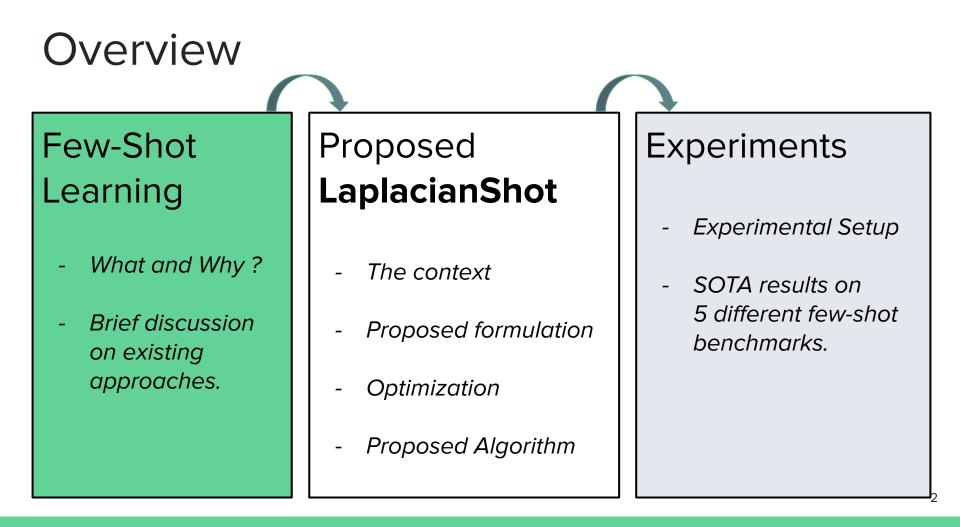


Laplacian Regularized Few Shot Learning (LaplacianShot)

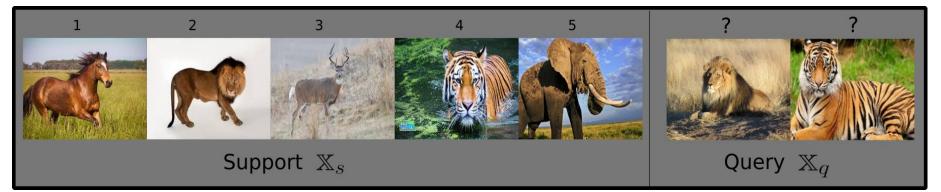
Imtiaz Masud Ziko, Jose Dolz, Eric Granger and Ismail Ben Ayed



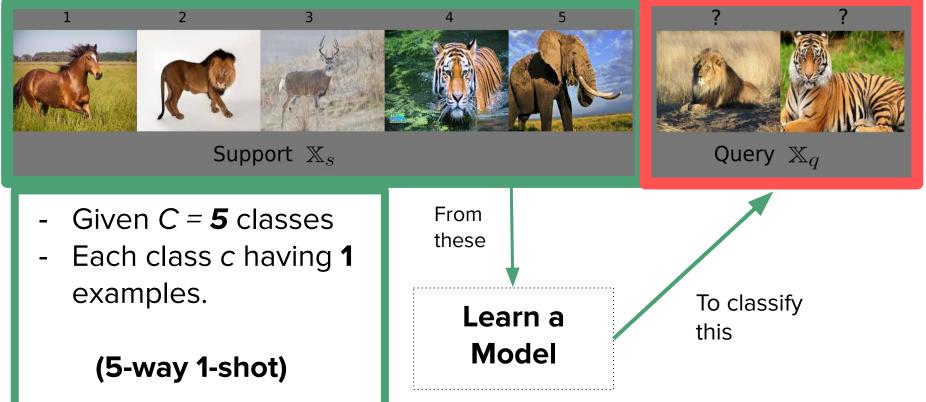
ETS Montreal



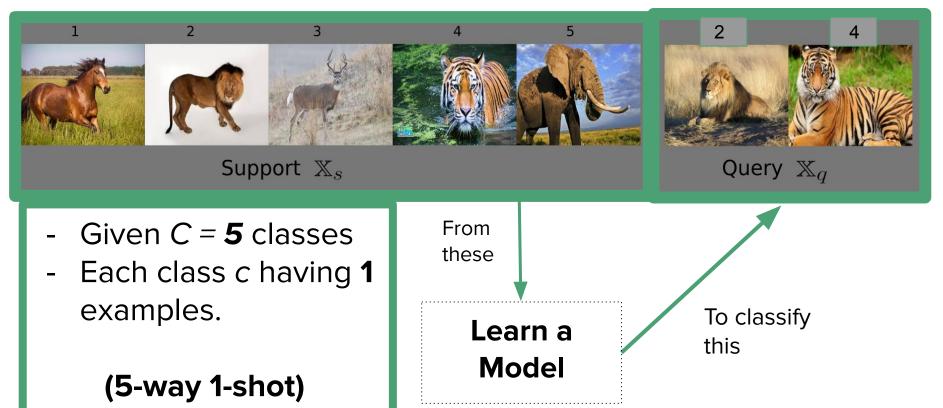
Few-Shot Learning (An example)



Few-Shot Learning (An example)



