Physics 101

Lecture 9

Linear Momentum and Collisions

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Linear Momentum and Collisions

- Conservation of Energy
- Momentum
- Impulse
- Conservation of Momentum
- 1-D Collisions
- 2-D Collisions
- The Center of Mass

Conservation of Energy

- $\Delta E = \Delta K + \Delta U = 0$ if conservative forces are the only forces that do work on the system.
- The total amount of energy in the system is constant.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

- $\Delta E = \Delta K + \Delta U = -f_k d$ if friction forces are doing work on the system.
- The total amount of energy in the system is still constant, but the change in mechanical energy goes into "internal energy" or heat.

$$-f_k d = \left(\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2\right) - \left(\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2\right)$$

Linear Momentum

- This is a new fundamental quantity, like force, energy. It is a vector quantity (points in same direction as velocity).
- The linear momentum **p** of an object of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$\vec{p} = m\vec{v}$$

- The terms momentum and linear momentum will be used interchangeably in the text
- Momentum depend on an object's mass and velocity

Linear Momentum

- Linear momentum is a vector quantity $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
  - Its direction is the same as the direction of the velocity
- The dimensions of momentum are ML/T
- The SI units of momentum are kg m / s
- Momentum can be expressed in component form:

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

Newton’s Law and Momentum

- Newton’s Second Law can be used to relate the momentum of an object to the resultant force acting on it

$$\vec{F}_{net} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$$

- The change in an object’s momentum divided by the elapsed time equals the constant net force acting on the object

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse

- When a single, constant force acts on the object, there is an **impulse** delivered to the object

$$\vec{I} = \vec{F}\Delta t$$

  - $\vec{\mathbf{I}}$ is defined as the *impulse*
  - The equality is true even if the force is not constant
  - Vector quantity, the direction is the same as the direction of the force

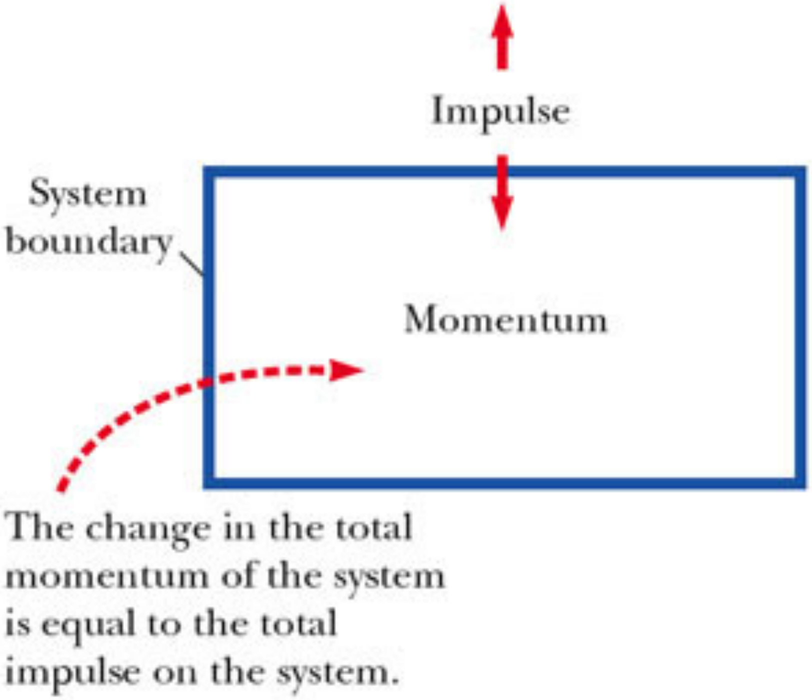
$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse-Momentum Theorem

- The theorem states that the impulse acting on a system is equal to the change in momentum of the system

$$\Delta\vec{p} = \vec{F}_{net}\Delta t = \vec{I}$$

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$



Calculating the Change of Momentum

$$\Delta p = p_{after} - p_{before}$$

$$= mv_{after} - mv_{before}$$

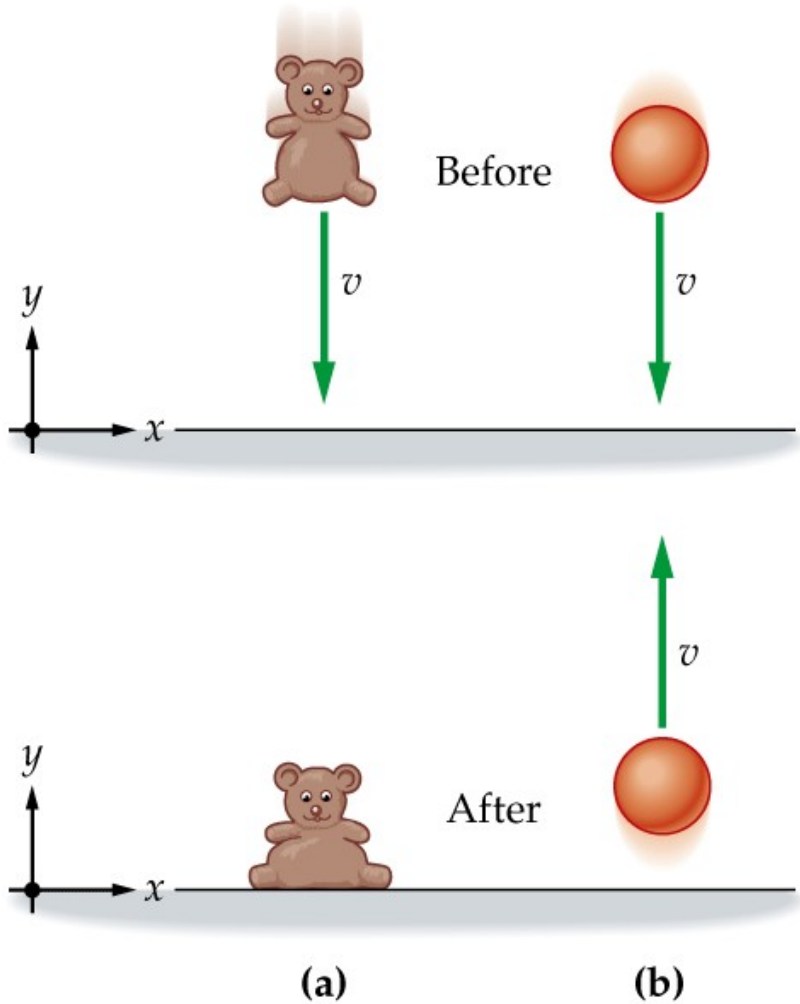
$$= m(v_{after} - v_{before})$$

For the teddy bear

$$\Delta p = m[0 - (-v)] = mv$$

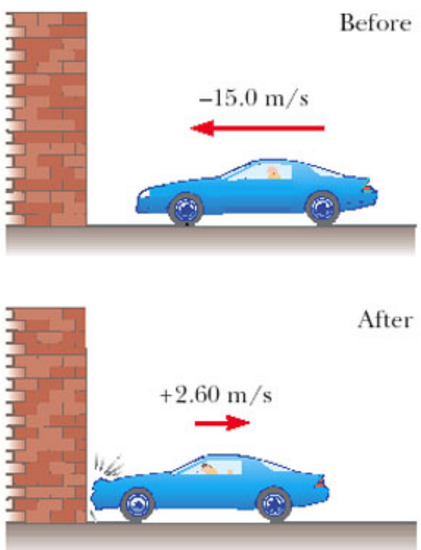
For the bouncing ball

$$\Delta p = m[v - (-v)] = 2mv$$



Ex1: How Good Are the Bumpers?

- In a crash test, a car of mass 1.5×10^3 kg collides with a wall and rebounds as in figure. The initial and final velocities of the car are v_i=-15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car



How Good Are the Bumpers?

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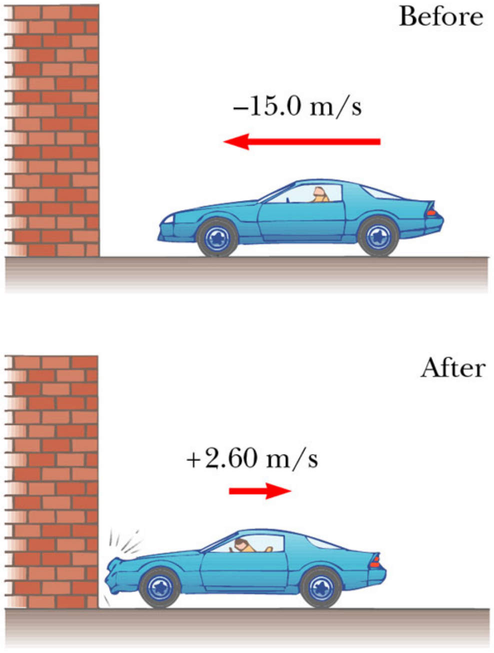
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car

$$p_i = mv_i = (1.5\times10^3 kg)(-15 m/s) = -2.25\times10^4 kg\cdot m/s$$

$$p_f = mv_f = (1.5\times10^3 kg)(+2.6 m/s) = +0.39\times10^4 kg\cdot m/s$$

$$I = p_f - p_i = mv_f - mv_i$$
$$= (0.39\times10^4 kg\cdot m/s) - (-2.25\times10^4 kg\cdot m/s)$$
$$= 2.64\times10^4 kg\cdot m/s$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{2.64\times10^4 kg\cdot m/s}{0.15 s} = 1.76\times10^5 N$$



Ex2: Impulse-Momentum Theorem

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the impulse imparted to the superball?

  (A) 0
  (B) 2 kg-m/s
  (C) 20 kg-m/s
  (D) 200 kg-m/s
  (E) 2000 kg-m/s

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

Ex3: Impulse-Momentum Theorem 2

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the force between the sidewalk and the superball?

