What about $\frac{1}{4}+\frac{1}{3}$? What about $\frac{5}{8}+\frac{1}{8}$?

To add fractions, we need to change the denominators to the *least common multiple*.
To reduce fractional answers, we need to find a *greatest common factor*.
That's what we'll study in this lesson.

Least Common Multiples

In a multiplication problem, the product (result of multiplication) is also called a **multiple** of the numbers or factors.

Example: What is the least common multiple of 6 and 9?

Multiples of 6 are: 6, 12, **18**, 24, 30, **36**, 42, 48, **54**, ...

Multiples of 9 are: 9, **18**, 27, **36**, 45, **54,** 63, ...

The numbers in bold are common multiples of 6 and 9.
That is, they are common to both lists of multiples.
The least (or lowest or smallest) common multiple of 6 and 9 is **18**.

☞ We do not call "zero" a multiple of any other number! So the *least common multiple* of two numbers is never 0. After all, the product is zero if any of the factors are zero.

☞ The letters **LCM** are often used for to abbreviate the words Least Common Multiple.

Example: What is the least common multiple of 12 and 24?

Multiples of 12 are: 12, **24**, 36, **48**, 60, **72**, …

Multiples of 24 are: **24**, **48**, **72**, …

The answer is easy if one number is a multiple of another.
The least common multiple is **24**.

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Example: Suppose two cars drive over a spot of wet paint.
One car's tire then leaves a mark every 2½ feet, and the other car's tire leaves a mark every 3 feet.
How far until the two cars leave a mark at the same spot?

Multiples of 2½ are: 2 ½, 5, 7½, 10, 12½, **15**, 17½, 20, 22½, 25, 27½, **30**

Multiples of 3 are: 3, 6, 9, 12, **15**, 18, 21, 24, 27, **30**

Do *any* two numbers *always* have a common multiple? Yes. The product of the numbers is a common multiple. It may or may not be the *least* common multiple.

By the way, if two numbers are both prime, then their product is always the *least* common multiple.

To add or subtract fractions, we change them to have a common denominator. Any common multiple will work, but using a denominator of the *least common multiple* gives you the smallest (and easiest) numbers to work with.

Example: Change these denominators to use the least common multiple for: $\frac{1}{8}+\frac{1}{6}$

Multiples of 6 are: 6, 12, 18, **24**

Multiples of 8 are: 8, 16, **24**

The least common multiple is 24, so now convert the fractions.

$\frac{1}{8}\times\frac{3}{3}=\frac{3}{24}$ and $\frac{1}{6}\times\frac{4}{4}=\frac{4}{24}$

Answer: $\frac{1}{8}+\frac{1}{6}=\frac{3}{24}+\frac{4}{24}$

Greatest Common Factors

Factors of a number are simply numbers that are multiplied together to get an answer or product. For example, $6 \times 9 = 54$. The number 54 is the product of the factors 6 and 9. The "product" means the answer of a multiplication.

We use multiplication to find the factors of a whole number.

Example: What is the greatest common factor of 12 and 18?

Factors of 12 are: **1**, **2**, **3**, 4, **6** and 12.

Factors of 18 are: **1**, **2**, **3**, **6**, 9 and 18.

The numbers in bold are common factors of 12 and 18.

Answer: The greatest common factor is **6**.

☞ The letters **GCF** are often used for to abbreviate the words "greatest common factor".

Do *any* two numbers *always* have a common factor? Yes! For example, if both numbers are prime then their only common factor is 1.

Prime and Composite Numbers

So far, we've looked at how factors make up the product of a number. Now let's look at some special products that have the smallest number of factors.

A *prime number* has exactly two factors, the number 1 and itself. It is not evenly divisible by any other whole number.

