

Lab 2: Vector Analysis

Objectives:

- to practice using graphical and analytical methods to add vectors in two dimensions

Equipment:

- Meter stick
- Ruler
- Protractor
- Force table
- Ring
- Pulleys with attachments
- String
- Hangers
- Weights
- Scale

If you are not familiar with graphical and analytical vector addition methods, read the Appendix to this lab first.

Exploration 1 Displacement

A bug starting at the corner of your lab table, walking along the edge of the table, makes its way along the length of the table, turns 135° , and walks 0.30m further and then stops. Take the bug's initial direction as the +x direction.

Exploration 1.1 You wish to determine the displacement vector from the origin (where the bug started) to the point where the bug stops. Represent each leg of the bug's journey by a vector. And add the two vectors: 1) graphically, using an appropriately scaled diagram, and 2) analytically, adding vector components. Report the magnitude and direction of the total vector displacement from the beginning to the end of the bug's journey. Do your two solutions agree? Is there any uncertainty in either of the solution methods? Estimate the uncertainty, if there is an uncertainty. Explain how you estimated the uncertainty. Show your work in the space on the next page.

Check your methods, answer and uncertainty estimates with your TA before continuing.

Exploration 2 Force

Force is a vector quantity. An object will remain at rest or, if the object is in motion, moving at constant velocity, if the vector sum of all the forces acting on it is zero. We will study a ring at rest in the center of a force table. In order for the ring to remain at rest, the net force (vector sum of all forces acting on the ring) must be zero. Examine the force table on your lab table and how the forces are applied. The magnitude of the force applied by each hanging mass is $F = mg$ where m is the sum of the hanging mass and the mass of the hanger in kilograms and $g = 9.8\text{m/s}^2$.

Exploration 2.1.a Suppose a force of magnitude $F_1 = 0.98\text{ N}$ is acting at 0° and another force of magnitude $F_2 = 1.96\text{ N}$ is acting at 90° on the disk. Find the vector sum of \mathbf{F}_1 and \mathbf{F}_2 two ways: 1) graphically and 2) analytically. Draw the vectors with the tail of the second at the head of the first for the graphical method. Draw the vectors with the tails together at the origin for the analytical solution. Show your work in the space below.

Check your vector drawings for each case, methods and answer with your TA before continuing.

Exploration 2.1.b Determine the magnitude and direction of a third force \mathbf{F}_3 that would need to be added to create a net force (vector sum of all forces) of zero on the ring.

Investigation 1 Net force zero

You are going to practice adding vectors in order to make the net force on the ring in the center of the force table zero.

Investigation 1.1 Three forces

Investigation 1.1.a Set up two forces \mathbf{F}_1 and \mathbf{F}_2 ($F_1 = F_2$) such that the angle between them is not 90° . Then determine by calculation the magnitude and direction of the third force \mathbf{F}_3 needed so that the net force on the ring is zero. Remember that the mass m is the sum of the hanging mass and the mass of the hanger. Show your work below.

Investigation 1.1.b Add the mass needed to create the third force and test your calculations.

Investigation 2.1 Four forces

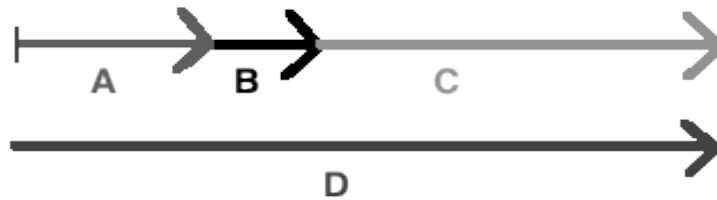
Investigation 2.1.a Set up three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 , such that only two forces are at right angle. Determine the fourth force \mathbf{F}_4 that causes zero net force on the system. Show your vector diagrams and calculations in the space below.

Investigation 2.1.b Add the mass needed to create the third force and test your calculations.

Appendix I

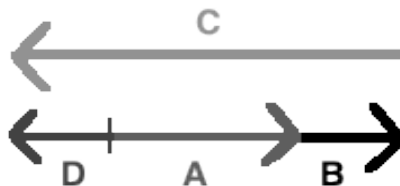
Quantities that can be specified by giving their magnitudes are called **SCALAR** quantities (temperature, length, mass and volume, for example, are scalar quantities). Quantities that require both a magnitude and a direction in their proper description are called **VECTOR** quantities. Examples of vector quantities are velocity, force, and acceleration. The sum of two scalar quantities is simply the mathematical sum of their values. For example, the sum of a volume of 3.0cm^3 and a volume of 5.0cm^3 is a volume of 8.0cm^3 . The mathematical treatment of vector quantities, however, is more complicated as they have both a magnitude and a direction.

Vectors can be represented by arrows. The addition and subtraction of vectors in one dimension is straightforward. If several vectors **A**, **B** and **C** are acting in the same direction, the arrows representing the vectors can be lined up with the tail of the second at the head of the first, the tail of the third at the head of the second, etc., as in the picture below.



Their sum is the vector $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$.

