# Laboratory #5:  RF Filter Design

## I.        OBJECTIVES

A.  Design a third order low-pass Chebyshev filter with a cutoff frequency of 330 MHz and 3 dB ripple with equal terminations of 50 $\Omega$ using:
    (a)  discrete components (pick reasonable values for the capacitors)
    (b)  What is the SWR of the filter in the passband (pick 100 MHz and 330 MHz)?

B.  Design a third order high-pass Chebyshev filter with a cutoff frequency of 330 MHz and 3 dB ripple with equal terminations of 50 $\Omega$ using:
    (a)  discrete components (pick reasonable values for the capacitors)
    (b)  What is the SWR of the filter in the passband (pick 330 MHz and 500 MHz)?
    (c)  What does the SWR do outside the passband?

## II.       INTRODUCTION

Signal filtering is often central to the design of many communication subsystems.  The isolation or elimination of information contained in frequency ranges is of critical importance.  In simple amplitude modulation (AM) radio receivers, for example, the user selects one radio station using a bandpass filter techniques.  Other radio stations occupying frequencies close to the selected radio station are eliminated.

In electronic circuits, active filter concepts using OpAmps were introduced.  One of the advantages of using active filters included the addition of some gain.  However, due to their limited gain-bandwidth product, active filters using OpAmps see little use in communication system design where the operational frequencies are orders of magnitude higher than the audio frequency range.

The two types of frequency selective circuit configurations most commonly used in communication systems are the passive *LC* filter (low, high, and bandpass responses) and the tuned amplifier (bandpass response).  *LC* ladder networks are commonly used as building blocks for passive filters at RF.  The values of the inductors and capacitors are varied depending on the type of filter, frequency  specifications, and terminations.   In this laboratory, passive *LC* filters at radio frequencies (RF) will be designed and tested.

Two common low-pass LC filter configurations are shown in Figures 1 (a) and (b).  Each "section" consists of an L-C pair, with each section corresponding to the order of the filter.  Two section (or second order) filters are shown in Figure 1.  Note that the values of the capacitors and inductors changes with varying input and output resistances.  Tabulated "normalized" values for the inductors and capacitors for varying termination ratios are available to the design engineer. The component values in the tables are normalized with respect to the termination ratio and cutoff frequency.  Generic representations of the *LC* low-pass filter are shown in Figure 2.
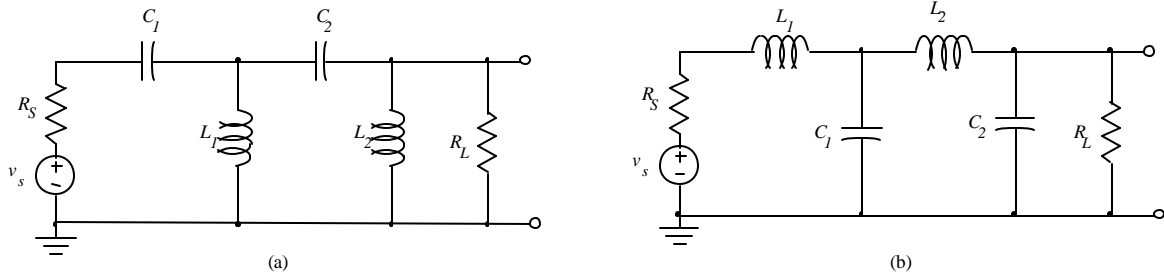
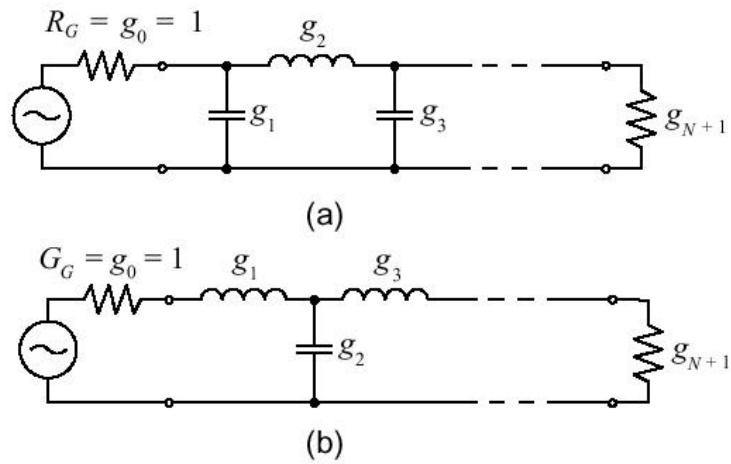Figure 1.  Two Ladder Network Configurations for *LC* Filters



Figure 2.  Two Generic Representations of Figure 1.