Example: What are the factors of the number 5**?**

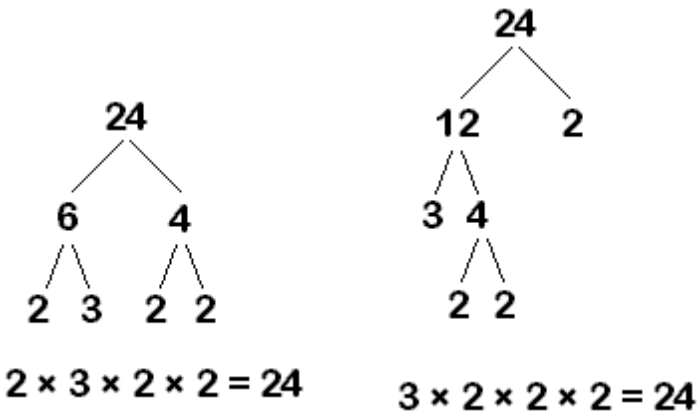
$1 \times 5 = 5$ The numbers 1 and 5 are the only factors of 5.

Therefore 5 is a prime number.

What is the opposite of *prime*? Any number greater than 1 that is not a prime number is called a *composite* number. The number 18 is a composite number because it has three factors: 2, 3, 3.

Any whole number greater than 1 can be written as the product of prime number factors. This is called *prime factoring*. One method of finding the prime factors of a whole number is to make a factor tree. The branches end in prime numbers.

Example:



☞ By definition, the number **1** is not a prime number. The smallest prime number is **2**.

Adding and Subtracting Fractions

Now we can apply these concepts to working with fractions.

To add or subtract fractions with the *same* denominator (bottom), simply add or subtract the numerators (top).

Example: $\frac{3}{11}+\frac{6}{11}=\frac{3+6}{11}=\frac{9}{11}$

Example: $\frac{15}{16}-\frac{12}{16}=\frac{(15-12)}{16}=\frac{3}{16}$

To add or subtract fractions with *different* denominators (bottom), we must change the fractions so the denominators match. You can use any matching denominator, but finding the denominator with the least common multiple gives you the simplest arithmetic.

Example: $\frac{2}{9}+\frac{5}{6}=?$

Step 1: First we must change the fractions so they have the same denominator. We start by finding the least common multiple of the numbers 6 and 9:

Multiples of 9 are 9, **18**, 27, **36**, ...
Multiples of 6 are 6, 12, **18**, 24, 30, **36**, ...

The least common multiple is therefore **18**.

Step 2: Change the fractions by multiplying the numerator and denominator by the factor that makes the denominator l8:

For the first fraction $\frac{2}{9}=\frac{2}{9}\times\frac{2}{2}=\frac{4}{18}$

For the second fraction $\frac{5}{6}=\frac{5}{6}\times\frac{3}{3}=\frac{15}{18}$

Step 3: Now we can add the two fractions:

$$\frac{4}{18}+\frac{15}{18}=\frac{(4+15)}{18}=\frac{19}{18}$$

Step 4: We always reduce our fractions, but this result has no common factors to both 19 and 18.

Improper Fractions

The answer to the above example, 19/18, is an *improper fraction*, because the numerator 19 is larger then the denominator 18. We convert this into a mixed number, which is a

number that contains a whole number. Do this by dividing 19 by 18 and showing the remainder as a fraction.

Example: $\frac{19}{18}$ is the same as 19 divided by 18.

18 divides into 19 once with remainder 1.

The mixed number is $1+\frac{1}{18}$, which is written as $1\frac{1}{18}$

Adding and Subtracting Mixed Fractions

To add or subtract mixed numbers simply add or subtract the whole numbers and fraction portion separately and then recombine to form the new mixed number.

Example: $2\frac{1}{5}+1\frac{1}{2}$

You say "two and one-fifth plus one and one-half".

