

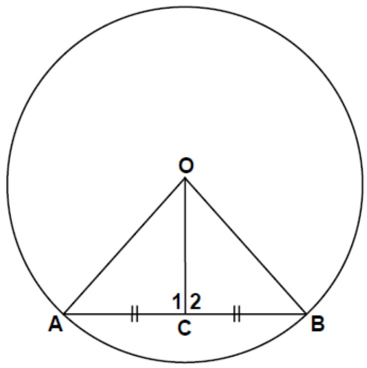
2. The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord *(Theorem 1 converse)*.

Given : Any circle O and C the midpoint of chord AB.

Required : Prove that OC ⊥ AB .

Construction : Draw OC , OA and OB .

Proof : In Δ AOC and Δ BOC :



1) AO = OB (radii)

2) OC is common

3) AC = BC (given)

$\therefore \ \Delta AOC \equiv \Delta BOC$ (S S S)

$\therefore \ \hat{C}_1 = \hat{C}_2 = 90°$ (straight line)

∴ OC ⊥ AB

3. The angle subtended by an arc (or chord) of a circle at the centre is double the angle it subtends at any point on the circle. (The central angle is double the inscribed angle subtended by the same arc.)

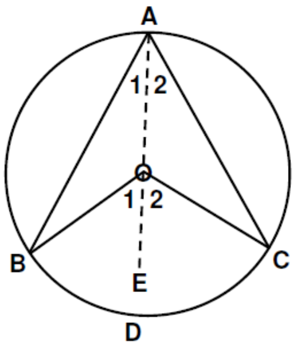


Figure 1

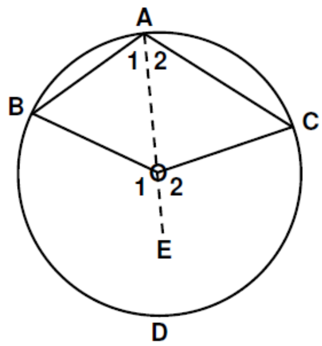


Figure 2

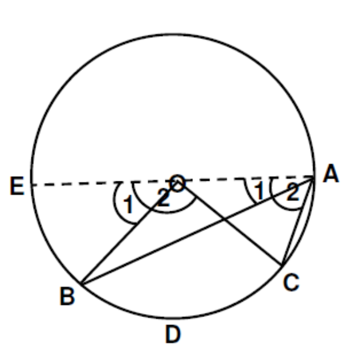


Figure 3

Given : Any circle O with central angle BOC and inscribed angle BAC , both subtended by an arc BDC.

Required : Prove that $B\hat{O}C = 2B\hat{A}C$

Construction : Join AO and produce to E.

Proof :

$\hat{A}_1 = \hat{B}$ (OA = OB = radius)

$\hat{O}_1 = B + \hat{A}_1$ (ext.∠ of Δ = sum of opp.)

$\hat{O}_1 = 2\hat{A}_1$

Similarly $\hat{O}_2 = 2\hat{A}_2$

$\therefore\ B\hat{O}C = 2B\hat{A}C$

In Figure 1 and 2

$\hat{O}_1 + \hat{O}_2 = 2\hat{A}_1 + 2\hat{A}_2$

$B\hat{O}C = 2(\hat{A}_1 + \hat{A}_2)$

$B\hat{O}C = 2B\hat{A}C$

In Figure 3

$\hat{O}_2 - \hat{O}_1 = 2\hat{A}_2 - 2\hat{A}_1$

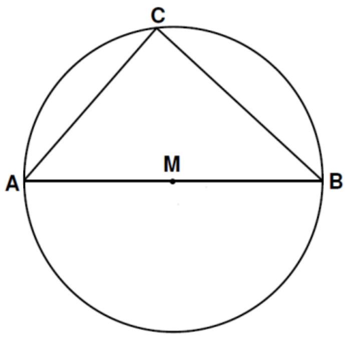
$\hat{O}_2 - \hat{O}_1 = 2(\hat{A}_2 - \hat{A}_1)$