Few-Shot Learning (An example)



Few-Shot Learning



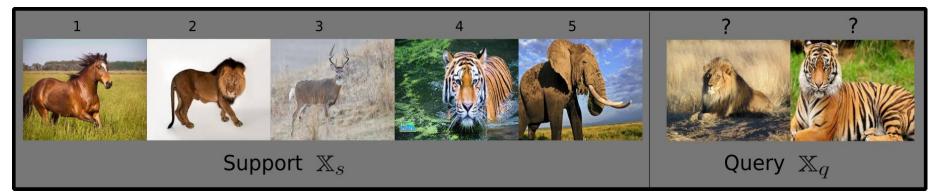
Support X_s

Query \mathbb{X}_q

Humans recognize perfectly with few examples



Few-Shot Learning



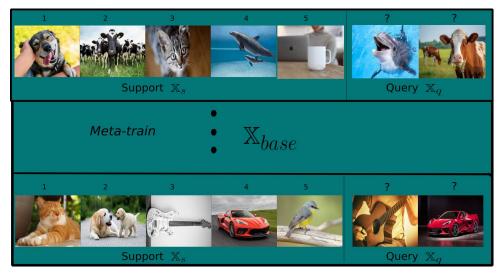
□ Modern ML methods generalize poorly

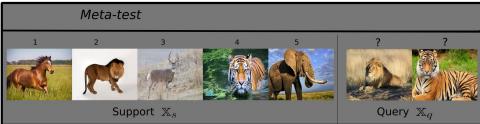
Need a better way.

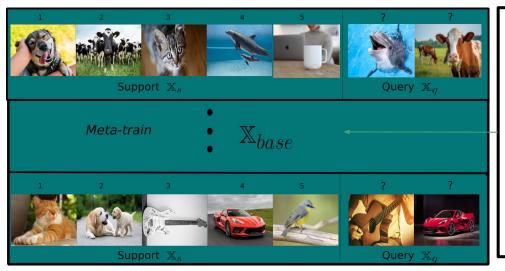
Few-shot learning

A very large body of recent works, mostly based on:

Meta-learning framework

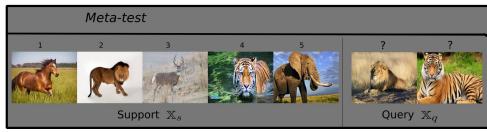


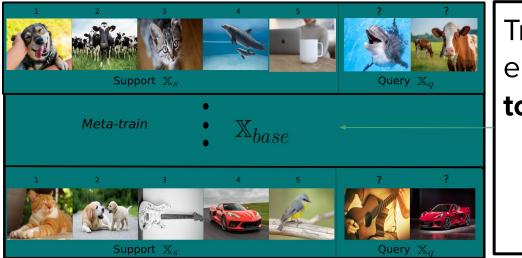




Training set with enough labeled data

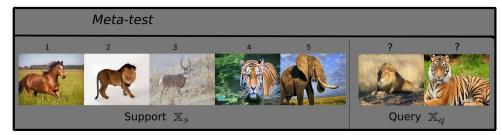
(base classes *different from the* test classes)

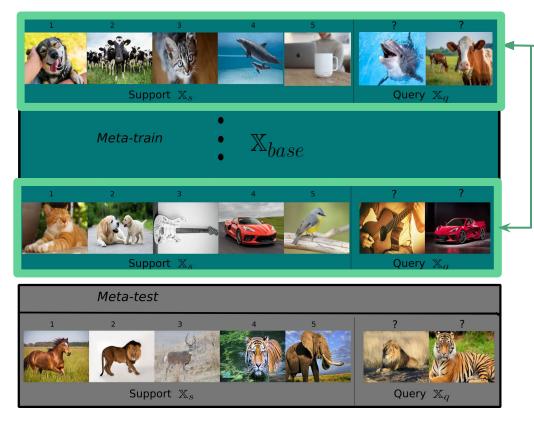




Training set with enough labeled data **to learn initial model**







Create episodes and do **episodic training** to learn **meta-learner**

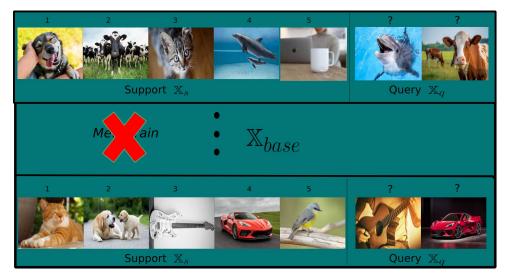
Vinyal et al, (Neurips '16), Snell et al, (Neurips '17), Sung et al, (CVPR ' 18), Finn et al, (ICML' 17), Ravi et al, (ICLR' 17), Lee et al, (CVPR' 19), Hu et al, (ICLR '20), Ye et al, (CVPR '20),...

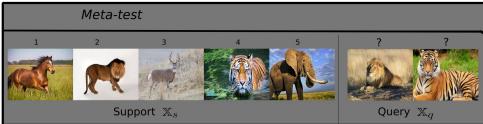
Taking a few steps backward . .

