For many years, it wasn't possible to know the exact position of ships and aircraft at any instant. The worldwide Global Positioning System (GPS) now makes it possible to answer the simple question 'Where am I?' almost instantaneously and with a high degree of precision. GPS technology does this by utilising atomic clocks that keep time to within a billionth of a second, in conjunction with a network of navigational satellites.
The system made its public debut in the 1991 Gulf War but has since found many applications in the civilian sector. GPS is now used for a vast array of military and civilian applications, including locating vessels lost at sea, assisting emergency vehicles to find their destinations, and helping freight and transport companies to keep track of their fleets. Cars can now be fitted with GPS devices that allow the driver to be directed to a requested street location.


### 7.1 Latitude, longitude and position

## Latitude and longitude

Great circles are drawn on the surface so that their centre is at the centre of the Earth. Small circles do not have their centre at the centre of the Earth.


A position on the Earth is described by its latitude and longitude. Latitude and longitude are both stated as angles, because they are measured by angles at the centre of the Earth between circles drawn on the Earth's surface. Meridians are great semicircles drawn between the North and South Poles. The Prime Meridian of longitude is the meridian passing through Greenwich, England.


Small circles


The Equator is also a great circle. Parallels of latitude are small circles drawn parallel to the Equator with their centres at the axis through the North and South Poles.


The latitude of a point is the angle between the Equator and the parallel of latitude passing through the point. It is measured north or south from the Equator.
The longitude of a point on the Earth is the angle between the Prime Meridian and a meridian passing through the point, measured east or west to make the angle less than $180^{\circ}$.


South Pole ( $90^{\circ} \mathrm{S}$ )


When the position of a point is stated, the latitude is stated first and then the longitudewithout a comma between them. So the position of Mexico City, for example, is $19^{\circ} \mathrm{N} 99^{\circ} \mathrm{W}$.

## Example 1

Use the following diagram to find the latitude and longitude of Brisbane. The Prime Meridian and Equator are both shown on the diagram.


## Solution

Brisbane is shown as about $153^{\circ}$ to the east of the Prime Meridian and about $27^{\circ}$ south of the Equator. The position of Brisbane is $27^{\circ} \mathrm{S} 153^{\circ} \mathrm{E}$.

## Positions of places on the Earth

Most people cannot measure angles of latitude and longitude for themselves. To find the latitude and longitude of a place, we actually look up the position in an atlas. More precise information can also be obtained from government mapping authorities.

## Example

## 2

Use an atlas to find the latitude and longitude of Hong Kong.

## Solution

Large atlases may have the precise positions of major cities shown in the index. Otherwise you can find the position on a map of Southeast Asia. The position is $22^{\circ} \mathrm{N} 114^{\circ} \mathrm{E}$.

## Exercise 7.1

 Latitude, fongitude and position7.1 1 a Are all small circles the same length?
b Which latitude is the greatest length?
c What is at latitude $90^{\circ} \mathrm{S}$ ?
d Are all longitude meridians the same length?
2 The $40^{\circ}$ meridian of longitude passes through the Atlantic Ocean. Would it be east or west?
3 Use the map provided to match each of the countries named below with its corresponding latitude from the following list. (Consult an atlas if you are unsure of the positions of countries.)
$\begin{array}{llllllll}5^{\circ} \mathrm{S} & 30^{\circ} \mathrm{S} & 70^{\circ} \mathrm{N} & 55^{\circ} \mathrm{N} & 40^{\circ} \mathrm{N} & 40^{\circ} \mathrm{S} & 15^{\circ} \mathrm{N} & 25^{\circ} \mathrm{N}\end{array}$

a Spain
b Greenland
c South Africa
d Papua New Guinea
e Thailand
f New Zealand
g Canada
h Pakistan

4 Use the map provided to match each of the countries named below with its corresponding longitude from the following list. (Consult an atlas if you are unsure of the positions of countries.)

```
115\mp@subsup{5}{}{\circ}\textrm{E}
```



5 State the latitude and longitude of each point shown on the diagrams below. The Prime Meridian and Equator are shown on each diagram.

b



e





6 Use an atlas to find the latitude and longitude of each of the following cities.
a New York
b Adelaide
e New Delhi
i Tokyo
f Perth, WA
j Quito
c Townsville
g Nairobi
d Kingston, Jamaica
h Berlin

## Modelling and problem solving

7 The angle between the vertical and the direction of the Sun is called the azimuth. At point $P$ on the diagram, the azimuth is $A^{\circ}$. At midday on 21 March and 21 September the Sun is directly overhead at the Equator.
At this time in Brisbane, the azimuth is $27^{\circ}$.
a What is the latitude of Brisbane?
b What is the azimuth for New York $\left(41^{\circ} \mathrm{N} 74^{\circ} \mathrm{W}\right)$ at the same time?


### 7.2 Speed of rotation on the Earth

The Earth rotates on its axis once a day, from west to east. This means the Sun appears to move from the east to the west. Points on the surface of the Earth travel around a circle while the Earth rotates. At the Equator, the circle is a great circle, but at other latitudes the circle of rotation is a small circle. The radius of the Earth averages 6371 km , but the radius of the equatorial circle is slightly bigger ( 6378 km ). The radius of a small circle of rotation depends on its latitude.

## Example 3

What is the radius of the circle of rotation for a point at latitude $40^{\circ} \mathrm{N}$ ?

## Solution

Start by drawing a diagram showing the Equator, the latitude angle and the radius of the Earth.
Next draw a sketch using the triangle to work out the radius of the small circle of rotation, $r$.


Calculate $\theta$ using alternate angles.
$r$ is adjacent to $\theta$ and $R$ is the hypotenuse. Use $\cos \theta$ to calculate $r$.

Rewrite using information from the sketch.
Rearrange and substitute for $R$.
Find $\cos 40^{\circ}$.
Evaluate and round.
State the result.


$$
\begin{aligned}
\theta & =40^{\circ} \\
\cos \theta & =\frac{\text { adjacent }}{\text { hypotenuse }} \\
\cos 40^{\circ} & =\frac{r}{R} \\
r & =6371 \times \cos 40^{\circ} \\
& =6371 \times 0.7660 \ldots \\
& \approx 4880 \mathrm{~km}
\end{aligned}
$$

The radius of the circle of rotation at latitude $40^{\circ} \mathrm{N}$ is about 4880 km .

Once you know the radius of the circle of rotation, you can work out the distance travelled in a day and the speed of rotation of a point on the surface of the Earth on that circle.

## Example

Calculate the circumference of the circle of rotation of a point at latitude $40^{\circ} \mathrm{N}$ and the speed of rotation of points on this circle of rotation.

## Solution

From Example 3, the radius of the circle of rotation at latitude $40^{\circ} \mathrm{N}$ is about 4880 km . Write down the rule for circumference ( $C$ ).

$$
\begin{aligned}
C & =2 \pi r \\
C & =2 \times \pi \times 4880 \mathrm{~km} \\
& \approx 30662 \mathrm{~km}
\end{aligned}
$$

Substitute for $r$.
Evaluate and round.
Any point on the Earth travels around it in 24 hours.

Write down the rule for speed.
Substitute for known values.
Evaluate and round.
State the result.

$$
\begin{aligned}
\text { Speed } & =\frac{\text { distance }}{\text { time }} \\
& \approx 30662 \mathrm{~km} \div 24 \mathrm{~h} \\
& \approx 1278 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

A point at latitude $40^{\circ} \mathrm{N}$ has a circle of rotation of about 30662 km circumference and rotates at about $1278 \mathrm{~km} / \mathrm{h}$.

## Exercise 7.2 Speed of rotation on the Earth

Calculate the radius of the circle of rotation, its circumference and the speed of rotation for these places.
a Brisbane $\left(27^{\circ} \mathrm{S} 153^{\circ} \mathrm{E}\right)$
b Townsville ( $19^{\circ} \mathrm{S} 147^{\circ} \mathrm{E}$ )
c Cape Canaveral $\left(28^{\circ} \mathrm{N} 81^{\circ} \mathrm{W}\right)$
d Bamaga on Cape York ( $11^{\circ} \mathrm{S} 142^{\circ} \mathrm{E}$ )
e Hobart ( $\left.43^{\circ} \mathrm{S} 147^{\circ} \mathrm{E}\right)$
f Jakarta $\left(6^{\circ} \mathrm{S} 107^{\circ} \mathrm{E}\right)$

## Investigation Speeds at different latitudes

Work in groups of two or three for this activity.
1 Calculate the speeds of rotation of points at different latitudes $0^{\circ}, 10^{\circ}, 20^{\circ}, 30^{\circ}, \ldots, 90^{\circ}$ on the Earth's surface.
2 Draw up a table for speeds at these latitudes.
Now consider the following questions.

- At what latitude is the speed the greatest?
- Between what latitudes does the speed change the most?
- What are the implications of your table for launching a spacecraft?
- Why do you think the Cape Canaveral rocket-launching facility is positioned in the southern American state of Florida?
- What would be a suitable location for launching spacecraft in Australia? Why?


### 7.3 Distances on the Earth

The shortest distance between two points is a straight line. However, we cannot tunnel through the Earth to make a straight-line path between positions on the surface of the Earth. The shortest distance on the surface of the Earth will be directly above the line going through the Earth. It will be a curve following a great circle route, because the straight line is a chord of the great circle directly above it.


The shortest distance between any two points on Earth is part of a great circle.

## Investigation Great circle routes

1 Visit the websites of major international airlines to see if their air routes are displayed. Other websites, such as the Great Circle Mapper, http://gc.kls2.com/, allow you to find great circle air routes by identifying the departure and destination locations. This website was used to find the route from Brisbane to London as shown.


2 Try to find the routes from Hawaii to Sydney, San Francisco and Tokyo. What shape are the routes as they appear on the map?
3 Find the route from London to San Francisco. Why does it seem to go off the top of the map?
Now work with a partner, using a small globe.
4 Use a piece of string to find the shortest surface routes between different places on the Earth. What do you find about the routes?
5 Compare with your findings from examination of maps of air routes. What do you conclude?

## Did you know?

The Earth is round but it is also very bis - it has a circumference of about 40000 km . This means that for distances on thie Farh's surface of less than 20 km we can treat the Earth as being flat and use flate Earth formulas such as Pythagoras's Theorem to find the distance bitween points:

Points that lie on the same meridian are on a great circle. We can calculate the distance between them using the difference in their latitudes. Great circles have a radius of about 6371 km , so their circumference is

$$
2 \times \pi \times 6371 \approx 40030 \mathrm{~km}
$$

Travel through an angle of 1 degree on a great circle will involve a distance of $1 / 360$ th of the circumference. Dividing 40030 km by 360 , we find that:

On a great circle

$$
1^{\circ} \approx 111 \mathrm{~km}
$$

## Example

Find the distance between Auckland Island $\left(51^{\circ} \mathrm{S} 166^{\circ} \mathrm{E}\right)$ and Noumea $\left(22^{\circ} \mathrm{S} 166^{\circ} \mathrm{E}\right)$. Find the time it would take to travel in a boat averaging $15 \mathrm{~km} / \mathrm{h}$.

## Solution

Since the two positions have the same longitude they are on the same meridian, which is a great circle. The angle travelled is given by the difference in latitudes.
Calculate the difference in latitudes.

$$
\begin{aligned}
\text { Latitude difference } & =(51-22)^{\circ} \\
& =29^{\circ} \\
\text { Distance } & \approx 111 \times 29 \mathrm{~km} \\
& =3219 \mathrm{~km} \\
\text { Speed } & =\frac{\text { distance }}{\text { time }} \\
15 \mathrm{~km} / \mathrm{h} & \approx \frac{3219 \mathrm{~km}}{\text { time }} \\
\text { Time } & \approx \frac{3219 \mathrm{~km}}{15 \mathrm{~km} / \mathrm{h}} \\
& \approx 215 \mathrm{~h}
\end{aligned}
$$

Calculate the distance.
Evaluate
Write down rule for speed.
Substitute known values.

Rearrange.
Evaluate and round.
State the result.
The distance is about 3219 km and the trip would about take about 215 hours (almost 9 days).

You have previously seen how to calculate the radius of a parallel of latitude as a circle of rotation. The radius can be used to calculate the distance for travel through 1 degree of longitude along a parallel of latitude. This is also given by the following formula.

On a parallel of latitude
$1^{\circ} \approx 111.2 \cos \theta \mathrm{~km} \quad$ where $\theta$ is the angle of latitude.

## Example

Find the distance along the parallel of latitude between Bowen ( $20^{\circ} \mathrm{S} 148^{\circ} \mathrm{E}$ ) and Port Hedland ( $20^{\circ} \mathrm{S} 119^{\circ} \mathrm{E}$ ).

## Solution

Write the formula.
Substitute the latitude.
Calculate the difference in longitude.
Calculate the distance for a difference of $29^{\circ}$ of longitude at a latitude of $20^{\circ}$.
Evaluate and round.
State the result.

$$
\begin{aligned}
\text { Distance for } 1^{\prime \prime} & \approx 111.2 \cos \theta \mathrm{~km} \\
& =111.2 \cos 20^{\circ} \mathrm{km} \\
\text { Longitude diff. } & =(148-119)^{\circ}=29^{\circ} \\
\text { Distance } & \approx 29 \times 111.2 \times \cos 20^{\circ} \mathrm{km} \\
& \approx 29 \times 111.2 \times 0.9397 \mathrm{~km} \\
& \approx 3030 \mathrm{~km}
\end{aligned}
$$

It is about 3030 km along the parallel of latitude from Bowen to Port Hedland.

## Exercise 7.3 Distances on the Earth

1 Find the distance between Perth $\left(32^{\circ} \mathrm{S} 116^{\circ} \mathrm{E}\right)$ and Beijing $\left(40^{\circ} \mathrm{N} 116^{\circ} \mathrm{E}\right)$.
2 Find the distance between Nakano, Japan ( $37^{\circ} \mathrm{N} 138^{\circ} \mathrm{E}$ ) and Adelaide ( $35^{\circ} \mathrm{S} 138^{\circ} \mathrm{E}$ ).
3 What is the distance from Quebec $\left(47^{\circ} \mathrm{N} 71^{\circ} \mathrm{W}\right)$ to Santiago, Chile $\left(34^{\circ} \mathrm{S} 71^{\circ} \mathrm{W}\right)$ ? How long would it take to fly at $600 \mathrm{~km} / \mathrm{h}$ between these cities?

4 What is the distance from Woomera ( $31^{\circ} \mathrm{S} 137^{\circ} \mathrm{E}$ ) to Kalgoorlie ( $31^{\circ} \mathrm{S} 121^{\circ} \mathrm{E}$ ) along the parallel of latitude?
5 What is the distance between the Indian cities Aurangabad $\left(20^{\circ} \mathrm{N} 75^{\circ} \mathrm{E}\right)$ and Puri $\left(20^{\circ} \mathrm{N}\right.$ $86^{\circ} \mathrm{E}$ ) along the parallel of latitude?

6 What is the distance between Montevideo $\left(35^{\circ} \mathrm{S} 56^{\circ} \mathrm{W}\right)$ and Canberra $\left(35^{\circ} \mathrm{S} 149^{\circ} \mathrm{E}\right)$ along the parallel of latitude?
7 Find the time taken for a crested tern to fly from Flinders Island ( $40^{\circ} \mathrm{S} 148^{\circ} \mathrm{E}$ ) to King Island $\left(40^{\circ} \mathrm{S} 144^{\circ} \mathrm{E}\right)$ if it flies at $10 \mathrm{~km} / \mathrm{h}$ along the parallel of latitude.


8 A plane flying at $400 \mathrm{~km} / \mathrm{h}$ travelled directly north for 6 hours and 40 minutes before making an emergency landing. If the plane took off from Hobart $\left(43^{\circ} \mathrm{S} 147^{\circ} \mathrm{E}\right)$, where did it make the landing?
9 A balloon floats at an average speed of $12 \mathrm{~km} / \mathrm{h}$ on the prevailing breezes from Bordeaux $\left(45^{\circ} \mathrm{N} 1^{\circ} \mathrm{W}\right)$ to Krasnodar, Russia $\left(45^{\circ} \mathrm{N} 39^{\circ} \mathrm{E}\right)$. If it was released at 8:00 am, what was the time in Bordeaux when it came down in Russia?

## Investigation Shortest distances

You have seen how to calculate distances along meridians and along parallels of latitude. The distance along a parallel of latitude is not actually the shortest distance between points. That is always along a great circle route. The formula below can be used to calculate the great circle distance $d$ between any two points $A\left(\operatorname{Lat}_{1} \mathrm{Lon}_{1}\right)$ and $B\left(\mathrm{Lat}_{2} \mathrm{Lon}_{2}\right)$.

$$
d \approx 111.2 \times \cos ^{-1}\left[\cos (\Delta \mathrm{Lon}) \cos \mathrm{Lat}_{1} \cos \mathrm{Lat}_{2}+\sin \mathrm{Lat}_{1} \sin \mathrm{Lat}_{2}\right]
$$

$\Delta \mathrm{Lon}$ is the difference between longitudes. $\Delta \mathrm{L}$ on must be less than $180^{\circ}$.
For example, the shortest distance between Flinders Island ( $40^{\circ} \mathrm{S} 148^{\circ} \mathrm{E}$ ) and Perth $\left(32^{\circ} \mathrm{S} 116^{\circ} \mathrm{E}\right)$ is given by:

$$
\begin{aligned}
d & \approx 111.2 \times \cos ^{-1}\left[\cos \left(148^{\circ}-116^{\circ}\right) \times \cos 40^{\circ} \times \cos 32^{\circ}+\sin 40^{\circ} \times \sin 32^{\circ}\right] \\
& \approx 111.2 \times \cos ^{-1}\left[\cos 32^{\circ} \times \cos 40^{\circ} \times \cos 32^{\circ}+\sin 40^{\circ} \times \sin 32^{\circ}\right] \\
& \approx 111.2 \times \cos ^{-1}[0.550928+0.340626] \\
& \approx 111.2 \times 26.930868 \\
& \approx 2995
\end{aligned}
$$

So the distance from Flinders Island to Perth is about 2995 km .
1 Work with a partner to calculate the shortest distances between different positions on the Earth. Compare the distance along the parallel of latitude with the shortest distance between:

- Bowen $\left(20^{\circ} \mathrm{S} 148^{\circ} \mathrm{E}\right)$ and Port Hedland $\left(20^{\circ} \mathrm{S} 119^{\circ} \mathrm{E}\right)$
- Shanghai $\left(31^{\circ} \mathrm{N} 121^{\circ} \mathrm{E}\right)$ and Jerusalem $\left(32^{\circ} \mathrm{N} 35^{\circ} \mathrm{E}\right)$
- Canberra $\left(35.5^{\circ} \mathrm{S} 149^{\circ} \mathrm{E}\right)$ and Montevideo $\left(35^{\circ} \mathrm{S} 56^{\circ} \mathrm{W}\right)$
- Los Angeles $\left(34^{\circ} \mathrm{N} 118^{\circ} \mathrm{W}\right)$ and Tokyo $\left(36^{\circ} \mathrm{N} 140^{\circ} \mathrm{E}\right)$

2 Discuss the differences in distance with your partner.

### 7.4 Dax

The week was originally part of the Jewish calendar but is now used throughout the world as a convenient unit of time. In different cultures the days have had different names. The present English names are derived from the names of the Sun, Moon, planets and Norse gods. The days of the week gradually cycle through the dates of the year, so that New Year's Day can fall on any day of the week, depending on the year. A perpetual calendar shows the days of the week for any date in any year. There are many perpetual calendar websites, such as www.calendarhome.com/tycl, which has a 10000 -year calendar.

## Investigation Days of the week

1 Work in groups of three or four to find the day of the week:
a for your birthday this year
b for your birthday next year
c on which you were actually born
d on which Christmas ( 25 December) will fall this year
e on which Australia Day (26 January) fell this year
f on which the summer solstice falls this year.
2 Use a perpetual calendar to check your answers to question 1.

## Did you krow?

The ancient Babylomans believed that the complete cycle of seasons lasted 360 days.
Because they dedured that seasons were like circles, they divided circles into the 360 sections we now real deggrees. Of course, we know chat a year (the gycle of the seasons) is a litale more than 365 days, but the slight eroo of the Batylonians was the basis of moden zeomethe

There are 365 days in a year. Because $365=52 \times 7+1$ and $366=52 \times 7+2$, the day of the week on which New Year's Day falls moves through the week as the years advance. If the previous year was a leap year, it advances by 2 days; otherwise it advances by 1 day.

## Example

## 7

In 2001, New Year's Day was a Monday. On what day of the week will it occur in 2007?

## Solution

It is easiest to work it out year by year.
2002-It moves on 1 day to Tuesday.
2003-It moves on 1 day to Wednesday.
2004-Thursday
2005-It moves on 2 days because the previous year is a leap year, so it will be Saturday.
2006-Sunday
2007-Monday
So New Year's Day in the year 2007 will be on a Monday.

## Exercise 7.4 Days of the week

1 Work out the day of the week for New Year's Day in:
a 2010
b 1990
c 1975
d 1970
e 1950

2 Work out the day of the week for Christmas Day in:
a 2009
b 1989
c 1974

3 Find the day of the week for your birthday this year. Work out the day for your birthday in:
a 2012
b 1993
c 2040

## 75 sumkise, sumset and the lengths, of daxs

The Earth rotates on an imaginary axis passing through the North and South Poles. This axis is tilted at about $23.5^{\circ}$ to the vertical. The tilt in the axis causes the seasons because it permits the Sun's rays to shine more directly and for longer periods on certain locations on the Earth's surface at different times of the year. In this way, when it is summer in the Northern Hemisphere the Sun's rays are more directly overhead and more intense than in the Southern Hemisphere, where it is winter at this time.


The length of the day changes during the year. The lengths of day and night are the same for an equinox. For the Southern Hemisphere, the autumn equinox occurs on 21 March and the spring (vernal) equinox occurs on 22 September. The longest and shortest days of the year occur at the summer and winter solstices- 22 December and 21 June respectively in the Southern Hemisphere.


The following table gives the approximate day length for various latitudes at different times of the year.

Approximate day length
Southern Hemisphere: read down

| Latitude | 22 September | 22 December | 21 March | 21 June |
| :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 12 h | 12.0 h | 12 h | 12.0 h |
| $10^{\circ}$ | 12 h | 12.6 h | 12 h | 11.4 h |
| $20^{\circ}$ | 12 h | 13.2 h | 12 h | 10.8 h |
| $30^{\circ}$ | 12 h | 13.9 h | 12 h | 10.1 h |
| $40^{\circ}$ | 12 h | 14.9 h | 12 h | 9.1 h |
| $50^{\circ}$ | 12 h | 16.3 h | 12 h | 7.7 h |
| $60^{\circ}$ | 12 h | 18.4 h | 12 h | 5.6 h |
| $70^{\circ}$ | 12 h | 2 months | 12 h | 0 h |
| $80^{\circ}$ | 12 h | 4 months | 12 h | 0 h |
| $90^{\circ}$ | 12 h | 6 months | 12 h | 0 h |
| Latitude | 21 March | 21 June | 22 September | 22 December |

## Example 8

What is the approximate day length in Camooweal $\left(20^{\circ} \mathrm{S} 138^{\circ} \mathrm{E}\right)$ in the middle of winter?

## Solution

Camooweal is in the Southern Hemisphere, so the middle of winter is at 21 June.
Look up the table of day lengths.
Day length $=10.8 \mathrm{~h}$

The exact lengths of the days change in a very complicated way between these extremes because of the fact that the speed of the Earth in its orbit is not constant. The lengths of the longest and shortest days depend on the latitude.

For a latitude $\theta<66.5^{\circ}$ :
Daylight minutes of shortest day $\approx 8 \times \cos ^{-1}(0.4338 \times \tan \theta)$
Daylight minutes of longest day $\approx 1440-8 \times \cos ^{-1}(0.4338 \times \tan \theta)$

These formulas do not take the width of the Sun's disc or the refraction of the atmosphere into account. A correction of about 10 minutes (longer) will account for these factors. At latitudes greater than $66.5^{\circ}$, the longest day is actually 24 hours and the shortest day is 0 hours. Hence the Arctic region is sometimes called the Land of the Midnight Sun.


Summer solstice, Mawson, Antarctica

## Example

## 9

What are the lengths of the shortest and longest days in Brisbane, at latitude $27^{\circ} \mathrm{S}$ ?

## Solution

State the rule for shortest day. Replace $\theta$ with $27^{\circ}$, evaluate and round.
Add 10-minute correction.
Convert to hours and minutes.
State the rule for longest day.
Replace $\theta$ with $27^{\circ}$, evaluate and round.
Add 10-minute correction.

Convert to hours and minutes.

$$
\begin{aligned}
\text { Shortest day } & \approx 8 \times \cos ^{-1}(0.4338 \times \tan \theta) \mathrm{min} \\
& \approx 618 \mathrm{~min} \\
\text { Corrected time } & \approx 618+10=628 \mathrm{~min} \\
& =10 \mathrm{~h} 28 \mathrm{~min} \\
\text { Longest day } & \approx 1440-8 \times \cos ^{-1}(0.4338 \times \tan \theta) \mathrm{min} \\
& \approx 1440-618 \mathrm{~min} \\
& =822 \mathrm{~min} \\
\text { Corrected time } & \approx 822+10 \mathrm{~min} \\
& =832 \mathrm{~min} \\
& =13 \mathrm{~h} \mathrm{52} \mathrm{~min}
\end{aligned}
$$

Solar noon is the time when the Sun is at its highest point in the sky. It is not always at 12 o'clock because of variation in the Earth's orbit and differences in longitude between places in the same time zone. However, solar noon is always halfway between sunrise and sunset. If you know the latitude and time when the Sun rises, from the length of the day you can work out the time when the Sun will set and the time when it will be highest in the sky.

## Example 10

On 3 April the Sun rises at 5:59 am in Brisbane and the day is 11 h 46 min in length. At what times are solar noon and sunset?

## Solution

Calculate the half-day length.

$$
\text { Length of } \begin{aligned}
\frac{1}{2} \text { day } & =\frac{1}{2} \text { of } 11 \mathrm{~h} 46 \mathrm{~min} \\
& =5 \mathrm{~h} 53 \mathrm{~min}
\end{aligned}
$$

Since solar noon is halfway through the day, it will occur 5 h 53 min after sunrise.
Calculate when solar noon occurs.

$$
\begin{aligned}
\text { Solar noon } & =5: 59 \mathrm{am}+5 \mathrm{~h} 53 \mathrm{~min} \\
& =11: 52 \mathrm{am} \\
\text { Sunset } & =5: 59 \mathrm{am}+11 \mathrm{~h} 46 \mathrm{~min} \\
& =5: 45 \mathrm{pm}
\end{aligned}
$$

Calculate when sunset occurs.

## Exercise 7.5 <br> 

1 Use the table on page 195 to find the approximate day length in:
a Brussels $\left(50^{\circ} \mathrm{N}\right)$ on 22 December
b Walgett $\left(30^{\circ} \mathrm{S}\right)$ on 21 June
c Quito $\left(0^{\circ}\right)$ in late June
d Flinders Island $\left(40^{\circ} \mathrm{S}\right)$ in the middle of the southern winter
e Bowen $\left(20^{\circ} \mathrm{S}\right)$ in the middle of the southern summer
f Cairo $\left(30^{\circ} \mathrm{N}\right)$ in the middle of the northern winter
g Oslo $\left(60^{\circ} \mathrm{N}\right)$ in the middle of the northern summer
2 a What are the lengths of the shortest and longest days in Hobart ( $43^{\circ} \mathrm{S}$ )?
b If sunrise is at 7:48 am on the shortest day in Hobart, at what times are solar noon and sunset?
3 a What are the lengths of the shortest and longest days on Cape York ( $11^{\circ} \mathrm{S}$ )?
b If sunrise is at $5: 25$ am on the longest day on Cape York, when is sunset?
4 What are the lengths of the shortest and longest days at the Equator?
5 What are the lengths of the shortest and longest days in London, England $\left(51^{\circ} \mathrm{N}\right)$ ?
6 What is the difference between the shortest and longest days in Anchorage, Alaska ( $61^{\circ} \mathrm{N}$ )?

## Modelling and problem solving

7 Which city receives more intense sunlight in June: Sydney ( $34^{\circ} \mathrm{S} 151^{\circ} \mathrm{E}$ ) or Quebec ( $47^{\circ} \mathrm{N} 71^{\circ} \mathrm{W}$ )? Why?
8 What would happen to the seasons if the Earth were tilted $40^{\circ}$ instead of its current $23.5^{\circ}$ ?
9 What would happen to the seasons if the Earth were tilted at $23.5^{\circ}$ in the opposite direction?

### 7.6 Time zones on the Earth

The side of the Earth facing the Sun is illuminated, so it is daylight on that side. As the Earth spins, different parts turn to face the Sun. The circle of illumination sweeps around the Earth once a day, bringing dawn at one edge and sunset at the other.

Late afternoon in the Australian summer


We find it convenient to have our clocks set so that dawn, solar noon and sunset are at about the same time each day. However, when it is dawn in Brisbane it is still dark in Toowoomba. It is dawn in Toowoomba about 4 minutes later. It is another 20 minutes before it is dawn in Townsville, and it is dawn in Melbourne about half an hour after Brisbane. It would be silly to have clocks set to different times in every town down the eastern coast of Australia.
Australia is divided into three time zones. In addition, some States change the clocks in summer to include daylight saving, but this is not a standard time zone. The eastern and western Australian standard time zones are part of an international system of time zones.
The Eastern Standard Time zone covers the whole of Queensland, New South Wales, Victoria and Tasmania. It is 10 hours ahead of Greenwich Mean Time. Internationally, Australian Eastern Standard Time is abbreviated to AEST. The Central Standard Time zone covers South Australia and the Northern Territory but it is not an international standard time zone. It is only half an hour behind Eastern Standard Time. The Western Standard Time zone is 2 hours behind Eastern Standard Time and covers Western Australia. It is 8 hours ahead of Greenwich and is an international standard time zone. Australian time zones are shown below.


Since $360^{\circ} \div 24=15^{\circ}$, each hour corresponds to a difference in longitude of $15^{\circ}$; and since $24 \times 60 \div 360^{\circ}=4$, each degree of longitude corresponds to a time difference of 4 minutes.

## $1 \mathrm{~h} \equiv 15^{\circ}$ of longitude $1^{\circ}$ of longitude $\equiv 4$ minutes

The world is divided into standard time zones with the clocks set 1 hour apart in neighbouring time zones.


The clocks in a time zone are set so that solar noon is at 12 o'clock in the middle of the zone. The date is changed at the International Date Line, which passes through the middle of the Pacific Ocean. All time zones are referred to Greenwich (at $0^{\circ}$ longitude). Areas with eastern longitudes are ahead of Greenwich, and areas with western longitudes are behind Greenwich. Throughout the world, time zone boundaries are modified to state and country boundaries. There are a few countries, such as Saudi Arabia, which do not use the appropriate standard time zone. A map of the time zones of different areas of the world is often given in an atlas. Time differences for various countries are also listed in the back of the White Pages telephone directory to help you avoid ringing people overseas at inappropriate times. It lists the time difference for each country in the form AEST - $x$ hours. For example, Denmark is AEST - 9 hours and the USA is AEST - 15-21 hours. The diagram on page 200 shows world time zones referred to Greenwich.

## Investigation Time zones and the telephone

The theoretical time difference between two places on the Earth can be worked out from the difference in longitude. Every degree of longitude makes a difference of 4 minutes. The difference in longitude between Townsville $\left(19^{\circ} \mathrm{S} 146^{\circ} \mathrm{E}\right)$ and Wellington, New Zealand $\left(41^{\circ} \mathrm{S} 174^{\circ} \mathrm{E}\right)$ is $174^{\circ}-146^{\circ}=28^{\circ}$. Thus the theoretical time difference between Townsville and Wellington is $28 \times 4$ minutes $=112$ minutes. The time difference shown in the White Pages is 2 hours.
1 Work in groups of three or four with an atlas and a copy of the White Pages to find the theoretical and actual time differences between your area and New York, London, Paris, Moscow, Tokyo, Jakarta, San Francisco and Johannesburg.
There are many websites that give the time and date of places relative to Greenwich Mean Time, such as www.timeanddate.com/worldclock/ and www.worldtimezone.com/.

2 Use a website of your choice to investigate the time differences between places of interest to you.


World times when it is $5: 26 \mathrm{am}$ on Thursday in Queensland.
Greenwich

The following example demonstrates that differences in time zones need to be taken into consideration when travelling long distances.

## Example 11

It takes $6 \frac{1}{2}$ hours to fly from Brisbane to Perth.
a If you take off at 8 am in Brisbane, at what time will you arrive in Perth?
b If you take off at 8 am in Perth, at what time will you arrive in Brisbane?

## Solution

> a Calculate the arrival time in Perth.
> This is Brisbane time.
> Perth is 2 hours behind Brisbane time.
> Adjust for the time difference.
> State the result.

b Calculate the arrival time in Brisbane. This is Perth time.
Brisbane is 2 hours ahead of Perth time.
Adjust for the time difference.
State the result.

$$
\begin{aligned}
\text { Arrival time } & =8 \mathrm{am}+6 \frac{1}{2} \mathrm{~h} \\
& =2: 30 \mathrm{pm} \\
\text { Perth time } & =2: 30 \mathrm{pm}-2 \mathrm{~h} \\
& =12: 30 \mathrm{pm}
\end{aligned}
$$

The flight will arrive in Perth at $\mathbf{2 : 3 0} \mathbf{~ p m}$ Brisbane time or 12:30 pm local time.

$$
\begin{aligned}
\text { Arrival time } & =8 \mathrm{am}+6 \frac{1}{2} \mathrm{~h} \\
& =2: 30 \mathrm{pm} \\
\text { Brisbane time } & =2: 30 \mathrm{pm}+2 \mathrm{~h} \\
& =4: 30 \mathrm{pm}
\end{aligned}
$$

The flight will arrive in Brisbane at 2:30 pm Perth time or 4:30 pm local time.

International air flights are sufficiently fast to cause problems with time zones. It can be confusing for air travellers when their flights cross the International Date Line. It is possible to land in Los Angeles 'before' taking off in Brisbane-provided you refer to local time. If you visit the website of Qantas (www.gantas.com.au/) or other international airlines you will be able to compare departure and arrival dates and times for overseas flights.

## Example 12

A flight from Brisbane to Los Angeles takes 17 hours including a stopover at Honolulu. If the flight leaves Brisbane at $1: 30 \mathrm{pm}$ on Sunday, when will it arrive in Los Angeles:
a in Brisbane time?
b in Los Angeles time?

## Solution

a Calculate the arrival time in Los Angeles. This is Brisbane time.
State the result.

$$
\begin{aligned}
\text { Arrival time } & =1: 30 \mathrm{pm} \text { Sun }+17 \mathrm{~h} \\
& =6: 30 \mathrm{am} \text { Mon }
\end{aligned}
$$

The flight will arrive in Los Angeles at 6:30 am on Monday-Brisbane time.
b Convert Brisbane time to Los Angeles (local) time. Los Angeles is on the western coast of the USA. Look up the time zone diagram on page 200.
Calculate the time difference $\quad$ Time difference $=(-8)-(+10)$ between Brisbane and Los Angeles. $\quad=\mathbf{- 1 8}$ hours
This means that Los Angeles is 18 hours behind Brisbane time.
Convert arrival time to local time.

$$
\begin{aligned}
\text { Arrival time } & =6: 30 \mathrm{am} \text { Mon }-18 \text { hours } \\
& =12: 30 \mathrm{pm} \text { Sun }
\end{aligned}
$$

State the result.
The flight will arrive in Los Angeles at 12:30 pm on Sunday-Los Angeles time.

Example 12 shows that travellers on the flight between Brisbane and Los Angeles arrive in Los Angeles an hour before leaving Brisbane (in local time)! Travelling across time zones may cause people to suffer from 'jet lag', which can make it difficult for them to adjust their sleep patterns.

## Exercise 7.6 lime zones on the Earth <br> You will need to refer to detailed time zone diagrams to answer the following questions. Depending on your knowledge of geography, you may also need to refer to a world atlas to find the time zones of various locations.

1 Calculate the time in Queensland when it is:
a 5 am in Perth, WA
b $7: 30 \mathrm{pm}$ in Adelaide
c 12 noon in Hobart
d $6: 15 \mathrm{pm}$ in London
e 10 pm in Auckland.

2 An email is sent at 9:00 am on Monday from Brisbane. Assuming that there are no transmission delays, work out the time and day when it will arrive in:
a Auckland
b London
c Berlin
d Los Angeles
e New York
f Tokyo
g Moscow
h Hyderabad, Pakistan.

## Modelling and problem solving

3 A plane leaves Brisbane at 7:00 am on Saturday for Narita airport, Tokyo. The direct flight takes 9 hours 30 minutes. When it arrives, what will be the day and time:
a in Brisbane?
b in Tokyo?

4 A plane leaves Sydney at 1:20 pm on Tuesday for Heathrow airport, London. The flight takes 25 hours 45 minutes including a stopover at Changi airport, Singapore. When it arrives, what will be the day and time:
a in Sydney?
b in London?

5 Answer the following questions in local time (i.e. the time at the destination).
a It takes $8 \frac{1}{2}$ hours to fly from Sydney to Honolulu. If a plane leaves Sydney at $4: 30 \mathrm{pm}$, at what time will it arrive in Honolulu? Remember the International Date Line!
b It takes 4 hours and 45 minutes to fly from San Francisco to Honolulu. If an American surfer leaves San Francisco at 8 am , at what time will her flight reach Honolulu?
c It takes 7 hours 50 minutes to fly from Hong Kong to Sydney. If a flight leaves Hong Kong at 8 pm , at what time will it arrive in Sydney?
d A direct flight from Los Angeles to Brisbane takes 12 hours and 25 minutes. At what time will a flight leaving Los Angeles at 10 am arrive in Brisbane?
e It takes $24 \frac{3}{4}$ hours, including stopovers, to fly from Frankfurt in Germany to Brisbane. When would a flight leaving Frankfurt at 7 am on Wednesday arrive in Brisbane?
f A flight leaves Brisbane at $4: 30 \mathrm{pm}$ on Tuesday and flies to Rome. When does it arrive if the flight takes 23 hours 45 minutes (including stopovers).

## New QMaths 11A CD-ROM

## Summary: chapter 7

## Summary 7

- Great circles are drawn on the surface so that their centre is at the centre of the Earth. Small circles do not have their centre at the centre of the Earth.

- Parallels of latitude are small circles drawn parallel (north and south) to the Equator ( $0^{\circ}$ ). Meridians of longitude are great semicircles drawn between the North and South Poles. The Prime Meridian ( $0^{\circ}$ ) passes through Greenwich, England. The position of a point on the Earth is stated in terms of degrees north or south of the Equator and east or west of the Prime Meridian.

- The Earth rotates on its axis once a day, from west to east. The speed of rotation of points on the surface of the Earth decreases the further they are away from the Equator.
- The radius of the Earth varies but averages 6371 km .
- The shortest distance on the surface of the Earth is part of a great circle. On a great circle, $1^{\circ} \approx 111 \mathrm{~km}$. On a parallel of latitude, $1^{\circ} \approx 111.2 \cos \theta$ where $\theta$ is the angle of latitude.
- The length of the day changes during the year. The lengths of day and night are the same for an equinox ( 21 March and 22 September), while the shortest and longest days occur at the summer and winter solstices ( 22 December and 21 June respectively in the Southern Hemisphere).
- The exact lengths of the days change in a very complicated way between these extremes because the speed of the Earth in its orbit is not constant. The lengths of the longest and shortest days vary depending on the latitude $(\theta)$. For a latitude $\theta<66.5^{\circ}$ :

Daylight minutes of shortest day $\approx 8 \times \cos ^{-1}(0.4338 \times \tan \theta)$
Daylight minutes of longest day $\approx 1440-8 \times \cos ^{-1}(0.4338 \times \tan \theta)$

- The world is divided into standard time zones with the clocks set 1 hour apart in neighbouring time zones. Each hour corresponds to a difference in longitude of $15^{\circ}$, and each $1^{\circ}$ of longitude corresponds to a time difference of 4 minutes. The date changes at the International Date Line, which passes through the middle of the Pacific Ocean. All time zones are referred to Greenwich (at $0^{\circ}$ longitude), with eastern longitudes 'ahead' of Greenwich and western longitudes 'behind' Greenwich.
- Australia is divided into three time zones: Eastern Standard, Central Standard and Western Standard. Some states change the clocks in summer to include daylight saving, but this is not a standard time zone.


## CHAPTER 7

## Exercise 7.1

1 a No-they become smaller as you move away from the Equator.
b The Equator is the longest.
c The South Pole
d Yes-they are all great semicircles.
2 West

| 3 a $40^{\circ} \mathrm{N}$ | b $70^{\circ} \mathrm{N}$ | c $30^{\circ} \mathrm{S}$ | d $5^{\circ} \mathrm{S}$ |
| :---: | :--- | :--- | :--- |
| e $15^{\circ} \mathrm{N}$ | f $40^{\circ} \mathrm{S}$ | g $55^{\circ} \mathrm{N}$ | h $25^{\circ} \mathrm{N}$ |
| 4 a $0^{\circ}$ | b $30^{\circ} \mathrm{E}$ | c $175^{\circ} \mathrm{E}$ | d $40^{\circ} \mathrm{W}$ |
| e $140^{\circ} \mathrm{E}$ | f $45^{\circ} \mathrm{W}$ | g $70^{\circ} \mathrm{W}$ | h $100^{\circ} \mathrm{W}$ |
| i $115^{\circ} \mathrm{E}$ | j $100^{\circ} \mathrm{E}$ |  |  |
| 5 a $50^{\circ} \mathrm{N} 78^{\circ} \mathrm{E}$ | b $20^{\circ} \mathrm{N} 35^{\circ} \mathrm{W}$ |  |  |
| c $72^{\circ} \mathrm{N} 40^{\circ} \mathrm{W}$ | d $40^{\circ} \mathrm{S} 95^{\circ} \mathrm{E}$ |  |  |
| e $0^{\circ} \mathrm{N} 86^{\circ} \mathrm{E}$ | f $35^{\circ} \mathrm{S} 0^{\circ} \mathrm{E}$ |  |  |
| g $52^{\circ} \mathrm{S} 100^{\circ} \mathrm{W}$ | h $60^{\circ} \mathrm{N} 28^{\circ} \mathrm{W}$ |  |  |
| 6 a $41^{\circ} \mathrm{N} 74^{\circ} \mathrm{W}$ | b $35^{\circ} \mathrm{S} 139^{\circ} \mathrm{E}$ |  |  |
| c $19^{\circ} \mathrm{S} 147^{\circ} \mathrm{E}$ | d $18^{\circ} \mathrm{S} 77^{\circ} \mathrm{W}$ |  |  |
| e $29^{\circ} \mathrm{N} 77^{\circ} \mathrm{E}$ | f $32^{\circ} \mathrm{S} 116^{\circ} \mathrm{E}$ |  |  |
| g $1^{\circ} \mathrm{S} 37^{\circ} \mathrm{E}$ | h $53^{\circ} \mathrm{N} 13^{\circ} \mathrm{E}$ |  |  |
| i $36^{\circ} \mathrm{N} 140^{\circ} \mathrm{E}$ | j $0^{\circ} \mathrm{S} 79^{\circ} \mathrm{W}$ |  |  |
| 7 a $27^{\circ} \mathrm{S}$ | b $41^{\circ}$ |  |  |

## Exercise 7.2

a $5677 \mathrm{~km}, 35667 \mathrm{~km}, 1486 \mathrm{~km} / \mathrm{h}$
b $6024 \mathrm{~km}, 37849 \mathrm{~km}, 1577 \mathrm{~km} / \mathrm{h}$
c $5625 \mathrm{~km}, 35345 \mathrm{~km}, 1473 \mathrm{~km} / \mathrm{h}$
d $6254 \mathrm{~km}, 39295 \mathrm{~km}, 1637 \mathrm{~km} / \mathrm{h}$
e $4659 \mathrm{~km}, 29276 \mathrm{~km}, 1220 \mathrm{~km} / \mathrm{h}$
f $6336 \mathrm{~km}, 39811 \mathrm{~km}, 1659 \mathrm{~km} / \mathrm{h}$

## Exercise 7.3

|  | 7992 km | 27992 km |
| :---: | :---: | :---: |
| 38991 km ; about 15 h |  |  |
| 4 | 1525 km | 51149 km |
| 6 | 14118 km | 734 h |
| $819^{\circ} \mathrm{S} 147^{\circ} \mathrm{E}$ (Townsville) |  |  |
|  | About 6 am, 11 days | after its relea |

## Exercise 7.4

| 1 a Friday | b Monday |
| :--- | :--- |
| c Wednesday | d Thursday |
| e Sunday |  |
| 2 a Friday | b Monday |
| c Wednesday |  |

3 Answers will vary-check a perpetual calendar.

## Exercise 7.5

1 a $7.7 \mathrm{~h} \quad$ b $10.1 \mathrm{~h} \quad$ c $12 \mathrm{~h} \quad \mathrm{~d} 9.1 \mathrm{~h}$
e 13.2 h f $10.1 \mathrm{~h} \quad \mathrm{~g} 18.4 \mathrm{~h}$
2 a $539 \mathrm{~min}=8 \mathrm{~h} 59 \mathrm{~min}$ and $921 \mathrm{~min}=15 \mathrm{~h}$ 21 min
b About $12: 17 \mathrm{pm}$ and $4: 47 \mathrm{pm}$
3 a $691 \mathrm{~min}=11 \mathrm{~h} 31 \mathrm{~min}$ and $769 \mathrm{~min}=12 \mathrm{~h} 49 \mathrm{~min}$
b 6:14 pm
4 They are the same: $730 \mathrm{~min}=12 \mathrm{~h} 10 \mathrm{~min}$.
$5471 \mathrm{~min}=7 \mathrm{~h} 51 \mathrm{~min}$ and $989 \mathrm{~min}=16 \mathrm{~h} 29 \mathrm{~min}$
$6824 \mathrm{~min}=13 \mathrm{~h} 44 \mathrm{~min}$
7 The sunlight is more intense at Quebec as it is in the Northern Hemisphere, having summer, because the Earth is tilted so that the Sun is more directly overhead there.
8 The seasons would be more pronounced. Latitudes greater than $40^{\circ}$ would be in 24 hours of darkness or sunlight at the winter and summer solstices respectively.
9 The seasons would be reversed. The winter and summer solstices would occur on 22 December and 21 June respectively in the Southern Hemisphere.

## Exercise 7.6

1 a 7 am $\begin{array}{ll}\text { c } 12 \text { noon } & \text { b } 8 \mathrm{pm} \\ \text { e } 8 \mathrm{pm} & \text { d 4:15 am the next day }\end{array}$
2 a 11:00 am Monday b 11:00 pm Sunday
c Midnight Sunday d 3:00 pm Sunday
e 6:00 pm Sunday
g 2:00 am Monday $8: 00 \mathrm{am}$ Monday
3 a $4: 30 \mathrm{pm}$ Saturday
b $3: 30 \mathrm{pm}$ Saturday
4 a $3: 05 \mathrm{pm}$ Wednesday
b 5:05 am Wednesday
5 a 5:00 am the same day
b $10: 45$ am the same day
c $5: 50$ am the next day
d $4: 25 \mathrm{pm}$ the next day
e $4: 45 \mathrm{pm}$ Thursday
f 7:15 am Wednesday