$\hat{O}_2 - \hat{O}_1 = 2B\hat{A}C$

$B\hat{O}C = 2B\hat{A}C$

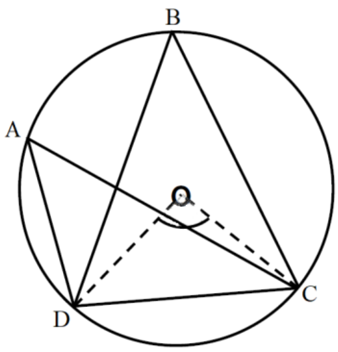
4. The inscribed angle subtended by the diameter of a circle is a right angle. (∠ in semi circle)

Given AB, the diameter of the circle passing through centre M.



In the sketch AB is the diameter of the circle.
Therefore $A\hat{C}B = 90°$ (angle in semi circle)

5. If the angles subtended by a chord (or arc) of the circle are on the same side of the chord (or arc), then the angles are equal. (Reason: ∠s in same segment)



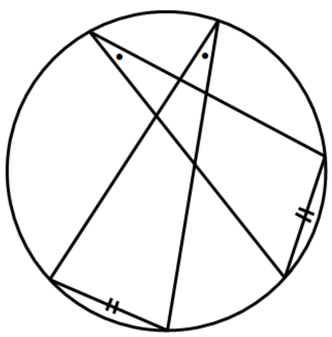
Given : Circle O with inscribed angles A and B subtended by chord DC.
Required : Prove that $\hat{A} = \hat{B}$
Construction : Draw central angle $D\hat{O}C$.

Proof : $\hat{A} = \frac{1}{2} D\hat{O}C$ (angle at centre = 2 × angle circumference)

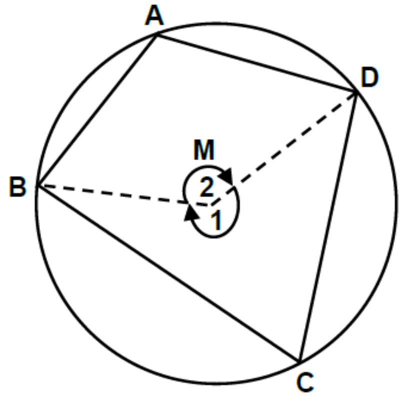
$\hat{B} = \frac{1}{2} D\hat{O}C$ (angle at centre = 2 × angle circumference)

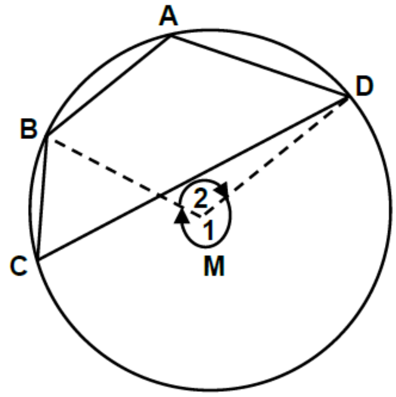
$\therefore \hat{A} = \hat{B}$

We can deduce from this theorem that if angles at the circumference of a circle are subtended by arcs (or chords) of equal length, then the angles are equal.



6. The opposite angles of a cyclic quadrilateral are supplementary (together 180º).





Given : Circle M with cyclic quadrilateral ABCD .

Prove : (a) $\hat{A} + \hat{C} = 180°$

(b) $A\hat{B}C + A\hat{D}C = 180°$

Construction : Draw BM and DM .

Proof : $\hat{M}_1 = 2\hat{A}$ (angle at central = 2 × angle circumference)

$\hat{M}_2 = 2\hat{C}$ (angle at central = 2 × angle circumference)

$\hat{M}_1 + \hat{M}_2 = 2(\hat{A} + \hat{C})$

But $\hat{M}_1 + \hat{M}_2 = 360°$

Therefore $\hat{A} + \hat{C} = 180°$

But $A\hat{B}C + A\hat{D}C + \hat{A} + \hat{C} = 360°$ (angles of quadrilateral)