If one of these vectors, say **C**, acts in a direction opposite to **A** and **B**, then the picture would look like



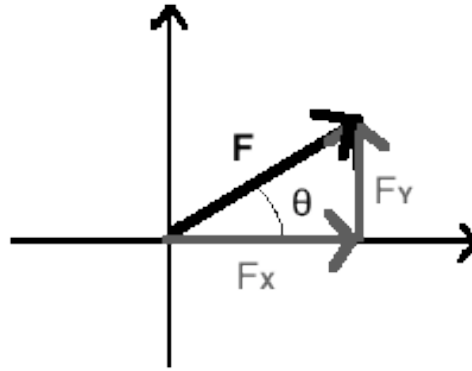
and **C** must be added with the opposite sign. The vector sum would be $\mathbf{D} = \mathbf{A} + \mathbf{B} - \mathbf{C}$.

When the vectors do not all point in the same direction, they must be added either 1) graphically or 2) analytically, as described below.

1. Graphical Method: A vector can be represented by a 'directed line' or an 'arrow'. The magnitude of the vector can be shown by choosing a convenient scale. For example, Fig. 1 shows two forces, $F_1 = 3.0\text{N}$ at 45° and $F_2 = 4.0\text{N}$ at 120° , originating from the same point and the scale is $1.0 \text{ inch} = 1.0\text{N}$. The same procedure can be used to represent a system of several vectors.

Fig. 2 illustrates the graphical method of vector addition of vectors F_1 and F_2 from Fig. 1. Starting at point P, draw the first vector F_1 in its own direction with its length proportional to its magnitude. Then draw the second vector F_2 beginning at the ending of the first vector, in its own direction, with its length proportional to its magnitude. The sum of F_1 and F_2 is the new vector R that is drawn from the beginning of the first vector (point P) to the end of the second vector (point Q). The vector R is called the **RESULTANT** with its magnitude proportional to the length PQ and direction shown by the arrow from the tail of F_1 to the head of F_2 . If three or more vectors act at a single point (Fig. 3), then their vector sum can be found in the same manner (Fig. 4). It is important to note that the magnitude and direction of the resultant R are independent of the order in which the original vectors are drawn (try it). It should be realized that graphical procedures usually introduce errors arising from inaccuracies in drawing various arrows representing the vectors.

2. Analytic Method: The analytic method of adding vectors involves using trigonometry to resolve a vector into its components. For a vector F directed at angle θ from the positive x- axis, the x and y components are $F_x = F\cos\theta$ and $F_y = F\sin\theta$, as shown in the diagram below.



The vector sum can be determined as follows. Let \mathbf{F}_1 and \mathbf{F}_2 represent the vectors to be added. Resolve each vector into its x and y components. Then the components of the resultant vector, \mathbf{R} , can be found by summing the x and y components of the vectors to be added:

$$R_x = F_{1x} + F_{2x} \text{ and } R_y = F_{1y} + F_{2y}$$

The magnitude, R , of the resultant vector is

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

and its direction with respect to +x axis is given by

$$\tan\theta = \frac{R_y}{R_x}; \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

where the vector components in the +x and +y directions have a positive algebraic sign while the components in the -x and -y directions have a negative algebraic sign.

**GRAPHICAL
REPRESENTATION
OF VECTORS
(as discussed in the text)**

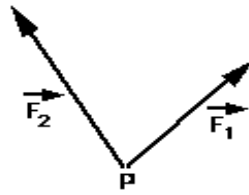
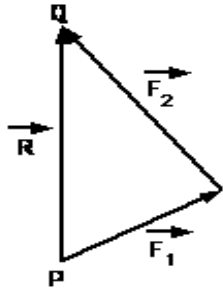


FIGURE 1



OR

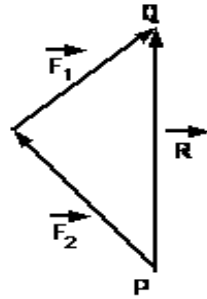


FIGURE 2

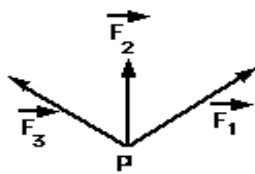


FIGURE 3

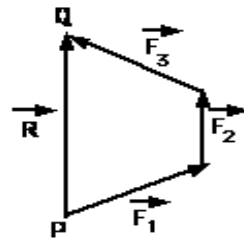


FIGURE 4

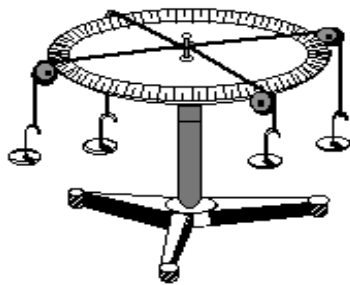


FIGURE 5

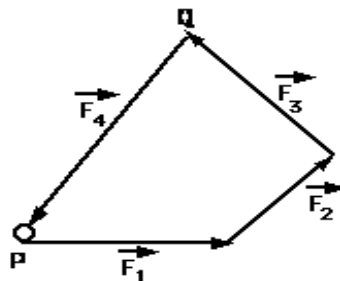


FIGURE 6