  (A) 0

  (B) 2 N

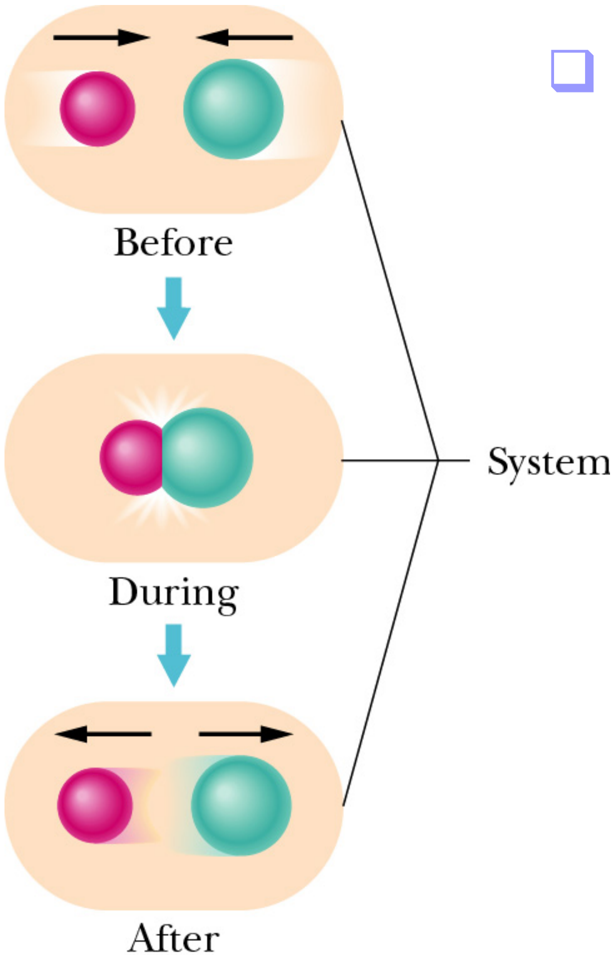
  (C) 20 N

  (D) 200 N

  (E) 2000 N

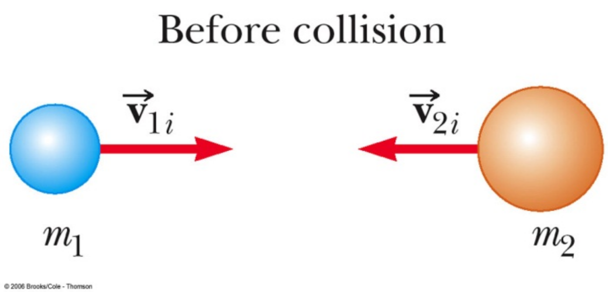
$$\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

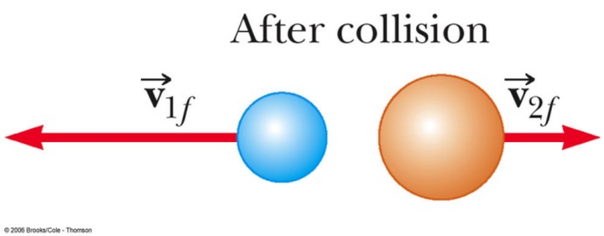
Conservation of Momentum

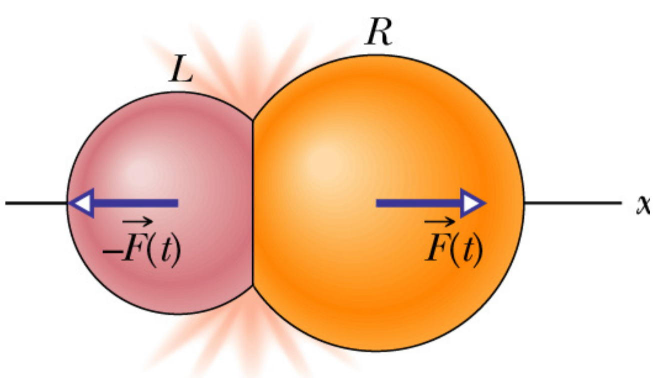


- In an isolated and closed system, the total momentum of the system remains constant in time.
  - Isolated system: no external forces
  - Closed system: no mass enters or leaves
  - The linear momentum of each colliding body may change
  - The total momentum *P* of the system cannot change.

Conservation of Momentum







- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since $\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$

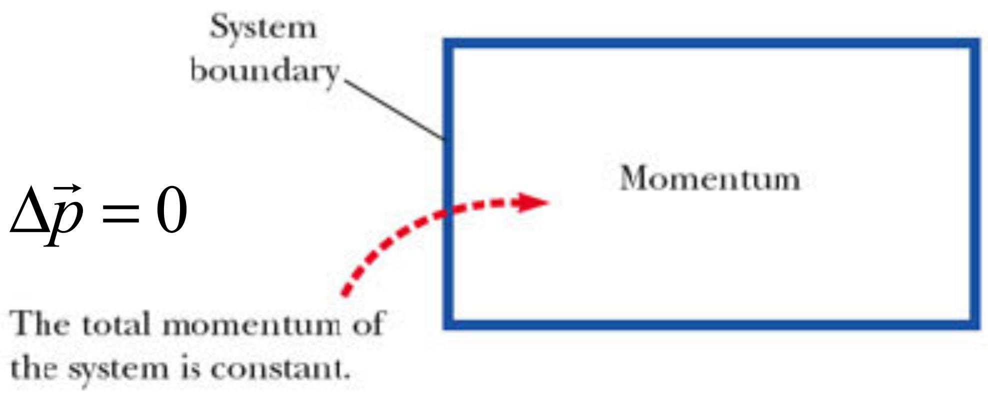
- Then $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$

- So $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Conservation of Momentum

- When no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$



- When $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$
- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Ex4: The Archer

- An archer stands at rest on frictionless ice and fires a 0.5-kg arrow horizontally at 50.0 m/s. The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

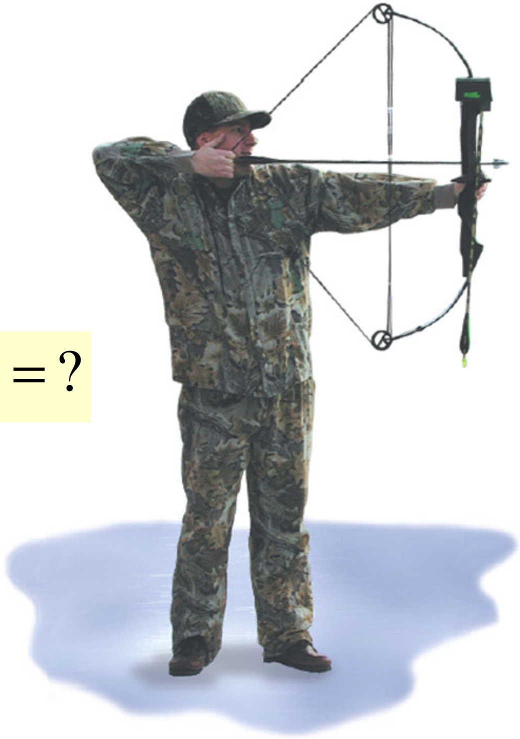
$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 = 60.0kg, m_2 = 0.5kg, v_{1i} = v_{2i} = 0, v_{2f} = 50m/s, v_{1f} = ?$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5kg}{60.0kg}(50.0m/s) = -0.417m/s$$



Ex5: Conservation of Momentum

- A 100 kg man and 50 kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1 m/s, at what speed does she recoil?
  - (A) 0
  - (B) 0.5 m/s
  - (C) 1 m/s
  - (D) 1.414 m/s
  - (E) 2 m/s

Types of Collisions

- Momentum is conserved in any collision
- **Inelastic collisions:** *rubber ball and hard ball*
  - Kinetic energy is not conserved
  - **Perfectly inelastic** collisions occur when the objects stick together
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved

Collisions Summary

- In an elastic collision, both momentum and kinetic energy are conserved
- In a non-perfect inelastic collision, momentum is conserved but kinetic energy is not. Moreover, the objects do not stick together
- In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types
- Momentum is conserved in all collisions

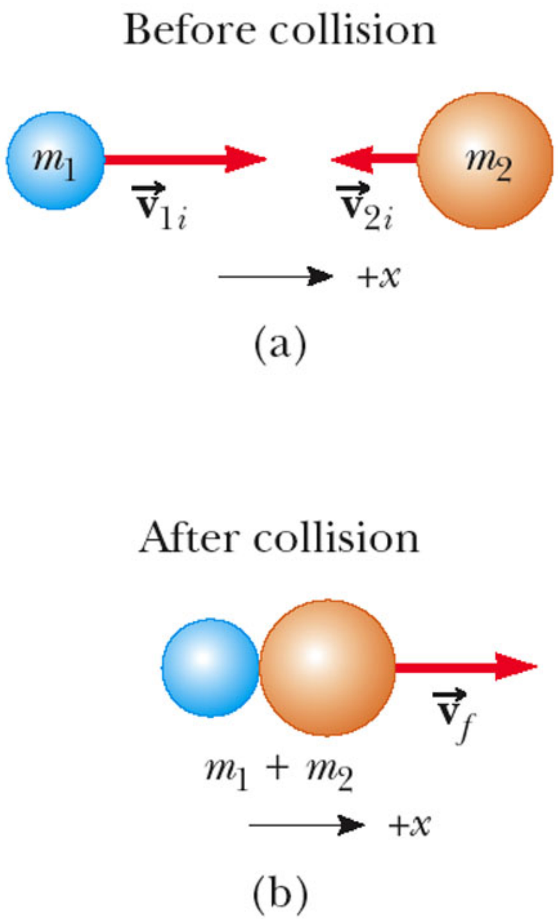
More about Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

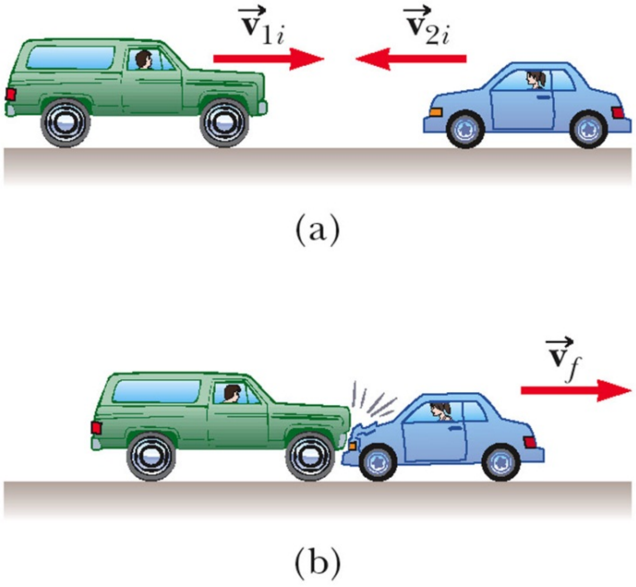
- Kinetic energy is NOT conserved



Ex6: An SUV Versus a Compact

- An SUV with mass 1.80×10^3 kg is travelling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is travelling westbound at -15.0 m/s. The cars collide head-on, becoming entangled.

