Lab 10 Simple Harmonic Motion

A study of the kind of motion that results from the force applied to an object by a spring

April 10, 2015

Print *Your Name*

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Print *Your Partners' Names*

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Instructions

Before lab, read the section titled *How to do this lab*, immediately below, and the *Introduction*. Then answer the Pre-Lab Questions on the last page of this handout. Hand in your answers as you enter the general physics lab.

How to do this lab

This lab has two parts.

Part I consists of ***Activities #1, #2, #3, and #4****. Work through Part I in the usual manner, and hand in your results at the end of your regularly scheduled lab period.*

Table of Contents

Simple Harmonic Motion

Equipment: Computer running LoggerPro 3.8
LabQuest
Motion Detector, on floor, facing upward
Wire basket to protect the Motion Detector
Dual Range force sensor
Digital scale
Spring able to provide simple harmonic motion with up to 0.5 kg mass
E.g., Cenco part number 75490N, having a nominal spring constant of 10 N/m.
If the spring is tapered, the small-diameter end of the spring is above the large-diameter end.
Loop of string connecting 50 g mass to spring (prevents rotational oscillation)
Masses: 50 g, 100 g, 150 g, 200 g, 250 g, 500 g (exact values are not critical)
Various clamps and rods for mounting as in Figure 1

Instructors: Align the motion detector under the spring with a plumb bob before the lab, and verify correct operation. Adjust the position of the motion detector if necessary.