Recently [Chen et al., ICLR'19, Wang et al., '19, Dhillon et al., ICLR'20]:

Simple baselines **outperform** the overly convoluted **meta-learning** based approaches.

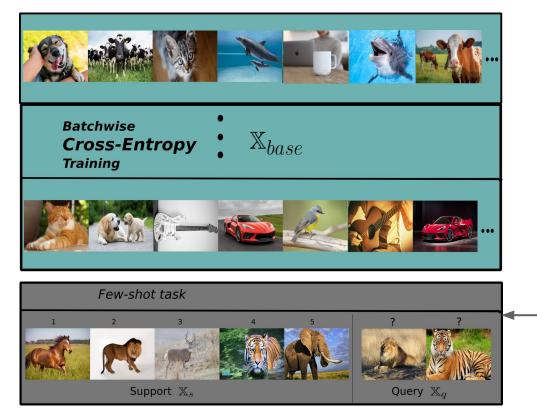
Baseline Framework





No need to *meta-train*

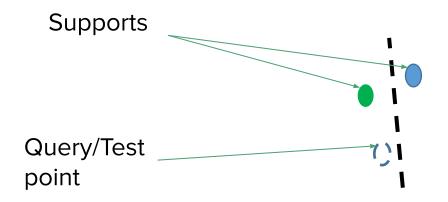
Baseline Framework



Simple conventional cross-entropy training

The approaches mostly differ **during inference**

Inductive vs Transductive inference

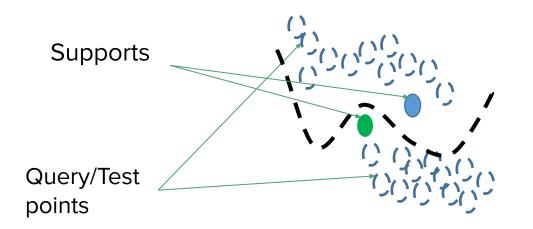


Examples

Vinayls et al., NEURIPS' 16 (Attention mechanism)

Snell et al., NEURIPS' 17 (Nearest Prototype)

Inductive vs Transductive inference



Transductive: Predict for all test points, instead of one at a time

Examples

Liu et. al., ICLR'19 (Label propagation)

Dhillon, ICLR'20 (Transductive fine-tuning)

Proposed LaplacianShot

Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$

- Latent Assignment matrix for N query samples:

$$\mathbf{Y} = [\mathbf{y}_q] \in \{0, 1\}^{N \times C}$$

- Label assignment for each query:

$$\mathbf{y}_q = [y_{q,1}, \dots, y_{q,C}]^t \in \{0, 1\}^C$$

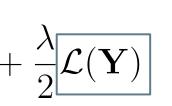
- And Simplex Constraints:

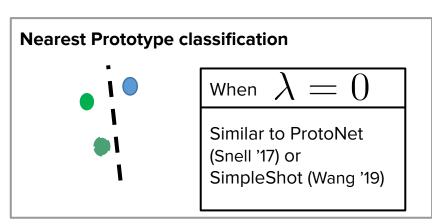
$$\mathbf{1}^t \mathbf{y}_q = 1$$

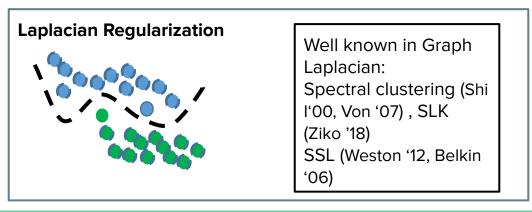
Proposed LaplacianShot

Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$







LaplacianShot Takeaways

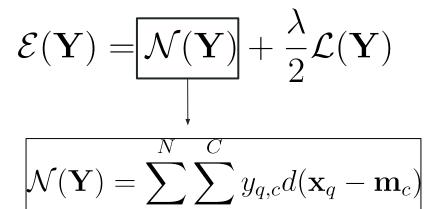
- \checkmark **SOTA results** without bell and whistles.
- ✓ Simple **constrained graph clustering** works very well.
- ✓ No network fine-tuning, neither meta-learning
- ✓ Model Agnostic
- ✓ **Fast** transductive inference: almost inductive time

LapLacianShot

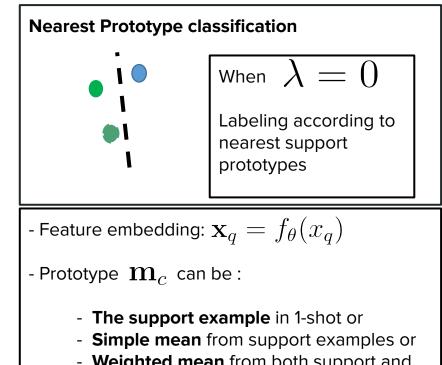
More Details

Proposed LaplacianShot

Laplacian-regularized objective:



q=1 c=1



- **Weighted mean** from both support and initially predicted query samples

Proposed LaplacianShot

Pairwise similarity

Laplacian-regularized objective:

objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2} \mathcal{L}(\mathbf{Y})$$

$$\mathcal{L}(\mathbf{Y}) = \sum_{q,p} \mathbf{w}(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$

q,p

Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$

$$\mathcal{N}(\mathbf{Y}) = \sum_{q=1}^{N} \sum_{c=1}^{C} y_{q,c} d(\mathbf{x}_q - \mathbf{m}_c)$$
$$\mathcal{L}(\mathbf{Y}) = \sum_{q=1}^{N} w(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$

