Lattice QCD approach to the strangeness S=-2 two-baryon system

Kenji Sasaki (*CCS, University of Tsukuba*)

for HAL QCD collaboration

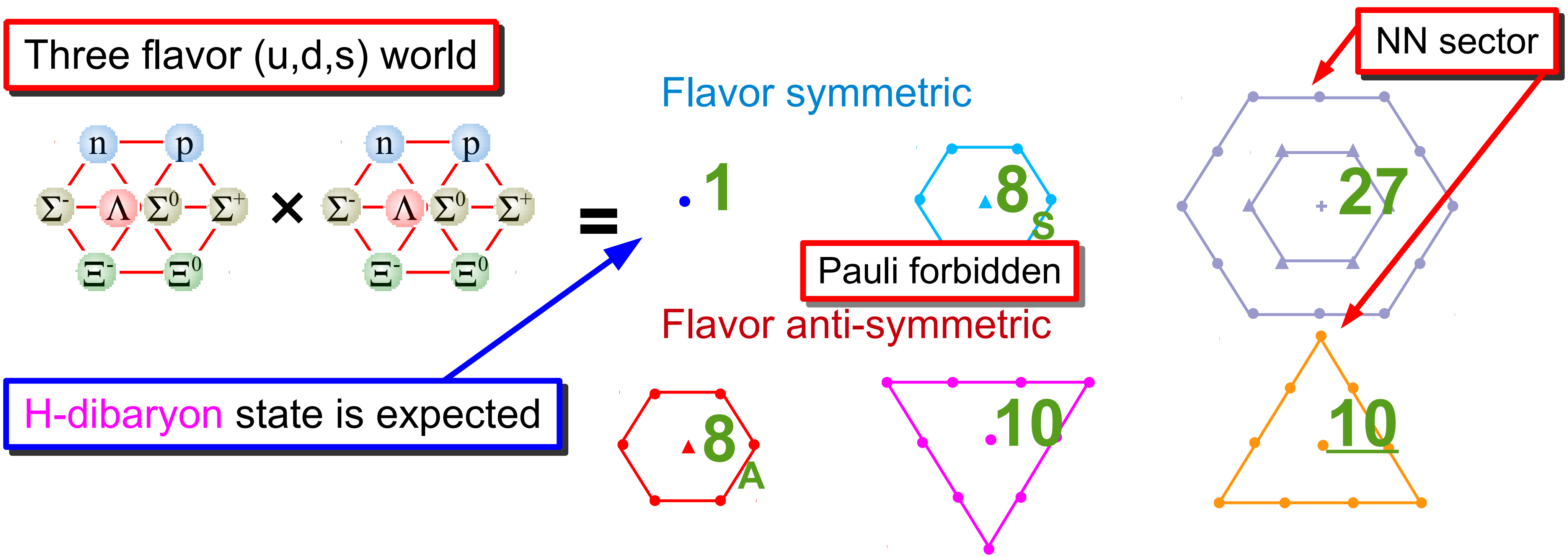
HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (*Univ. of Tsukuba*)	**B. Charron** (*Univ. of Tokyo*)	**T. Doi** (*RIKEN*)	**F. Etminan** (*Univ. of Tsukuba*)	**T. Hatsuda** (*RIKEN*)
Y. Ikeda (*Tokyo Inst. Tech.*)	**T. Inoue** (*Nihon Univ.*)	**N. Ishii** (*Univ. of Tsukuba*)	**K. Murano** (*RIKEN*)	**H. Nemura** (*Univ. of Tsukuba*)
M. Yamada (*Univ. of Tsukuba*)				

Introduction

It is interesting to study baryon-baryon interactions with S=-2

It brought us the deeper understanding of BB interaction.



- All irreducible representations are involved in S=-2 BB system
- Experimentally difficult to investigate

"H-dibaryon"

- R.L. Jaffe predict the flavor singlet (uds)x(uds) state with J=0.

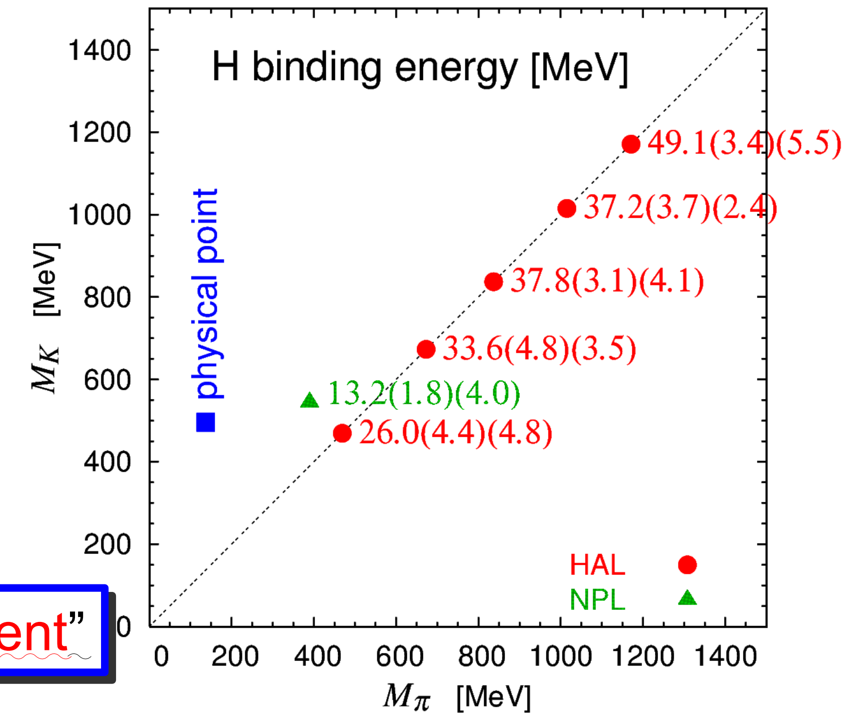
Recent Lattice QCD studies

- HAL QCD: SU(3) limit

  BE = 26MeV $m\pi$ = 470MeV

- NPLQCD: SU(3) breaking

  BE = 13MeV $m\pi$ = 390MeV



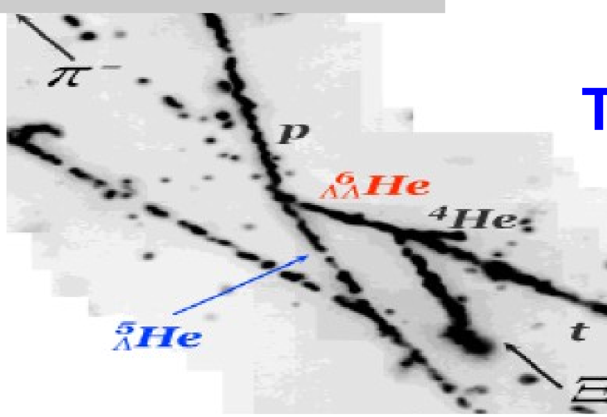
Experimental constraint by the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV



Toward the physical point

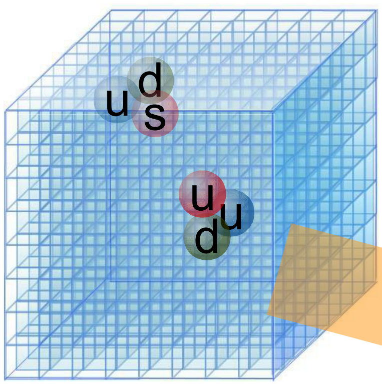
QCD to hadronic interactions

HAL QCD method can derive baryon-baryon potential directly from QCD

QCD Lagrangian

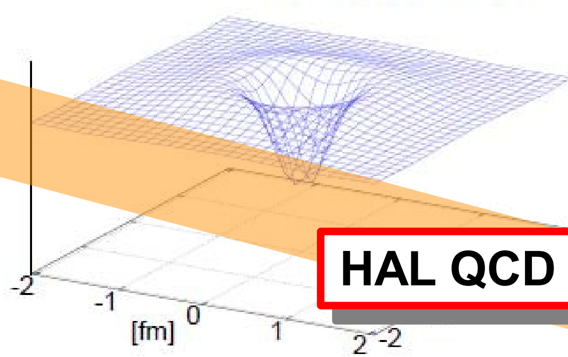
$$L_{QCD}=\bar{q}(i\gamma_\mu D^\mu - m)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation



2. Put NBS wave function in Schroedinger eq

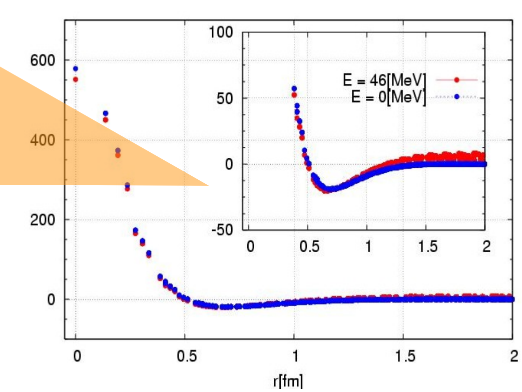
NBS wave function



The potential through our method reproduce the phase shift by QCD

HAL QCD method

BB interaction (potential)



1. Measure NBS wave function on the lattice

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99** (2007) 022001

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi_\nu(E, t-t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x}+\vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle$$

E : Total energy of system

ν : other observables which needs to form the complete set

Four point correlator

$$F_{B_1 B_2}(\vec{r}, t) = \langle 0 [T [B_1(\vec{r}, t) B_2(0, t) (\bar{B}_2 \bar{B}_1)_{t_0}] | 0 \rangle = \sum_n A_n \Psi_n e^{-E_n t}$$

Local composite interpolating operators

$$p = udu \quad n = udd \quad \Xi^0 = sus \quad \Xi^- = sds$$

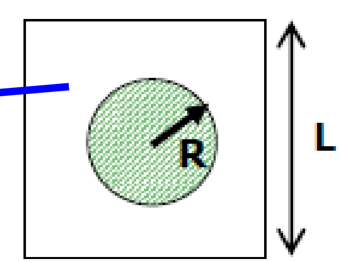
$$\Lambda = -\sqrt{\frac{1}{6}}[dsu + sud - 2uds] \qquad B = \epsilon^{abc}(q_a^T C \gamma_5 q_b) q_c$$

$$\Sigma^+ = -usu \quad \Sigma^0 = -\sqrt{\frac{1}{2}}[dsu + usd] \quad \Sigma^- = -dsd$$

NBS wave function has a same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

$$\Psi(t - t_{0,} \vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$



Coupled channel Schrödinger equation

Preparation for the NBS wave function

Two-channel coupling case

$$\Psi^{\alpha}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\alpha}(t,\vec{r}) | E \rangle$$
$$\Psi^{\beta}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\beta}(t,\vec{x}) | E \rangle$$

The same "in" state

Inside the interaction range

In the *leading order of velocity expansion* of non-local potential,

Coupled channel Schrödinger equation.

Factorization of interaction kernel

$$\left(\frac{p_\alpha^2}{2\mu_\alpha} + \frac{\nabla^2}{2\mu_\alpha}\right)\psi^{\alpha}(\vec{x},E) = V^{\alpha}{}_{\alpha}(\vec{x})\psi^{\alpha}(\vec{x},E) + V^{\alpha}{}_{\beta}(\vec{x})\psi^{\beta}(\vec{x},E)$$

μ_α *: reduced mass*

p_α : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of *R*.

$$R_I^{B_1 B_2}(t,\vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t,\vec{x}+\vec{r}) B_2(t,\vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1+m_2)t}$$

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})x \\ V^{\beta}{}_{\alpha}(\vec{r})x^{-1} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t}) R_{I1}^{\alpha}(\vec{r},E) & (\frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t}) R_{I2}^{\beta}(\vec{r},E) \\ (\frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t}) R_{I1}^{\alpha}(\vec{r},E) & (\frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t}) R_{I2}^{\beta}(\vec{r},E) \end{pmatrix} \begin{pmatrix} R_{I1}^{\alpha}(\vec{r},E) & R_{I1}^{\beta}(\vec{r},E) \\ R_{I2}^{\alpha}(\vec{r},E) & R_{I2}^{\beta}(\vec{r},E) \end{pmatrix}^{-1}$$

$$x = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

N. Ishii *et al.* [HAL QCD Collab.], Phys. Lett. B **712** (2012) 437

Interactions in SU(3) limit and dibaryon

Setup for SU(3) limit situation

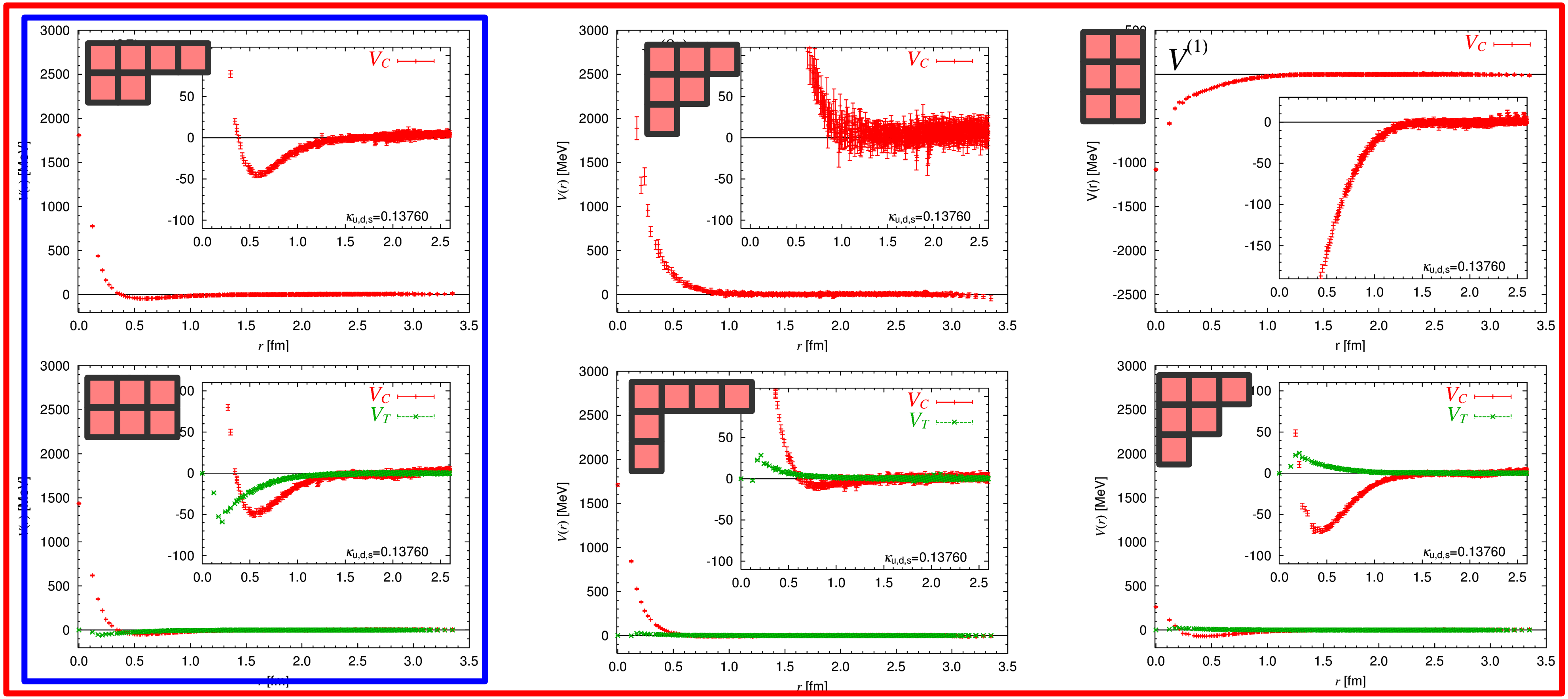
- 3 flavor gauge configurations generated by DDHMC/PHMC code.
  - We appreciate PACS-CS collab for giving us their code sets.
  - RG improved gauge action & O(a) improved clover quark action
  - β = 1.83, a^{-1} = 1.631 [*GeV*], 32^3x32 lattice, L = 3.872 [*fm*].
  - Five values of κ_{uds} are considered.
- Flat wall source is considered to produce S-wave B-B state.
- Numerical simulations are curried out at T2K-Tsukuba.

κ_{uds}	# of confs	M_ps [MeV]	M_b [MeV]
0.13660	420	1170.9(7)	2274(2)
0.13710	360	1015.2(6)	2031(2)
0.13760	480	836.8(5)	1749(1)
0.13800	360	672.3(6)	1484(2)
0.13840	720	468.6(7)	1161(2)

T.Inoue (HAL QCD collab.) PRL106 (2011) 162002

B-B potentials in SU(3) limit

m_π= 837MeV

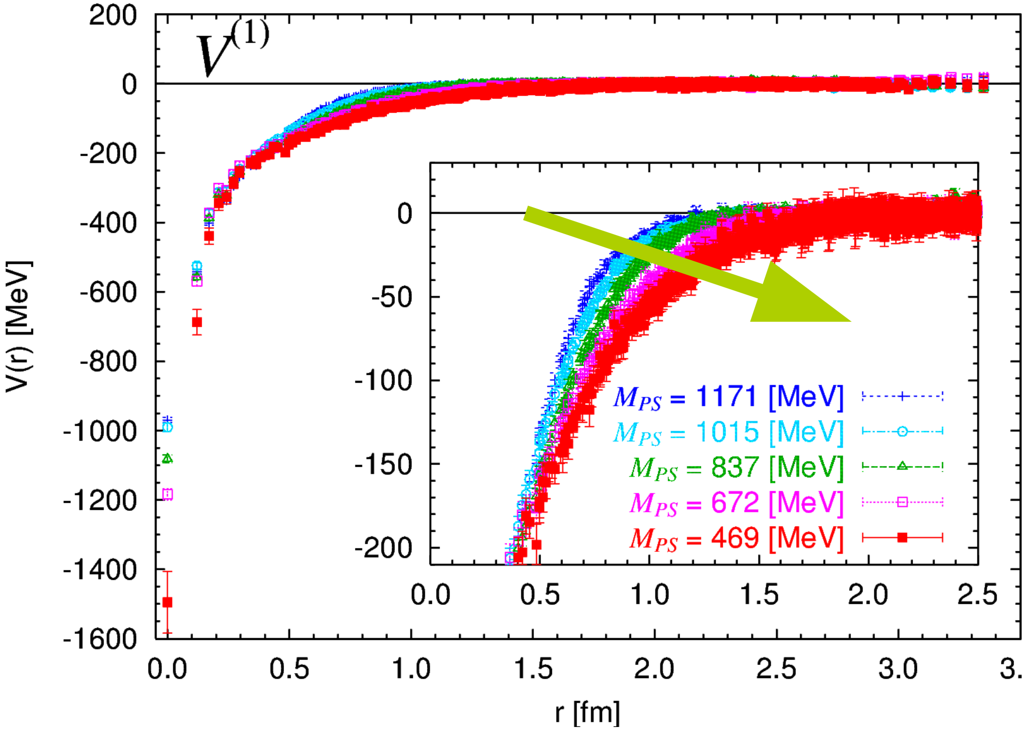


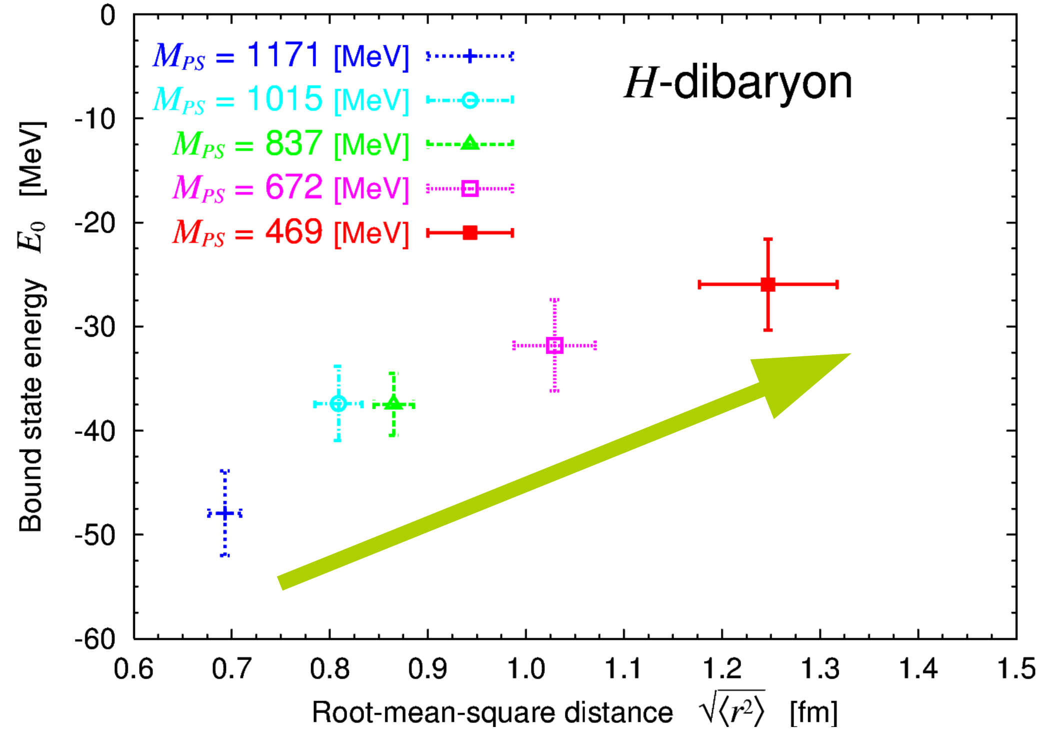
Two-flavors

Three-flavors

- Quark Pauli principle can be seen at around short distances
  - No repulsive core in flavor singlet state
  - Strongest repulsion in flavor 8s state

Looking for H-dibaryon in SU(3) limit





- Potential in flavor singlet channel is getting more attractive as decreasing quark masses
- There is a 6q bound state in this mass range with SU(3) symmetry.

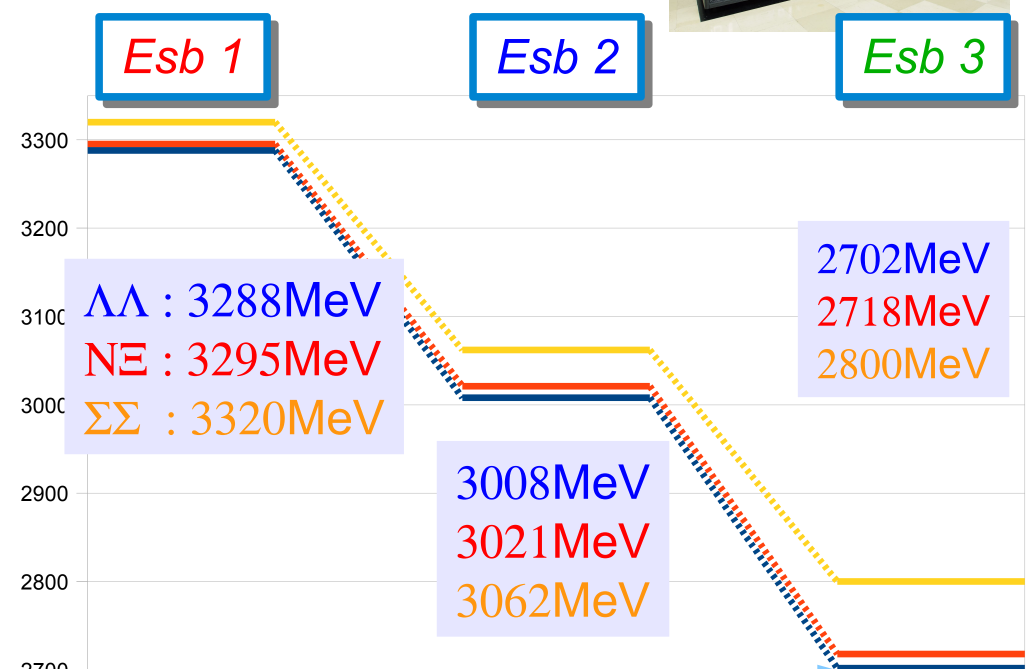
Results in SU(3) broken world

Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved clover quark action
  - β = 1.90, a^{-1} = 2.176 [*GeV*], 32^3x64 lattice, L = 2.902 [*fm*].
  - κ_s = 0.13640 is fixed, κ_{ud} = 0.13700, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter

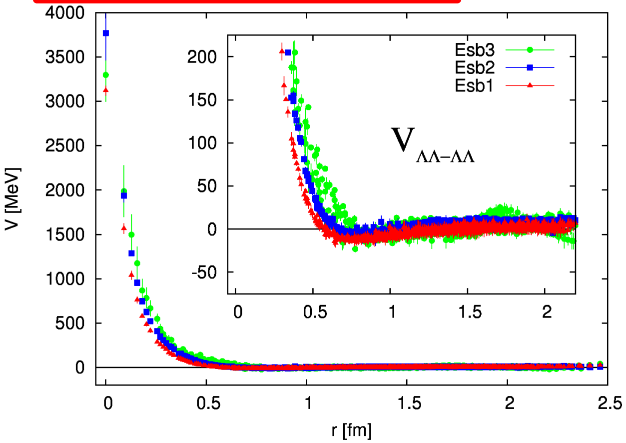


SU(3) breaking effects becomes larger

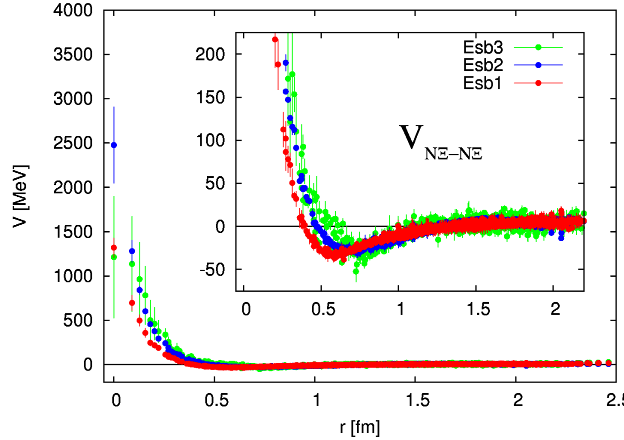
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

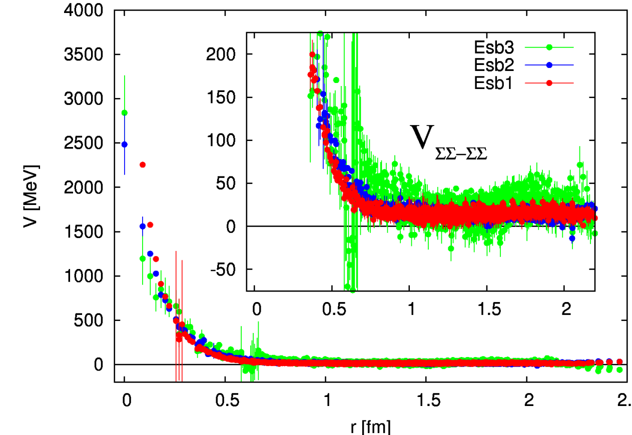
Diagonal elements



shallow attractive pocket



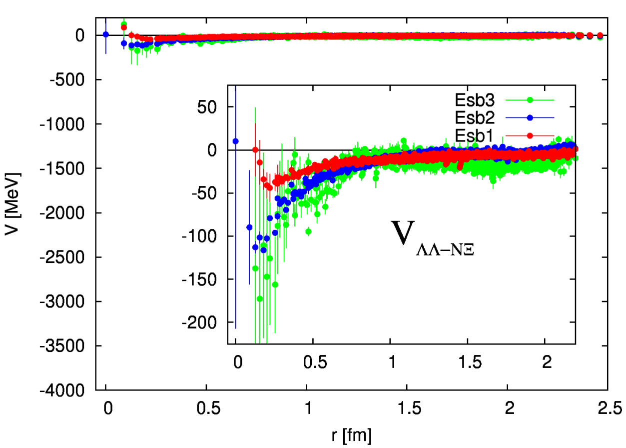
Deeper attractive pocket

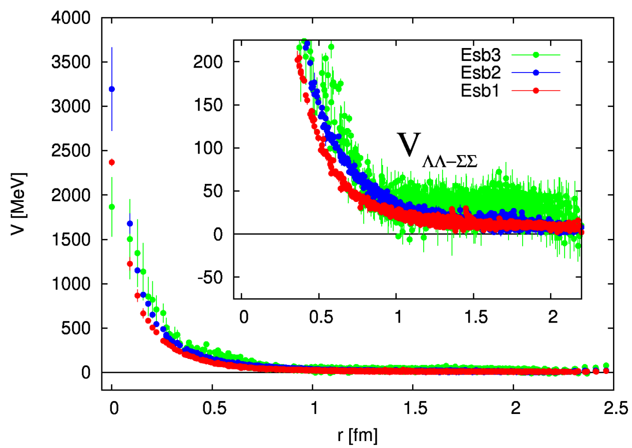


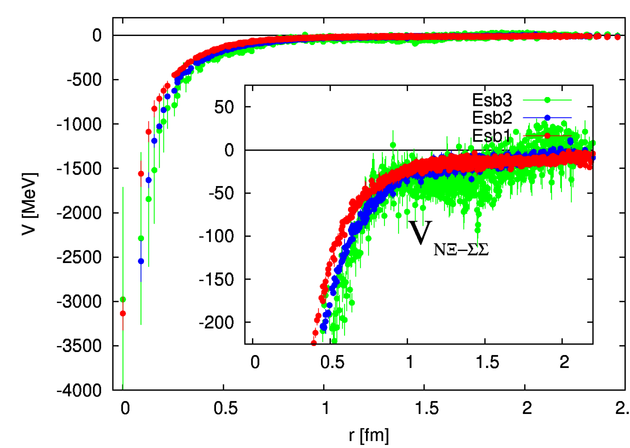
Strongly repulsive

All channels have repulsive core

Off-diagonal elements







Relatively weaker than the others

Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (IR) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

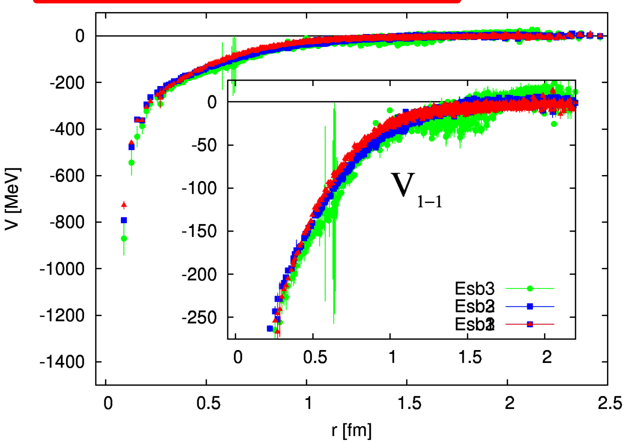
Off-diagonal part of the potential matrix in the SU(3) IR basis
would be an effective measure of the SU(3) breaking effect.

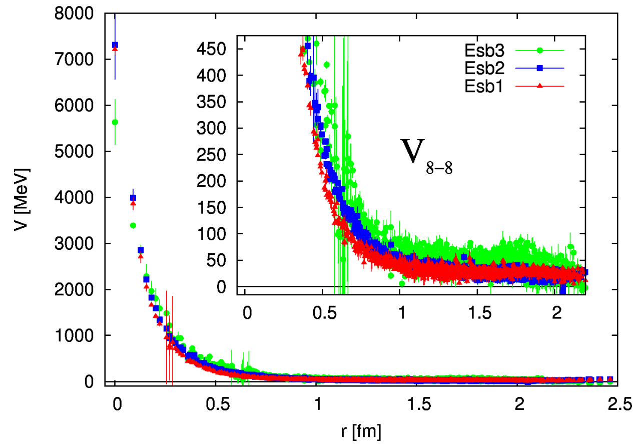
We will see how the SU(3) symmetry of potential will be broken
by changing the u,d quark masses lighter.

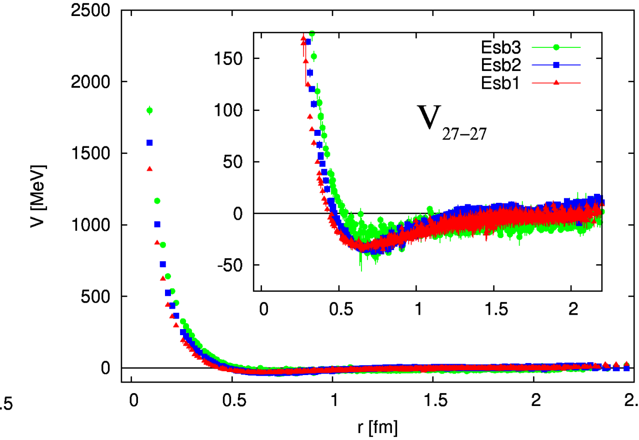
1, 8_s, 27 (I=0) $^1S_0$ channel

Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

Diagonal elements



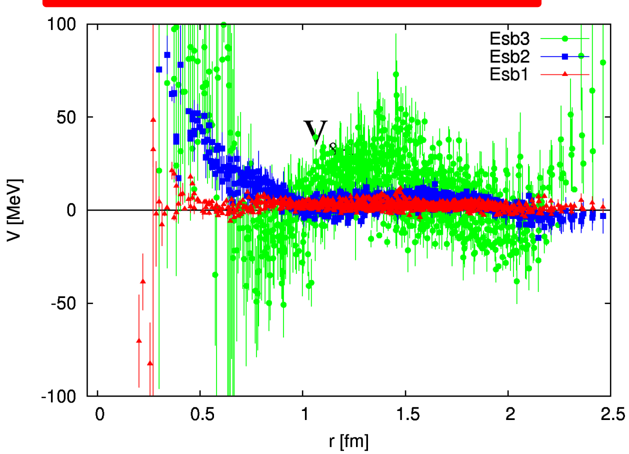


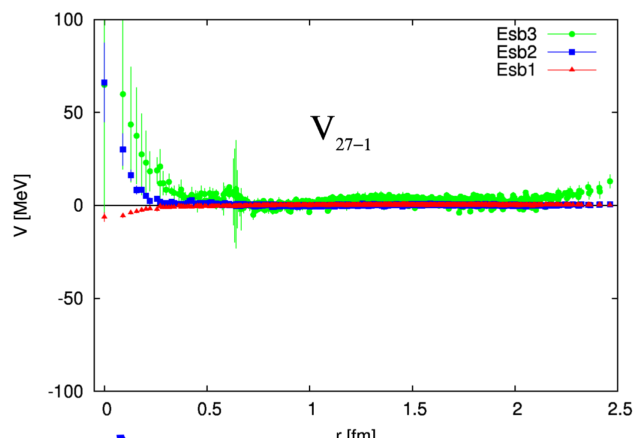


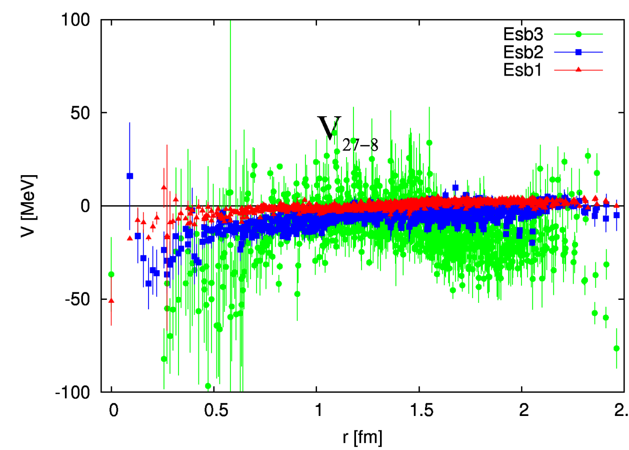
Strongly attractive
H-dibaryon channel

Pauli blocking effect

Off-diagonal elements





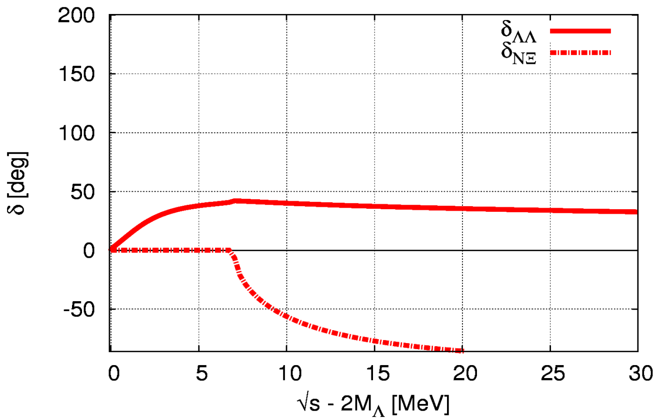


Mixture of singlet and octet
Is relatively larger than the others

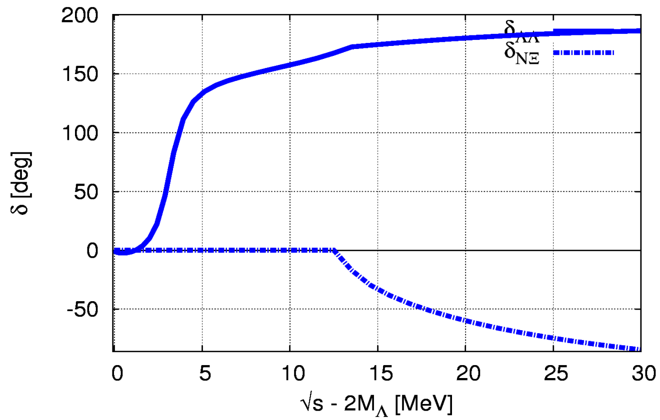
27 plet does not mix so much to the other representations

$\Lambda\Lambda$ and $N\Xi$ phase shifts

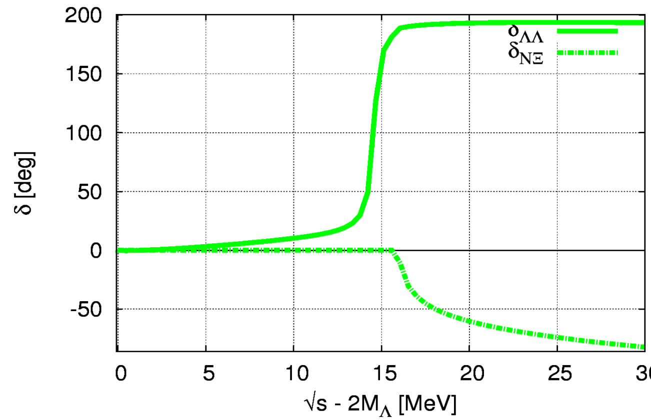
Esb1 : m_π= 701 MeV



Esb2 : m_π= 570 MeV



Esb3 : m_π= 411 MeV



Preliminary!

- Esb1:
  - Bound H-dibaryon
- Esb2:
  - H-dibaryon is near the $\Lambda\Lambda$ threshold
- Esb3:
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold..

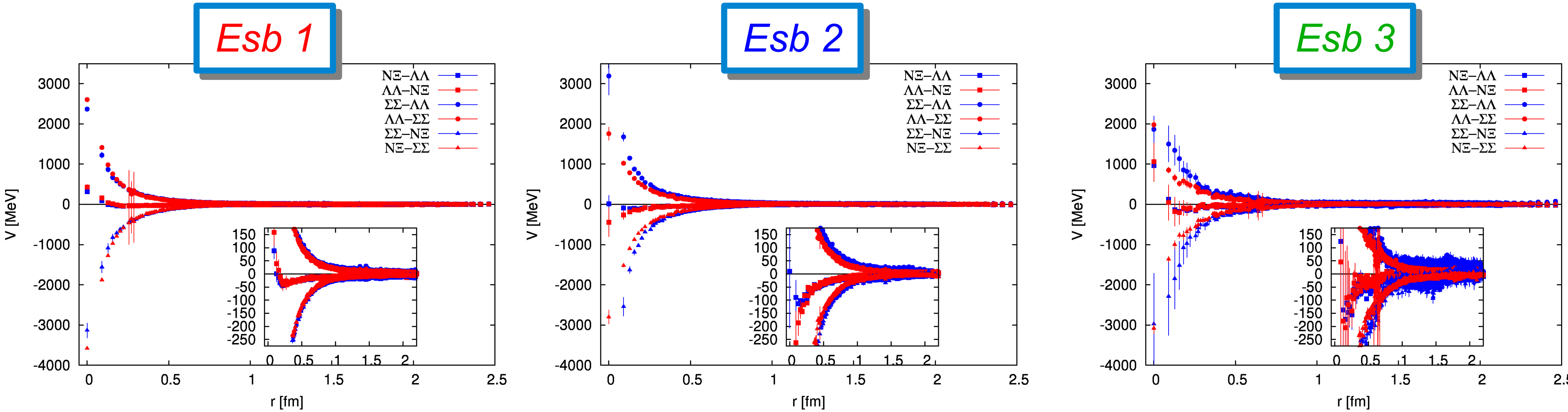
- We can see the clear resonance shape in $\Lambda\Lambda$ phase shifts for Esb2 and 3.
- The "binding energy" of H-dibaryon from $N\Xi$ threshold becomes smaller as decreasing of quark masses.

Summary and outlook

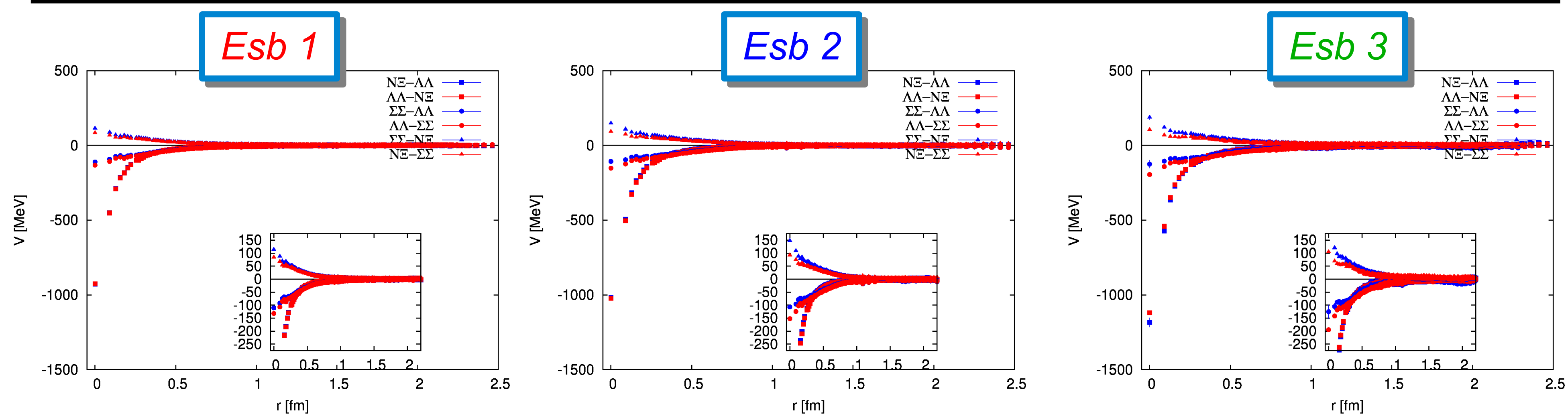
- We have investigated the S=-2 BB system from lattice QCD.
- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.
- Potentials are derived from NBS wave functions calculated with PACS-CS configurations
- Quark mass dependence of potentials can be seen not in long range region but in shot distances as an enhancement of repulsive core.
- Small mixture between different SU(3) irreps can be seen as the flavor SU(3) breaking effect.
- SU(3) breaking effects are still small even in $m\pi/mK=0.65$ situation but it would be change drastically at physical situation $m\pi/mK=0.28$.

Backup slides

Harmiticity check for $^1S_0$ I=0



Harmiticity check for $^3S_1$ I=1



Hermiticity is roughly fine, but we need more statistics

Isospin combinations of BB operator

$\Lambda\Lambda,\ p\Xi^{-},\ n\Xi^{0},\ \Sigma^{+}\Sigma^{-},\ \Sigma^{0}\Sigma^{0},\ \Lambda\Sigma^{0}$

I=0 operstors

$$\Sigma\,\Sigma = +\sqrt{\frac{1}{3}}\Sigma^{+}\Sigma^{-} - \sqrt{\frac{1}{3}}\Sigma^{0}\Sigma^{0} + \sqrt{\frac{1}{3}}\Sigma^{-}\Sigma^{+}$$

$$N\,\Xi = +\sqrt{\frac{1}{2}}p\,\Xi^{-} - \sqrt{\frac{1}{2}}n\,\Xi^{0}$$

I=1 operstors

$$\Sigma\,\Sigma = +\sqrt{\frac{1}{2}}\Sigma^{+}\Sigma^{-} - \sqrt{\frac{1}{2}}\Sigma^{-}\Sigma^{+}$$

$$N\,\Xi = +\sqrt{\frac{1}{2}}p\,\Xi^{-} + \sqrt{\frac{1}{2}}n\,\Xi^{0}$$

I=2 operstors

$$\Sigma\,\Sigma = +\sqrt{\frac{1}{6}}\Sigma^{+}\Sigma^{-} + \sqrt{\frac{4}{6}}\Sigma^{0}\Sigma^{0} + \sqrt{\frac{1}{6}}\Sigma^{-}\Sigma^{+}$$

SU(3) breaking effects

To see the SU(3) breaking effects

The potentials in the SU(3) irreducible representation (IR) basis are fitted.

The potential in IR basis transformed into the baryon basis.

SU(3) Clebsh-Gordan coefficients

$$U^t \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix} U \rightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix}$$

Transformation with and without the off-diagonal components

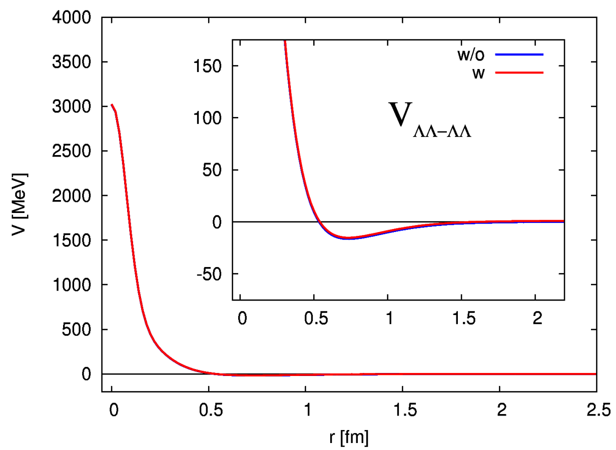
- Potential matrix with (w) off-diagonal part :
  SU(3) breaking effects are involved in the potentials

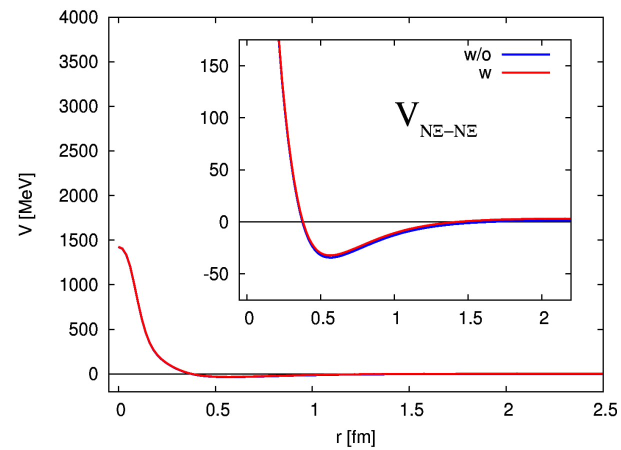
- Potential matrix without (w/o) off-diagonal part :
  SU(3) symmetry is assumed

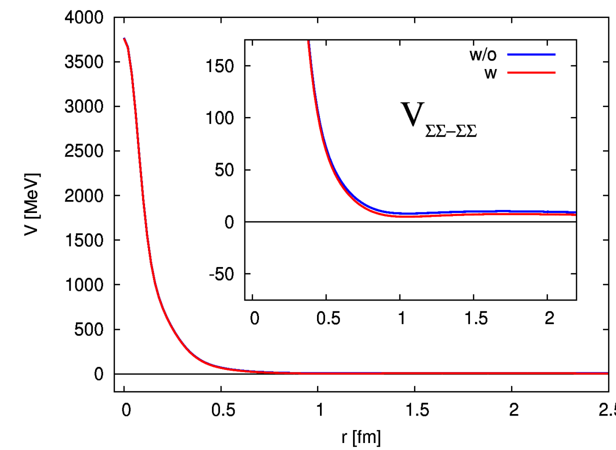
SU(3) breaking effects on phase shift

Esb 1

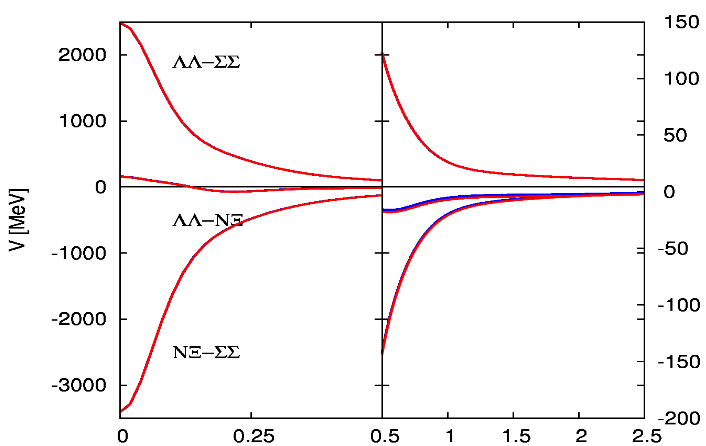
Diagonal part of potential matrix







Off-diagonal part of potential matrix

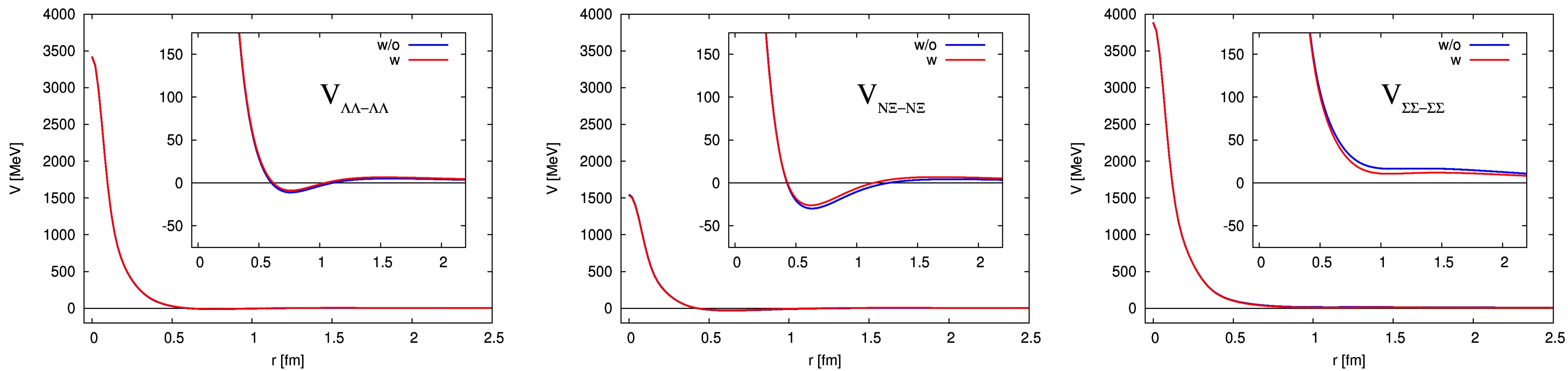


- SU(3) breaking effects are quite small.
- Small deviation can be seen in $N\Xi$ potential.

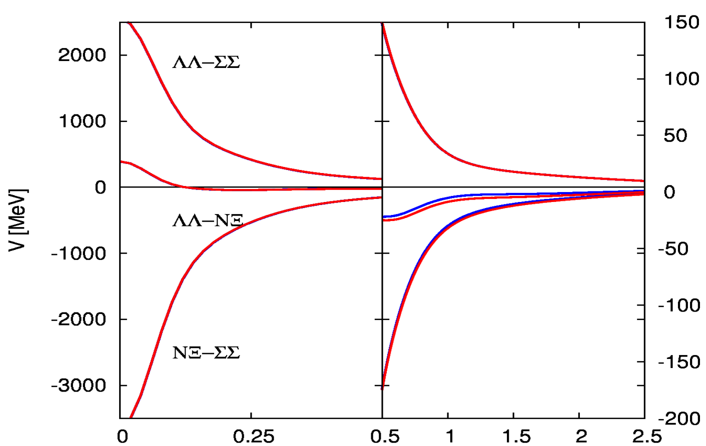
SU(3) breaking effects on phase shift

Esb 2

Diagonal part of potential matrix



Off-diagonal part of potential matrix

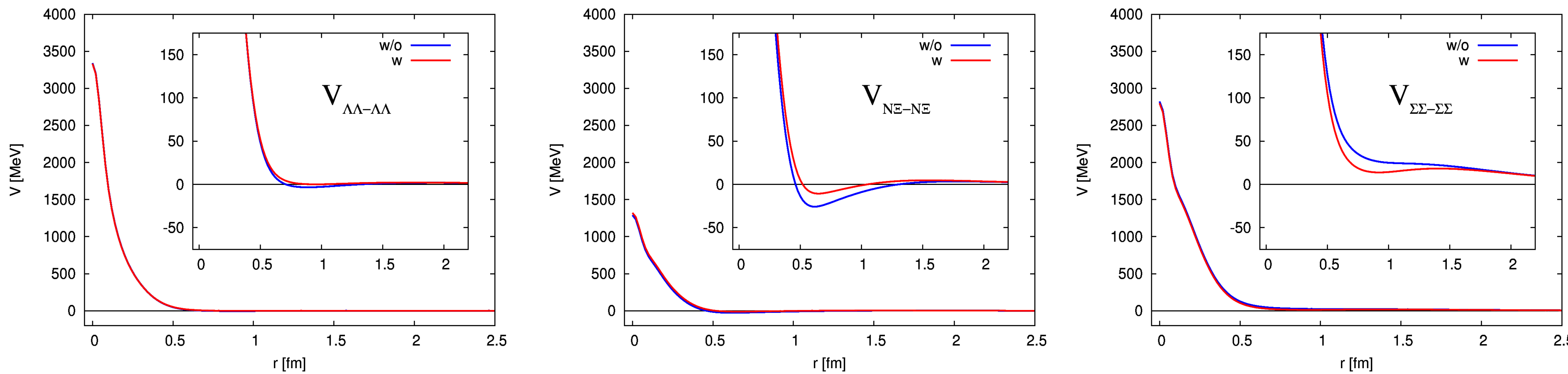


- SU(3) breaking effects are still small.
- Deviations can be seen in NΞ, ΣΣ and ΛΛ–NΞ transition potentials.

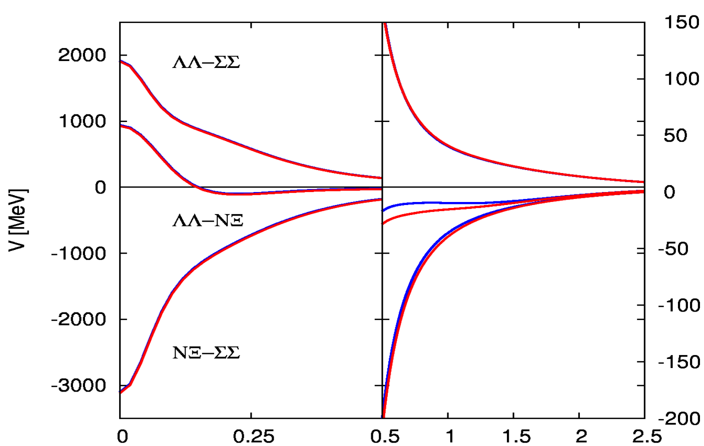
SU(3) breaking effects on phase shift

Esb 3

Diagonal part of potential matrix



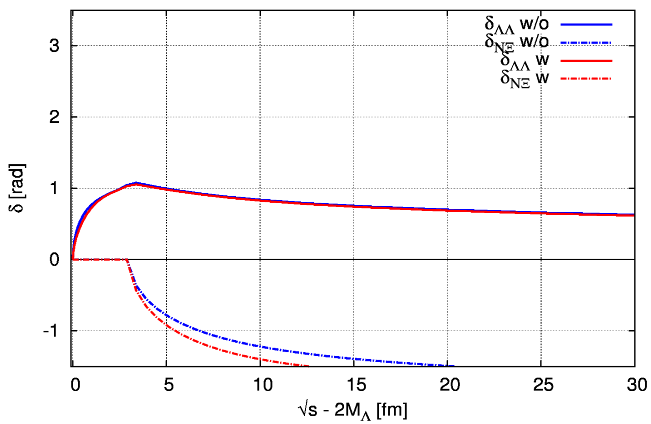
Off-diagonal part of potential matrix



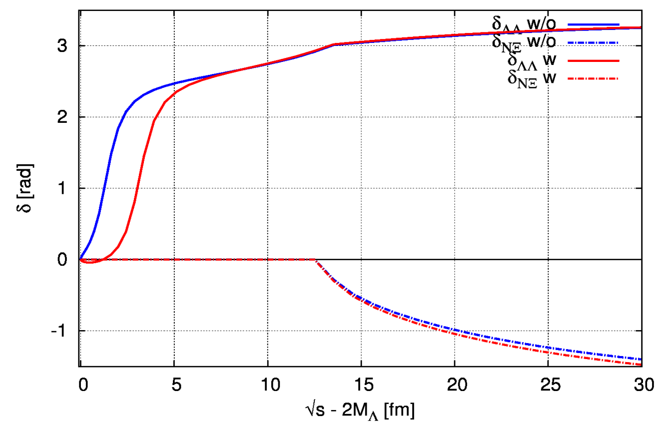
- SU(3) breaking effects become larger.
- Except for the ΛΛ–ΣΣ transition potential,
  - differences of the potentials are visible.

SU(3) breaking effects on phase shift

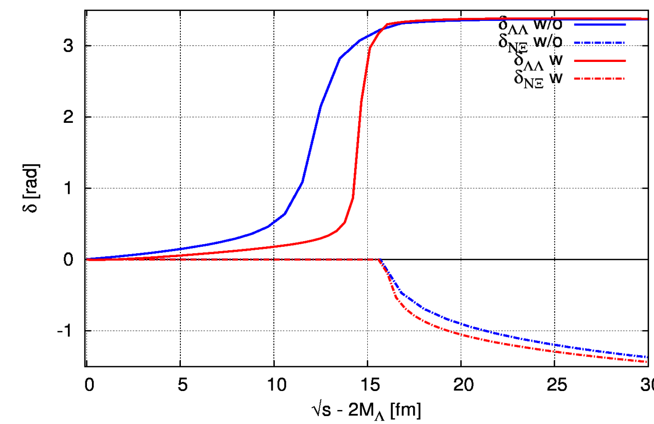
Esb 1



Esb 2



Esb 3



Preliminary!

- Esb1:
  - No difference between two types of potentials.
  - Bound H-dibaryon
- Esb2:
  - $\Lambda\Lambda$ phase shift is visibly changed
  - H-dibaryon is near the $\Lambda\Lambda$ threshold
- Esb3:
  - $\Lambda\Lambda$ phase shifts are visibly changed.
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold..

Effective Schrodinger equation with E-independent potential

$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$ **[START] local but E-dep pot. (L³xL³ dof)**

(1) We assume ψ(x; E) for different E is linearly independent with each other.

(2) ψ(x; E) has a "left inverse" as an integration operator as

$E \equiv 2\sqrt{m_N^2 + k^2}$

$$\int d^3x \tilde{\psi}(\vec{x};E')\psi(\vec{x};E) = 2\pi\delta(E - E')$$

(3) K(x; E) can be factorized as

$$K(\vec{x};E) = \int \frac{dE'}{2\pi} K(\vec{x};E') \times \int d^3y \tilde{\psi}(\vec{y};E')\psi(\vec{y};E)$$

$$= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x};E')\tilde{\psi}(\vec{y};E') \right\} \psi(\vec{y};E)$$

$\equiv m_N U(\vec{x},\vec{y})$

(4) We are left with an effective Schrodinger equation with an E–independent potential U.

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y\, U(\vec{x},\vec{y})\psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. (L³xL³ dof)

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\bar{q}}(\bar{x}) \equiv \left\langle 0 \middle| N(\bar{x})\, N(\bar{0}) \middle| N(\bar{q}) N(-\bar{q}), in \right\rangle$$

complete set

$$1 = \int \frac{d^3 p}{(2\pi)^3 2E_N(\bar{p})} |N(\bar{p})\rangle\langle N(\bar{p})| + \cdots$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E_N(\bar{p})} \langle 0 | N(\bar{x}) | N(\bar{p})\rangle \left\langle N(\bar{p}) \middle| N(\bar{0}) \middle| N(\bar{q}) N(-\bar{q}), in \right\rangle + I(\bar{x})$$

$$Z^{1/2} e^{i\bar{p}\cdot\bar{x}} \qquad disc. + Z^{1/2} \frac{T(\bar{p};\bar{q})}{m_N^2 - (2E_N(\bar{q}) - E_N(\bar{p}))^2 + \bar{p}^2 - i\varepsilon}$$

$$= Z\left(e^{i\bar{q}\cdot\bar{x}} + \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_N(\bar{p})} \frac{T(\bar{p};\bar{q})}{4E_N(\bar{q})\cdot(E_N(\bar{p}) - E_N(\bar{q}) - i\varepsilon)} e^{i\bar{p}\cdot\bar{x}} \right)$$

Integral is dominated by the on-shell contribution $E_N(\bar{p}) \approx E_N(\bar{q})$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\bar{q})}{2|\bar{q}|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

$$= Z\left(e^{i\bar{q}\cdot\bar{x}} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

$$\psi_{\bar{q}}(\bar{x}) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}$$

This is analogous to a non-rela. wave function

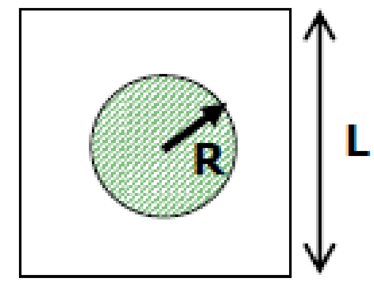
Energy indep. potential in coupled channel S.E.

Inside the interaction range, we can define the interaction kernel

$$\begin{pmatrix} p^2+\nabla & q^2+\nabla \\ p^2+\nabla & q^2+\nabla \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{x},E) & \psi_a^b(\vec{x},E) \\ \psi_a^b(\vec{x},E) & \psi_a^b(\vec{x},E) \end{pmatrix} = \begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix} \xrightarrow{|x|>R} 0$$

Factorization of the interaction kernel

$$\begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix} = \int dy \begin{pmatrix} U_a^a(\vec{x},\vec{y}) & U_b^a(\vec{x},\vec{y}) \\ U_a^b(\vec{x},\vec{y}) & U_b^b(\vec{x},\vec{y}) \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{y},E) & \psi_b^a(\vec{y},E) \\ \psi_a^b(\vec{y},E) & \psi_b^b(\vec{y},E) \end{pmatrix}$$



$$\int dx \begin{pmatrix} \tilde{\psi}_a^a(\vec{x},E') & \tilde{\psi}_b^a(\vec{x},E') \\ \tilde{\psi}_a^b(\vec{x},E') & \tilde{\psi}_b^b(\vec{x},E') \end{pmatrix}\begin{pmatrix} \psi_a^a(\vec{x},E) & \psi_b^a(\vec{x},E) \\ \psi_a^b(\vec{x},E) & \psi_b^b(\vec{x},E) \end{pmatrix} = 2\pi\delta(E-E')$$

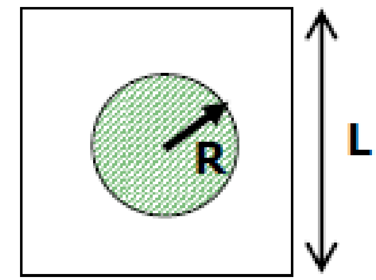
$$\begin{pmatrix} U_a^a(\vec{x},\vec{y}) & U_b^a(\vec{x},\vec{y}) \\ U_a^b(\vec{x},\vec{y}) & U_b^b(\vec{x},\vec{y}) \end{pmatrix} = \int \frac{dE}{2\pi}\begin{pmatrix} K_a^a(\vec{x},E) & K_b^a(\vec{x},E) \\ K_a^b(\vec{x},E) & K_b^b(\vec{x},E) \end{pmatrix}\begin{pmatrix} \tilde{\psi}_a^a(\vec{y},E) & \tilde{\psi}_b^a(\vec{y},E) \\ \tilde{\psi}_a^b(\vec{y},E) & \tilde{\psi}_b^b(\vec{y},E) \end{pmatrix}$$

Energy independent potential in Schrödinger equation.

Schrödinger equation

- Define the energy-independent potential in Schrödinger equation (most general form)

$$\left(\frac{k^2}{2\mu}-H_0\right)\Psi(\vec{x})=\int U(\vec{x},\vec{y})\Psi(\vec{y})d^3y$$



- Recent development : Time dependent method.

We replace ψ to *R* defined below

$$\partial_t R_\alpha(\vec{x},E) \equiv \partial_t\left(\frac{A\Psi_\alpha(\vec{x},E)e^{-Et}}{e^{-m_A t}e^{-m_B t}}\right) \propto -\frac{p_\alpha^2}{2\mu_\alpha}R_\alpha(\vec{x},E)$$

- Performing the derivative expansion for the interaction kernel

$$\left(-\frac{\partial}{\partial t}-H_0\right)R(\vec{x})=\int U(\vec{x},\vec{y})R(\vec{y})d^3y$$

- Taking the leading order of derivative expansion of non-local potential

$$U(\vec{x},\vec{y}) \simeq V_0(\vec{x})\delta(\vec{x}-\vec{y}) + V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y}) \cdots$$

- Finally local potential was obtained as

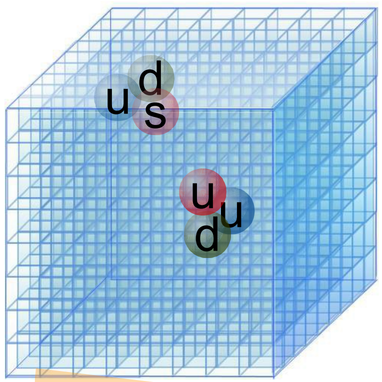
$$V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{v})}+\frac{1}{2\mu}\frac{\nabla^2 R(\vec{x})}{R(\vec{x})}$$

QCD to hadronic interactions

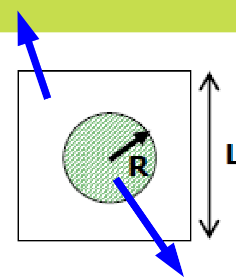
Lattice QCD simulation can connect the fundamental QCD with nuclear physics

$$L_{QCD}=\bar{q}(i\gamma_{\mu}D^{\mu}-m)q+\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu}$$

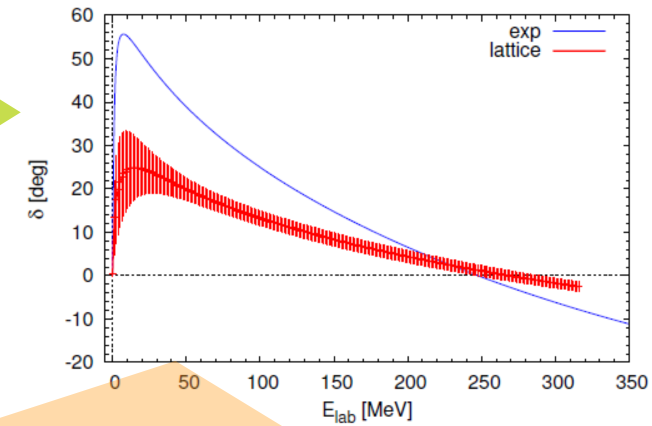
Lattice QCD simulation



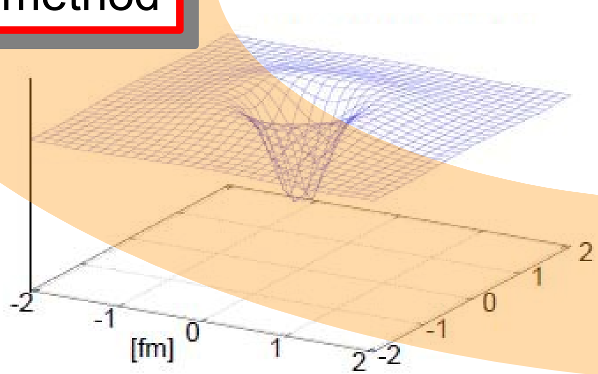
Luscher's finite volume method



BB scattering phase shift

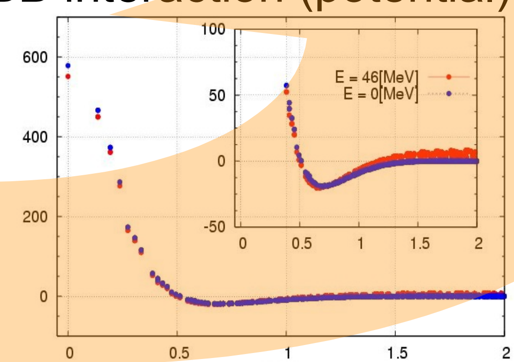


HAL QCD method



NBS wave function

BB interaction (potential)

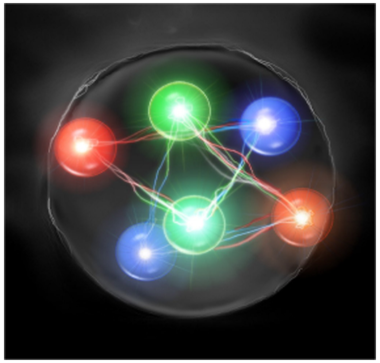


The potential is proper for the phase shift by QCD

N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99** (2007) 022001

"H-dibaryon"

- R.L. Jaffe predict the flavor singlet (uds)x(uds) state with J=0.
  - Bag model calculation
    - Strongly attractive color-magnetic interaction
  - Quark model calculation
    - Strongly attractive color-magnetic interaction
    - Free from quark Pauli blocking effect in flavor singlet channel



Importance of quark degrees of freedom

- Short range repulsion in BB interaction is result of Pauli principle for the substructures of baryons and depend on their flavor structures

Otsuki, Tamagaki, Yasuno PTPS (1965)578
Oka, Shimizu and Yazaki NPA464 (1987)

"H-dibaryon"

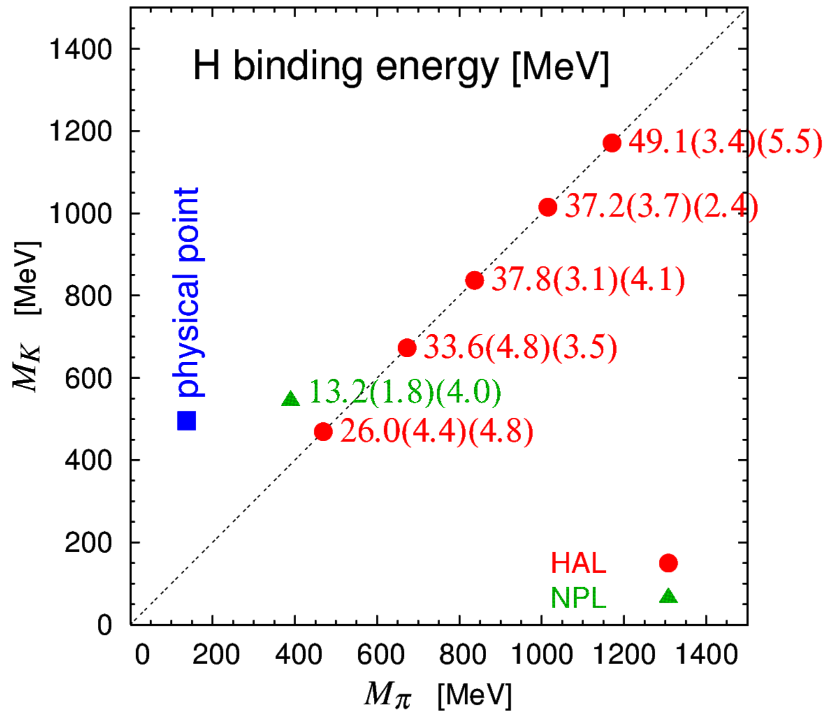
Recent Lattice QCD studies

- HAL QCD: SU(3) limit

  BE = 26MeV m_π = 470MeV

- NPLQCD: SU(3) breaking

  BE = 13MeV m_π = 390MeV



Experimental constraint

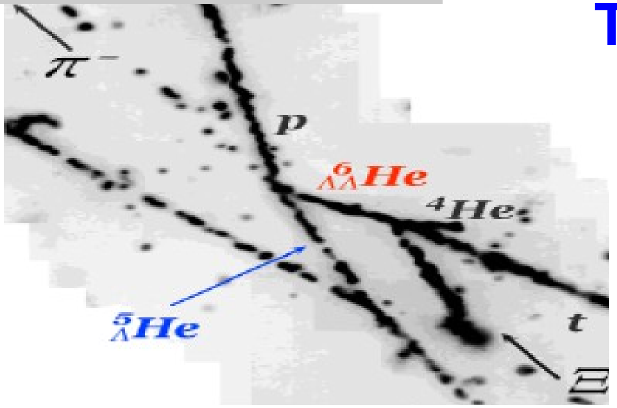
Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV



Toward the physical point