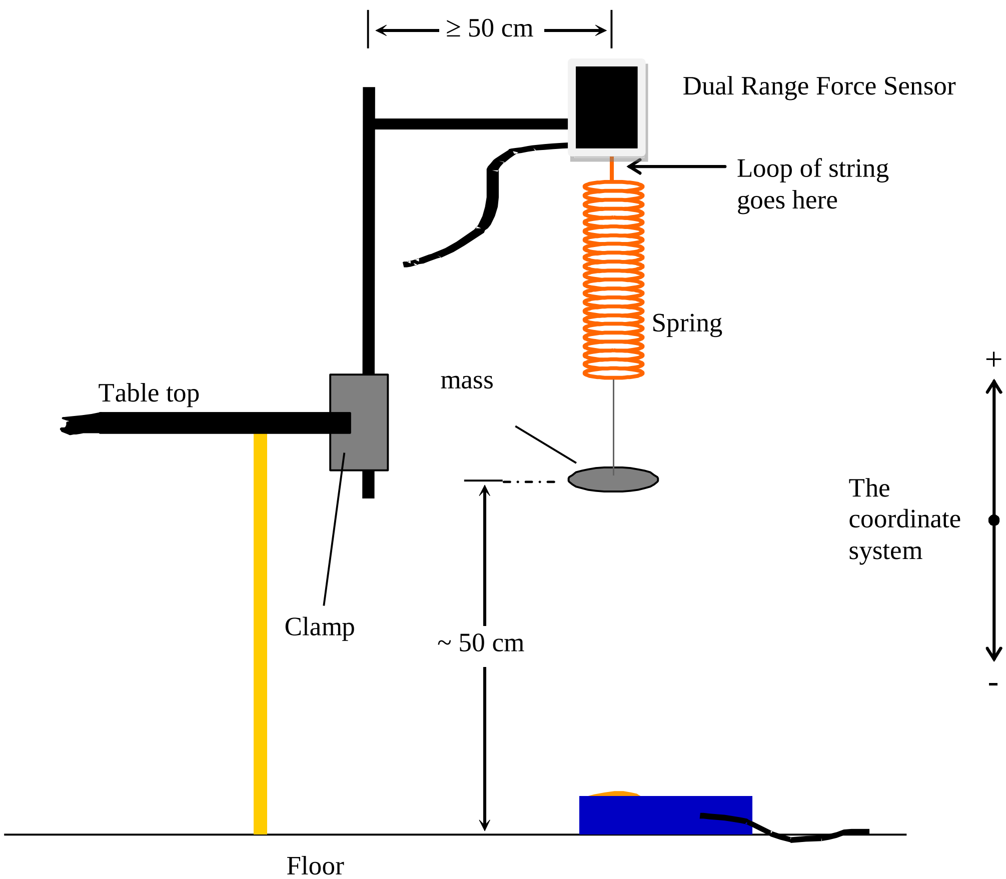

Figure 1: Experimental setup to study oscillating mass plus spring systems

0. Introduction

Abstract This Introduction contrasts forces that are not constant with the constant forces previously experienced and describes the kind of motion resulting from the non-constant force exerted by a spring.

0.1 *Hooke's Law contrasted with constant forces*

In previous activities, you encountered objects that moved when little or no force acted on them and objects that moved when a constant force acted on them. Those activities illustrated the facts that the speed of an object does not change when the net force acting on that object is zero, and the speed of an object changes with constant acceleration when a constant net force acts on the object.

The activities today involve the motion of an object when a *variable* force – a force which continually changes – acts on an object. The cause of the variable force will be a spring.

The force from an ideal spring is proportional to how much the spring is stretched or compressed; this is *Hooke's Law*. Expressed as a formula:

$$F = -ky$$

The ***minus sign*** is present in the formula because the force is always opposite to the direction the spring is stretched or compressed. For example, if y is an upward displacement, the force is downward.

0.2 *Simple harmonic motion and the formula that describes it*

If you hang a mass from an ideal spring and set the mass in vertical motion, the mass moves up and down in what is known as *simple harmonic motion*, with the vertical position y related to time t by the following.[*]

$$y = A\sin(2\pi f t + \varphi) \qquad \text{or}$$

$$y = A\sin(\omega t + \varphi) \qquad \text{(in which } \omega = 2\pi f\text{)}$$

In the above relations, y is the vertical displacement from the equilibrium position. A is the amplitude of the motion, the maximum distance from the equilibrium position. f is the frequency of oscillation in hertz (1 hertz equals 1 complete up-and-down cycle every second; Hz is the standard abbreviation for hertz). t is the time, and φ is a phase constant. ω, as you can see from its definition, is derived from f. ω is called the *angular frequency* and is measured in radians per second. You will get experience with A and f in the activities in this handout. The phase constant φ does not affect the nature of the motion and will not be of interest. ω will be ignored in favor of the equivalent and intuitively more meaningful oscillation frequency f.

Whereas f is the number of complete up-and-down cycles every second, $T = 1/f$ is the number of seconds for one complete up-and-down cycle. T is called the period of the vibration.

10 cm

Example If a mass on a spring has 4 complete up-and-down cycles every second, the frequency is $f = 4$ Hz and the period is $T = 1/4$ s $= 0.25$ s.

[*] Often one sees the cosine function used instead of the sine function: $y = A\cos(\omega t + \varphi)$, for example. That the same motion is described whether sine or cosine is used can be seen from the fact that the graphs of sine and cosine are identical except that the cosine graph is shifted to the left by 90° compared to the sine graph. Logger Pro provides a fit to simple harmonic motion data using the sine function but not using the cosine function, so this handout uses the sine function in order to be consistent with Logger Pro.

0.3 *Frequency is related to mass m and spring constant k*

Using the expression $y = A\sin(2\pi f t + \varphi)$ for the displacement y of a mass m oscillating at the end of a spring with spring constant k, it is possible to show (this is most easily done using calculus) that there should be the following relation between f, k, and m.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

By squaring both sides of this relation and rearranging the terms, one can obtain the following.

$$(4\pi^2 f^2) = k \cdot \left(\frac{1}{m}\right)$$

Comparing the above with the slope-intercept form of a straight line,

$$y = m \cdot x + b,$$

one sees that plotting $y = (4\pi^2 f^2)$ on the y-axis versus $x = (1/m)$ on the x-axis should yield a straight line with slope k, the spring constant, and intercept $b = 0$. This is the content of Activity #4.

