Lecture 03
Semiconductor Physics

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ILOS

- **In this section, we will learn:**
  - The basic properties of semiconductors and, in particular, silicone – the material used to make most modern electronic circuits.
  - How doping a pure silicon crystal dramatically changes electrical conductivity – the fundamental idea in underlying the use of semiconductors in the implementation of electronic devices.
  - The two mechanisms by which current flows in semiconductors – drift and diffusion charge carriers.
  - The structure and operation of the *pn* junction – a basic semiconductor structure that implements the diode and plays a dominant role in semiconductors.

Agenda

- ❑ **Current Flow in Semiconductors**
- ❑ **The pn Junction with Open-Circuit Terminals (Equilibrium)**
- ❑ **The pn Junction with Applied Voltage**
- ❑ **Capacitive Effects in the pn Junction**

Intrinsic Semiconductors

- **Q:** Why can thermal generation not be used to affect meaningful current conduction?
    - **A:** Silicon crystal structure described previously is not sufficiently conductive at room temperature.
        - Additionally, a dependence on temperature is not desirable.
- **Q:** How can this "problem" be fixed?

doping – is the intentional introduction of impurities into an extremely pure (intrinsic) semiconductor for the purpose changing carrier concentrations.

Doped Semiconductors

A semiconductor material that has been subjected to the doping process is called an extrinsic material.

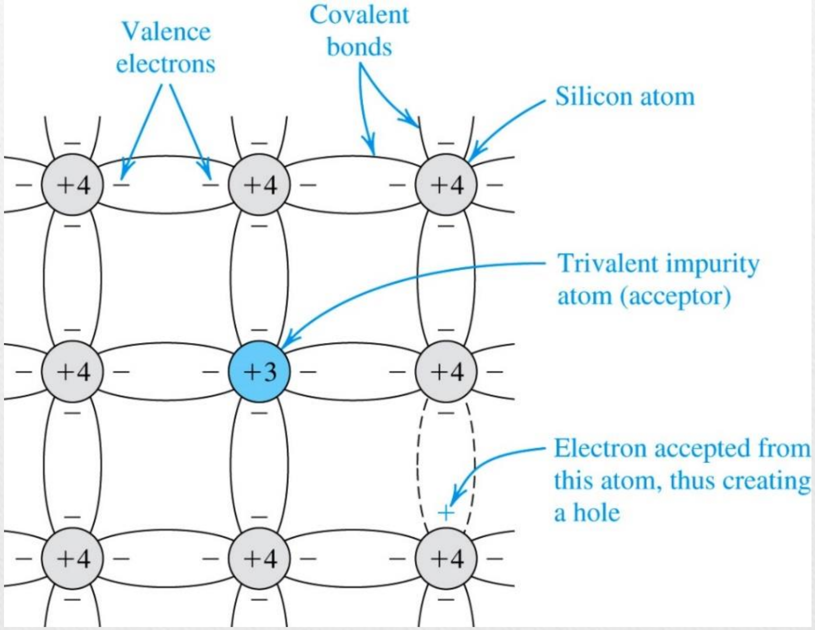
p-type semiconductor

- Silicon is doped with element having a valence of 3.
- To increase the concentration of holes (p).
- One example is boron, which is an acceptor.

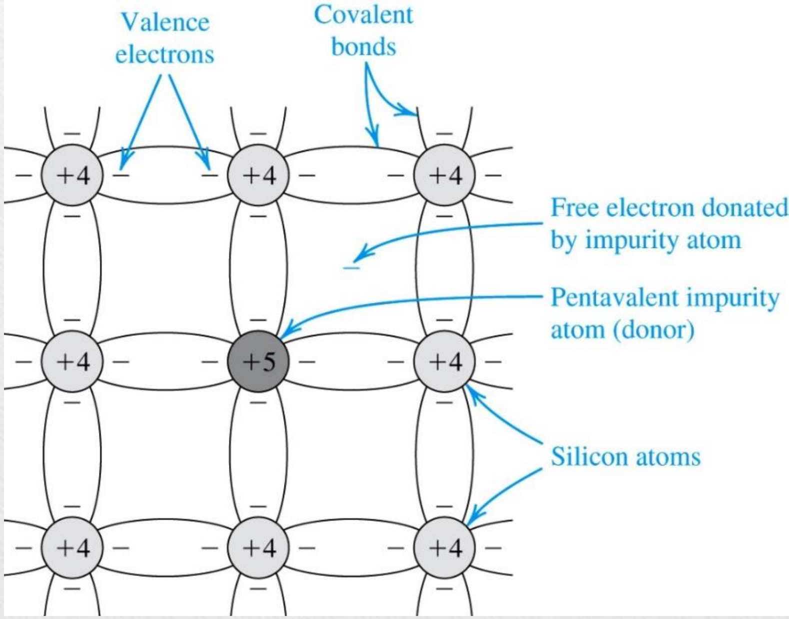
- **n-type semiconductor**
  - Silicon is doped with element having a valence of 5.
  - To increase the concentration of free electrons (n).
  - One example is phosophorus, which is a donor.