The closeness of the impedance match between the source resistance $R_S$ and filter input resistance $R_{fin}$ is frequency expressed as a return loss defined as:

$$A_\rho = 20\log\left|\frac{1}{\rho}\right| \quad . \tag{4}$$

Normalized Butterworth (Maximally Flat), Linear Phase, and Chebyshev *LC* lowpass filters are presented in tabular form on the following pages.  The normalized inductors and capacitors are denormalized using:

$$C = \frac{C_n}{2\pi f_C R} \tag{5}$$

and
$$L = \frac{L_n R}{2\pi f_C} \quad , \tag{6}$$

where   $C_n$ is the normalized capacitor value,

$L_n$ is the normalized inductor value,

$C$ is the denormalized (actual) capacitor value,

$L$ is the denormalized (actual) inductor value,

and     $R$ is the final load resistor.

The Coefficients for the Low-Pass Butterworth, Linear Phase, Chebyshev (3 db ripple) and Chebyshev (0.5 db ripple) filters are shown in Talbes 5-2, 5-3, 5-4(a), and 5-4(b), repsectively.

**Table 5-2**  Coefficients for maximally flat low-pass filter ($N$ = 1 to 10)

| N | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.0000 | 1.0000 | | | | | | | | | |
| 2 | 1.4142 | 1.4142 | 1.0000 | | | | | | | | |
| 3 | 1.0000 | 2.0000 | 1.0000 | 1.0000 | | | | | | | |
| 4 | 0.7654 | 1.8478 | 1.8478 | 0.7654 | 1.0000 | | | | | | |
| 5 | 0.6180 | 1.6180 | 2.0000 | 1.6180 | 0.6180 | 1.0000 | | | | | |
| 6 | 0.5176 | 1.4142 | 1.9318 | 1.9318 | 1.4142 | 0.5176 | 1.0000 | | | | |
| 7 | 0.4450 | 1.2470 | 1.8019 | 2.0000 | 1.8019 | 1.2470 | 0.4450 | 1.0000 | | | |
| 8 | 0.3902 | 1.1111 | 1.6629 | 1.9615 | 1.9615 | 1.6629 | 1.1111 | 0.3902 | 1.0000 | | |
| 9 | 0.3473 | 1.0000 | 1.5321 | 1.8794 | 2.0000 | 1.8794 | 1.5321 | 1.0000 | 0.3473 | 1.0000 | |
| 10 | 0.3129 | 0.9080 | 1.4142 | 1.7820 | 1.9754 | 1.9754 | 1.7820 | 1.4142 | 0.9080 | 0.3129 | 1.0000 |

**Table 5-3**  Coefficients for linear phase low-pass filter ($N$ = 1 to 10).

| N | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.0000 | 1.0000 | | | | | | | | | |
| 2 | 1.5774 | 0.4226 | 1.0000 | | | | | | | | |
| 3 | 1.2550 | 0.5528 | 0.1922 | 1.0000 | | | | | | | |
| 4 | 1.0598 | 0.5116 | 0.3181 | 0.1104 | 1.0000 | | | | | | |
| 5 | 0.9303 | 0.4577 | 0.3312 | 0.2090 | 0.0718 | 1.0000 | | | | | |
| 6 | 0.8377 | 0.4116 | 0.3158 | 0.2364 | 0.1480 | 0.0505 | 1.0000 | | | | |
| 7 | 0.7677 | 0.3744 | 0.2944 | 0.2378 | 0.1778 | 0.1104 | 0.0375 | 1.0000 | | | |
| 8 | 0.7125 | 0.3446 | 0.2735 | 0.2297 | 0.1867 | 0.1387 | 0.0855 | 0.0289 | 1.0000 | | |
| 9 | 0.6678 | 0.3203 | 0.2547 | 0.2184 | 0.1859 | 0.1506 | 0.1111 | 0.0682 | 0.0230 | 1.0000 | |
| 10 | 0.6305 | 0.3002 | 0.2384 | 0.2066 | 0.1808 | 0.1539 | 0.1240 | 0.0911 | 0.0557 | 0.0187 | 1.0000 |

**Table 5-4 (a)**   Chebyshev filter coefficients; 3 dB filter design ($N = 1$ to 10)

| N | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|
| 1 | 1.9953 | 1.0000 | | | | | | | | | |
| 2 | 3.1013 | 0.5339 | 5.8095 | | | | | | | | |
| 3 | 3.3487 | 0.7117 | 3.3487 | 1.0000 | | | | | | | |
| 4 | 3.4389 | 0.7483 | 4.3471 | 0.5920 | 5.8095 | | | | | | |
| 5 | 3.4817 | 0.7618 | 4.5381 | 0.7618 | 3.4817 | 1.0000 | | | | | |
| 6 | 3.5045 | 0.7685 | 4.6061 | 0.7929 | 4.4641 | 0.6033 | 5.8095 | | | | |
| 7 | 3.5182 | 0.7723 | 4.6386 | 0.8039 | 4.6386 | 0.7723 | 3.5182 | 1.0000 | | | |
| 8 | 3.5277 | 0.7745 | 4.6575 | 0.8089 | 4.6990 | 0.8018 | 4.4990 | 0.6073 | 5.8095 | | |
| 9 | 3.5340 | 0.7760 | 4.6692 | 0.8118 | 4.7272 | 0.8118 | 4.6692 | 0.7760 | 3.5340 | 1.0000 | |
| 10 | 3.5384 | 0.7771 | 4.6768 | 0.8136 | 4.7425 | 0.8164 | 4.7260 | 0.8051 | 4.5142 | 0.6091 | 5.8095 |