(a) Find the speed of the entangled cars after the collision.
(b) Find the change in the velocity of each car.
(c) Find the change in the kinetic energy of the system consisting of both cars.



An SUV Versus a Compact

(a) Find the speed of the entangled cars after the collision.

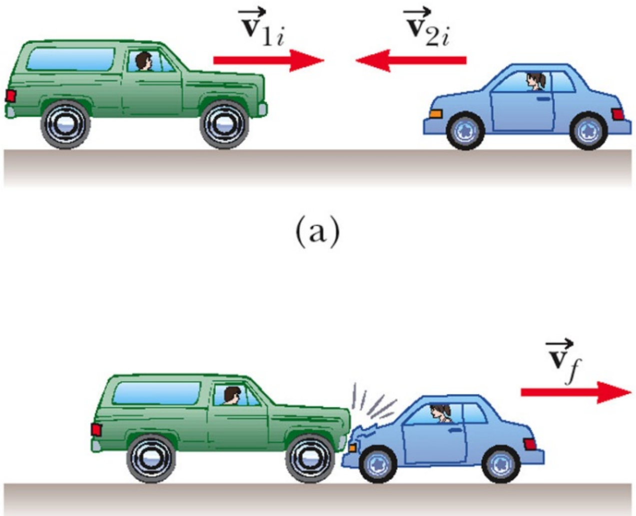
$$m_1 = 1.80\times10^3 kg, v_{1i} = +15 m/s$$
$$m_2 = 9.00\times10^2 kg, v_{2i} = -15 m/s$$

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$v_f = +5.00 m/s$$



(a)
(b)

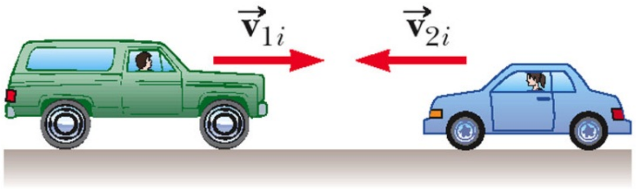
An SUV Versus a Compact

(b) Find the change in the velocity of each car.

$$m_1 = 1.80\times10^3 kg, v_{1i} = +15 m/s$$
$$m_2 = 9.00\times10^2 kg, v_{2i} = -15 m/s$$

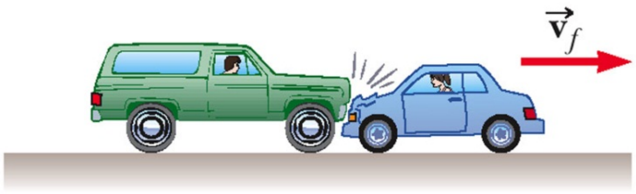
$$v_f = +5.00 m/s$$

$$\Delta v_1 = v_f - v_{1i} = -10.0 m/s$$
$$\Delta v_2 = v_f - v_{2i} = +20.0 m/s$$



(a)

$$m_1\Delta v_1 = m_1(v_f - v_{1i}) = -1.8\times10^4 kg\cdot m/s$$
$$m_2\Delta v_2 = m_2(v_f - v_{2i}) = +1.8\times10^4 kg\cdot m/s$$



(b)

$$m_1\Delta v_1 + m_2\Delta v_2 = 0$$

An SUV Versus a Compact

(c) Find the change in the kinetic energy of the system consisting of both cars.

$$m_1 = 1.80\times10^3 kg, v_{1i} = +15 m/s$$

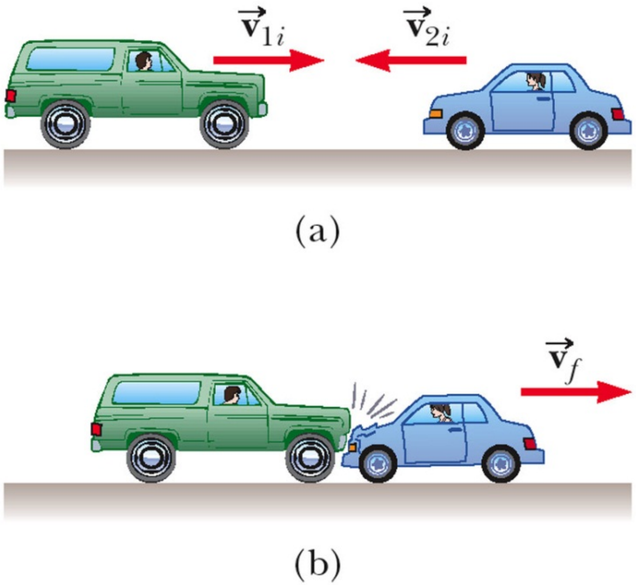
$$m_2 = 9.00\times10^2 kg, v_{2i} = -15 m/s$$

$$v_f = +5.00 m/s$$

$$KE_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = 3.04\times10^5 J$$

$$KE_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = 3.38\times10^4 J$$

$$\Delta KE = KE_f - KE_i = -2.70\times10^5 J$$

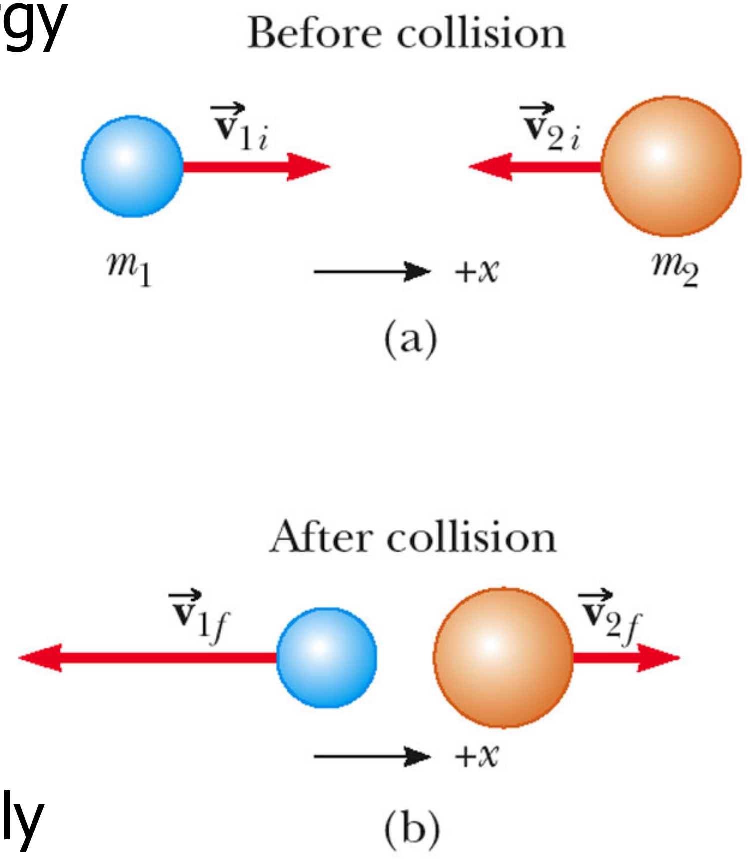


More About Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$



- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Solve the equations simultaneously

Elastic Collisions

- A simpler equation can be used in place of the KE equation

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Ex7: Conservation of Momentum

- An object of mass m moves to the right with a speed v. It collides head-on with an object of mass $3m$ moving with speed $v/3$ in the opposite direction. If the two objects stick together, what is the speed of the combined object, of mass $4m$, after the collision?