1. Activity #1: The nature of the force exerted by a stretched spring

Abstract Get a feel for spring force and the kind of motion it produces.

The system under investigation today is a simple mass hanging from a spring. The apparatus is shown in Figure 1.

1.1 Add a 200 g mass to the 50 g mass

1.2 The bar in the right margin of page 3 shows you how big 10 cm is. Set the mass vibrating up and down by raising the mass upward about 10 cm, holding it motionless, and then letting go. This way of putting the mass in motion ensures that the mass will not fall off the 50 g mass and do damage to the Motion Detector or your foot.

Q 1 Describe the motion of the mass.

When you release the mass, the mass begins to fall. The spring brings the mass to a stop and pulls it back up to its original starting position, whereupon the mass descends again. This repeats over and over for a long period of time. Now clap your hands each time the mass reaches the lowest point in its movement. You should notice that the time interval between claps is unchanging. This is periodic motion, i.e. the same motion occurs over and over again. A mass on a spring is just one example of a system that periodically vibrates or oscillates. Children swinging, whistles, and grandfather clocks are other examples.

The force applied by the spring depends on how much it is stretched or compressed. Remove the mass from the 50 g mass, and grab the bottom of the 50 g mass. Slowly pull down while noting the strength and direction of the force you feel from the spring. Please be careful not to pull so hard that the spring gets permanently deformed or the Force Sensor is damaged.

Q 2 Describe the force you feel as a result of pulling the spring.

It is this kind of a force that leads to simple harmonic motion, described by the quantitative relations in the *Introduction*.

2. Activity #2: Force constant of a spring.

Abstract Determine how strong a spring is by measuring the spring constant *k* as the slope of the graph of *Force* versus *Displacement*, a direct application of Hooke's Law.

2.1 Suspend a 50 g mass from the bottom of the spring.

2.2 With the mass ***but nothing else*** suspended from the spring, ensure the 50 g mass is motionless, and the spring is not quivering.

2.3 ***Caution*** *To accurately determine the position of the 50 g mass it is important that the spring and mass be motionless, but, as you may now have realized, it is impossible to completely eliminate the motion of the spring and mass.*

2.4 Run Logger Pro..

2.5 *Calibrate the motion detector*

2.5.1 First, use the room temperature provided by your lab instructor for the usual Motion Detector calibration.

2.5.2 Second, click the **Zero** button, and zero only the Displacement (by clicking the "Zero Displacement" button). There must be no mass on the 50 g mass, and the spring and the 50 g mass must be as nearly motionless as possible.

2.5.3 Collect data, and verify that you get a straight line at Displacement 0.0 m. If you do, proceed to 2.6. If your straight line *does not fall on the 0.0 m line, continue with the immediately following paragraphs*, paragraph 2.5.4 ff, which perform the calibration manually.

2.5.4 Click the Displacement versus Time window, so that it is selected.

2.5.5 Click **Analyze → Statistics**, and write down the value of **Mean**.
__________ m

2.5.6 Click **Experiment → Set up sensors → Dig/Sonic1**

2.5.7 Locate the value in the input box labeled **SetOffset(m)**, and subtract **Mean** from it. *Example* **Offset(m)** = –1.0779 and **Mean** = 0.003. The calculation is

Offset(m) – **Mean** = (–1.0779) – (0.003) = –1.0809

2.5.8 Enter the result of your calculation into the **SetOffset(m)** input box.

2.5.9 Click **OK**.

2.5.10 Collect data. The Displacement versus Time graph should now fall on the 0.0 m line. If it does not, it is likely that the subtraction was done incorrectly.

2.6 ***Explanation of this Motion Detector Calibration*** With this calibration, the Motion Detector reads displacements relative to the current bottom of the 50 g mass. i.e., the bottom of the 50 g mass defines $y = 0.0$ m. *Down* (toward the Motion Detector) is the negative direction, and *up* is the positive direction.

