LAB 4: PULSE SHAPING AND MATCHED FILTERING

I. OVERVIEW

The objective of this laboratory session is to introduce the basics of pulse shaping and matched filtering designs in digital communication systems. At the transmitter, we focus on pulse shaping; while at the receiver, we focus on matched filtering. Pulse shaping is the process of shaping pulses to be transmitted based on the symbols generated via modulation (Lab 3). The goal is to make the signal suitable to be transmitted through the communication channel mainly by limiting its effective bandwidth.

In the first part of this lab, we build a pulse shaping virtual instrument (VI) that is capable of shaping pulses by lowpass filtering. In the second part, we build a matched filter to the pulse shape to maximize the signal to noise ratio before symbol detection.

A general block diagram of a transmitter is given in Fig. 1 below:

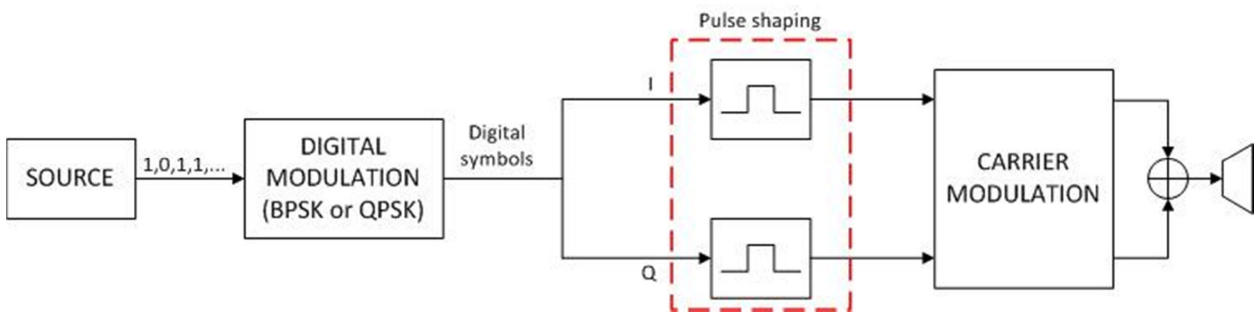

Fig. 1 – Transmitter block diagram

A general block diagram of a receiver is given in Fig. 2 below:

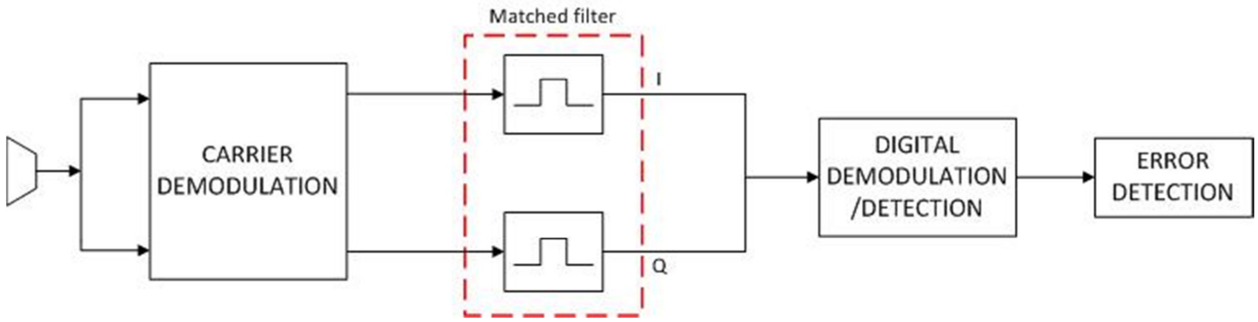

Fig. 2 – Receiver block diagram

In this lab session we implement the Pulse shaping part at the transmitter and the Matched filter part at the receiver.

PART 1: PULSE SHAPING

In communications, digital signals need to be mapped to an analog waveform in order to be transmitted over the channel. The mapping process is accomplished in two steps: (i) Mapping from source bits to complex symbols (also known as constellation points), which we learned about in Lab 3 – Part 1: Modulation. (ii) Mapping from complex symbols to analog pulse trains, which is studied in this part.

In our pulse shaping scheme, we introduce a set of complex-valued symbols as the input. These symbols are mapped from a bit stream via digital modulation (Lab 3 – Part 1). The bit stream may represent any data format (e.g., text, image, voice, video, etc). These complex-valued symbols are passed through the pulse shaping filter. A representation of this process is shown in Fig. 3 below.