The word "and" means addition, so we can write this as:

$$2+\frac{1}{5}+1+\frac{1}{2}$$

We can add the whole numbers, then get a common denominator and add:

$$2+1+\frac{1}{5}+\frac{1}{2}=3+\frac{2}{10}+\frac{5}{10}=3+\frac{7}{10}$$

Answer: $3\frac{7}{10}$

Another way is to add or subtract mixed numbers is to convert the mixed numbers into improper fractions, perform the operation and then change the answer back (if necessary) to a mixed number.

Example: $2\frac{1}{5}+1\frac{1}{2}$ (same as above)

Using this method, we first convert mixed fractions to improper fractions.

$$\frac{11}{5}+\frac{3}{2}$$

Next we change them to a common denominator and add.

$$\frac{22}{10}+\frac{15}{10}=\frac{22+15}{10}=\frac{37}{10}$$

Finally change back to a mixed fraction. 10 goes into 37 just three times, with a remainder of 7:

Answer: $3\frac{7}{10}$ (same as above!)

Hardest Addition in the World

Here is a math problem that might be the hardest fractions to add. Something about the pattern just makes you want to add numbers going straight across. But don't! Follow the same procedure we always use.

Example: $\frac{2}{5}+\frac{5}{2}=$

Do the 2's cancel each other? Not in addition!

Find a common denominator and then add.

$$\frac{2}{5}+\frac{5}{2}=\frac{4}{10}+\frac{25}{10}=\frac{4+25}{10}=\frac{29}{10}$$

Finding LCM and GCF – The Korean Way

Don't you wish there was an easier way to compute common multiples and greatest factors? There is! A parent showed me the Korean way. It gives you both LCM and GCF at the same time.

Find the GCF and LCM of 6 and 18.

1. Write 6 and 18 side by side.
   Leave some room around it for long division.

$$\overline{)\ 6,\ 18}$$

2. Think of *any* common factor.
   Let's use 2.

$$\begin{array}{r} 3,\ 9 \\ 2\overline{)\ 6,\ 18} \end{array}$$

3. Divide 6 and 18 by the common factor.
   Write the result above, as in regular division.

4. Look at the results, and find another common factor.
   Let's use 3, and continue the long division of both numbers.

$$\begin{array}{r} 1,\ 3 \\ 3\overline{)\ 3,\ 9} \\ 2\overline{)\ 6,\ 18} \end{array}$$

5. If you can think of 6 right away, that's great.
   But you can start with any common factor and get the same result.

6. Now all the common factors are listed on the left.
   Together these make the GCF!
   GCF = $2 \times 3 = 6$

7. The remaining factors (which are *not* common) are listed across the top.
   Take *all* the factors together to make the LCM!
   LCM = $2 \times 3 \times 1 \times 3 = 18$

$$\begin{array}{r} 1,\ 3 \\ 3\overline{)\ 3,\ 9} \\ 2\overline{)\ 6,\ 18} \end{array}$$

Example: Find the GCF and LCM of 16 and 18.

$$\begin{array}{r} 8,\ 9 \\ 2\overline{)16,\ 18} \end{array}$$

GCF = 2

LCM = $2 \times 8 \times 9 = 144$

Example: Find the GCF and LCM of 9 and 18.

$$\begin{array}{r} 1,\ 2 \\ 3\overline{)\ 3,\ 6} \\ 3\overline{)\ 9,\ 18} \end{array}$$

GCF = $3 \times 3 = 9$

LCM = $3 \times 3 \times 1 \times 2 = 18$

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Large Prime Numbers

People have always been searching for ever-larger prime numbers. Since computers became available in the 50's they have accelerated the search.

The largest known prime is $2^{6,972,593}-1$, which is a decimal number with more than two million digits! A large DEC Alpha computer ran for two weeks just to verify it.

The largest known prime was recently discovered last year by hobbyists cooperating on the Internet. They are contributing idle cpu time (the milliseconds between mouse clicks) and cpu time which is often wasted by screen savers. This loose group of more than 21,000 computers world-wide is arguably the world's most powerful computing system! Collectively it is running at 1839 billion floating point operations per second (gigaflops).