2.7 *Calibrate the Force Sensor*

2.7.1 Access the Force Sensor calibration window in the usual way, and click **Perform Now**.

2.7.2 Ensure the 50 g mass is motionless, and the spring is not quivering. Then enter 0 for the force (0.0 N). 0 N is correct because the net force on *spring plus 50 g mass* is zero. The net force on the SFS is not zero, but we are calibrating the SFS to read the net force on *spring plus 50 g mass*.

2.7.3 Add 500 grams to the 50 g mass. Ensure the mass is motionless and the spring is not quivering, and enter 4.9 for the force (4.9 N). This is because the 500 gram weight exerts a force of 4.9 N on the spring.

2.7.4 The force sensor is now calibrated to read *force applied to the spring plus the 50 g mass*. Note that we are treating the spring and the 50 g mass as if they were a single object.

2.8 *Explanation of the coordinate system used in this experiment*

2.8.1 When you place a weight on the 50 g mass, the mass goes down (a negative displacement, according to the Motion Detector). Simultaneously, the spring exerts an upward force on the 50 g mass. This force is recorded as positive by the SFS. In agreement with $F = -ky$, negative displacements of the 50 g mass correspond to positive spring forces on the 50 g mass.

2.8.2 ***Zero displacement*** and ***zero force*** is when no EXTRA weight is applied to the 50 g mass.

2.9 *Verify the calibrations are all correct*

2.9.1 With the mass as nearly motionless as possible, click **COLLECT,** and wait until the time runs out. The graphs of Displacement versus time and of Force versus time should both be horizontal lines very near to zero.

2.9.2 Use the **Analyze → Statistics** option on the Displacement versus Time plot. The value MEAN should be within about 0.004 m (4 mm) of 0.0 m. If not, ask your lab instructor if you need to retake the reading.

2.9.3 Using the **Analyze → Statistics** option on the Force versus Time plot should show a net force on the spring plus 50 g mass of very nearly 0.0 N.

2.9.4 Place a 200 g mass on the mass. Click **COLLECT**, and use the **Analyze-Statistics** option on the Force graph to determine the force on the spring plus 50 g mass. The value should be near 1.96 N.
Write the value you obtained here. __________ N

2.9.5 Remove the 200 g weight, but leave the mass attached to the spring.

2.10 *Get Force versus Displacement data*

2.10.1 Put a 50 g slotted mass on the 50 g mass, collect data, and use the **Analyze → Statistics** option to determine the Displacement of the 50 g slotted mass and the spring force in response to the addition of the 50 g slotted mass. Enter the results into ***Column 2*** and ***Column 3*** in the 50 g row of Table 1.

2.10.2 Repeat 2.10.1 for the remaining masses in Table 1.

Column 1 *Additional Mass* *(in addition to 50 g mass)*		***Column 2*** *Displacement* *(due to Additional Mass: subtract 0 g displacement)*	***Column 3*** *Force* *(due to Additional Mass)*
(grams)	(kg)	(meters, negative)	(newtons, positive)
0	0.000	0.000	0.00
50	0.050		
100	0.100		
150	0.150		
200	0.200		
250	0.250		

Table 1

2.11 *Determine the spring constant of the spring*

2.11.1 Using LoggerPro, make a graph of *Spring Force* (***Column 3***, positive) versus ***Distance*** *Spring Stretched* (the **positive magnitude** of the Displacement in ***Column 2***) from the data in Table 1. Arrange the columns in the spreadsheet as in Table 1.

2.11.2 Right-click a data point, and use the **Curve fit** option to add a straight line fit *with intercept zero* to the graph and to display the equation of the fit on the graph. Use the manual fit option, set b = 1, and adjust the slope to minimize RMSE(root mean square error).