 $\mathcal{L}(\mathbf{1}) = \sum_{q,p} w(\mathbf{x}_q, \mathbf{x}_p) ||\mathbf{y}_q - \mathbf{y}$

Tricky to optimize due to:

Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$

$$\mathcal{N}(\mathbf{Y}) = \sum_{q=1}^{N} \sum_{c=1}^{C} y_{q,c} d(\mathbf{x}_q - \mathbf{m}_c)$$

$$\mathcal{L}(\mathbf{Y}) = \sum_{q,p} w(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$

Tricky to optimize due to:

Simplex/Integer Constraints.

$$\mathbf{Y} = [\mathbf{y}_q] \in \{0, 1\}^{N \times C}$$
$$\mathbf{1}^t \mathbf{y}_q = 1$$

Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$

$$\mathcal{N}(\mathbf{Y}) = \sum_{q=1}^{N} \sum_{c=1}^{C} y_{q,c} d(\mathbf{x}_{q} - \mathbf{m}_{c})$$

$$\mathcal{L}(\mathbf{Y}) = \sum_{q,p} w(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$

Tricky to optimize due to:



Laplacian over discrete variables.

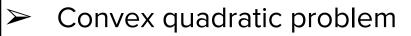
Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$

$$\mathcal{N}(\mathbf{Y}) = \sum_{q=1}^{N} \sum_{c=1}^{C} y_{q,c} d(\mathbf{x}_{q} - \mathbf{m}_{c})$$
$$\mathcal{L}(\mathbf{Y}) = \sum_{q=1}^{N} w(\mathbf{x}_{q} \cdot \mathbf{x}_{p}) \|\mathbf{y}_{q} - \mathbf{y}_{p}\|^{2}$$

$$\mathcal{L}(\mathbf{Y}) = \sum_{q,p} w(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$

Relax integer constraints:



- Require solving for the N×C variables all together
 - Extra projection steps for the simplex constraints

X

Laplacian-regularized objective:

$$\mathcal{E}(\mathbf{Y}) = \mathcal{N}(\mathbf{Y}) + \frac{\lambda}{2}\mathcal{L}(\mathbf{Y})$$

$$\mathcal{N}(\mathbf{Y}) = \sum_{q=1}^{N} \sum_{c=1}^{C} y_{q,c} d(\mathbf{x}_q - \mathbf{m}_c)$$
$$\mathcal{L}(\mathbf{Y}) = \sum w(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$

We do:

✓ Concave relaxation

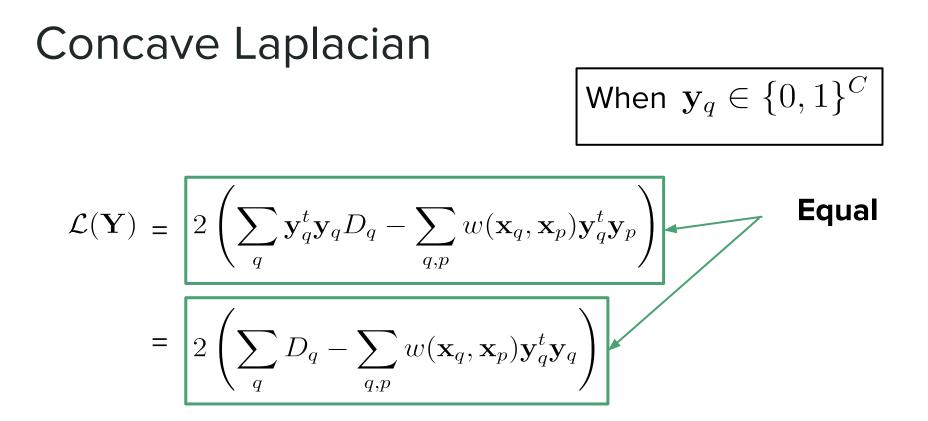
 ✓ Independent and closed-form updates for each assignment variable