Mersenne primes, such as this latest discovery, are probably the oldest type studied -- Euclid discussed them as early as 350 B.C. – they are central to an area of mathematics known as number theory.

Primes have practical applications. Virtually everyone who's done a Web transaction has used one. Primes fuel cryptography, the field behind the secret coding of computer communication as well as communication between governments.

They're also found on Wall Street, where smaller primes propel risk-management theories and computational finance. Biology looks at prime numbers when it studies such questions as why cicadas have 13- and 17-year life cycles. Primes even show up in DNA sequences.

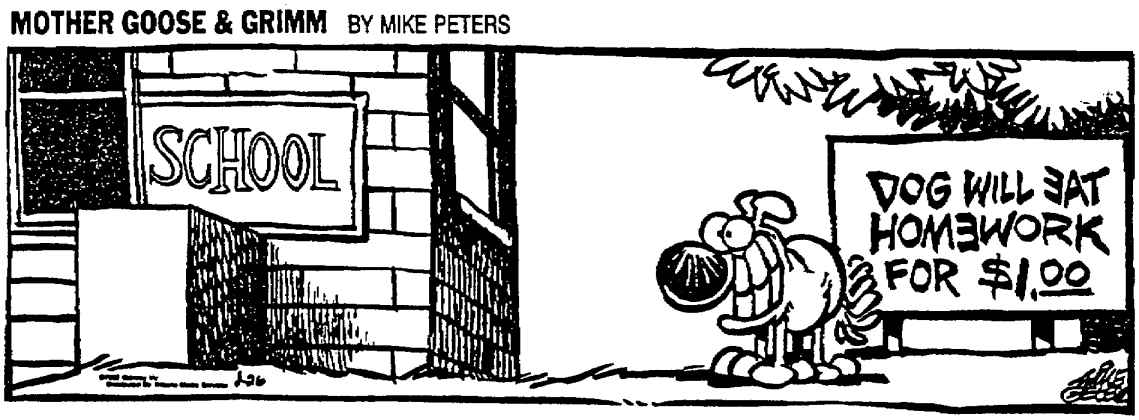
Eventually, even household appliances with computerized components could be harnessed for the prime search. Just think - your toaster could be generating cash slowly as it sits there. It's not really that far-out of an idea.

There is even a cash prize for the first prime found with more than ten million digits.

You can read more about it at http://www.entropia.com/ips. Your coach, Barry Hansen, contributes his idle cpu time to GIMPS (the great internet prime search). Look for his account name on GIMPS as "**barryh**". He is currently around 95th position out of fifteen thousand teams, and has been working on it for about three years.

Vocabulary

- *Product* - The result of a multiplication: *factor* × *factor* = *product*
- *Factor* - Whole numbers (not fractions) that you can multiply together to get an answer. Every number has at least two factors: 1 and itself. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- *Prime factoring* - Finding the factors of a number, where each factor is itself a prime number. For example, the prime factors of 12 are $2 \times 2 \times 3$.
- *Least common multiple* - The least common multiple (LCM) of two or more numbers is the smallest number that can be evenly divided by those numbers. A common multiple will always exist by multiplying the numbers together, but it may not be the *least* common multiple.
  In fact, one way of finding the LCM is to multiply the given numbers, then dividing the product by the largest common factor of the two numbers. For example, the least common multiple of 15 and 25 can be found: $15 \times 25 \div 5 = 75$.
- *Greatest common factor* - The greatest common factor (GCF) is the largest number that is a factor of two or more other numbers. A common factor always exists. If the numbers are relatively prime, the only common factor is 1.
- *What's bigger?* The LCM and GCF are often confused with each other. For any two numbers, is the LCM bigger than the GCF? Yes! Always examine these math problems closely for the word *multiple* (which is bigger than the numbers) and the word *factor* (which is smaller than the starting numbers).
- *Prime number* - A number that can only be the product of 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, … In fact, the only even number that is prime is the number 2. Note that zero is not called a prime number.
- *Composite number* - Any number greater than 1 that is not a prime number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SCHOOL
DOG WILL EAT HOMEWORK FOR $1.00