2.11.3 The slope of the line is the spring constant, k, of the spring.
Record the value of the spring constant here. k = _______ N/m

2.12 Also **Record** the value of the spring constant k as the value k_2 on the first page of Part II of this lab.

2.13 Print one copy of the LoggerPro graph for each member of your group.

Q 3 If the spring stretches from 0.15 m to 0.35 m, by how much does the force exerted by the spring increase? Show how you obtained your answer.

3. Activity #3: Introduction to Simple Harmonic Motion.

Abstract Analyze the motion of a mass on a spring qualitatively and quantitatively.

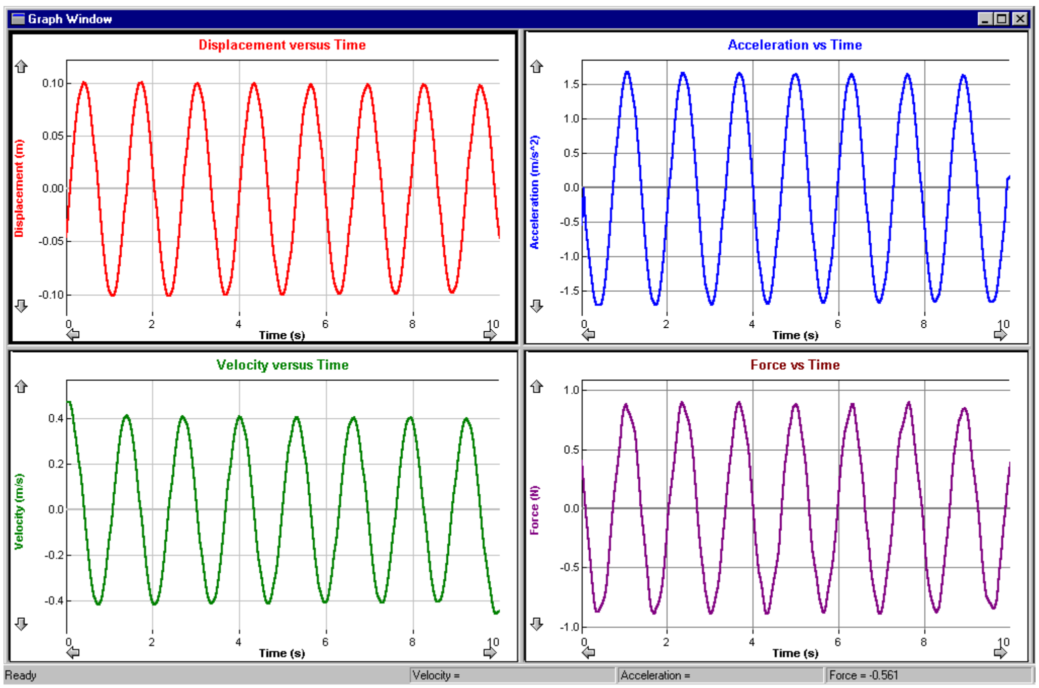

Figure 2 Sample data for a 250 g oscillating mass.

3.1 Now you are to collect data for an oscillating mass. The screen should show four panes – the top left displaying Displacement versus Time, the bottom left displaying Velocity versus Time, the top right displaying Acceleration versus Time, and the bottom right displaying Force versus Time. See Figure 2.

3.2 Place a 250 g mass on the 50 g mass. After making sure the spring and mass are as nearly motionless as possible, click the **Zero** button, and choose the **Zero All Sensors** option. By doing this you have directed Logger Pro to measure displacements with respect to the equilibrium position at which the 250 g mass hangs and to measure the net force (including gravity) on the 250 g mass plus spring.

3.3 Raise the mass ***no more that 10 cm***, and carefully release it. The mass should move smoothly up and down, with no side-to-side motion, and the spring should not be quivering. After a few oscillations have passed, begin collecting data. Sample data is shown in Figure 2.

3.4 Fit the Displacement versus Time curve to a sine function using **Analyze → Curve Fit… → Sine**.

3.5 Print one copy of your Logger Pro screen for each member of your group. The printout must show the result of the curve fit.