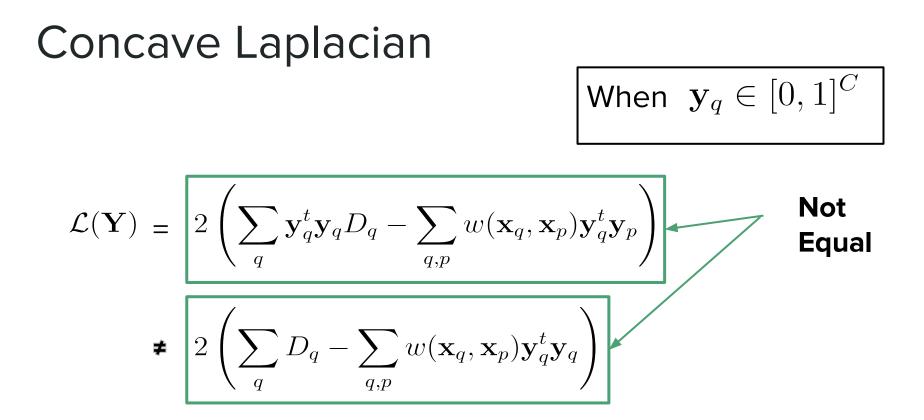
q.p

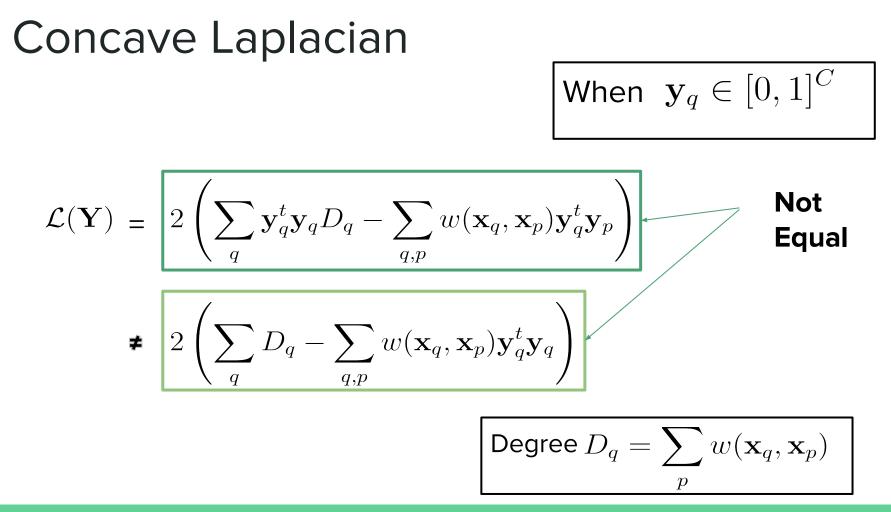
 \checkmark Efficient bound optimization

Concave Laplacian

$$\mathcal{L}(\mathbf{Y}) = \sum_{q,p} w(\mathbf{x}_q, \mathbf{x}_p) \|\mathbf{y}_q - \mathbf{y}_p\|^2$$







Concave Laplacian

Remove constant terms

$$\mathcal{L}(\mathbf{Y}) = 2\left(\sum_{q} D_{q} - \sum_{q,p} w(\mathbf{x}_{q}, \mathbf{x}_{p})\mathbf{y}_{q}^{t}\mathbf{y}_{q}\right)$$

Concave Laplacian

$$\mathcal{L}(\mathbf{Y}) = -2\sum_{q,p} w(\mathbf{x}_q, \mathbf{x}_p) \mathbf{y}_q^t \mathbf{y}_p$$

Concave for PSD matrix
$$\mathbf{W} = [w(\mathbf{x}_q, \mathbf{x}_p)]$$

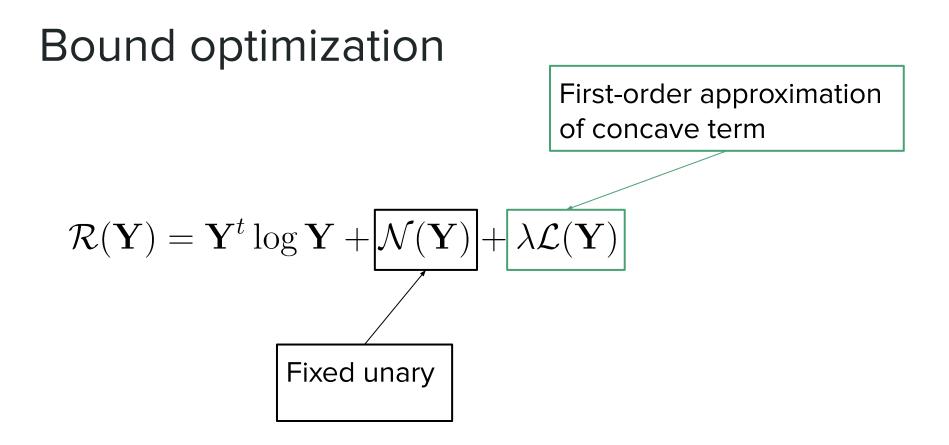
Concave-Convex relaxation

Putting it altogether

$$\mathcal{R}(\mathbf{Y}) = \mathbf{Y}^t \log \mathbf{Y} + \mathcal{N}(\mathbf{Y}) + \lambda \mathcal{L}(\mathbf{Y})$$

Convex barrier function:

- Avoids extra dual variables for $\mathbf{y}_q \ge 0$
- Closed- form update for the simplex constraint duel



Bound optimization

We get **Iterative tight upper bound:**

Iteratively optimize:
$$\mathbf{Y}^{i+1} = rgmin_{\mathbf{Y}}\mathcal{B}_i(\mathbf{Y})$$

$$B_i(\mathbf{Y}) = \sum_{q=1}^{N} \mathbf{y}_q^t (\log(\mathbf{y}_q) + \mathbf{a}_q - \lambda \mathbf{b}_q^i)$$