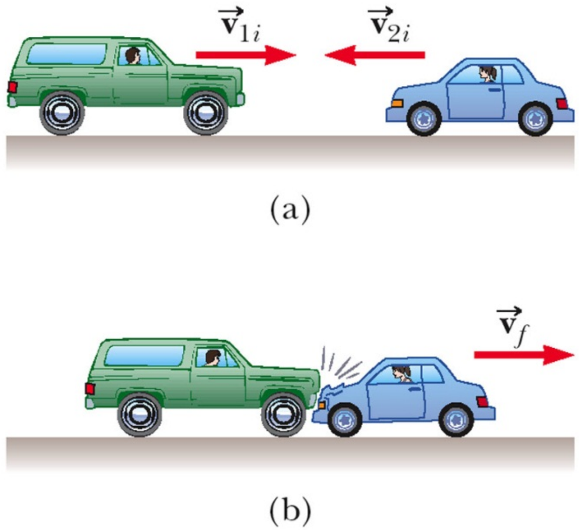
  (A) 0

  (B) v/2

  (C) v

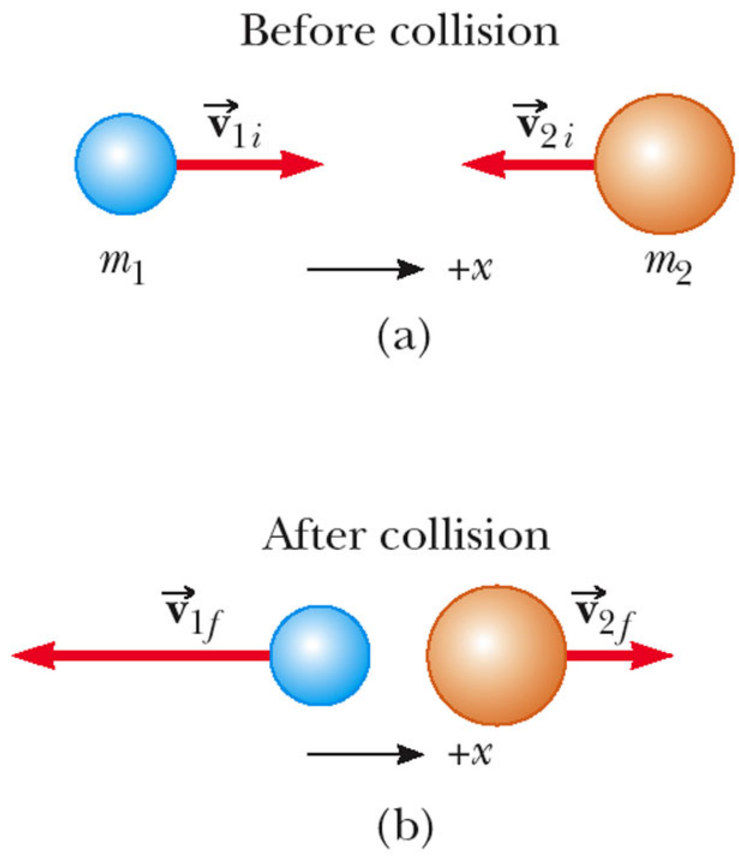
  (D) 2v

  (E) 4v



Problem Solving for 1D Collisions, 1

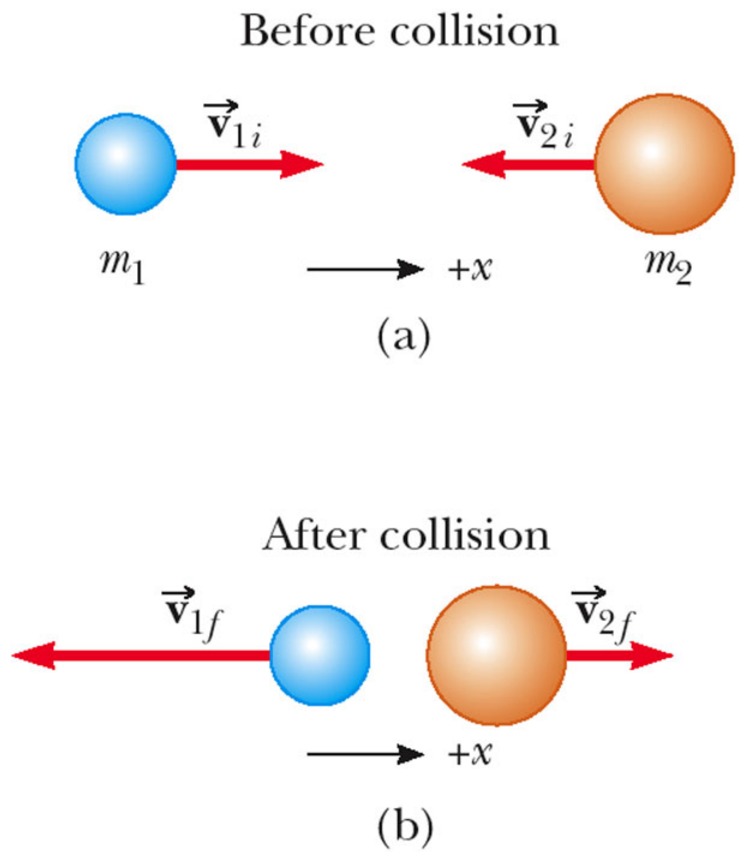
- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Problem Solving for 1D Collisions, 2

- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

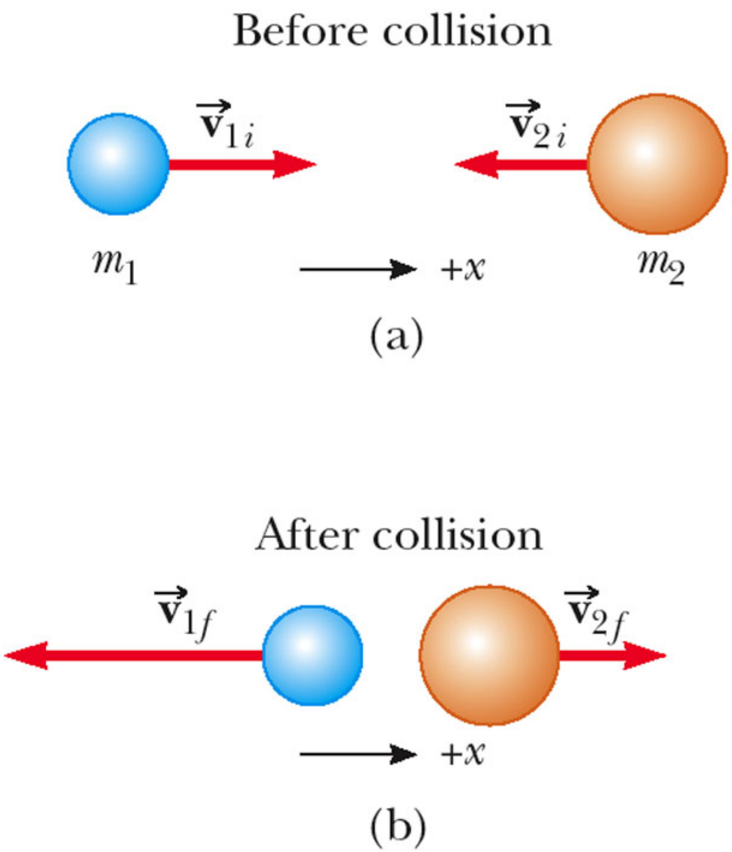


Problem Solving for 1D Collisions, 3

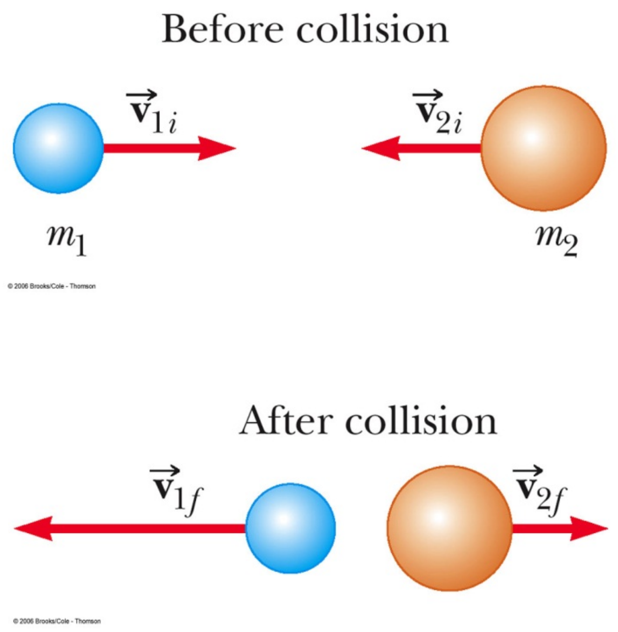
- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

- **Solve:** the resulting equations simultaneously



One-Dimension vs Two-Dimension

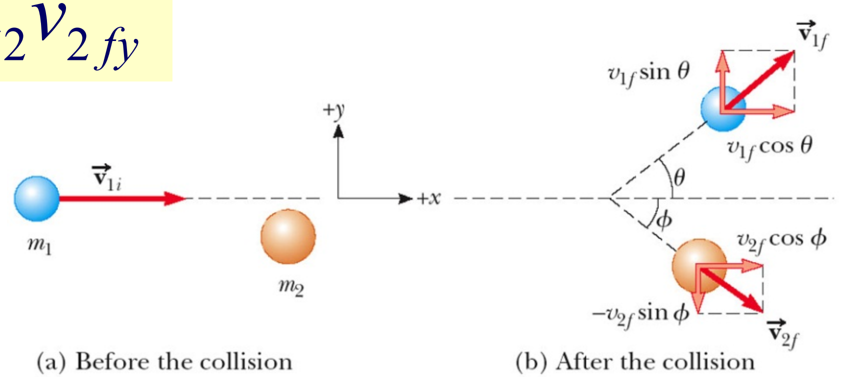


Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



Two-Dimensional Collisions

- The momentum is conserved in all directions
- Use subscripts for
  - Identifying the object
  - Indicating initial or final values
  - The velocity components

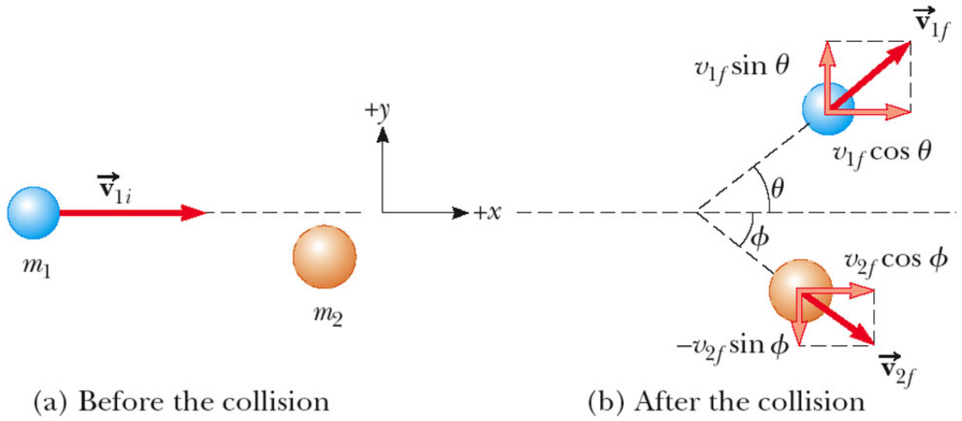
$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

Glancing Collisions

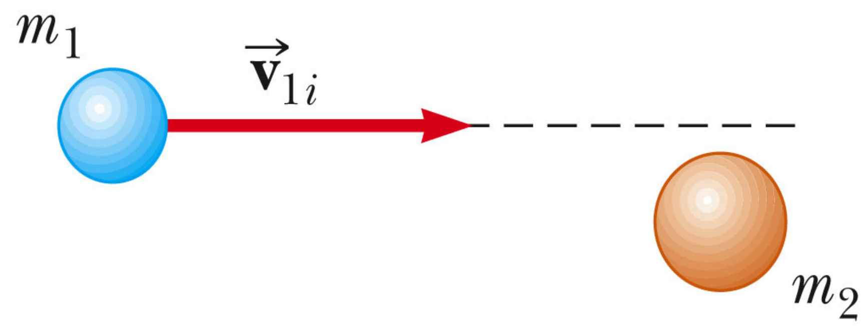


- The "after" velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

2-D Collision, example

- Particle 1 is moving at velocity $\vec{v}_{1i}$ and particle 2 is at rest
- In the *x*-direction, the initial momentum is $m_1 v_{1i}$
- In the *y*-direction, the initial momentum is 0