3.6 *Print a copy of the Displacement versus Time graph for use with Part II*

You need to do this only if you are assigned to do Part II of this lab.

3.6.1 Click on the Displacement versus Time graph to make it active.

3.6.2 Click the printer icon.

3.6.3 Click **Yes**.

3.6.4 Print one copy of the Displacement versus Time graph for each member of your group. Save the printout for when you do Part II (if you do).

3.7 *Interpreting your curve fit*

3.7.1 Logger Pro fits your Displacement versus Time data to a function of the following form.

$$\mathbf{y} = \mathbf{A}*\sin(\mathbf{B}*\mathbf{x} + \mathbf{C}) + \mathbf{D}$$

3.7.2 The formula from the *Introduction* that describes the motion of the mass on the spring is the following.

$$y = A\sin(2\pi f t + \varphi)$$

3.7.3 From comparing these two functions, you can obtain the following.

- **y** corresponds to the vertical displacement of the mass from the equilibrium position, y.
- **x** corresponds to the time t.
- The constant **A** corresponds to A, the amplitude of the oscillation. The amplitude is the maximum positive displacement from the equilibrium position, and its units are meters.
- The constant **B** corresponds to $2\pi f$, $\mathbf{B} = 2\pi f$. Thus, you can calculate the frequency of the oscillation f from the relation $f = \mathbf{B}/2\pi$. (**B** is also known as the angular frequency of the oscillation, in radians per second, and is represented by the symbol $\omega = 2\pi f$.)
- **B** is in radians per second. Then $f = \mathbf{B}/2\pi$ is in vibrations per second (or Hz, for hertz).
- The constant **C** is called the phase constant and is usually known as φ. It determines the initial displacement of the mass at time $t = 0$ s. The initial displacement of the mass at time $t = 0$ s depends on exactly when you click the **Start** button and is not particularly interesting.

- The constant **D** is the distance from the equilibrium position of the mass to the origin of the Motion Sensor's coordinate system. Because you chose the **Zero Motion Detector** option in step 3.1, **D** should be very close to zero.

Q 4 Determine the frequency, f, of the oscillation, in hertz (1 hertz = 1 vibration per second).

3.8 You will find Logger Pro's **Analyze → Examine** option helpful when answering the following questions.

Q 5 Refer to your Logger Pro graph in responding to this question. Qualitatively (in words; no numbers or formulae) describe the relation between Displacement and velocity. For example, where in the full cycle is the mass when the velocity is zero?

Q 6 Refer to your Logger Pro graph in responding to this question. Qualitatively (in words; no numbers or formulae) describe the relationship between the force and the acceleration.

Q 7 What physics theory (a famous equation!) explains the relationship between the force and the acceleration?

Q 8 Refer to your Logger Pro graph in responding to this question. Is the speed of the mass a maximum or a minimum when the mass is at the equilibrium position?

Q 9 Refer to your Logger Pro graph in responding to this question. Qualitatively (in words; no numbers or formulae) describe the relationship between the displacement and the acceleration.

Q 10 Refer to your Logger Pro graph in responding to this question. Qualitatively (in words; no numbers or formulae) describe the relationship between velocity and force.

Q 11 Refer to your Logger Pro graph in responding to this question. Determine the amplitude A of the oscillation in the graph of Displacement versus Time. Please circle the region on the printed graph that gave you this information.

3.9 **Record** the value of the amplitude A in the appropriate space on the first page of Part II of this lab.

3.10 Finally, determine the time Δt between measurements in the Logger Pro graphs, and **Record** the value in the appropriate space on the first page of Part II of this lab. To determine Δt, from the main menu: **Setup → Data Collection → Sampling** tab; then look for the value of **seconds/sample**. Δt will be a small value, something like 0.05 seconds.