Homework

1) Find the greatest common factor for each of the following number sets:

a) GCF of 12, 16 *Example:* $12 = \underline{2}\times\underline{2}\times 3$
$16 = \underline{2}\times\underline{2}\times 2\times 2$ $GCF = 2 \times 2 = 4$

b) GCF of 9, 12 =

c) GCF of 8, 24 =

d) GCF of 36, 48 =

e) GCF of 15, 35 =

f) GCF of 36, 60 =

2) Find the least common multiple for each of the following sets of numbers:

a) LCM of 6 and 18 *Example:* *6, 12, <u>18</u>, 24, 30, <u>36</u>, …*
<u>18</u>, <u>36</u>, 54, 72, … *LCM = 18*

b) LCM of 5 and 7 =

c) LCM of 12 and 20 =

d) LCM of 9 and 12 =

e) LCM of 6 and 10 =

f) LCM of 16 and 18 =

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3) Find the least common multiple for these sets of three numbers:

a) LCM of 3, 4 and 6 =

b) LCM of 3, 6 and 9 =

c) LCM of 6, 9 and 12 =

d) LCM of 10, 15 and 20 =

4) Write *even* if the product of these numbers would be even, or *odd* if the product would be an odd number:

a) even × odd = _____

b) odd × odd = _____

c) even × even = _____

5) Solve the problem by finding the least common multiple:

a) Mrs. Winston is buying pencils and pink erasers. She wants to have the same number of each. Pencils are sold in packages of 12. Pink erasers are sold in packages of 9. How many packages of each should she buy?

b) Amy, Bradley and Chris are hiking the same trail. Amy rests every 4 kilometers, Bradley rests every 3 kilometers and Chris rests every 2 kilometers. Where is the first rest stop where all three hikers stop together?

6) Solve these problems:

a) How many prime numbers are there between 1 and 50?
(Hint: List the prime numbers.)

b) What fraction of the numbers from 1 to 50 are prime numbers?
(Hint: It is the number of primes, divided by 50.)

7) Solve these problems regarding proper fractions: Always reduce results to simplest terms.

a) $\frac{6}{32}+\frac{12}{32}=$ *Example:* $\frac{6+12}{32}=\frac{18}{32}$ *reduces to* $\frac{9}{16}$

b) $\frac{8}{9}-\frac{1}{9}=$

c) $\frac{5}{12}+\frac{7}{24}=-$

d) $\frac{2}{3}-\frac{1}{6}=$

e) $\frac{4}{5}-\frac{1}{10}=$

f) $\frac{7}{3}+\frac{3}{7}=$

g) $\frac{1}{8}-\frac{1}{16}=$

h) $\left(\frac{1}{3}+\frac{1}{4}\right)-\frac{1}{12}=$

8) Solve these problems involving mixed fractions. Show your work. Always reduce fractions.

a) $2\frac{1}{3}+2\frac{2}{3}=$

b) $6\frac{3}{8}+\frac{3}{8}=$

c) $1\frac{4}{5}-1\frac{1}{10}=$

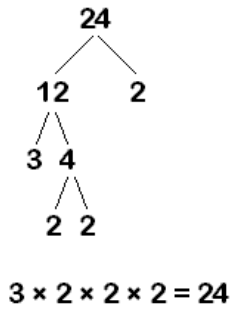
d) $\left(\frac{1}{2}+5\frac{3}{4}\right)-6\frac{1}{4}=$

e) $\frac{23}{13}+8\frac{1}{4}=$

9) Draw the factor tree for these numbers.

a) 24

Example:



$3 \times 2 \times 2 \times 2 = 24$

b) 30

c) 468