EE485 Introduction to Photonics

Laser Operation

1. **Rate equations**
2. **Gain media**
3. **Steady-state laser operation**
4. **Laser line broadening**
5. **Pulsed operation**

Reading: Pedrotti[3], Chapter 26

Ref: Verdeyen, “Laser Electronics,” 3rd ed. Sec. 9.4-9.5

Transition Cross Section

Stimulated emission cross section

$$\sigma = B_{21} g(\nu') h\nu' / c$$

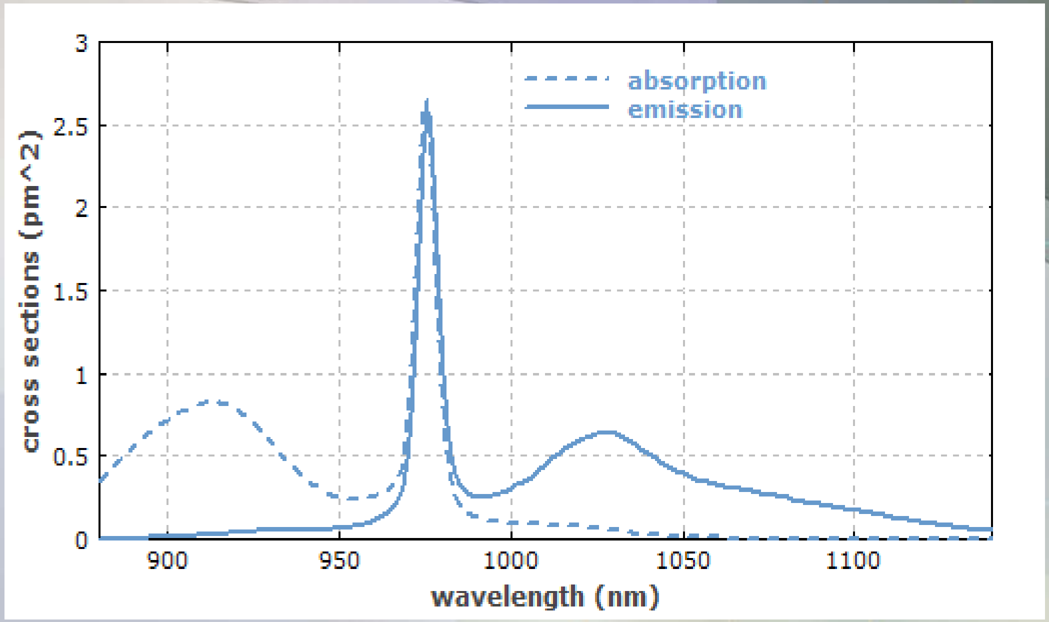
Absorption cross section

$$\sigma_{abs} = (g_2 / g_1)\sigma$$

Absorption coefficient

$$\alpha = -\sigma(N_2 - N_1)$$

$$\frac{dI}{dz} = -\alpha I$$



Effective absorption and emission cross sections of ytterbium-doped germanosilicate glass, as used in the cores of ytterbium-doped fibers.

Exercise: The cross section σ, for a transition from the ground state to an excited state that is resonant with an EM field of λ = 808 nm, for a neodymium (Nd) atom doped into a YAG (yttrium aluminum garnet) crystal is ~3×10^{-20} cm^2. Assume that the dopant density of Nd in the YAG crystal is 10^{20} atoms/cm^3 and that the YAG crystal itself is transparent to 808-nm light. A diode laser with λ = 808 nm is used to pump an Nd:YAG laser rod.

(a) Estimate the small-signal absorption coefficient for the Nd:YAG crystal.

(b) Estimate the depth of penetration into to the Nd:YAG crystal at which the intensity of the diode laser beam attenuates to 1/*e* of its initial value.

(c) One can define saturation intensity $I_{S,abs}$ for an absorptive medium as the intensity for which the loss coefficient α is reduced by a factor of 2 from its small-signal value. Find the expression for $I_{S,abs}$.

Four-Level System

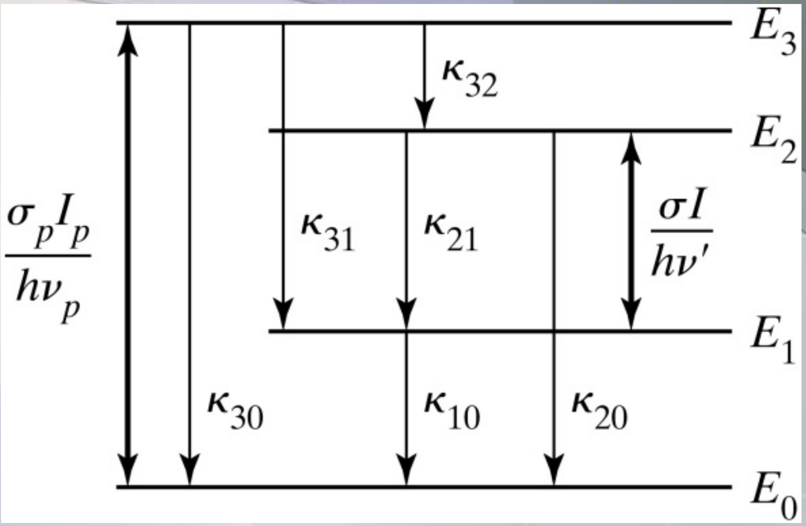
Rate equations:

$$\frac{dN_3}{dt} = -\kappa_3 N_3 - \frac{\sigma_P I_P}{h\nu_P}(N_3 - N_0)$$

$$\frac{dN_2}{dt} = \kappa_{32} N_3 - \kappa_2 N_2 - \frac{\sigma I}{h\nu'}(N_2 - N_1)$$

$$\frac{dN_1}{dt} = \kappa_{31} N_3 + \kappa_{21} N_2 - \kappa_{10} N_1 + \frac{\sigma I}{h\nu'}(N_2 - N_1)$$

$$\frac{dN_0}{dt} = \kappa_{30} N_3 + \kappa_{20} N_2 + \kappa_{10} N_1 + \frac{\sigma_P I_P}{h\nu_P}(N_3 - N_0)$$



$$\frac{d}{dt}\left(N_0 + N_1 + N_2 + N_3\right) = 0$$

$$\kappa_3 = \kappa_{32} + \kappa_{31} + \kappa_{30} \qquad \kappa_2 = \kappa_{21} + \kappa_{20}$$

Gain coefficient $\gamma = \sigma(N_2 - N_1) = \sigma N_{inv}$

$$\frac{dI}{dz} = \gamma I$$

The gain coefficient depends on intensity *I* since the population inversion is, in general, dependent on the intensity.