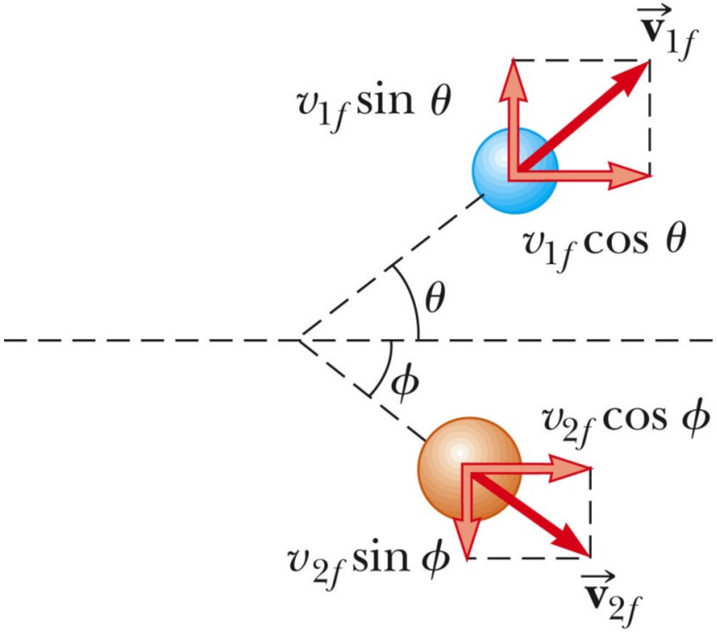
(a) Before the collision

2-D Collision, example cont

- After the collision, the momentum in the *x*-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- After the collision, the momentum in the *y*-direction is $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$

$$m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

$$0 + 0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$



(b) After the collision

- If the collision is elastic, apply the kinetic energy equation

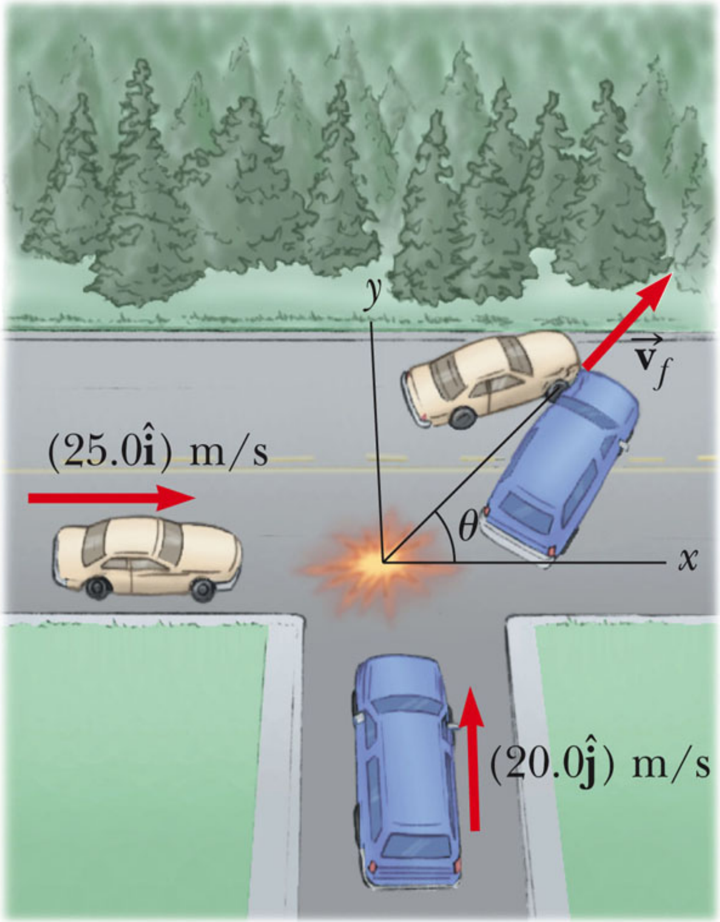
$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Ex8: Collision at an Intersection

- A car with mass 1.5×10^3 kg traveling east at a speed of 25 m/s collides at an intersection with a 2.5×10^3 kg van traveling north at a speed of 20 m/s. Find the magnitude and direction of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision and assuming that friction between the vehicles and the road can be neglected.

$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$

$$v_{cix} = 25m/s, v_{viy} = 20m/s, v_f = ?\ \theta = ?$$



Collision at an Intersection

$$m_c = 1.5\times10^3\,\text{kg}, m_v = 2.5\times10^3\,\text{kg}$$

$$v_{cix} = 25\,\text{m/s}, v_{viy} = 20\,\text{m/s}, v_f = ?\,\theta = ?$$

$$\sum p_{xi} = m_c v_{cix} + m_v v_{vix} = m_c v_{cix} = 3.75\times10^4\,\text{kg}\cdot\text{m/s}$$

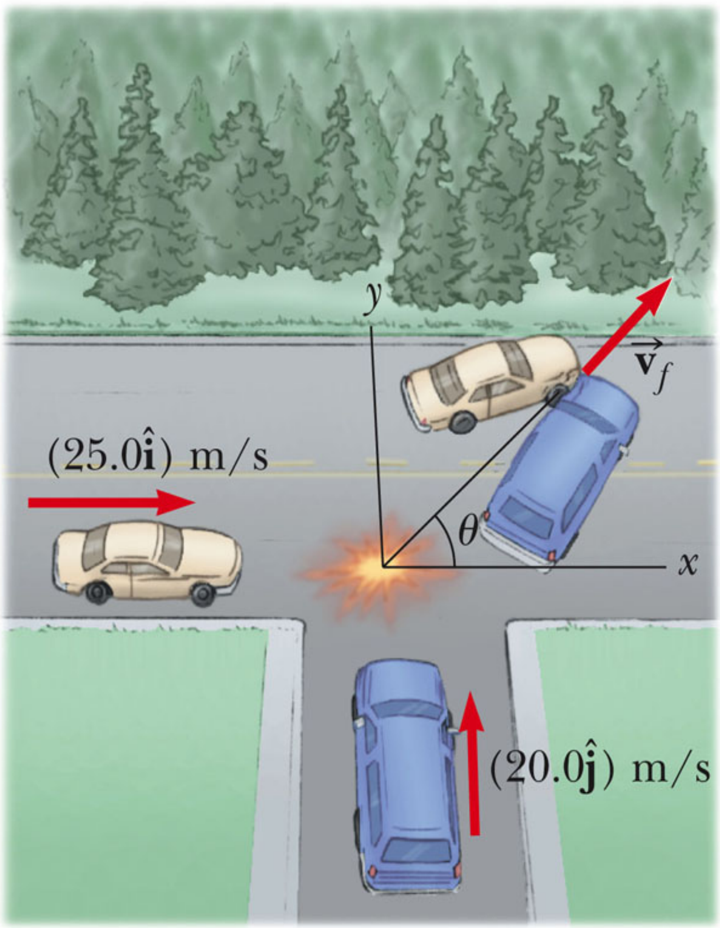
$$\sum p_{xf} = m_c v_{cfx} + m_v v_{vfx} = (m_c + m_v) v_f \cos\theta$$

$$3.75\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \cos\theta$$

$$\sum p_{yi} = m_c v_{ciy} + m_v v_{viy} = m_v v_{viy} = 5.00\times10^4\,\text{kg}\cdot\text{m/s}$$

$$\sum p_{yf} = m_c v_{cfy} + m_v v_{vfy} = (m_c + m_v) v_f \sin\theta$$

$$5.00\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \sin\theta$$



Collision at an Intersection

$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$

$$v_{cix} = 25m/s, v_{viy} = 20m/s, v_f = ?\ \theta = ?$$

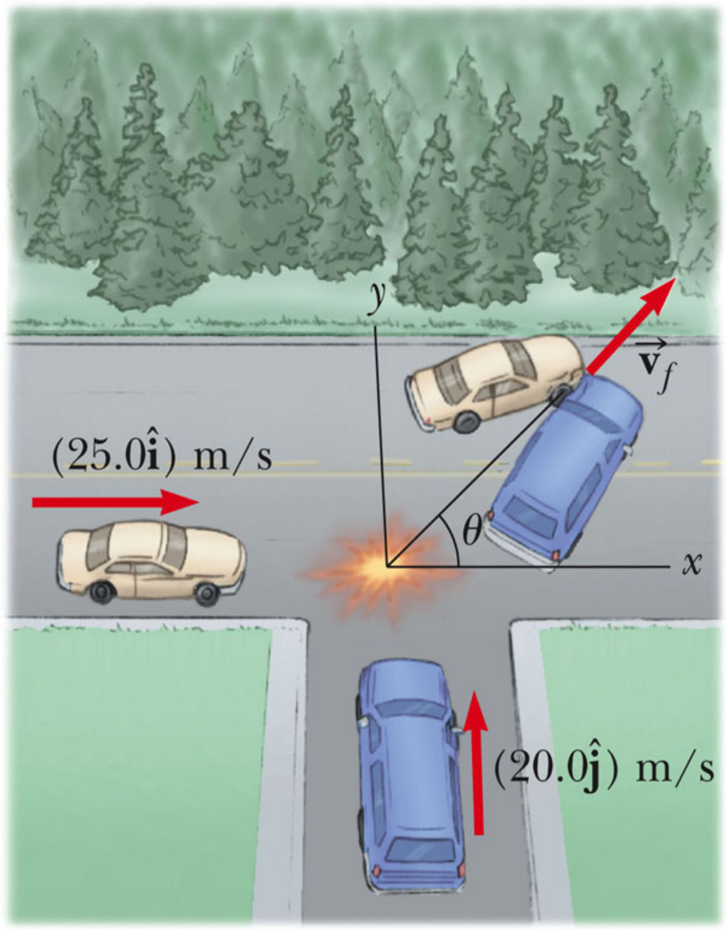
$$5.00\times10^4 \text{kg}\cdot\text{m/s} = (4.00\times10^3 \text{kg}) v_f \sin\theta$$

$$3.75\times10^4 \text{kg}\cdot\text{m/s} = (4.00\times10^3 \text{kg}) v_f \cos\theta$$

$$\tan\theta = \frac{5.00\times10^4 kg\cdot m/s}{3.75\times10^4 kg\cdot m/s} = 1.33$$

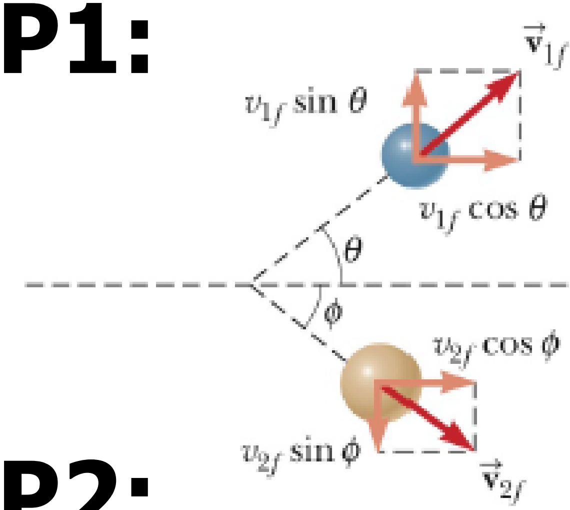
$$\theta = \tan^{-1}(1.33) = 53.1^\circ$$

$$v_f = \frac{5.00\times10^4 \text{kg}\cdot\text{m/s}}{(4.00\times10^3 \text{kg})\sin 53.1^\circ} = 15.6 \text{m/s}$$



A puck with mass $m_2 = 0.3kg$, initially at rest on a horizontal, frictionless surface, is struck by a puck of mass $m_1 = 0.2kg$ moving initially along the x axis with a speed of $v_{1_i} = 2.0m/s\hat{\imath}$. After the collision, the puck with mass $m_1 = 0.2kg$ has a speed of $v_{1_f} = 1.0m/s$ at an angle of $\theta = 53.0^\circ$ to the positive x axis (see figure below). (7P)

P1:

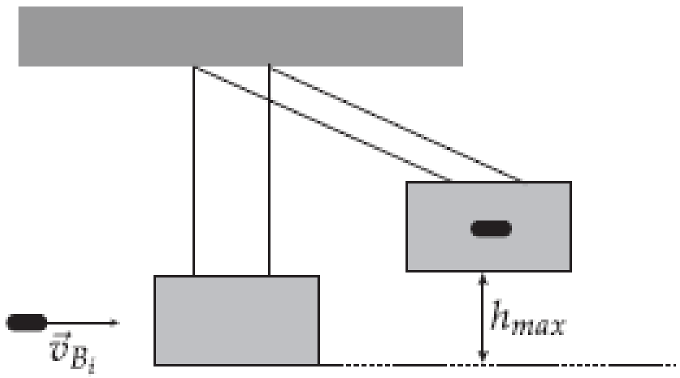


(a) Determine the magnitude v_{1_f} and direction ϕ of the velocity of the puck of mass $m_1 = 0.3kg$ after the collision.