Fig. 3 – A representation of the pulse shaping process

Oversampling:

Oversampling is the process of sampling a signal with a significantly higher sampling frequency than indicated by the Nyquist-Shannon sampling theorem. This theorem states that if the highest frequency of a function is *B* Hertz, the signal can be perfectly reconstructed from samples taken at time intervals equal to or less than $\frac{1}{2B}$. Time domain and frequency domain representations of oversampling are provided in Fig. 4 below:

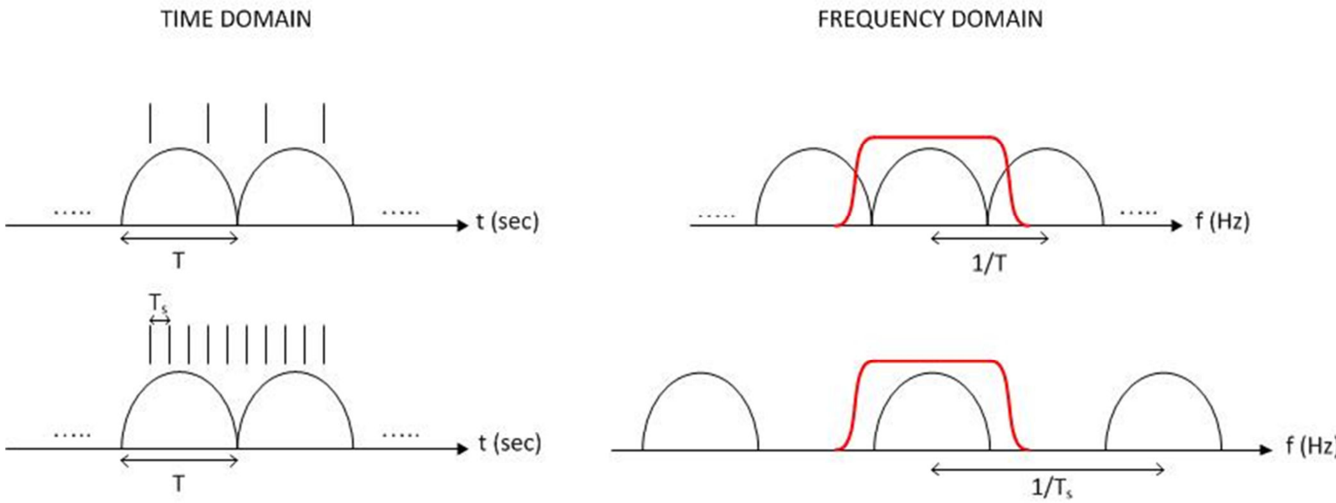

Fig. 4 - Time domain and frequency domain representations of oversampling

Sampling generates a periodic spectrum. However, of interest is only the fundamental spectrum at baseband. Oversampling increases the repetition interval of the spectra, facilitating the filtering of the undesired mirror images of the spectrum.

Pulse shaping:

Pulse shaping filter must be chosen carefully not to introduce intersymbol interference. Some of the commonly used pulse shaping filters are listed below:

- **(i)** **Rectangular pulse shape:** This pulse shape has poor spectral properties with high sidelobes.
- **(ii)** **Sinc pulse shape:** Theoretically, the sinc filter has ideal spectral properties, as the Fourier transform of a sinc function is an ideal lowpass spectrum. However, a sinc pulse is non-causal, hence not realizable.
- **(iii)** **Raised-cosine pulse:** This is a pulse widely used in practice. The pulse shape and the excess bandwidth can be controlled by changing the roll-off factor (0≤α≤1, where 0 means no excess bandwidth, and 1 means maximum excess bandwidth). The frequency-domain expression of raised-cosine filter is given in (1.1):

$$G_{rc}(f)=\begin{cases} T, & 0\le |f| \le \frac{(1-\alpha)}{2T} \\ \frac{T}{2}\left[1+\cos\frac{\pi T}{\alpha}\left(|f|-\frac{1-\alpha}{2T}\right)\right], & \frac{(1-\alpha)}{2T}\le\frac{(1+\alpha)}{2T} \\ 0, & |f|>\frac{(1+\alpha)}{2T} \end{cases} \qquad (1.1)$$

The frequency responses of raised-cosine pulses with different roll-off factors are shown in Fig. 5 below:

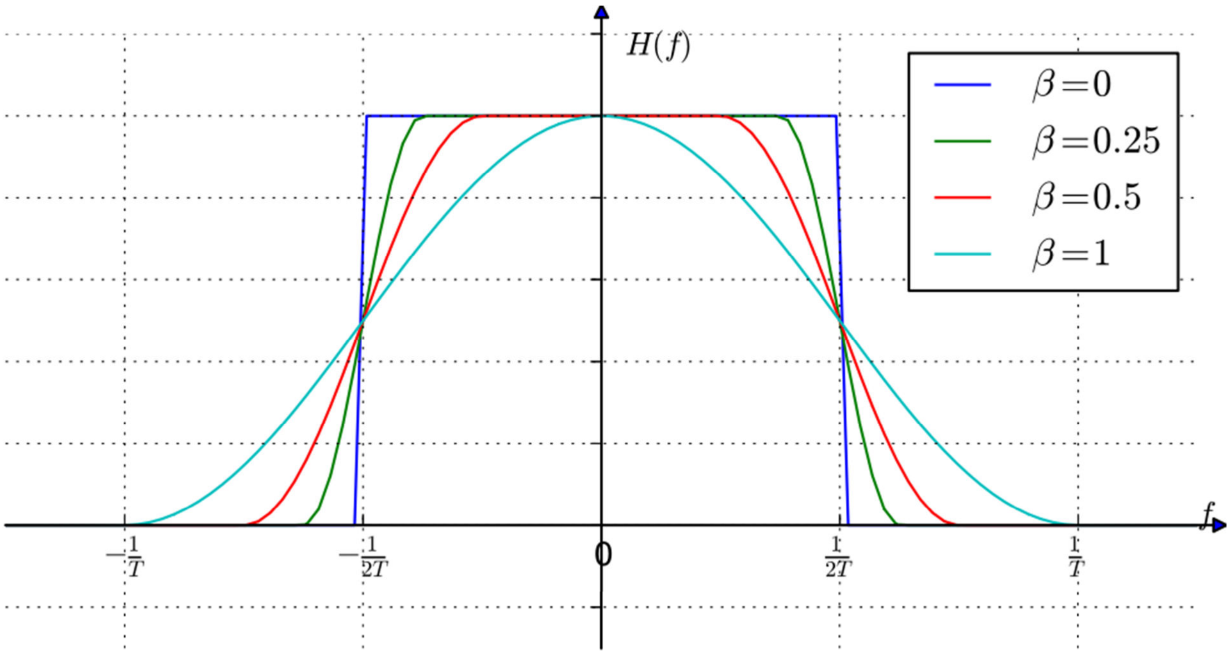

Fig. 5 – Frequency response of a raised-cosine filter with different roll-off factors (denoted as β) [1]

(iv) **Root raised-cosine pulse (RRC):** The total effective filter of the transmission system is the combination of transmit and receive filter $g_{TX} * g_{RX}$, where $*$ is convolution. This effective filter (and not the individual filters) must fulfill the Nyquist criterion. We can achieve this goal if both filters have a transfer function that is equal to the square root of that of the raised cosine filter. Such a filter is therefore called a root raised cosine (RRC). The combination of both RRC filters then becomes a raised cosine and thus fulfills the Nyquist criterion. Furthermore, since the filters are real-valued and symmetric, the RRC is its own matched filter [2]. The impulse response of the RRC filter is given in (1.2):

$$g_{RRC}(t) = \frac{\frac{4\alpha}{\pi}\cos\left(\pi(1+\alpha)\frac{t}{T}\right) + (1-\alpha)\sin\left(\pi(1-\alpha)\frac{t}{T}\right)}{1-\left(4\alpha\frac{t}{T}\right)^2} \qquad (1.2)$$

(v) **Gaussian pulse:** The impulse response of this filter is a Gaussian function. Gaussian pulses have good spectral properties (low spectral sidelobes).

PART 2: MATCHED FILTERING

The receiver's RF front-end receives analog pulse trains. The information bits need to be recovered from these pulse trains. This is accomplished in two steps: (i) Mapping from analog pulse trains to constellation points, which is studied in this part. (ii) Mapping from complex symbols to bits, which we learned about in Lab 3 – Part 2: Detection.

The received analog signals are matched filtered to create the output complex waveform. Then the samples are detected (Lab 3 – Part 2). A representation of this process is shown in Fig. 6 below.

Fig. 6 – A representation of the matched filtering process

Matched filter design:

A matched filter maximizes the signal to noise ratio (SNR) at its output. Characteristic of the matched filter at the receiver should be complex conjugate of the one at the transmitter in order to fulfill Nyquist criteria.

If an RRC filter used at the transmitter, the same filter can be used as it is in the receiver since RRC filter is its own matched filter (as explained earlier).

[1] Figure reference: http://en.wikipedia.org/wiki/Raised-cosine_filter (Last accessed: February 2013).
[2] http://www.nari.ee.ethz.ch/commth/teaching/rwc/handouts/labmanual.pdf (Last accessed: March 2013).