Therefore $A\hat{B}C + A\hat{D}C = 180°$

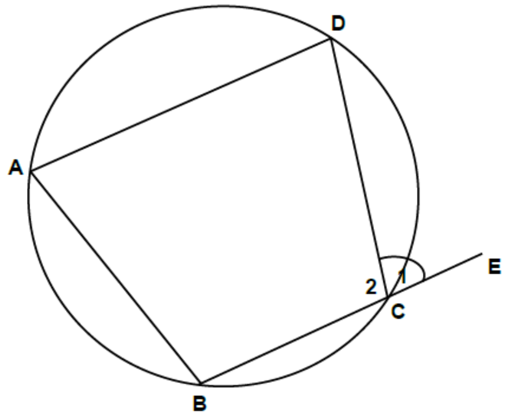
7. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Cyclic quad ABCD with BC produced to D.

$\hat{C}_1 + \hat{C}_2 = 180°$ (sum angles st. line)

$\hat{A} + \hat{C}_2 = 180°$ (proved)

$\hat{A} = \hat{C}_1$



When asked to prove that a quadrilateral is a cyclic quadrilateral:

* the sum of a pair of opposite interior angles is 180º

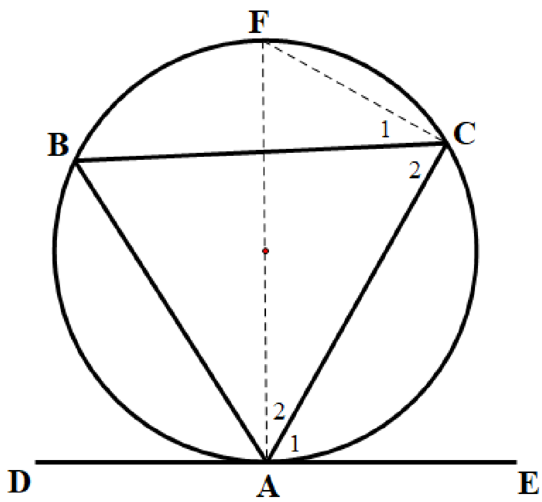
OR

* an exterior angle of the quadrilateral is equal to the opposite interior angle

OR

* a side of the quadrilateral subtends equal angles at the opposite vertices (angles in the same segment)

8. The angle between a tangent and a chord is equal to the angle in the alternate segment. (Tangent chord theorem)



Given : DE is a tangent to circle centre O at A. B and C are points on the circle.

Prove : $\hat{A}_1 = \hat{B}$

Construction : Diameter AOF, connect F and C.

Proof : $\hat{A}_1 + \hat{A}_2 = 90°$ (Radius ⊥ Tangent)

$F\hat{C}A = 90°$ (∠ in semi ⊙)

$\hat{F} = 90° - \hat{A}_2$ (sum ∠s triangle)

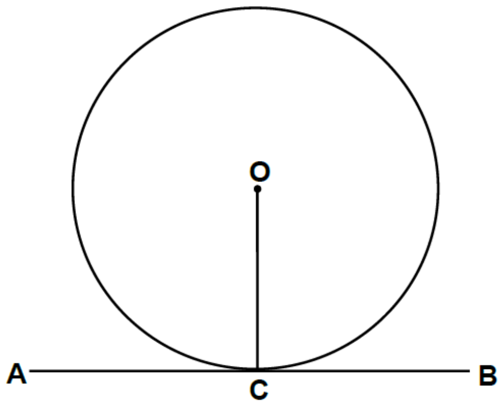
and $\hat{A}_1 = 90° - \hat{A}_2$

$\therefore \hat{A}_1 = \hat{F}$

but $\hat{B} = \hat{F}$ (∠s in same segment)

$\therefore \hat{A}_1 = \hat{B}$

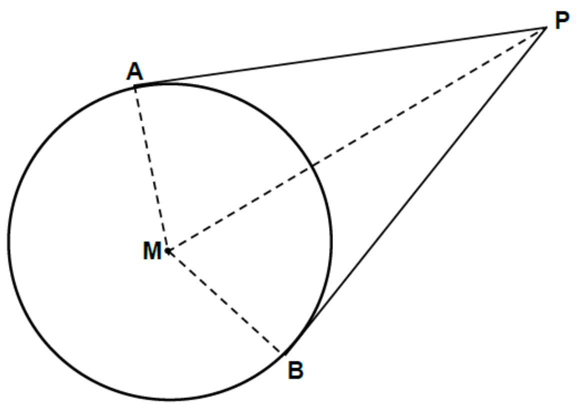
9. A tangent to a circle is perpendicular to the radius at the point of contact.