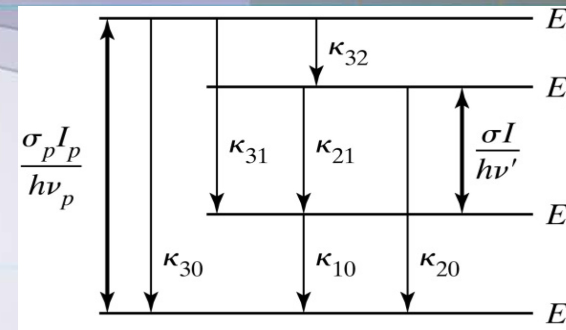
Exercise: 4-level amplifying system

$\kappa_{32} = 10^8$/s, $\kappa_{21} = 1000$/s, $\kappa_{10} = 10^8$/s, $\kappa_{30} = \kappa_{31} = \kappa_{20} \sim 0$

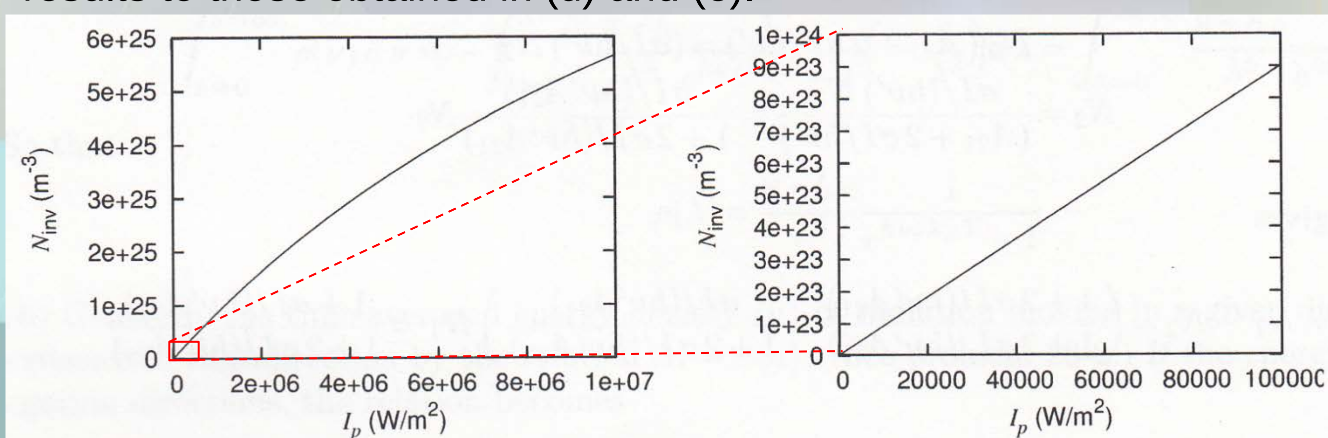
$\sigma_P = 3\text{x}10^{-19}\ \text{cm}^2$, $\sigma = 10^{-18}\ \text{cm}^2$

$\lambda_{30} = 400$ nm, $\lambda_{21} = 600$ nm (in free space)

$N_T = 1.5\text{x}10^{26}/\text{m}^3$



(a) Rate equations for the population densities of the levels.
(b) Steady-state small-signal population inversion as a function of the pump intensity.
(c) Pump intensity I_P required to sustain a steady-state population inversion.
(d) Pump intensity I_P required to sustain a small-signal gain coefficient of 0.01/cm.
(e) Pump intensity I_P required to sustain a small-signal gain coefficient of 1/cm.
(f) Compare N_0 to N_1, N_2, and N_3 for the pump intensities in (d) and (e).
(g) Use the ideal four-level gain medium relation together with the definition of the effective density under undepleted pump approximation, estimate the pump intensity required to sustain a small-signal gain coefficient of 0.01/cm and 1/cm. Compare these results to those obtained in (d) and (e).



For $I_p = 1104\ \text{W/m}^2$:
$N_0 = 1.49990 \times 10^{26}/\text{m}^3$
$N_1 = 1.000 \times 10^{17}/\text{m}^3$
$N_2 = 1.000 \times 10^{22}/\text{m}^3$
$N_3 = 1.000 \times 10^{17}/\text{m}^3$

For $I_p = 1.1 \times 10^5\ \text{W/m}^2$:
$N_0 = 1.490 \times 10^{26}/\text{m}^3$
$N_1 = 1.000 \times 10^{19}/\text{m}^3$
$N_2 = 1.000 \times 10^{24}/\text{m}^3$
$N_3 = 1.000 \times 10^{19}/\text{m}^3$