Where:

$$\mathbf{a}_{q} = [a_{q,1}, \dots, a_{q,C}]^{t}; \ a_{q,c} = d(\mathbf{x}_{q}, \mathbf{m}_{c})$$

 $\mathbf{b}_{q}^{i} = [b_{q,1}^{i}, \dots, b_{q,C}^{i}]^{t}; \ b_{q,c}^{i} = \sum_{p} w(\mathbf{x}_{q}, \mathbf{x}_{p})y_{p,c}^{i}$

Bound optimization

Independent **upper bound:**

$$B_i(\mathbf{Y}) = \sum_{q=1}^{N} \mathbf{y}_q^t (\log(\mathbf{y}_q) + \mathbf{a}_q - \lambda \mathbf{b}_q^i)$$

Bound optimization

Minimize Independent **upper bound:**

$$\min_{\mathbf{y}_q \in \nabla_C} \mathbf{y}_q^t (\log(\mathbf{y}_q) + \mathbf{a}_q - \lambda \mathbf{b}_q^i), \, \forall q$$

KKT conditions brings **closed form updates:**
$$\mathbf{y}_q^{i+1} = \frac{\exp(-\mathbf{a}_q^i + \lambda \mathbf{b}_q^i)}{\mathbf{1}^t \exp(-\mathbf{a}_q^i + \lambda \mathbf{b}_q^i)} \ \forall q$$

LaplacianShot Algorithm

Input: $\mathbb{X}_{s}, \mathbb{X}_{q}, \lambda, f_{\theta}$ **Output:** Labels $\in \{1, .., C\}^N$ for \mathbb{X}_a Get prototypes \mathbf{m}_c . Compute $\mathbf{a}_q \ \forall \mathbf{x}_q \in \mathbb{X}_q$. a Initialize i = 1. \mathbf{b}_q^i Initialize $\mathbf{y}_q^i = \frac{\exp(-\mathbf{a}_q)}{\mathbf{1}^t \exp(-\mathbf{a}_q)}$. repeat Compute \mathbf{y}_q^{i+1} $\mathbf{y}_{a}^{i} \leftarrow \mathbf{y}_{a}^{i+1}$. $\mathbf{Y} = [\mathbf{y}_a^i]; \; \forall q.$ i = i + 1. until $\mathcal{B}_i(\mathbf{Y})$ does not change $l_q = \arg \max \mathbf{y}_q; \ \forall \mathbf{y}_q \in \mathbf{Y}.$ $Labels = \{l_q\}_{q=1}^N$

$$\frac{q}{\mathbf{y}_{q}^{i+1}} = \frac{\left[a_{q,1}, \dots, a_{q,C}\right]^{t}; \ a_{q,c} = d(\mathbf{x}_{q}, \mathbf{m}_{c})}{\mathbf{1}^{t} = \left[b_{q,1}^{i}, \dots, b_{q,C}^{i}\right]^{t}; \ b_{q,c}^{i} = \sum_{p} w(\mathbf{x}_{q}, \mathbf{x}_{p})y_{p,c}^{i}}$$
$$\mathbf{y}_{q}^{i+1} = \frac{\exp(-\mathbf{a}_{q}^{i} + \lambda \mathbf{b}_{q}^{i})}{\mathbf{1}^{t} \exp(-\mathbf{a}_{q}^{i} + \lambda \mathbf{b}_{q}^{i})} \ \forall q$$

Datasets:

- 1. *Mini-*ImageNet
- 2. Tierd-ImageNet
- 3. CUB 200-2001
- 4. Inat

Generic Classification

minilmageNet splits: 64 base, 16 validation and 20 test classes *tieredImageNet splits:* 351 base, 97 validation and 160 test classes

Fine-Grained Classification

Splits: 100 base, 50 validation and 50 test classes

Datasets:

- 1. *Mini-*ImageNet
- 2. *Tierd-ImageNet*
- 3. CUB 200-2001
- 4. Inat

Evaluation protocol:

- **5**-way **1**-shot/**5**-shot .
- 15 query samples per class (N=75).
- Average accuracy over 10,000
 few-shot tasks with 95%
 confidence interval.

Datasets:

- 1. Mini-ImageNet
- 2. Tierd-ImageNet
- 3. CUB 200-2001
- 4. Inat

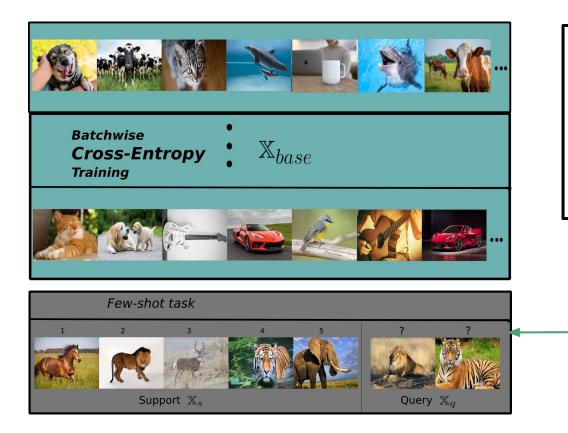
- More realistic and challenging
- Recently introduced (Wertheimer& Hariharan, 2019)
- Slight class distinction
- Imbalanced class distribution with variable number of supports/query per class

Datasets:

- 1. Mini-ImageNet
- 2. Tierd-ImageNet
- 3. CUB 200-2001
- 4. Inat

Evaluation protocol:

- 227-way multi-shot .
- Top-1 accuracy averaged over the test images Per Class.
- Top-1 accuracy averaged over all the test images (Mean)



We do Cross-entropy training with base classes

LaplacianShot during inference

Results (Mini-ImageNet)

Methods	Network	1-shot	5-shot
MAML [Finn et al., 2017]	$\operatorname{ResNet-18}$	49.61 ± 0.92	65.72 ± 0.77
Chen [Chen et al., 2019]	$\operatorname{ResNet-18}$	51.87 ± 0.77	75.68 ± 0.63
RelationNet [Sung et al., 2018]	$\operatorname{ResNet-18}$	52.48 ± 0.86	69.83 ± 0.68
MatchingNet [Vinyals et al., 2016]	$\operatorname{ResNet-18}$	52.91 ± 0.88	68.88 ± 0.69
ProtoNet [Snell et al., 2017]	$\operatorname{ResNet-18}$	54.16 ± 0.82	73.68 ± 0.65
Gidaris [Gidaris and Komodakis, 2018]	$\operatorname{ResNet-15}$	55.45 ± 0.89	70.13 ± 0.68
SNAIL[Mishra et al., 2018]	$\operatorname{ResNet-15}$	55.71 ± 0.99	68.88 ± 0.92
AdaCNN [Munkhdalai et al., 2018]	$\operatorname{ResNet-15}$	56.88 ± 0.62	71.94 ± 0.57
TADAM [Oreshkin et al., 2018]	$\operatorname{ResNet-15}$	58.50 ± 0.30	76.70 ± 0.30
CAML [Jiang et al., 2019]	$\operatorname{ResNet-12}$	59.23 ± 0.99	72.35 ± 0.71
TPN [Yanbin et al., 2019]	$\operatorname{ResNet-12}$	59.46	75.64
TEAM [Qiao et al., 2019]	$\operatorname{ResNet-18}$	60.07	75.90
MTL [Sun et al., 2019]	$\operatorname{ResNet-18}$	61.20 ± 1.80	75.50 ± 0.80
VariationalFSL [Zhang et al., 2019]	$\operatorname{ResNet-18}$	61.23 ± 0.26	77.69 ± 0.17
Transductive tuning [Dhillon et al., 2020]	$\operatorname{ResNet-12}$	62.35 ± 0.66	74.53 ± 0.54
MetaoptNet[Lee et al., 2019]	$\operatorname{ResNet-18}$	62.64 ± 0.61	78.63 ± 0.46
SimpleShot [Wang et al., 2019]	$\operatorname{ResNet-18}$	63.10 ± 0.20	79.92 ± 0.14
$\mathbf{\nabla}$ CAN+T [Hou et al., 2019]	$\operatorname{ResNet-12}$	67.19 ± 0.55	80.64 ± 0.35
LaplacianShot (ours)	$\operatorname{ResNet-18}$	$\textbf{72.11}\pm0.19$	82.31 ± 0.14

Results (Mini-ImageNet)

Methods	Network	1-shot	$5 ext{-shot}$
Qiao [Qiao et al., 2018]	WRN	59.60 ± 0.41	73.74 ± 0.19
LEO [Rusu et al., 2019]	WRN	61.76 ± 0.08	77.59 ± 0.12
ProtoNet [Snell et al., 2017]	WRN	62.60 ± 0.20	79.97 ± 0.14
CC+rot[Gidaris et al., 2019]	WRN	62.93 ± 0.45	79.87 ± 0.33
MatchingNet [Vinyals et al., 2016]	WRN	64.03 ± 0.20	76.32 ± 0.16
FEAT [Ye et al., 2020]	WRN	65.10 ± 0.20	81.11 ± 0.14
Transductive tuning [Dhillon et al., 2020]	WRN	65.73 ± 0.68	78.40 ± 0.52
SimpleShot [Wang et al., 2019]	WRN	65.87 ± 0.20	82.09 ± 0.14
SIB [Hu et al., 2020]	WRN	70.0 ± 0.6	79.2 ± 0.4
BD-CSPN [Liu et al., 2019]	WRN	70.31 ± 0.93	81.89 ± 0.60
LaplacianShot (ours)	WRN	74.86 ± 0.19	84.13 ± 0.14

Results (Tiered-ImageNet)

Methods	Network	1-shot	$5 ext{-shot}$
MetaoptNet[Lee et al., 2019]	$\operatorname{ResNet-18}$	65.99 ± 0.72	81.56 ± 0.53
SimpleShot [Wang et al., 2019]	$\operatorname{ResNet-18}$	69.68 ± 0.22	84.56 ± 0.16
CAN+T [Hou et al., 2019]	ResNet-12	73.21 ± 0.58	84.93 ± 0.38
LaplacianShot (ours)	ResNet-18	$\textbf{78.98} \pm 0.21$	86.39 ± 0.16
Meta SGD [Li et al., 2017]	WRN	62.95 ± 0.03	79.34 ± 0.06
LEO [Rusu et al., 2019]	WRN	66.33 ± 0.05	81.44 ± 0.09
FEAT [Ye et al., 2020]	WRN	70.41 ± 0.23	84.38 ± 0.16
CC+rot[Gidaris et al., 2019]	WRN	70.53 ± 0.51	84.98 ± 0.36
SimpleShot [Wang et al., 2019]	WRN	70.90 ± 0.22	85.76 ± 0.15
Transductive tuning [Dhillon et al., 2020]	WRN	73.34 ± 0.71	85.50 ± 0.50
BD-CSPN [Liu et al., 2019]	WRN	78.74 ± 0.95	86.92 ± 0.63
LaplacianShot (ours)	WRN	$\textbf{80.18} \pm 0.21$	87.56 ± 0.15

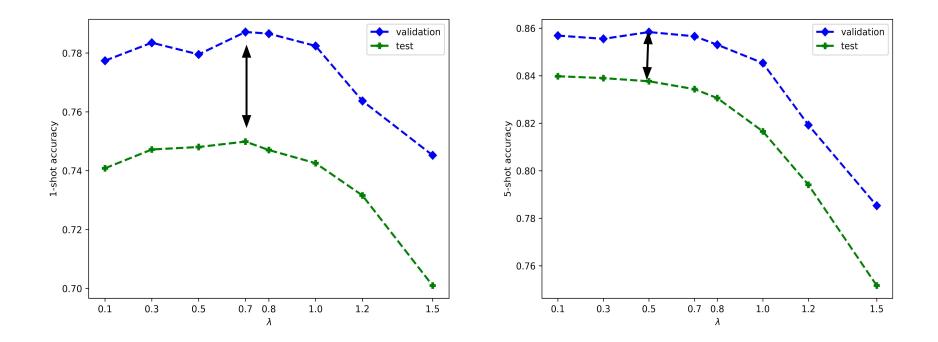
Results (CUB)

Methods	Network	Notwork CUB		miniImagenet $ ightarrow$ CUB	
Wethous	Inetwork	1-shot	5-shot	1-shot	5-shot
MatchingNet [Vinyals et al., 2016]	ResNet-18	73.49	84.45	-	53.07
MAML [Finn et al., 2017]	$\operatorname{ResNet-18}$	68.42	83.47	_	51.34
ProtoNet [Snell et al., 2017]	$\operatorname{ResNet-18}$	72.99	86.64	-	62.02
RelationNet [Sung et al., 2018]	$\operatorname{ResNet-18}$	68.58	84.05	-	57.71
Chen [Chen et al., 2019]	$\operatorname{ResNet-18}$	67.02	83.58	-	65.57
SimpleShot [Wang et al., 2019]	$\operatorname{ResNet-18}$	70.28	86.37	48.56	65.63
LaplacianShot(ours)	ResNet-18	80.96	88.38	55.46	66.33
				Cross Domain	

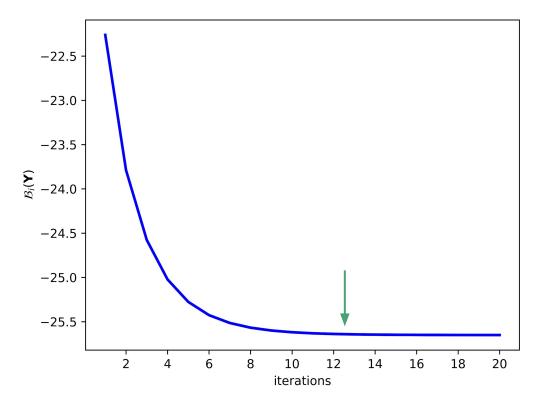
Results (iNat)

Methods	Network	Per Class	Mean
SimpleShot [Wang et al., 2019]	$\operatorname{ResNet-18}$	55.80	58.56
LaplacianShot	$\operatorname{ResNet-18}$	62.80	66.40
PN+BF+fsL+CP [Wertheimer and Hariharan, 2019]	ResNet-50	46.04	51.25
SimpleShot [Wang et al., 2019]	$\operatorname{ResNet-50}$	58.45	61.07
LaplacianShot	$\operatorname{ResNet-50}$	65.96	69.13
SimpleShot [Wang et al., 2019]	WRN	62.44	65.08
LaplacianShot	WRN	71.55	74.97

Ablation: Choosing λ



Ablation: Convergence



Ablation: Average Inference time

Methods	Network	inference time (s)
SimpleShot [Wang et al., 2019]	WRN	0.009
Transductive tuning [Dhillon et al., 2020]	WRN	20.7
LaplacianShot	WRN	0.012

Transductive

LaplacianShot Takeaways

- \checkmark **SOTA results** without bell and whistles.
- ✓ Simple **constrained graph clustering** works very well.
- ✓ No network fine-tuning, neither meta-learning
- ✓ Model Agnostic: during inference with any training model and gain up to 4/5%!!!
- ✓ **Fast** transductive inference: almost inductive time

Thank you

Code On: https://github.com/imtiazziko/LaplacianShot