(b) Find the magnitude of kinetic energy transferred away or transformed to other forms of energy in the collision.

P2:

A 28g bullet travelling at 230 m/s hits a 3.6 kg block of wood and remains in it. The block+bullet swings up to a maximum height h_{max} as shown in the figure below. Find h_{max}. (5P)



P3:

On a frictionless horizontal air table, puck A with mass $m_A = 0.250\text{kg}$ is moving toward puck B with mass $m_B = 0.350\text{kg}$, which is initially at rest. Just after collision, puck A has a velocity of $v_{A_f} = 0.120\text{m/s}$ to the left, and puck B has a velocity of $v_{B_f} = 0.650\text{m/s}$ to the right.

(a) What is the speed of the puck A just before the collision?

(b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

P4:

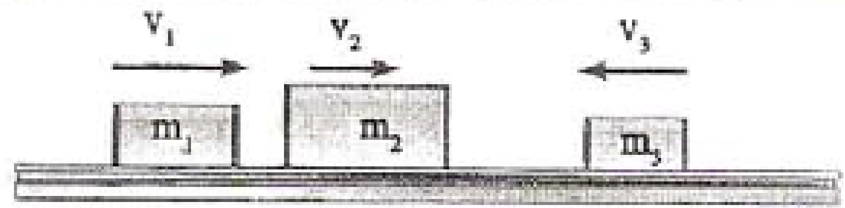
a) A 0.140-kg baseball is dropped and reaches a speed of 1.20 m/s just before it hits the ground. It rebounds with a speed of 1.00 m/s. What is the change of the ball's momentum? **(2 points)**

b) A 1.0 kg object travelling east at 1.0 m/s collides head on with a 2.0 kg object travelling west at 2.0 m/s. Find the velocity of two objects just after collision if the collision is perfectly inelastic. **(2 points)**

c) A 900-kg car traveling east at 15.0 m/s collides with a 750-kg car traveling north at 20.0 m/s. The cars stick together. (i) What is the speed of the wreckage just after the collision? (ii) In what direction does the wreckage move just after the collision? **(5 points)**

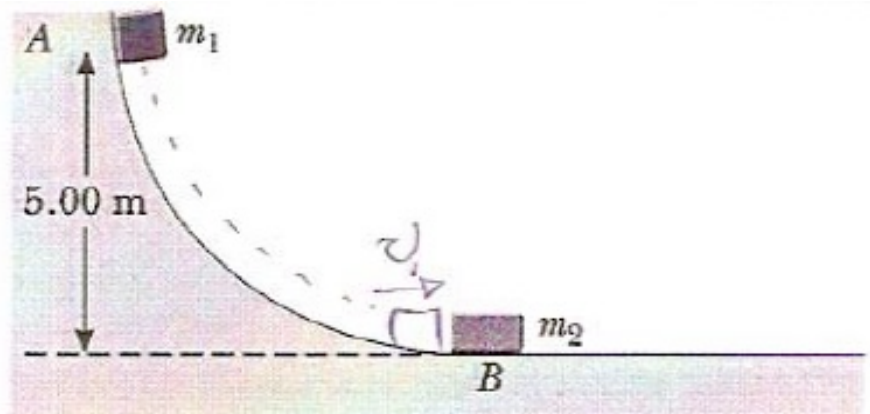
P5:

2. Three carts of masses $m_1 = 6kg$, $m_2 = 9kg$ and $m_3 = 5kg$ move on a frictionless, horizontal track with speeds $v_1 = 3m/s$ to the right, $v_2 = 2m/s$ to the right and $v_3 = 5m/s$ to the left as shown in the figure. The carts stick together after colliding. Find the final **velocity** of the carts. **(4 points)**



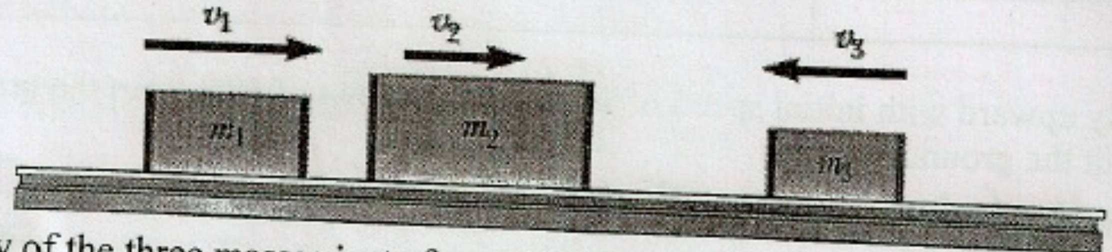
P6:

P-4: Two blocks $m_1 = 4kg$ and $m_2 = 8kg$ are free to slide along the frictionless, wooden track shown in the figure. The block of mass $m_1 = 4kg$ is released from the position shown, at height $h = 5m$ above the flat part of the track. After some time the blocks collide elastically at point B. Find the speeds of the blocks after the collision. **(9 points)**



P7:

P3: Three masses $m_1 = 5$ kg, $m_2 = 10$ kg, and $m_3 = 3$ kg move on a frictionless, horizontal track with speeds of $v_1 = 4$ m/s to the right, $v_2 = 3$ m/s to the right, and $v_3 = 5$ m/s to the left, as shown below. They stick and move together after colliding.

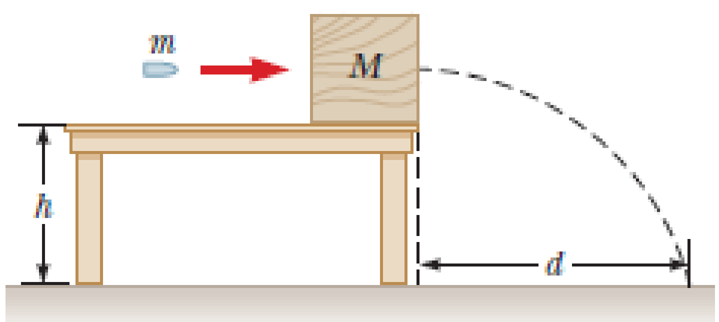


a) Find the velocity of the three masses just after collision. **(4pts)**

b) Find how much kinetic energy is lost during this inelastic collision. **(4pts)**

P8:

1. **Review.** A bullet of mass $m = 8.00$ g is fired into a block of mass $M = 250$ g that is initially at rest at the edge of a table of height $h = 1.00$ m (Fig. P9.81). The bullet remains in the block, and after the impact the block lands $d = 2.00$ m from the bottom of the table. Determine the initial speed of the bullet.



P9:

A truck of mass $m_1 = 6000kg$ is travelling at a velocity of $\vec{v}_{1_0} = 25\hat{i}\, m/s$ collides with a car of mass $m_2 = 1200kg$ travelling at a velocity $\vec{v}_{2_0} = 20\hat{i}\, m/s$. The velocity of the truck immediately after the collision is $\vec{v}_1 = 22\hat{i}\, m/s$.

(a) Calculate the velocity of the car, immediately after the collision.

(b) If the collision lasted for 0.4s, what is the average force exerted by the car on the truck?

(c) Calculate the lost kinetic Energy.

P10:

A 1.0 kg object travelling at 1.0 m/s collides head on with a 2.0 kg object initially at rest. Find the velocity of each object after impact if the collision is perfectly elastic.

P1 A 1.2 kg ball drops vertically onto a floor, hitting with a speed of 25 m/s. It rebounds with an initial speed of 10 m/s. (a) What impulse acts on the ball during the contact? (b) If the ball is in contact with the floor for 0.020 s, what is the magnitude of the average force on the floor from the ball?

ANS:

a) $\vec{J} = 42\hat{j}\ kgm/s$

b) $\vec{F}_{avg} = 2100\hat{j}\ N$

P2 In the overhead view of Fig. 9-54, a 300 g ball with a speed v of 6.0 m/s strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for 10 ms. In unit-vector notation, what are (a) the impulse on the ball from the wall and (b) the average force on the wall from the ball?

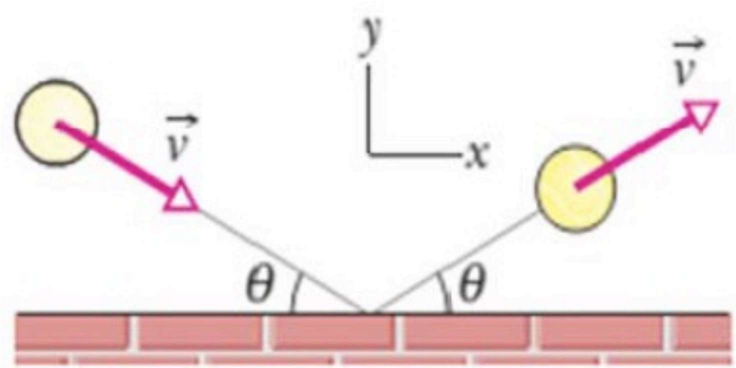


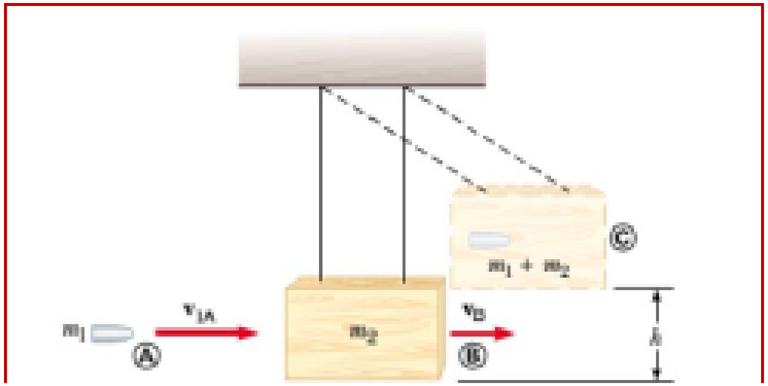
Figure 9-54 Problem 38.

ANS:

a) $\vec{J} = 1.8\hat{j}\ kgm/s$

b) $\vec{F}_{avg} = -180\hat{j}\ N$

The ballistic pendulum is an apparatus used to measure the speed of a fast-moving projectile, such as a bullet. A bullet of mass $m_1 = 20g$ with $v_{1A} = 200m/s$ is fired into a large block of wood of mass $m_2 = 480g$ suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height h. Find the max height, $h = ?$



ANS:

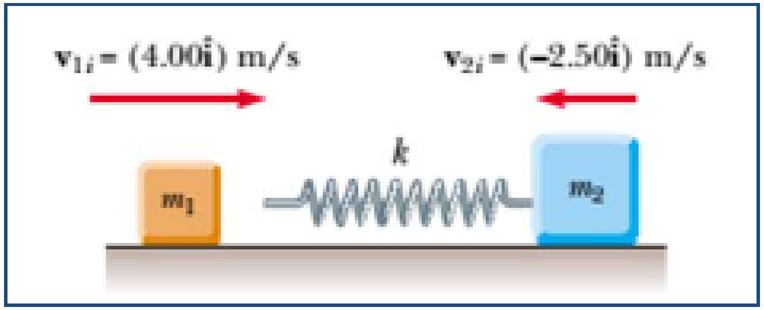
$v = 8m/s$

$h = 3.3m$

P5

A block of mass m_1=1.6kg initially moving to the rightwith a speed of 4m/s on a frictionless horizontal track collides with a spring attached to a second block of mass m_2=2.1kg initially moving to the left with a speed of 2.5 m/s, as shown in Figure. The spring constant is 600 N/m. (Suppose that the collision is elastic.)

a) Find the velocities of the two blocks after the collision.
b) Prove that during the collision the kinetic energy is conserved.
c) The maximum compression would occur when the two blocks are moving with the same velocity. What is the maximum compression of the spring during the collision?



ANS:

a) $\vec{v}_{1f} = -3.38\hat{\imath} m/s$ & $\vec{v}_{2f} = 3.12\hat{\imath} m/s$

b) $\Delta K = 0$

c) $v_{com} = 0.311\ m/s$; $x_{max} = 0.25m$

P6

60. A small block of mass $m_1 = 0.500$ kg is released from rest at the top of a curve-shaped frictionless wedge of mass $m_2 = 3.00$ kg, which sits on a frictionless horizontal surface as in Figure P9.60a. When the block leaves the wedge, its velocity is measured to be 4.00 m/s to the right, as in Figure P9.60b. (a) What is the velocity of the wedge after the block reaches the horizontal surface? (b) What is the height h of the wedge?

ANS:

$a)\ \vec{v}_{1f} = -0.67\hat{\imath} m/s$

$b)\ h = 0.95m$

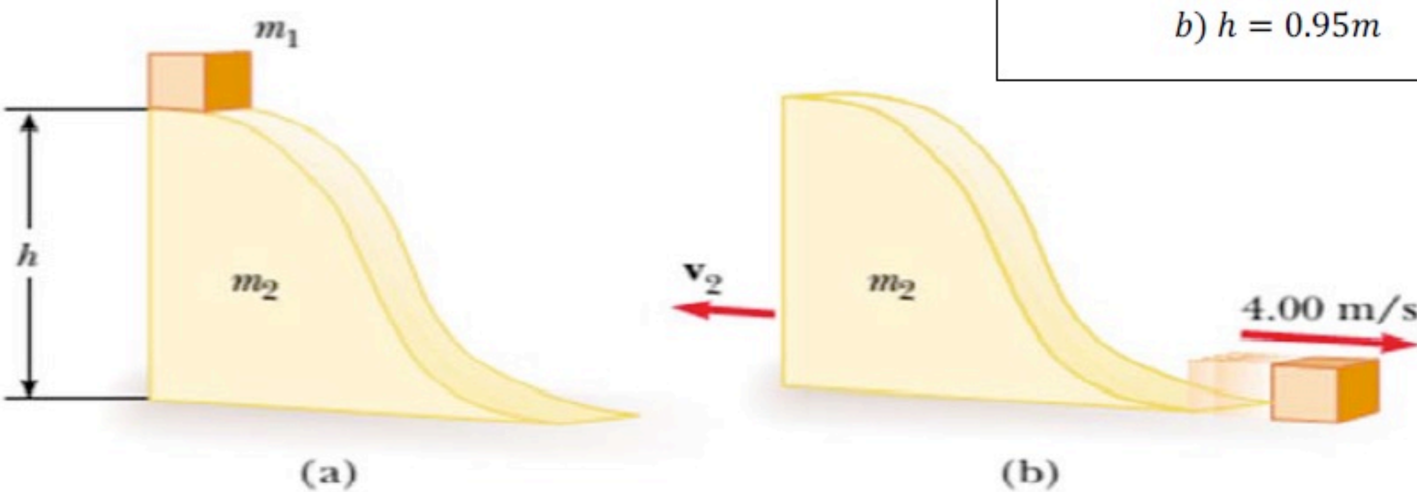


Figure P9.60

P7 GO In Fig. 9-59, a 10 g bullet moving directly upward at 1000 m/s strikes and passes through the center of mass of a 5.0 kg block initially at rest. The bullet emerges from the block moving directly upward at 400 m/s. To what maximum height does the block then rise above its initial position?

ANS:

$$h_{max} = 7.35cm$$

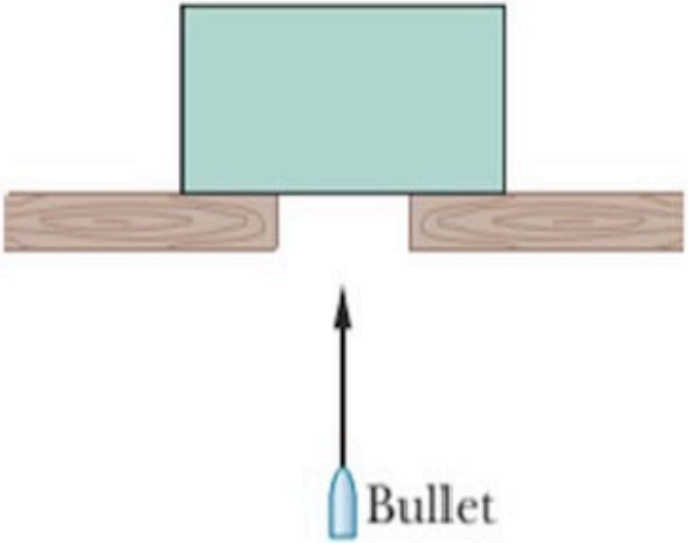
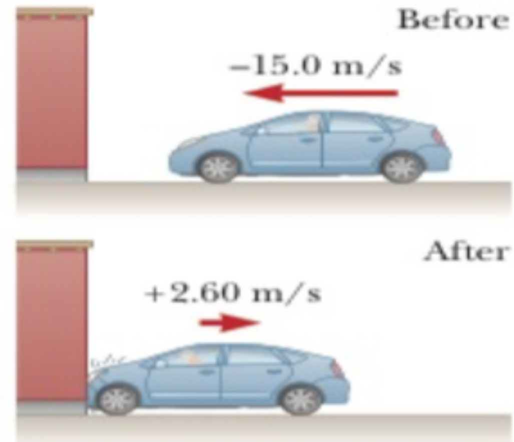
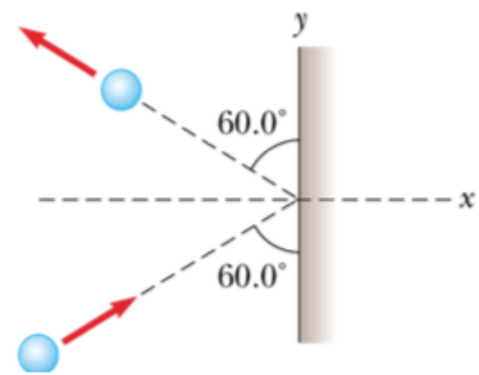


Figure 9-59 Problem 52.

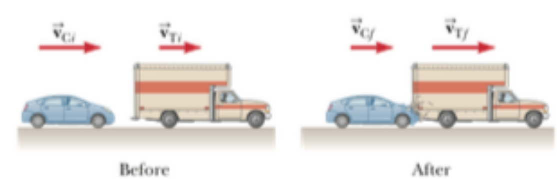
1) In a particular crash test, a car of mass $1500kg$ collides with a wall, as shown in the figure. The initial and final velocities of the car are $\vec{v}_i = -15\hat{\imath}\, {}^{m}/_{s}$ and $\vec{v}_f = 2.6\hat{\imath}\, {}^{m}/_{s}$, respectively. If the collusion lasts for $0.15s$, find the impulse caused by the collusion and the average force exerted on the car.



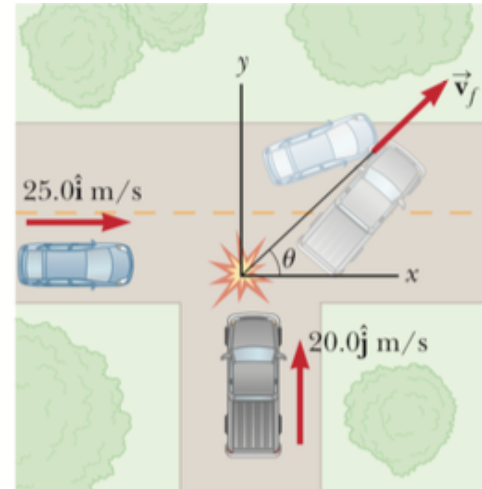
2) A $3kg$ steel ball strikes a wall with a speed of $10\, {}^{m}/_{s}$ at an angle of 60° with the surface. It bounces off with the same speed and angle. If the ball is in contact with the wall for $0.2s$, what is the average force exerted by the wall on the ball?



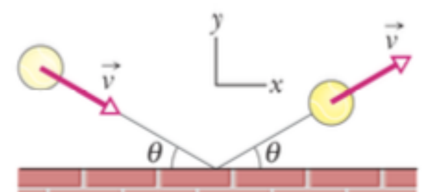
3) An $1800kg$ truck stopped at a traffic light is struck from the rear by a $900kg$ car, and that two become entangled, moving along the same path as that of the originally moving car. If the car were moving at $20\, {}^{m}/_{s}$ before the collision, what is the velocity of the entangled car-truck system after the collision?



4) A 1500kg car travelling east with a speed of 25 $^{m}/_{s}$ collides at an intersection with a 2500kg van travelling north at a speed of 20 $^{m}/_{s}$, as shown in the Figure. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that they stick together.



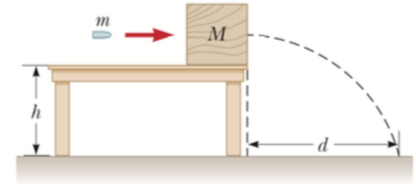
5) In the overhead view of the Figure, a 300g ball with a speed of v of 6 $^{m}/_{s}$ strikes a wall at an angle θ of 30° and then rebounds with the same speed and angle. It is in contact with the wall for *10ms.*



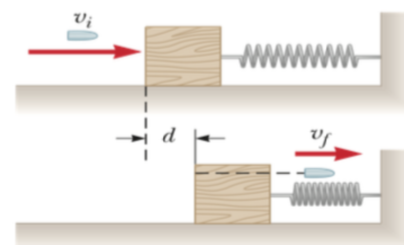
a) What is the impulse on the ball from the wall in unit vector notation?

b) What is the average force on the wall from the ball in unit vector notation?

7) A bullet of mass $m = 8g$ is fired into a block of mass $M = 250g$ that is initially at rest at the edge of a table of height $h = 1m$. The bullet remains in the block, and after the impact the block lands $d = 2m$ from the bottom of the table. Determine the initial speed of the bullet.



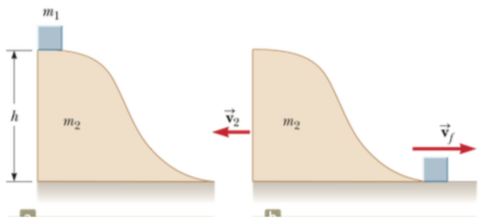
8) A $5g$ bullet moving with an initial speed of $v_i = 400\,{}^{m}/_{s}$ is fired into and passes through a $1kg$ blockas shown in the Figure. The block, initially at rest on a frictionless, horizontal surface, is connected to a spring with force constant $900\,{}^{N}/_{m}$. The block moves $d = 5cm$ to the right after impact being brought to rest by the spring.



a) Find the speed at which the bullet emerges from the block and
b) Find the amount of initial kinetic energy of the bullet is converted into internal energy in the block bullet system during the collusion.

9) Two blocks of masses $m_1 = 2kg$ and $m_2 = 4kg$ are released from rest at a height of $h = 5m$ on a frictionless track as shown in the Figure. When they meet on the level portion of the track, they undergo a head-on, elastic collusion. Determine the maximum heights to which m_1 and m_2 rise on the curved portion of the track after the collusion.

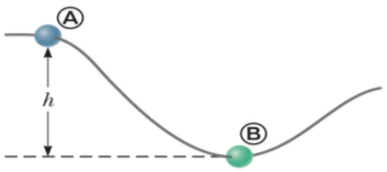
10) A small block of mass $m_1 = 0.5kg$ is released from rest at the top of a frictionless, curve-shaped wedge of mass $m_2 = 3kg$, which sits on a frictionless, horizontal surface as shown in the Figure. When the block leaves the wedge, its velocity is measured to be $4\,{}^{m}/_{s}$ to the right.



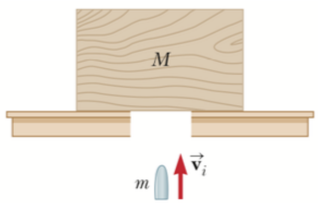
a) What is the velocity of the wedge after the block reaches the horizontal surface?

b) What is the height h of the wedge?

11) A $0.4kg$ blue bead slides on a frictionless, curved wire, starting at point A in the Figure, where $h = 1.5m$. At point B, the blue bead collides elastically with a $0.6kg$ green bead at rest. Find the maximum height the green bead rises as it moves up the wire.



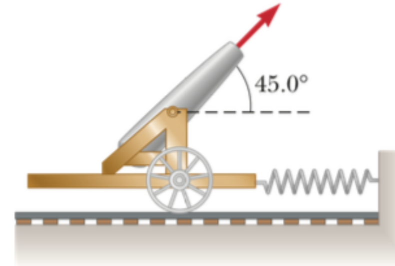
12) A $1.25kg$ wooden block rests on a table over a large hole as in the Figure. A $5g$ bullet with an initial velocity v_i is fired upward into the bottom of the block and remains in the block after the collusions. The block and bullet rise to a maximum height of $22cm$.



a) Describe how you would find the initial velocity of the bullet.

b) Calculate the initial velocity of the bullet from the information provided.

13) A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initiallt unstretched snd with force constant $k = 2\text{x}10^4\ ^N/_m$, as shown in the Figure. The cannon fires a $200kg$ projectile at a velocity of $125\ ^m/_s$ directed 45° above the horizontal.



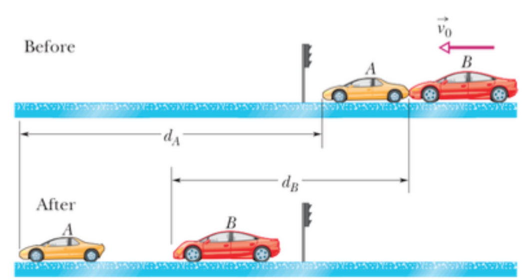
a) Assuming that the mass of the cannon and its carriage is $5kg$, find the recoil speed of the cannon.

b) Determine the maximum extension of the spring.

c) Find the maximum force the spring exerts on the carriage.

d) Consider the system consisting of the cannon, carriage, and projectile. Is the momentum of this system conserved during the fireing? Why or why not?

14) In the "before" part of the Figure, car *A* (mass *1100kg*) is stopped at a traffic light when it is rear-ended by car *B* (mass *1400kg*). Both cars then slides with locked wheels until the frictional force from the slick road (with a low μ_k of 0.13) stops them, at distances $d_A = 8.2m$ and $d_B = 6.1m$.

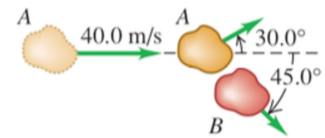


a) What is the speed of car *A* just after the collusion?

b) What is the speed of car *B* just after the collusion?

c) Assuming that linear momentum is conserved during the collision, find the speed of car *B* just before the collision.

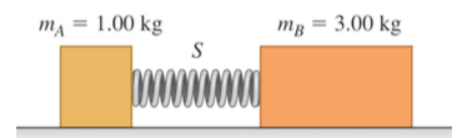
15) Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid A, which was initially travelling at $40\,{}^{m}/_{s}$, is deflected 30° from its original direction, while asteroid B, which was initially at rest, travels at 45° to the original direction of A.



a) Find the speed of each asteroid after the collision.

b) What fraction of the original kinetic energy of asteroid A dissipates during this collision?

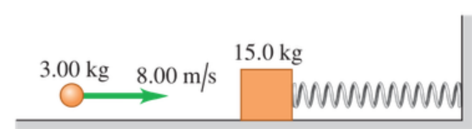
16) Block A has a mass of $1kg$, and block B has mass $3kg$. The blocks are forced together, compressing a spring S between them; then the system is released from rest on a level, frictionless surface. The spring, which has negligible mass, is not fastened to either block and drops to the surface after it has expanded. Block B acquires a speed of $1.2\,{}^{m}/_{s}$.



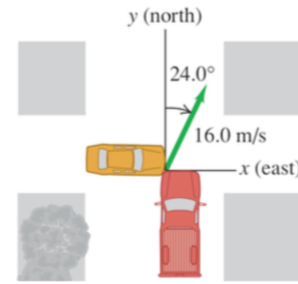
a) What is the final speed of block A?

b) How much potential energy was stored in the compressed spring?

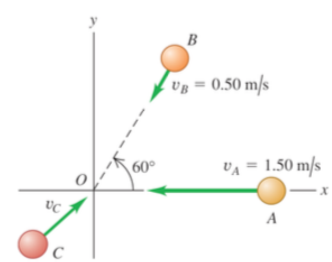
17) A $15kg$ block is attached to a very light horizontal spring of force constant $500\,{}^{N}/_{m}$ and is resting on a frictionless horizontal table. Suddenly it is stuck by a $3kg$ stone travelling horizontally at $8\,{}^{m}/_{s}$ to the right, whereupon the stone rebounds at $2\,{}^{m}/_{s}$ horizontally to the left. Find the maximum distance that the block will compress the spring after the collision.



18) At an intersection, a yellow subcompact car with mass *950kg* travelling east collides with a red pickup truck of mass *1900kg* that is travelling due north and has run a red light. The two vehicles stick together as a result of the collision, and the wreckage slides at $16\,{}^{m}/{}_{s}$ in the direction 24° east of north. Calculate the speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; you can ignore friction forces between the vehicles and the wet road.



19) Spheres *A* (mass 0.02*kg*), *B* (mass 0.03*kg*), and *C* (mass 0.05*kg*) are approaching the origin as they slide on a frictionless air table. The initial velocities of *A* and *B* are given in the Figure. All three spheres arrive to the origin at the same time and stick together.



a) What must the *x*- and *y*-components of the initial velocity of *C* be if all three objects are to end up moving at $0.5\,{}^{m}/{}_{s}$ in the +*x*-direction after the collision?

b) If *C* has the velocity found in part (a), what is the change in the kinetic energy of the system of three spheres as a result of the collision?

20) A $5kg$ chunk of ice is sliding at $12\,{}^{m}/_{s}$ on the floor of an ice covered valley when it collides with and sticks to another $5kg$ chunk of ice that is initially at rest. Since the valley is icy, there is no friction. After the collision, how high above the valley floor will the combined chunks go?