Steady-State Ring Laser Output

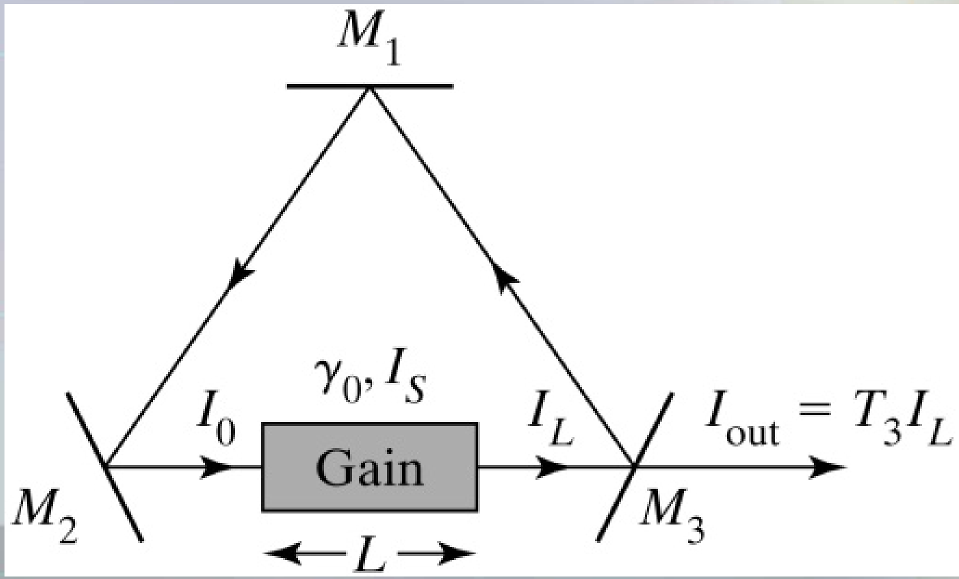
Consider a ring laser cavity. Mirrors M_1 and M_2 have reflectances $R_1 = R_2$. Let the gain medium be homogeneously broadened and have length L = 10 cm and a saturation intensity (at the lasing frequency) of $I_s = 2000$ W/cm^2.

(a) Find the threshold gain coefficient if $R_1 = R_2 = 1$, $R_3 = 0.95$, and $T_3=0.05$.

(b) If the small-signal gain coefficient is twice the threshold value, find the intensity of the output.

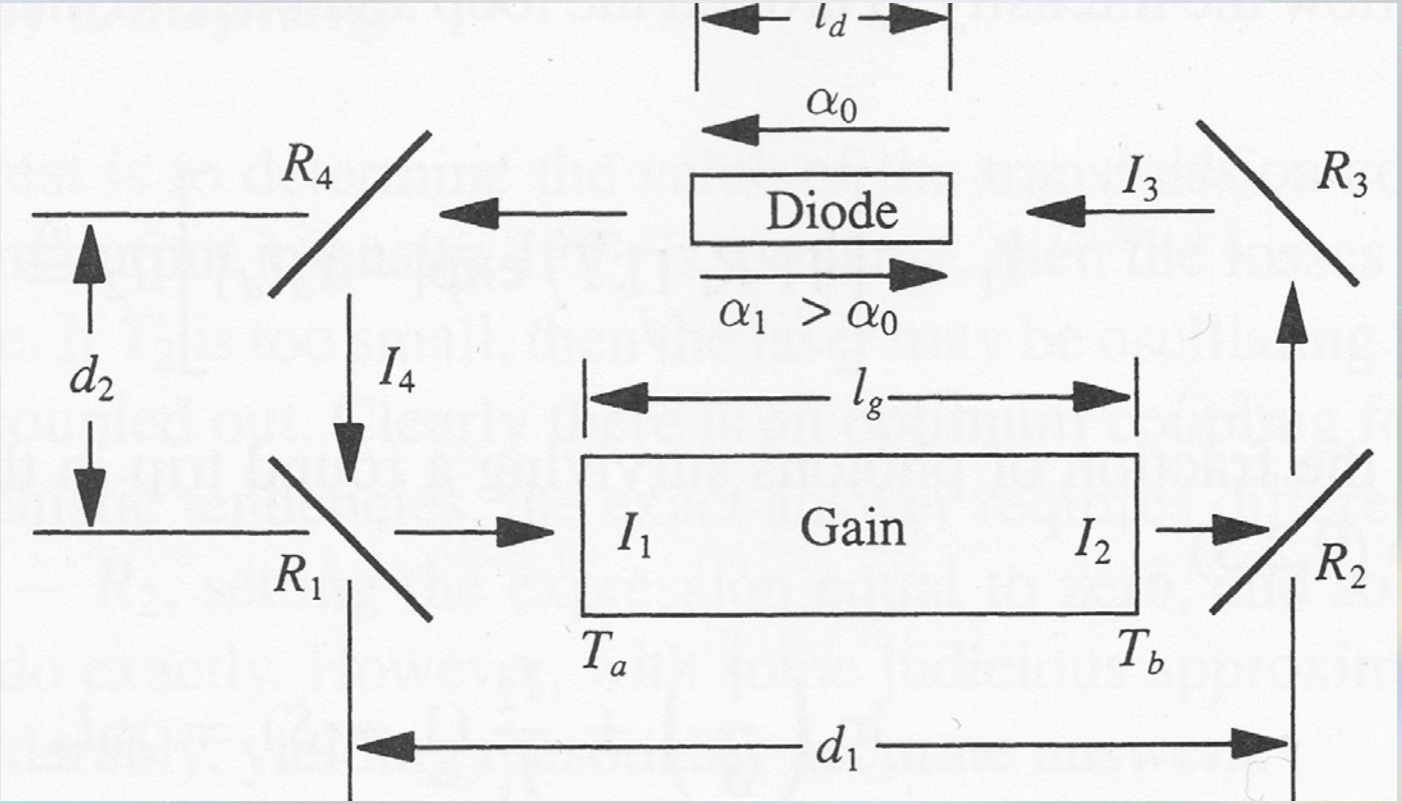
(c) Assuming that the laser beam has cross sectional area A = 0.1 cm^2, find the output power of the laser.

(d) Assuming that the overall efficiency of this laser system is 5%, find the pump power required to operate this laser system.



Exercise: Four-Mirror Ring Laser Cavity

Analyze the intensity change during a round trip in a four-mirror ring laser cavity.