E.g. if OC is a radius and tangent AB touches the circle at C, then AB ⊥ OC.

** There is no need to learn the proof for this theorem.

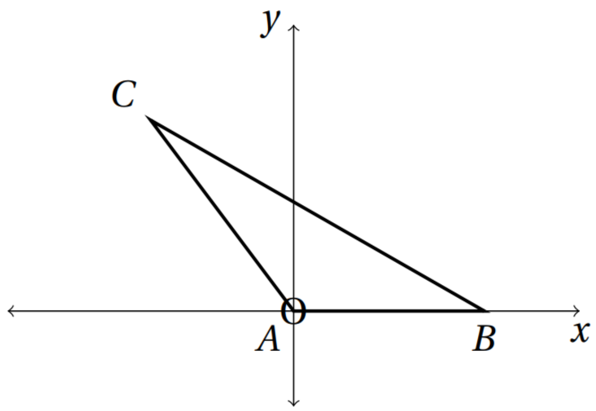
10. Two tangents drawn to a circle from the same point outside the circle, is equal in length.



** There is no need to learn the proof for this theorem.

Grade 11 – Sine, Cosine and Area Rules

Memorise the following diagram – we will use this to prove each rule.



We place the triangle so that A is at the origin, and B is on the *x*-axis. As usual, we say that the length of BC is a, AC is b, and AB is c.

This means that we can find the coordinates of A, B, and C:

A = (0; 0)
B = (c; 0)
C = (b cos A ; b sin A)

Area Rule Proof:

$$Area\ of\ \Delta ABC = \frac{1}{2} \times base \times perpendicular\ height$$

$$Area\ of\ \Delta ABC = \frac{1}{2} \times c \times b\sin A$$

$$Area\ of\ \Delta ABC = \frac{1}{2} bc.\sin A$$

Similarly, we can rearrange the triangle in the diagram to obtain:

$$Area\ of\ \Delta ABC = \frac{1}{2} ab.\sin C$$

$$Area\ of\ \Delta ABC = \frac{1}{2} ac.\sin B$$

Sine Rule Proof:

Using the Area Rule, we know that: $\frac{1}{2}bc.\sin A = \frac{1}{2}ac.\sin B = \frac{1}{2}ab.\sin C$

We are now required to divide throughout by $\frac{1}{2}abc$ so that we only have sines on the top:

$$\frac{\frac{1}{2}bc.\sin A}{\frac{1}{2}abc} = \frac{\frac{1}{2}ac.\sin B}{\frac{1}{2}abc} = \frac{\frac{1}{2}ab.\sin C}{\frac{1}{2}abc}$$

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Note that in an exam it is acceptable to prove the Sine Rule without proving the Area Rule first – however, you must say that you are using the Area Rule, and you should include your diagram.

Cosine Rule Proof:

The Cosine Rule is proved by looking at the lengths of the sides of ΔABC.

Using the distance formula that we have from analytical geometry we get that:

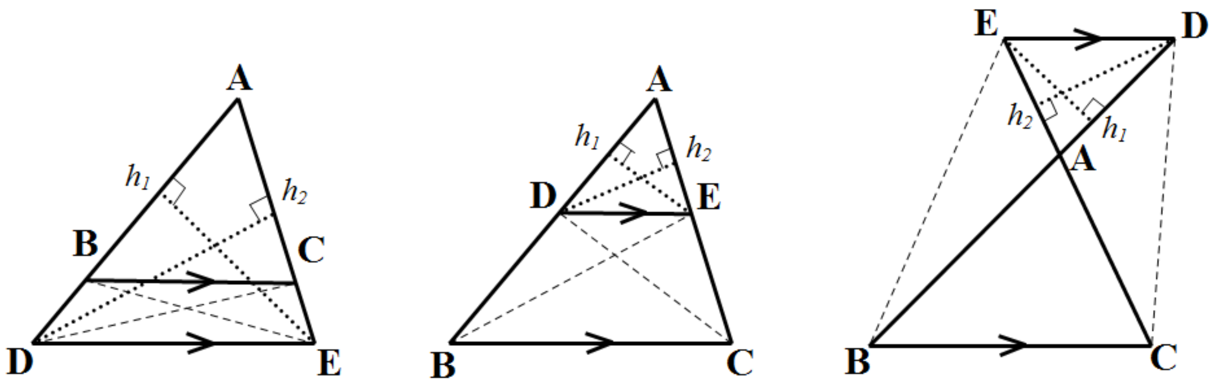
$$a = \sqrt{(b\cos A - c)^2 + (b\sin A - 0)^2}$$
$$a = \sqrt{b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A}$$
$$a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$$
$$a^2 = b^2(\cos^2 A + \sin^2 A) + c^2 - 2bc\cos A$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Similarly, we can find: $b^2 = a^2 + c^2 - 2ac\cos B$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Grade 12 – Proportional Geometry Proofs

1. A line drawn parallel to one side of a triangle divides the other two sides in the same proportion.



Given : ΔABC with $DE \parallel BC$

Required : Prove that $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Draw h_1 from E perpendicular to AD, and h_2 from D perpendicular to AE.

Draw BE and CD.

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}.\ AD\ .\ h_1}{\frac{1}{2}.\ DB.\ h_1} = \frac{AD}{DB}$$

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED} = \frac{\frac{1}{2}.AE.h_2}{\frac{1}{2}.EC.h_2} = \frac{AE}{EC}$$

but Area of ΔBDE = Area of ΔCED (equal base, equal height)

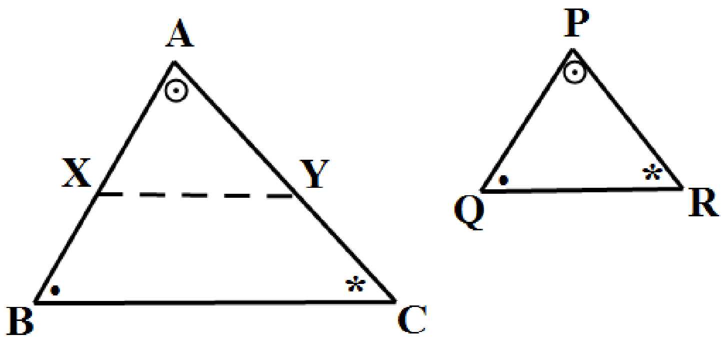
$$\therefore \quad \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED}$$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

Converse: proportion theorem

If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.

2. If the corresponding angles of two triangles are equal, then the corresponding sides are in proportion.



Given : ΔABC and ΔPQR with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$

Prove : $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

Construction : E and F on AB, AC respectively such that AE = PQ and AF = PR.
Join EF.

Proof : In ΔAXY and ΔPQR

AX = PQ given

$\left\{ \begin{array}{ll} AY = PR & \text{given} \\ \hat{A} = \hat{P} & \text{given} \end{array} \right\}$

$\therefore \Delta AXY \equiv \Delta PQR$ SAS

$\therefore A\hat{X}Y = \hat{Q}$ Δ's congruent

$\therefore A\hat{X}Y = \hat{B}$ $\hat{Q} = \hat{B}$ given

$\therefore XY \parallel BC$ corresponding $\angle$'s =

$\frac{AB}{AX} = \frac{AC}{AY}$ Prop Int theorem XY || BC

$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$ AX = PQ and AY = PR

Similarly it can be proved that $\frac{AC}{PQ} = \frac{BC}{QR}$

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB || CD, then the alternate angles are equal.	alt ∠s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp ∠s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int ∠s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS

If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS

THEOREM STATEMENT	**ACCEPTABLE REASON(S)**
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2^{nd} side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord

The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}$⊙
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord

QUADRILATERALS

The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ||
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ||m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite