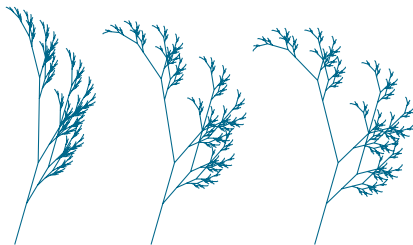


# Learning SciPy for Numerical and Scientific Computing

Francisco Blanco-Silva

University of South Carolina



# SOME BASIC PRINCIPLES IN SCIENTIFIC COMPUTING

## A PERSONAL PERSPECTIVE

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- ▶ **Trust the Computer Scientist:** Writing low-level code from scratch seldom guarantees best results.
- ▶ **Find a reliable way to communicate** through software.
- ▶ **Big guns:** Solving these problems usually require massive amounts of calculations and are often executed on supercomputers or distributed computing platforms.

# COMPUTATIONAL RESOURCES

## SOFTWARE

- ▶ Optimized Algebra Libraries
  - ▶ BLAS
  - ▶ LAPACK



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- ▶ with PyLab: `ipython + NumPy + SciPy + matplotlib`

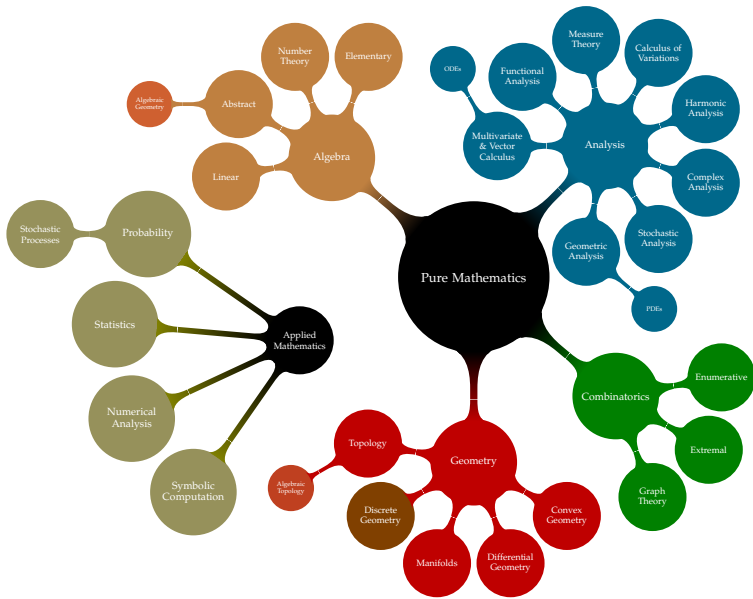
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    - ▶ with PyLab: `ipython + NumPy + SciPy + matplotlib`
    - ▶ with `scikits` and `Pandas` on top of that

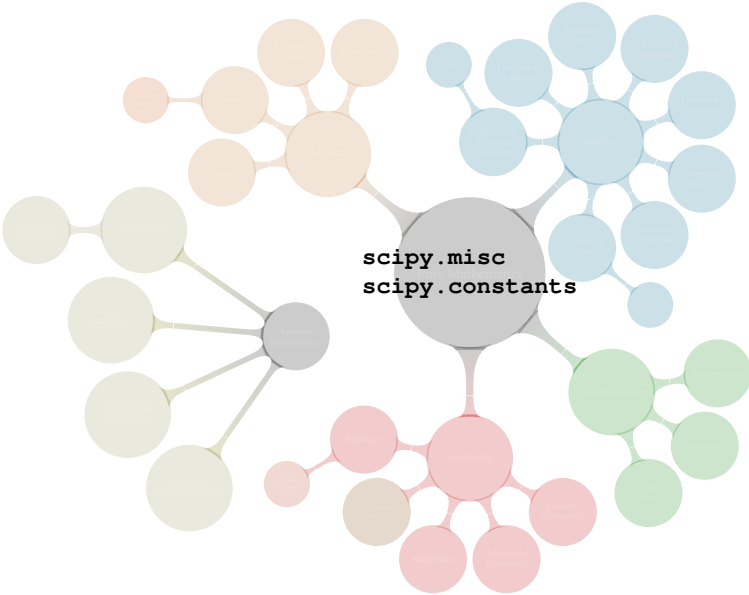
# THE STRUCTURE OF SciPy

SIMILARITY TO THE DIFFERENT AREAS OF MATHEMATICS



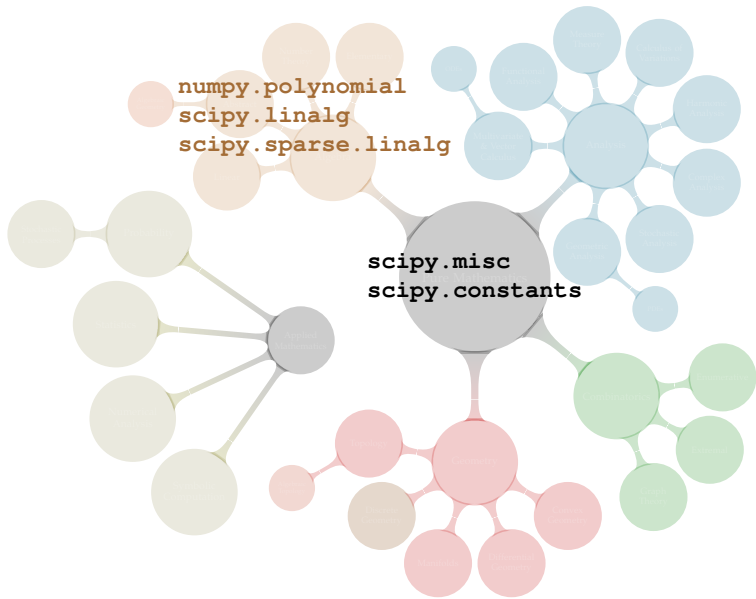
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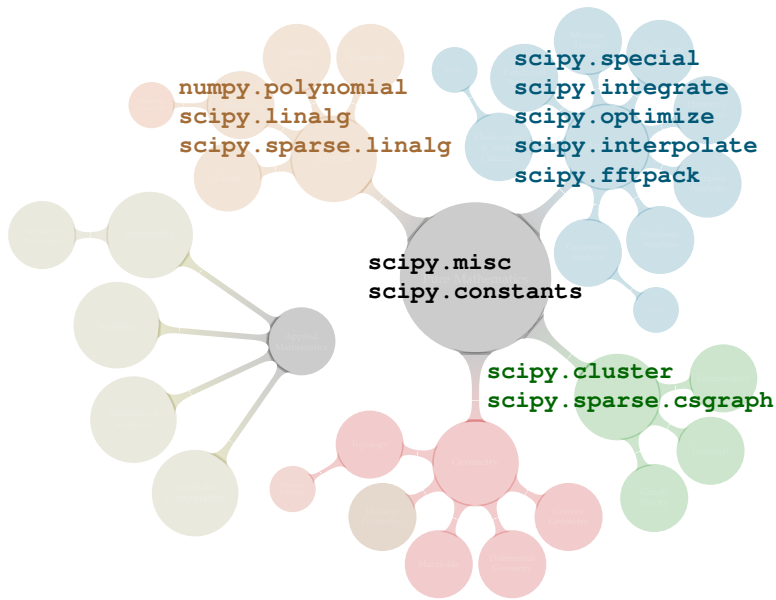


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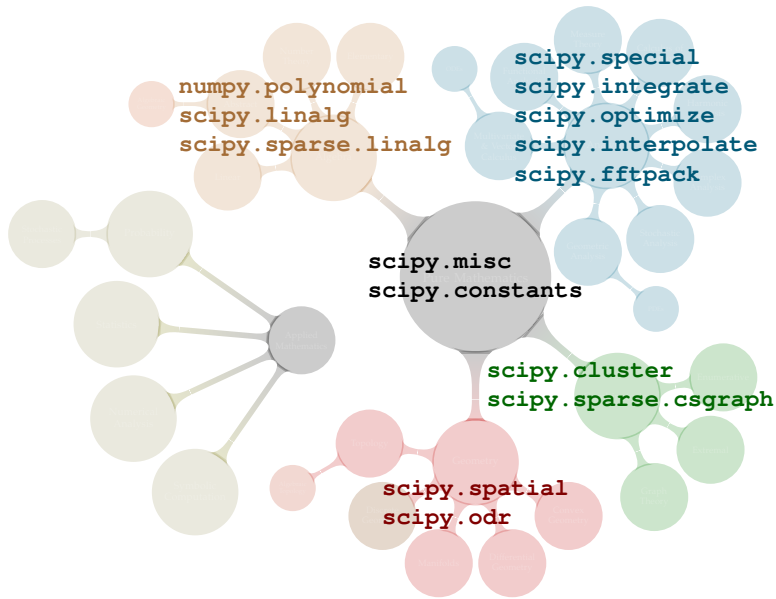


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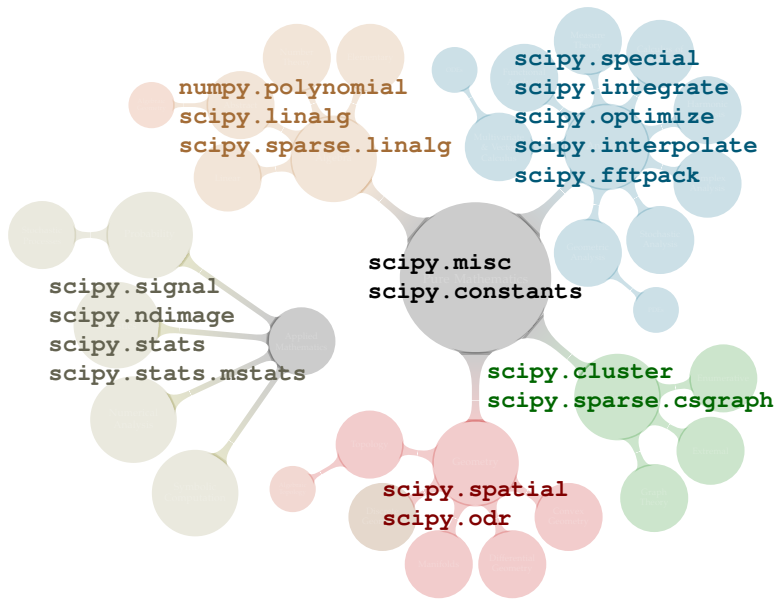


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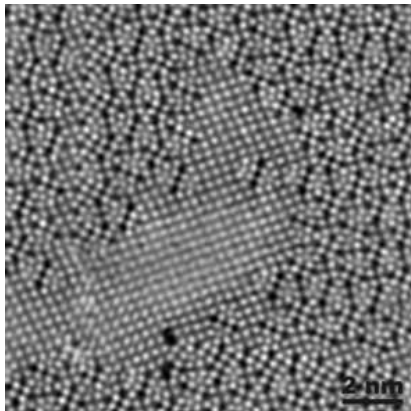
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## SCI-PY IN ACTION

EXTRACT THE STRUCTURAL MODEL OF A MOLECULE OF  $Nb_4W_{13}O_{47}$

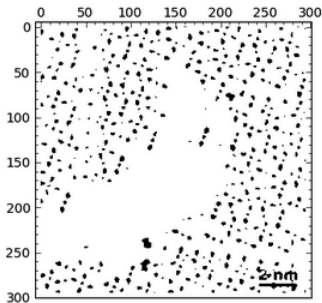


# SCIPLY IN ACTION

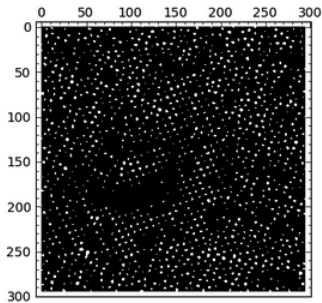
## COMPUTATION OF STRUCTURAL MODELS

We take the following (naïve) approach:

- **Segmentation** of the atoms by thresholding and morphological operations.



`img>0.2`



`img>0.7`

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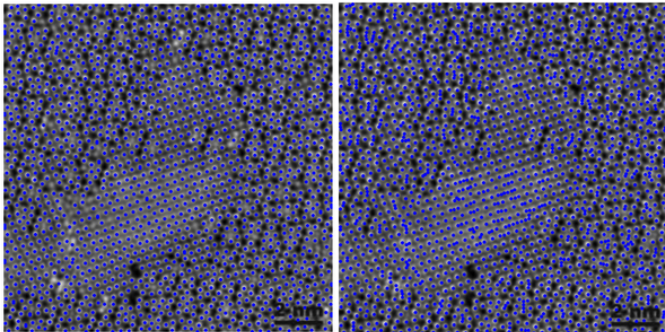
- ▶ **Segmentation** of the atoms by thresholding and morphological operations.
- ▶ **Connected component labeling** to extract each atom for posterior examination.

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We take the following (naïve) approach:

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- ▶ **Computation of the centers of mass** of each label identified as an atom.



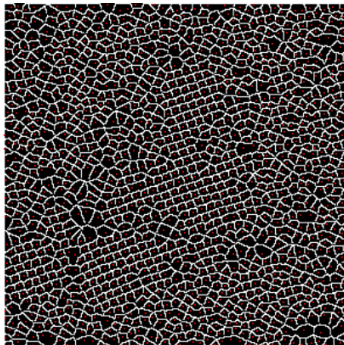


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- ▶ **Segmentation** of the atoms by thresholding and morphological operations.
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- ▶ **Computation of the centers of mass** of each label identified as an atom.
- ▶ **Computation of the Voronoi diagram** of the lattice formed by the previous points.



# SCIPLY IN ACTION

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4  from scipy.ndimage import binary_opening, label, center_of_mass,
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5  # Load the image
6  img = scipy.misc.imread('NbW-STEM.png')
7  # Apply a threshold to segment atoms
8  BWatoms = (img > 0.62)
9  # Perform a binary operation to eliminate outliers
10 BWatoms = binary_opening(BWatoms, structure=numpy.ones((2,2)))
11 # Segmentation of each atom
12 structuring_element = [[0,1,0],[1,1,1],[0,1,0]]
13 segmentation, segments = label(BWatoms, structuring_element)
14 # Computation of centers of mass of each atom
15 coords = center_of_mass(img, segmentation, range(1, segments+1))
16 xcoords = array([x[0] for x in coords])
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## FOR MORE INFORMATION, EXAMPLES, IDEAS, ...

**Francisco Blanco-Silva**  
Useless math is useful math. It can be used to generate more useless math.

About me Curriculum Vitae Research Teaching Problem Solving LaTeX

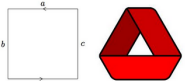
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### Möbius bands and Klein bottles

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Consider a long and narrow strip of paper, in which we glue the two narrower edges together after performing a half-twist of the strip, hence forming a funny-looking loop. Topologically, this is equivalent to identifying two parallel edges of a square in the proper way. Therefore, we can see the Möbius band as the quotient space of  $\mathbb{C}_2$  with the following equivalence relation: Given  $(x_1, x_2), (y_1, y_2) \in \mathbb{C}_2$ , we write  $(x_1, x_2) \sim (y_1, y_2)$  if

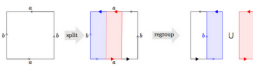
- $x_1 = y_1$  and  $x_2 = y_2$ , or
- $x_2 = -1$  and  $x_1 = 1 - y_1$ .



In a similar fashion, we construct a new surface by identification of two parallel edges of a square to form a cylinder, and a posterior identification of the remaining two edges (which turned into two circles after the first identification). The second identification will be performed after a "half-twist" (which unfortunately can only be realized in a fourth dimension!). The equivalence relation in the square  $\mathbb{C}_2$  is as follows: Given  $(x_1, x_2), (y_1, y_2) \in \mathbb{C}_2$ , we define  $(x_1, x_2) \sim (y_1, y_2)$  provided one of the following conditions are satisfied:

- $x_1 = y_1$  and  $x_2 = y_2$ , or
- $x_1 y_1 = -1$  and  $x_2 = y_2$ , or
- $x_2 y_2 = -1$  and  $x_1 = 1 - y_1$ .


A diagram representing this quotient space—which we denote  $\mathbb{K}$  and call Klein bottle—is shown below, together with an interesting way to split and recover the space: Notice how by cutting these strips in the manner shown, and adjoining two of the strips through the proper edge, we can see the Klein's bottle as the union of two Möbius bands.



**Miscellaneous**

All the images included in this presentation are generated with the aid of the package tikz in

blancosilva.wordpress.com



COMMUNITY EXPERIENCE DISTILLED

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**PACKT** open source  
PUBLISHING