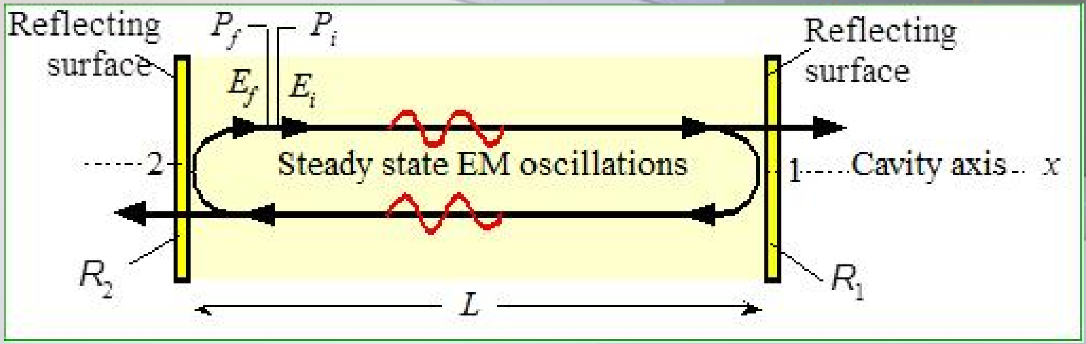
Two-mirror Linear Laser Cavity

Threshold gain

$$\gamma_{th} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right)$$

Output intensity

$$I_{out} = \frac{T_2 I_S}{2}\frac{\gamma_0(2L) - \ln\left(\frac{1}{R_1R_2}\right)}{(1-\sqrt{R_1R_2})(1+\sqrt{R_2/R_1})}$$

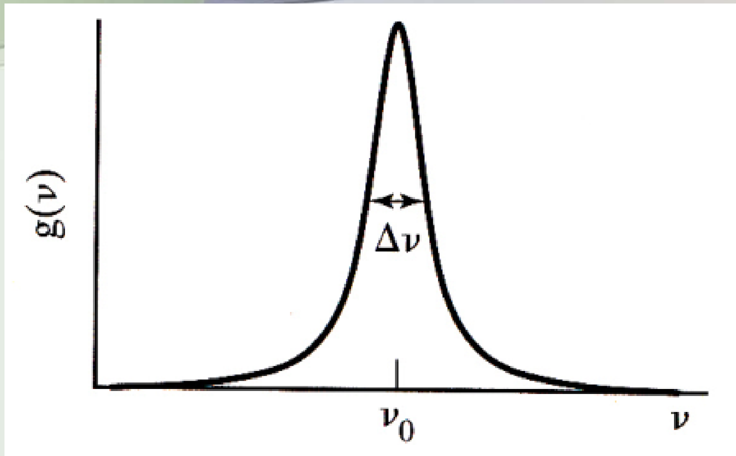


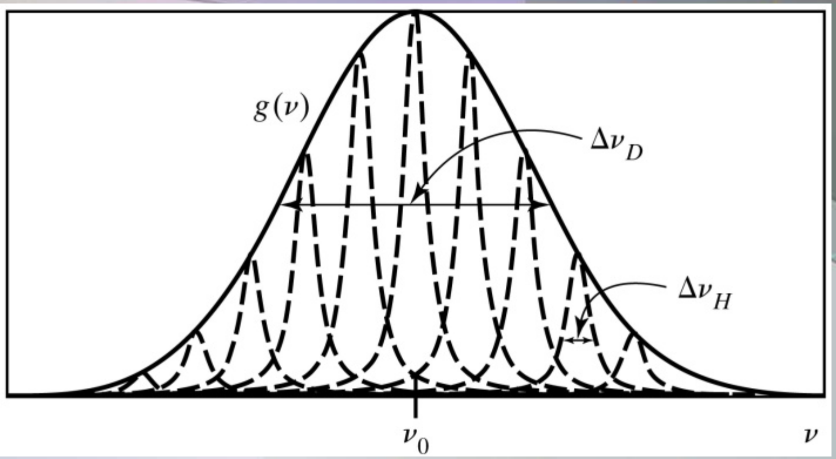
Exercise: Assume saturation intensity I_S = 10 W/cm^2 and α = 0. Design a laser cavity by picking a high reflectivity R_1 for the back mirror and a cavity length L, then write a computer program to plot I_{out} versus R_2 for R_2 ranging from 0.1 to 1. Do this consider two situations:

(1) The laser medium is pumped at a rate that leads to a small-signal gain γ_0 coefficient 5 times of the threshold gain (therefore it's a function of R_2).

(2) The laser medium is pumped at a rate that leads to a fixed small-signal gain coefficient γ_0 5 times of the threshold gain value when R_2= 0.2. Discuss the difference in the results.

Homogeneous and Inhomogeneous Broadening





$$g(\nu)=\frac{\Delta\nu_H}{2\pi\left[(\nu-\nu_0)^2+(\Delta\nu_H/2)^2\right]}$$

$$g(\nu)=\left(\frac{4\ln(2)}{\pi\Delta\nu_D^2}\right)e^{-4\ln(2)[(\nu-\nu_0)/\Delta\nu_D]^2}$$

$$\Delta\nu_D=\left(\frac{8k_BT}{Mc^2}\ln(2)\right)^{1/2}\nu_0$$

Exercise: A typical Ar^+ laser has a cavity length of 1 m and a center wavelength λ_0 = 488 nm. The atomic mass of an Ar atom is M = 6.64×10^{-26} kg. Assume the temperature of the gas under the operating condition is 3000 K. How many longitudinal modes can exist within the FWHM of the inhomogeneous broadened lineshape function?

Relaxation Oscillation

Laser turn-on and approach to steady state.

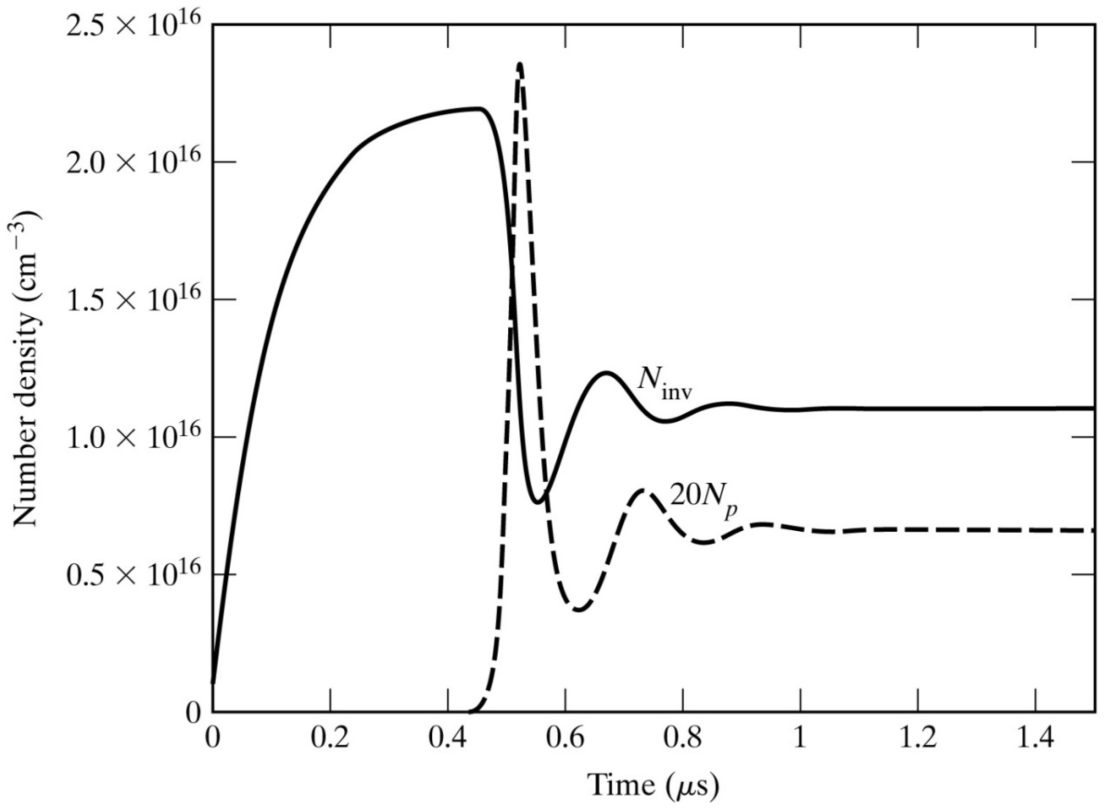
$$I = h\nu' c N_P$$

$$\frac{dN_2}{dt} = R_{p2} - \kappa_2 N_2 - \frac{\sigma I}{h\nu'} N_2 = R_{p2} - \kappa_2 N_2 - \sigma c N_P N_2$$

$$\frac{dN_P}{dt} = -\Gamma N_P + \frac{V_g}{V_c} \sigma c N_P N_2$$

Γ : Cavity loss rate

V_g / V_c : Ratio of the gain cell length to the perimeter of the laser cavity



$$\sigma = 10^{-18}\,\text{cm}^2$$

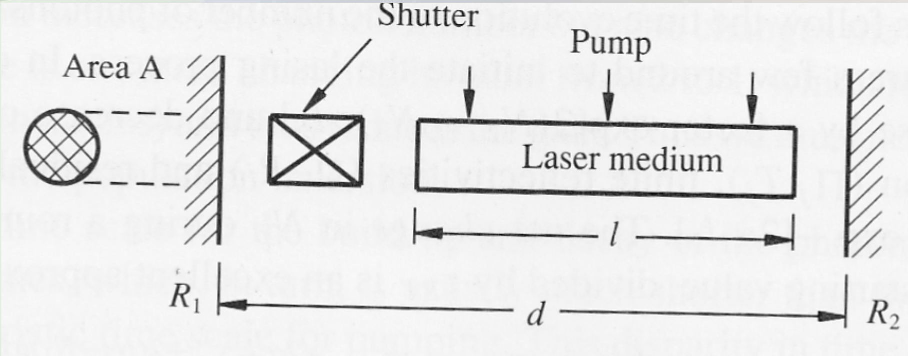
$$\Gamma = 10^8\,\text{s}^{-1}$$

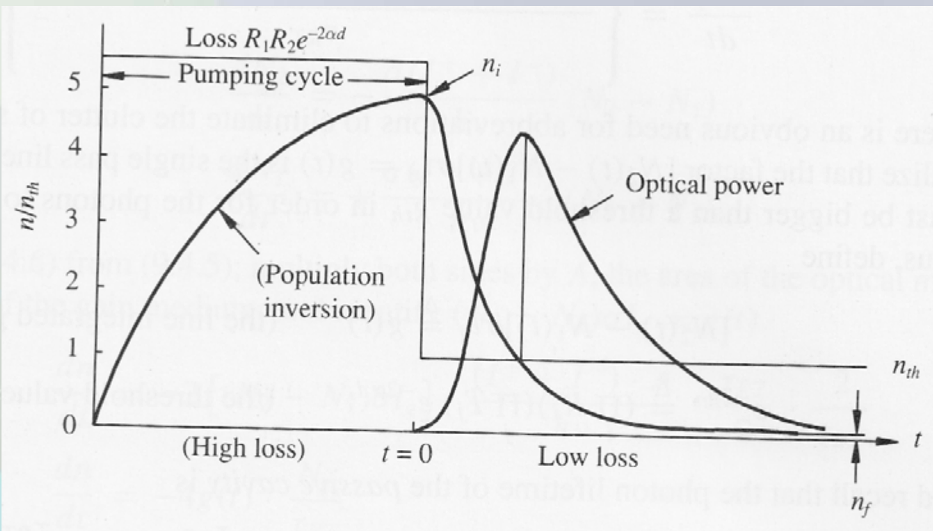
$$\kappa_2 = 10^7\,\text{s}^{-1}$$

$$\gamma_0 / \gamma_{th} = 2$$

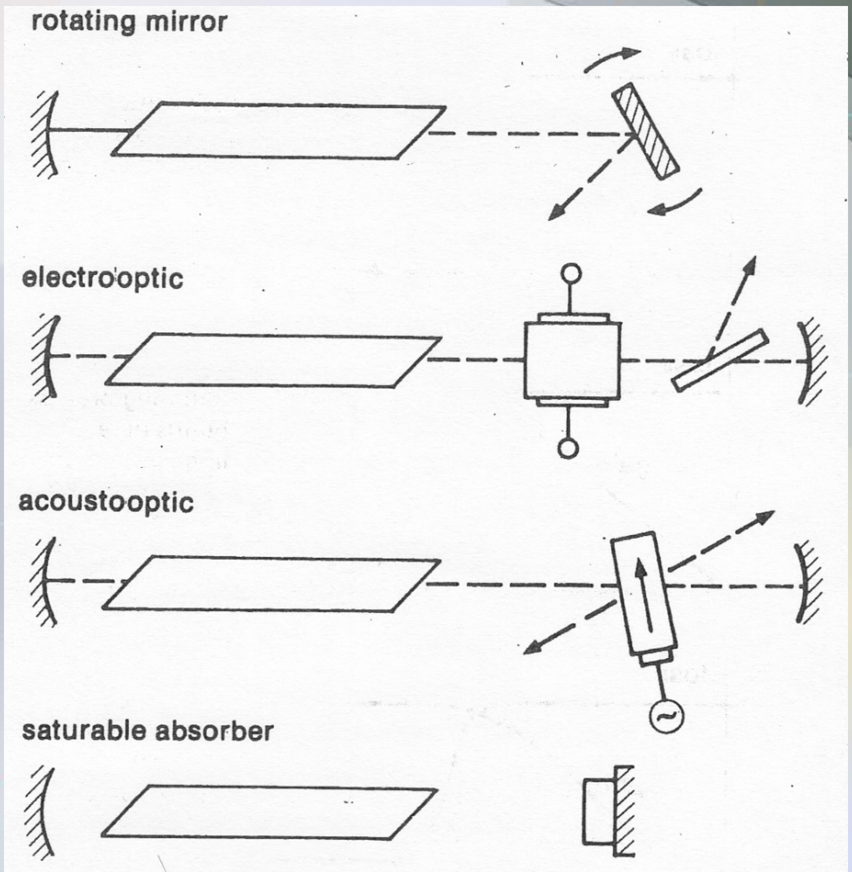
$$V_g / V_c = 0.3$$

Q-Switching



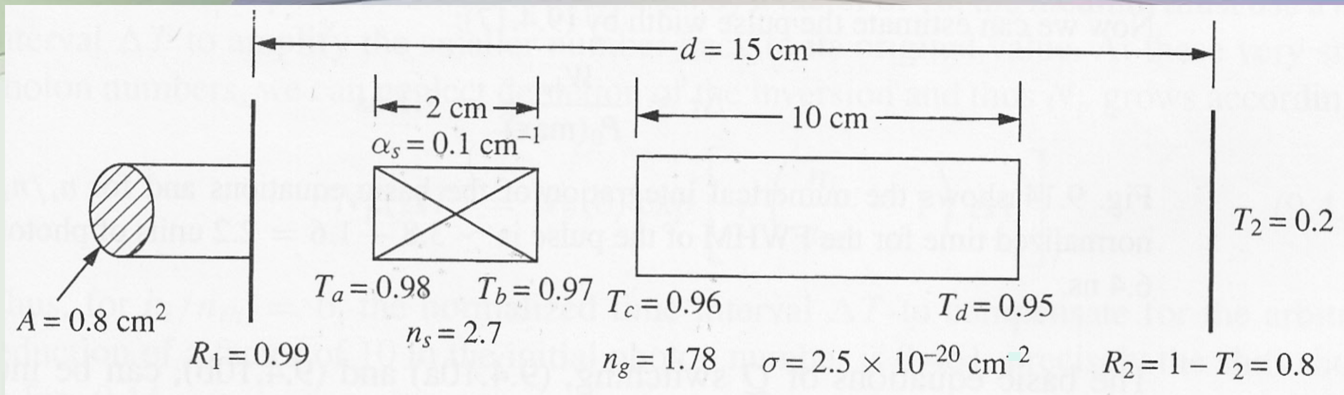


Laser Q-switching techniques



Ref: (1) Verdeyen, “Laser Electronics,” Prentice Hall (2) Siegman , “Lasers,” University Science Books

Exercise: Q-Switched Laser



$$\frac{dN_P}{dt} = \frac{N_P}{\tau_P}\left\{\frac{n}{n_{th}} - 1\right\}$$

$$\frac{dn}{dt} = -2\frac{n}{n_{th}} \cdot \frac{N_P}{\tau_P}$$

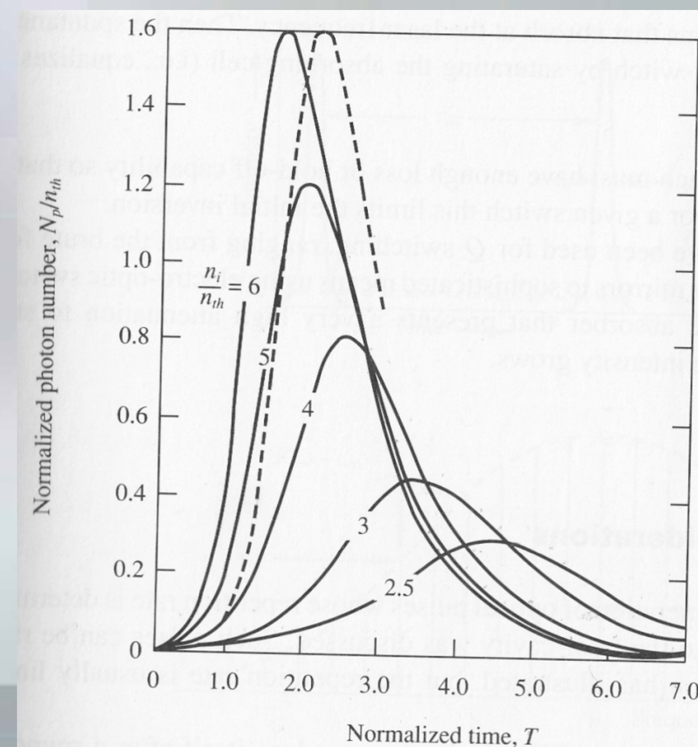
Assume a ruby laser (λ = 694.3 nm) to be pumped to four times threshold. For simplicity we assume equal degeneracy of the lasing states even though that is not true for ruby. We also assume a residual attenuation of 0.1 cm^{-1} by the switch medium even when it is in its high transmission state. The laser is operated at Q-switching mode. Calculate:

(a) Threshold gain, inversion density and total # of inverted atoms at threshold

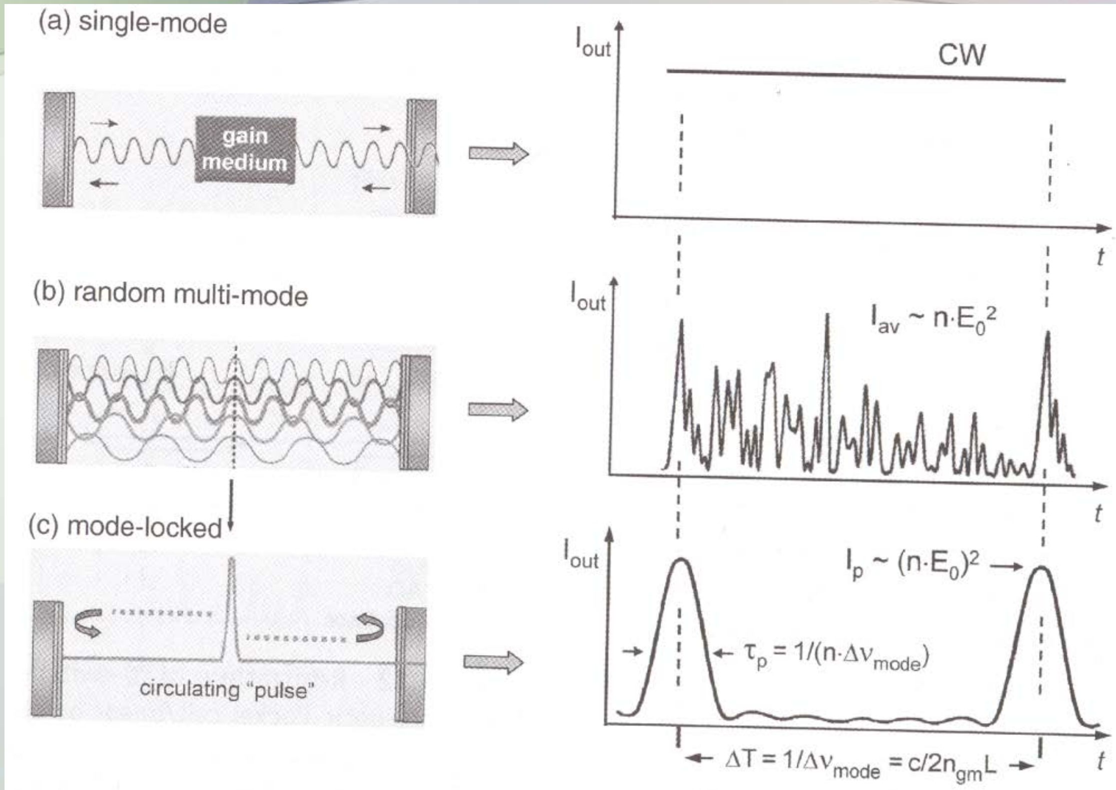
(b) Maximum photon # in the cavity

(c) Photon lifetime in the cavity

(d) Output power at the maximum of the pulse



Mode-Locked Laser



$$E(t) = E_0(t)\exp(i\omega_0 t)$$

$$E(t) = \sum_{-(N-1)/2}^{+(N-1)/2} E_n(t)\exp\left[i(\omega_0 + n\omega_c)t + \phi_n(t)\right]$$

$$E(t) = E_0 e^{i\omega_0 t}\frac{1-\exp(iN\omega_c t)}{1-\exp(i\omega_c t)}$$

$$I(t) = \frac{|E(t)|^2}{2\eta} = \frac{E_0^2}{2\eta}\frac{\sin^2(N\omega_c t/2)}{\sin^2(\omega_c t/2)}$$

$$T = \tau_{RT} = 2d/c$$

$$\Delta t_P \approx \frac{1}{\Delta\nu}$$

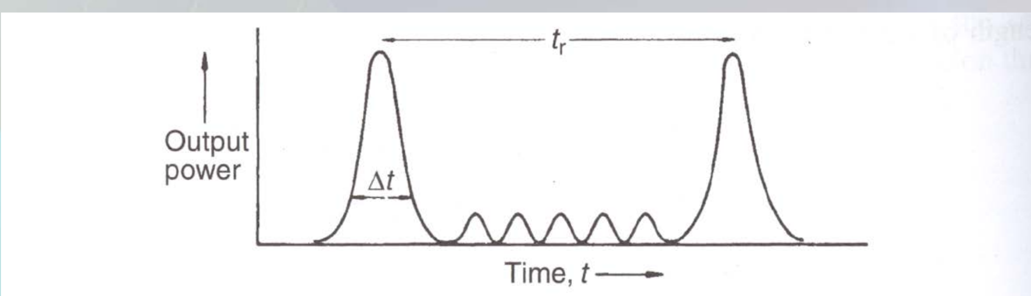
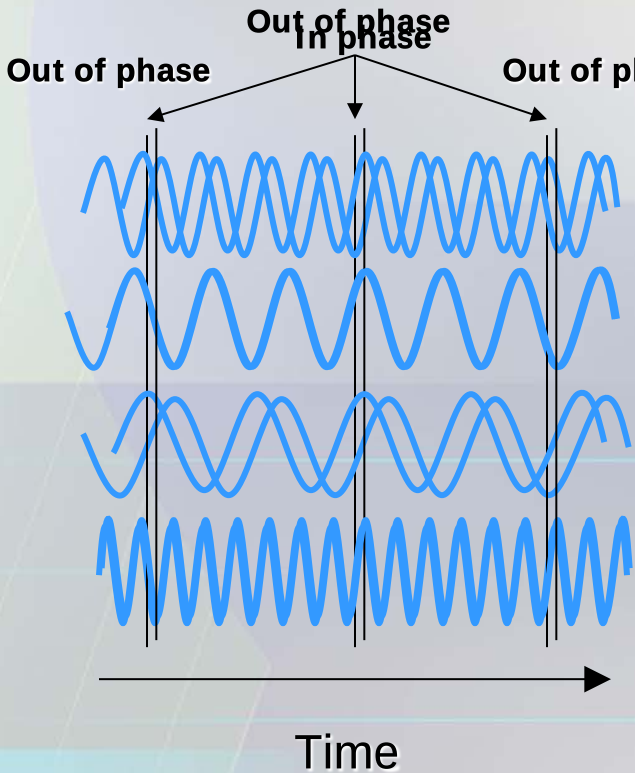