**Table 5-4 (b)**   Chebyshev filter coefficients; 0.5 dB filter design ($N = 1$ to 10)

| N | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ |
|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|
| 1 | 0.6986 | 1.0000 | | | | | | | | | |
| 2 | 1.4029 | 0.7071 | 1.9841 | | | | | | | | |
| 3 | 1.5963 | 1.0967 | 1.5963 | 1.0000 | | | | | | | |
| 4 | 1.6703 | 1.1926 | 2.3661 | 0.8419 | 1.9841 | | | | | | |
| 5 | 1.7058 | 1.2296 | 2.5408 | 1.2296 | 1.7058 | 1.0000 | | | | | |
| 6 | 1.7254 | 1.2479 | 2.6064 | 1.3137 | 2.4758 | 0.8696 | 1.9841 | | | | |
| 7 | 1.7372 | 1.2583 | 2.6381 | 1.3444 | 2.6381 | 1.2583 | 1.7372 | 1.0000 | | | |
| 8 | 1.7451 | 1.2647 | 2.6564 | 1.3590 | 2.6964 | 1.3389 | 2.5093 | 0.8796 | 1.9841 | | |
| 9 | 1.7504 | 1.2690 | 2.6678 | 1.3673 | 2.7939 | 1.3673 | 2.6678 | 1.2690 | 1.7504 | 1.0000 | |
| 10 | 1.7543 | 1.2721 | 2.6754 | 1.3725 | 2.7392 | 1.3806 | 2.7231 | 1.3485 | 2.5239 | 0.8842 | 1.9841 |

The transformation between Low-Pass and other types of filters, including High-Pass, are provided in tabular form in Figure
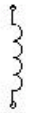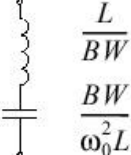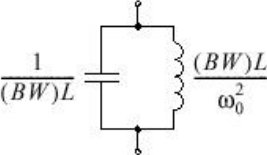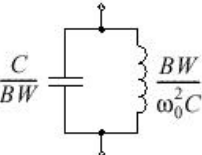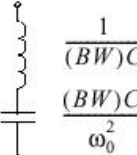
| Low-pass prototype | Low-pass | High-pass | Bandpass | Bandstop |
|---|---|---|---|---|
| $L = g_k$ | $\dfrac{L}{\omega_c}$ | $\dfrac{1}{\omega_c L}$ | $\dfrac{L}{BW}$  $\dfrac{BW}{\omega_0^2 L}$ | $\dfrac{1}{(BW)L}$  $\dfrac{(BW)L}{\omega_0^2}$ |
| $C = g_k$ | $\dfrac{C}{\omega_c}$ | $\dfrac{1}{\omega_c C}$ | $\dfrac{C}{BW}$  $\dfrac{BW}{\omega_0^2 C}$ | $\dfrac{1}{(BW)C}$  $\dfrac{(BW)C}{\omega_0^2}$ |

Figure 3. Transformations to Actual Capacitor and Inductor Values from the Low-Pass Prototype Normalized Component

As is shown in Figure 3 for high pass filter design, the capacitors and inductors are interchanged, and a transformation is applied to the values of the components. The transformed value takes into account the terminations and the cutoff frequency.

### III.    PROCEDURE

A.    *Hand analysis of discrete element filters*
- Design the filters using discrete elements.
- Using a Smith chart, determine the SWR of each filter using hand analysis.

B.    *Modeling of discrete element filters using Agilent ADS*
- Model the circuit using ADS and determine $|S_{11}|$, $|S_{21}|$, and $|S_{22}|$ on grid plots. Also plot $|S_{11}|$ on a Smith chart and compare to the hand analysis.
- In the OUT portion of the file, plan to plot $|S_{11}|$ on a grid and Smith charts.
- Sweep the frequency of operation across the design frequency (to about 900 MHz)
- Repeat the plots of $|S_{11}|$
- Remember to define the circuit as a two-port and terminate with the specified load and a perfect match at the input.

C.    *Prototype the low-pass filter designed*
- Prototype the Chebyshev low-pass filter designed
- Compare experimental results to theory and simulation

D.    *Repeat Procedures (A) through (C) for a Chebyshev High-Pass Filter*

E.    *Comment On Your Results*