3.11 If you do not have time to complete Activity #4, proceed immediately to section 0, on page 13.

4. Activity #4: Determine how the angular frequency depends on the mass

Abstract Determine how strong a spring is by measuring the relationship between frequency f, mass m, and spring constant k, an application of the theoretical prediction described in section 0.3.

4.1 Weigh your spring on the digital scales.

4.1.1 Write the mass of the spring here. M_{spring} = ______ kg

4.1.2 Also **Record** the value of M_{spring} in the appropriate blank space on the first page of Part II of this lab.

4.2 Use the results from Activity #3 to fill in the row for 250 g in Table 2.

4.3 Complete Table 2 by repeating 3.1 - 3.4 and *Q 4* for 50 g, 100 g, 150 g, and 200 g.

Explanation For the next analysis, it is necessary that the mass m take into account everything in motion when the added mass, the 50 g mass, and the spring oscillate up and down. That is why to get the values of m in the $M_{effective}$-column of Table 2 you sum the added mass, the spring mass, and the mass of the 50 g mass (0.05 kg). However, the spring is special because one of its ends – the upper end – does not move at all, while the other end – the lower end – moves just as much as the 50 g mass and additional mass; and, in between its ends, different sections of the spring move in intermediate amounts. For reasons that go beyond what can be explained here, the correct way to take into account the kind of motion that the spring has is to include – not the spring's entire mass – but only one-third of the spring's mass. That is why one-third of the spring mass is summed along with the additional mass and the mass of the 50 g mass to get $M_{effective}$, the *total effective mass* that is in motion during the oscillations.

M_{weight} *(mass in addition to 50 g)*		$M_{effective} = M_{weight} + \frac{1}{3} M_{\text{spring}} + 0.05$	f
(grams)	(kilograms)	(kilograms)	(hertz)
50	0.050		
100	0.100		
150	0.150		
200	0.200		
250	0.250		

Table 2

4.4 *Determine the spring constant k from the frequency data*

M_{Weight} (mass in addition to 50 g)	$M_{effective}$	f	$1/M_{effective}$ (on the x-axis)	$4\pi^2 f^2$ (on the y-axis)
(kilograms)	(kilograms)	(hertz)	(kilograms^{-1})	(seconds^2)
0.050				
0.100				
0.150				
0.200				
0.250				

Table 3 The form of the spreadsheet for determining the spring constant in Activity #4

4.4.1 In LoggerPro, make a table like that in Table 3, using spreadsheet formulae to fill in the rightmost two columns. The best way to get a value of π is by using the parameters box in New calculated column. E.g., $4\pi^2 f^2$ is written 4*pi^2*f (assuming labeled the frequency f).

4.4.2 Make a graph of $4\pi^2 f^2$ (on the y-axis) versus $1/M_{effective}$ (on the x-axis). Putting the graph on the same page as the data table saves paper when printing.

4.4.3 Add a linear fit with intercept set to zero and with the equation of the fit displayed on the graph (select Curve fit, then choose proportional.).

4.4.4 If necessary, edit the equation of fit to obtain at least three significant digits.

4.4.5 Write the spring constant here. k = _____ N/m

4.4.6 Print a copy of the spreadsheet for everybody in your group.

4.5 Also **Record** the value of the spring constant k as the value k_4 on the first page of Part II of this lab.

4.6 You have determined the spring constant for your spring statically (in Activity #2, where there was no motion and you used Hooke's Law) and dynamically (in Activity #4, where the spring and masses were in oscillation and you used the theoretical relation between f, m, and k.). Compare your two values of k by completing Table 4.

k_2, from Activity #2 (2.11.3)	k_2 = _____ N/m
k_4, from Activity #4 (4.4.5)	k_4 = _____ N/m
$k = (k_2+k_4)/2$, the average of k_2 and k_4	k = _____ N/m
$\Delta k = k_4 - k_2$, the discrepancy	Δk = _____ N/m
The discrepancy as a percent: $\Delta k/k \times 100$	_____ %

Table 4

If the discrepancy $\Delta k/k$ is more than a 3% or 5%, consult your lab instructor.