Figure 9.5 Suppression of five out of seven axial cavity modes by mode locking

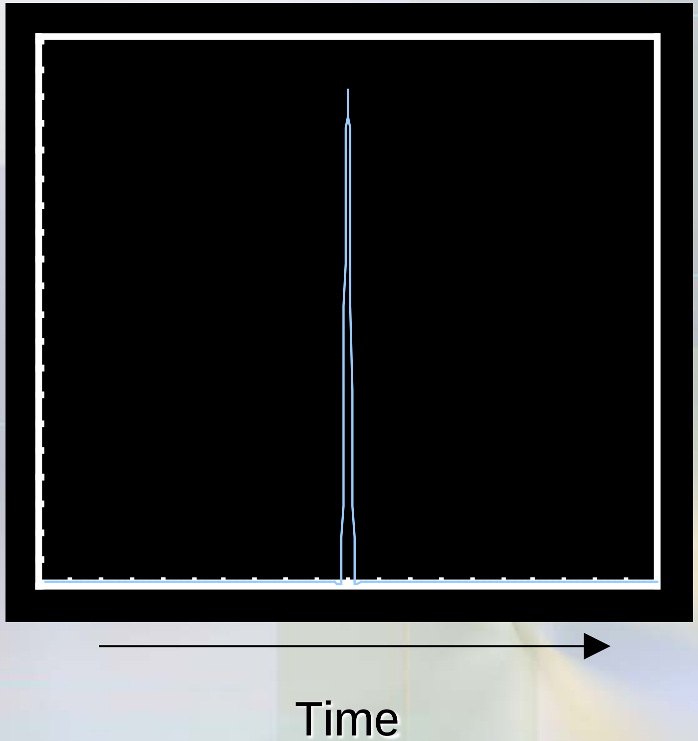
Mode-Locking

RANDOM phases for all the laser modes

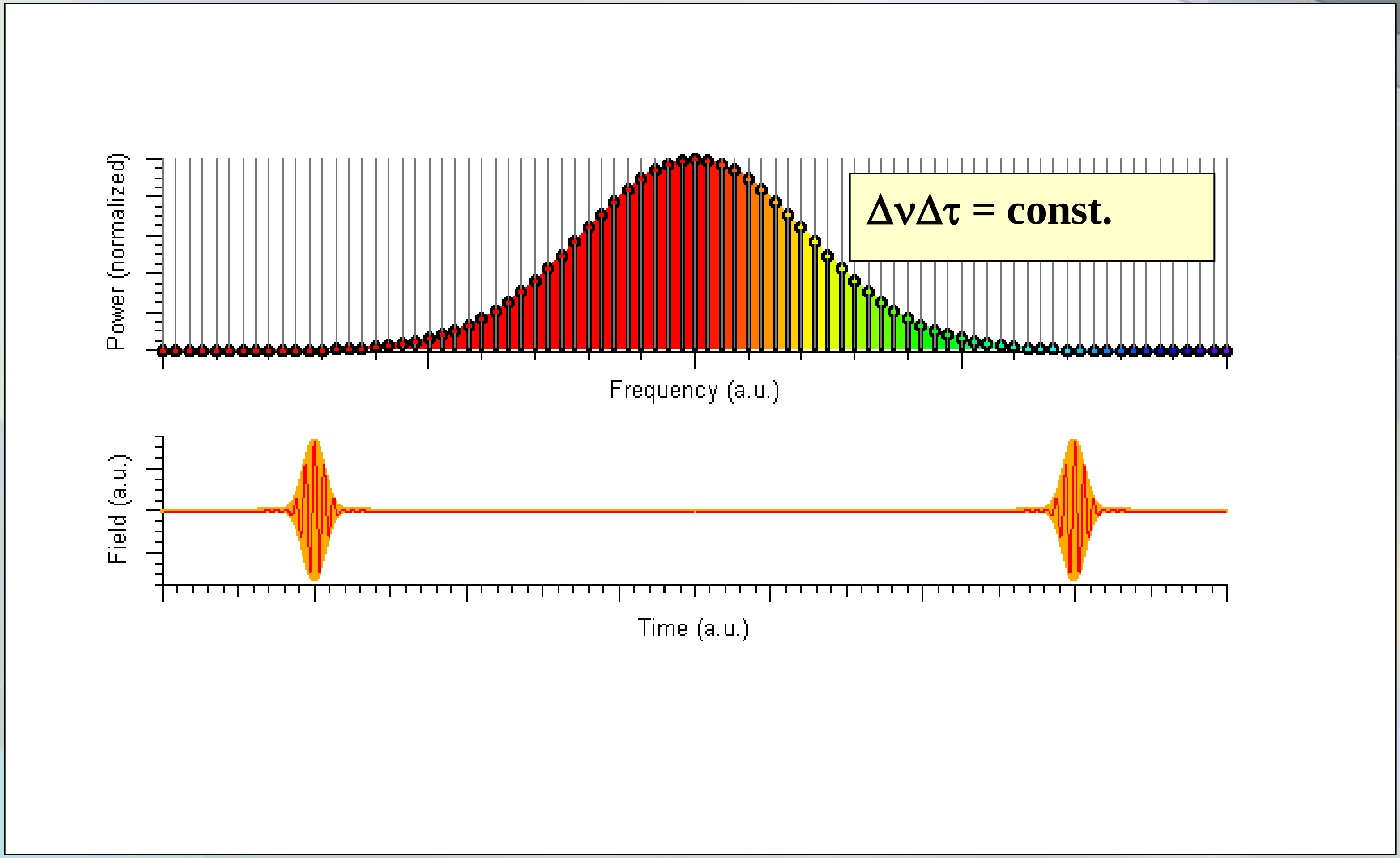
LOCKED phases for all the laser modes



Irradiance vs. Time



Bandwidth vs. Pulsewidth



Various Types of Laser Output