Leaving Certificate Mock Exams 2020

Maths (Higher Level)

Paper 1

Monday 2nd March 2020

9.30 a.m. – 12.00 p.m.

Time: 2 ½ hrs

Name:____

Instructions

There are two sections in this examination paper.

Section A	Concepts and Skills	150 marks 6 questions
Section B	Contexts and Applications	150 marks 3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the Formulae and Tables booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include supporting work.
You may lose marks if the appropriate units of measurement are not included, where relevant. You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

SECTION A **Concepts and Skills** **150 marks**

Question 1 **25 Marks**

(a) Show that $x = 2p$ is a root of $x^3 - 4px^2 + p^2x + 6p^3 = 0$ and find the other two roots in terms of p.

(b) Solve for x and y if

$$(x-1)^2 + (y+2)^2 = 13 \qquad and$$

$$2x - y - 5 = 0$$

Question 2 **25 Marks**

(a) The sum of the first three terms of a geometric series is 63 and their product is 1728. Find the value of each of the three terms?

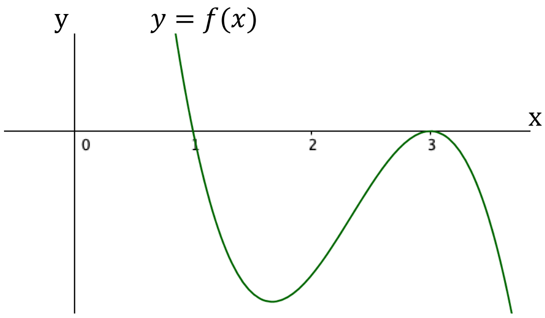
(b) The sum of the first three terms of an arithmetic series is 27 and their product is 704. Find the value of each of the three terms?

Question 3 — 25 Marks

(a) The graph to the right shows

$$f(x) = ax^3 + bx^2 + cx + d$$

Find the values of a, b, c and $d \in \mathbb{R}$ if the curve crosses the y-axis at $(0,18)$?



(b) Find the area enclosed by the curve and the x-axis

Question 4 25 Marks

A function $f(x)$ is defined for ***positive real numbers*** by

$$f(x) = \frac{\ln x}{x}$$

(a) Find $f'(x)$ and $f''(x)$

(b) Show that the graph of $y = f(x)$ has a maximum turning point at $(e, 1/e)$

(c) Draw a rough graph of $y = f(x)$ in the space provided.

(d) Deduce that $x^e \leq e^x$ for all $x \in \mathbb{R}, x > 0$

Question 5 25 Marks

Let

$$z = \frac{8}{\sqrt{3} + i}$$

be a complex number.

(a) Show that z can be written in the form

$$z = 2\sqrt{3} - 2i$$

(b) Hence write z in the form $r(\cos\theta + i\sin\theta)$

(c) Hence or otherwise evaluate z^5 in the form $2^n\left(a\sqrt{3} + b\,i\right)$ $\quad n \in \mathbb{N}$ *and* $a, b \in \mathbb{Z}$

Question 6 — 25 Marks

A poker is placed in a hot furnace and heated to a temperature of 1600°C. It is taken out of the furnace and plunged into a water bath at a temperature of 4 °C. After a time of 30 secs the temperature has dropped to 100°C. It is known that the temperature of the poker at any given time can be found using the function

$$Y(t) = T_o + a\,e^{bt}$$

where $\boldsymbol{T_o}$ **is the outside temperature** and t is time measured in **minutes**.

(a) Find the values of a and b to two decimal places.

(b) The poker is deemed safe to touch once the temperature of the poker has dropped below 40°C. Find to ***the nearest second*** how much time passes until the poker is safe to touch?

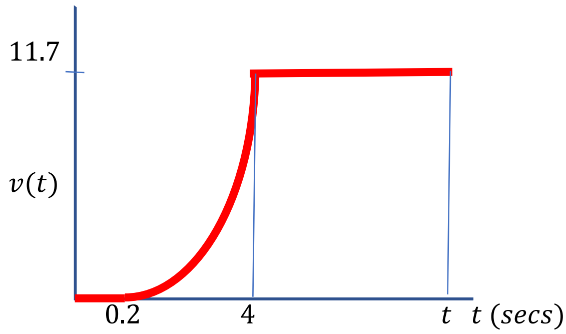
(c) Find the **rate** at which the poker is cooling down after a time of 2 minutes.

SECTION B **Contexts and Application** **150 marks**

Question 7 **40 Marks**

The graph to the right shows the speed of a sprinter at any given time in the 200m final of an athletics competition. The speed of the runner (in m/s) at any given time (t) is given by the function

$$v(t)=\begin{cases} 0 & t<0.2\ secs \\ 1.9e^{0.25at-0.1}-1.9 & 0.2\le t\le 4\ secs \\ 11.7 & t>0.4 \end{cases}$$



(a)

Using the fact that the speed of the runner after 4 secs is approximately 11.7 m/s find the value of 'a' to the **nearest whole number.**

(b) Find the distance travelled by the runner during the ***accelerating portion*** of the race.

Hint: You may assume that

$$\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+c$$

(c) Find the finishing time for the race to two decimal places?

(d) What was the average acceleration of the sprinter while they were accelerating?

Question 8 50 Marks

An airline purchases a new aircraft for a price of €250 million and decides to pay back the loan in **quarterly payments (every 3 months)** over a period of 20 years in equal payments. The money is borrowed at a fixed APR of 2.4% for the duration of the loan.

(a) Find the value of each of the quarterly payments if the first payment is made at the end of the first quarter and the last is made on the 20^{th} anniversary of signing the agreement.

(b) Find the total amount which the airline has paid for the aircraft once the loan has been repaid.

The airline currently carries an average of 10000 customers per month on this aircraft at an average price of €100 per customer journey. The company expects passenger numbers to grow at a compound rate of 0.5% per month over the next 20 years.

(c) Express the number of customers carried by the aircraft during the first 5 months of operation as a geometric series.

(d) Hence estimate the total number of customers which the aircraft will carry (passenger journeys) over the next 20 years and hence the total revenue (income) produced by this aircraft over the twenty year period. You may assume that the average price of a customer journey remains fixed during the 20 year period.

(e) In your opinion (ignoring other costs) do you feel that the aircraft was a profitable purchase for the airline.

Question 9 60 Marks

(a)

Eamon pours pancake batter onto a pan at a constant rate of $2\ cm^3/sec$. The pancake is of uniform thickness of approximately 2mm.

(i)
Find the **rate** at which the radius of the pancake is changing when the radius in 12 cm (in terms of π)?

(ii) Find the rate at which the perimeter of the pancake is changing when the radius is 12 cm? Leave your answer as a fraction.

(b) Let $f(x) = x^3 - 3x^2 - 6x + 8 \quad x \in \mathbb{R}$

(i) Find the turning points of the function leaving all coordinates to **TWO** decimal places where appropriate.

(ii) Find the coordinates of the point of inflection of the curve.

(iii) Find the equation of the tangent drawn through the inflection point of the curve?

Additional Extra Paper

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