5. When you are done with Part I ...

5.1 Hand in the following.

5.1.1 Part I of this handout, with questions answered and blanks filled in

5.1.2 Your LoggerPro graph of magnitude of force versus magnitude of displacement from Activity #2 (see 2.13)

5.1.3 One of the copies of your Logger Pro graph from Activity #3 (see 3.5)

5.1.4 The LoggerPro plot from Activity #4 (see 4.4.6)

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Pre-Lab Questions

Print *Your Name*

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Read the *Introduction* to this handout, and answer the following questions before you come to General Physics Lab. Write your answers directly on this page. When you enter the lab, tear off this page and hand it in.

1. Draw a line about 10 cm long in the box.

2. Write Hooke's Law as a formula and explain it in words. In particular, explain the minus sign.

3. Write two general equations: (a) the kinematic equation (refer to your physics text) that specifies the height y of a falling object in terms of the time since it started falling, and (b) the equation that specifies the height y of a mass on a spring in terms of the time since it started oscillating.

4. Write the equation that relates f, m, and k.

5. Refer to Figure 1 on page 2 and paragraph 2.8 on page 6. Explain the coordinate system used in Activity #2. Which is the positive direction for displacement, and which is the positive direction for the force of the spring on the mass?

6. In Activity #2, if you pull down on the SFS sensor arm, the SFS produces a positive reading. How can one justify the statement that the SFS takes *up* to be the positive direction for the force the spring applies to the mass?

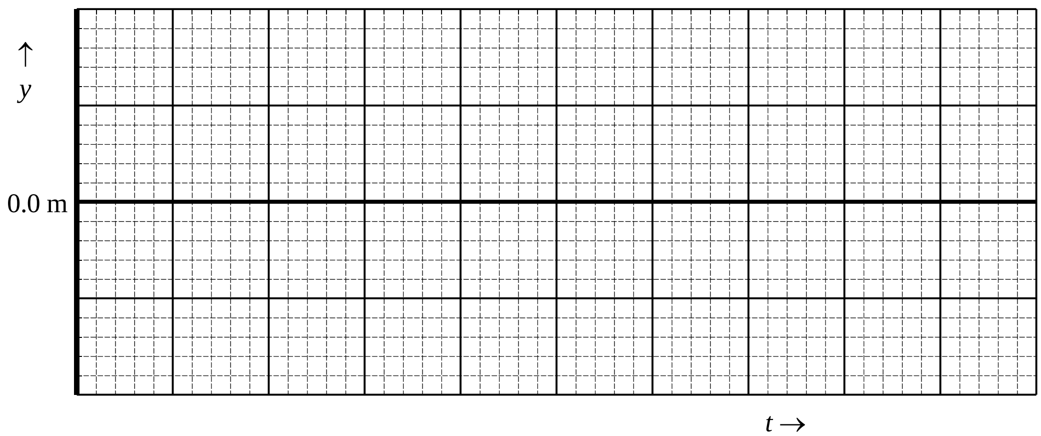
7. A mass on a spring bobs up and down, completing 2 full cycles every second. What are the period and frequency of the mass? (The correct answer uses Newton's Third Law but *not* $F = -ky$.)

8. Refer to section 3.7 on page 9. Write the formula used to calculate the frequency of oscillation f from the value of the constant **B**.

9. Again refer to section 3.7 on page 9. What are the units of **B** and of f?

10. Once more, refer to section 3.7 on page 9. Why is the value of **C** = φ not interesting?

11. Visualize a mass bobbing up and down at the end of a spring, and draw a qualitatively correct graph of the displacement y of the mass as a function of time t. Assume the motion of the mass is centered on 0.0 m.



12. For the mass in the previous problem, carefully draw a graph of the velocity of the mass versus time. Your lab instructor must be able to see, by comparing the graph in this problem with the graph in the previous problem, at what heights the mass comes to a stop.