Doped Semiconductors

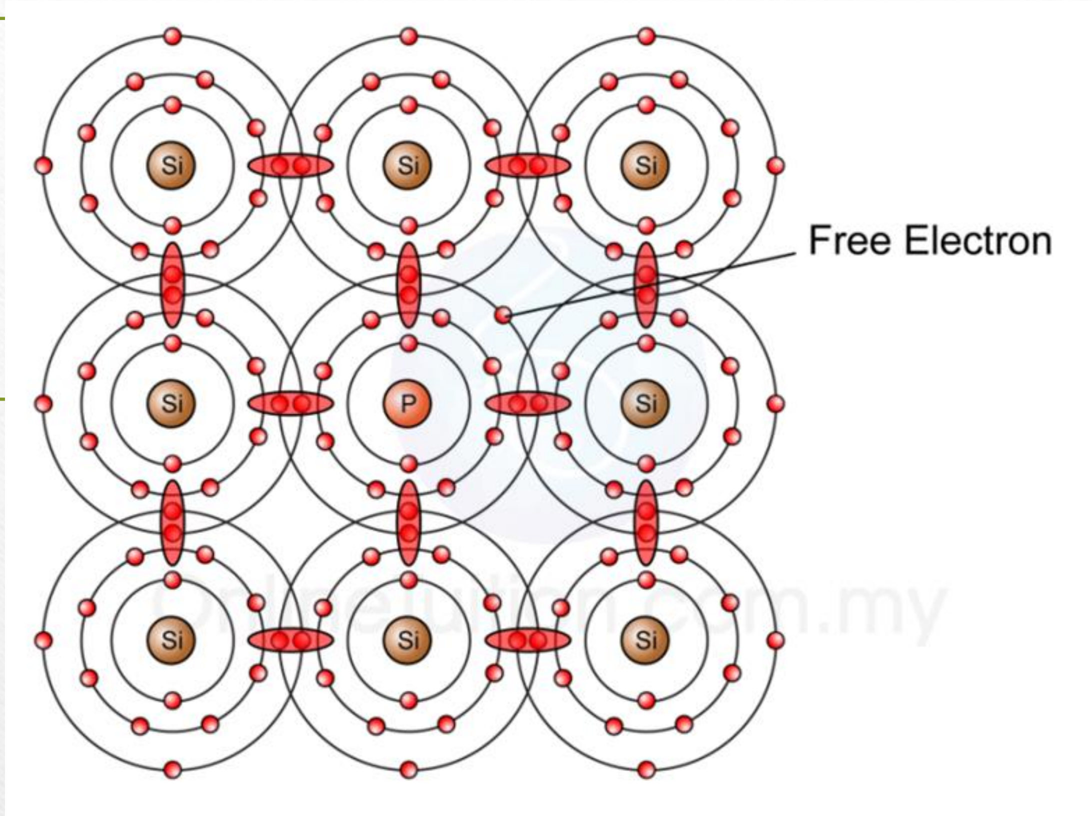
- ***p*-type semiconductor**



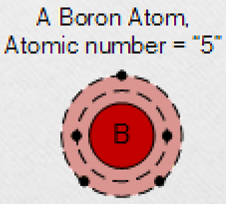
- ***n*-type semiconductor**



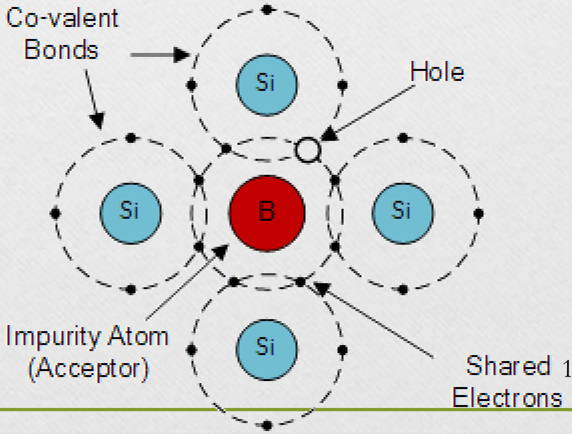
***N*-type**



***p*-type**



Boron atom showing 3 electrons in its outer valence shell (L)



P-Type Semiconductor

Doped Semiconductors

***p*-type doped semiconductor**

- If N_A is much greater than n_i …
  - concentration of acceptor atoms is N_A
- Then the concentration of holes in the *p*-type is defined as below.

$$\overbrace{(p_p) \approx (N_A)}^{\text{they will be equal...}}$$

number holes in *p*-type

number acceptor atoms

Doped Semiconductors

_n_-type doped semiconductor

- If N_D is much greater than n_i…
  - concentration of donor atoms is N_D
- Then the concentration of electrons in the n-type is defined as below.

$$\overbrace{\underset{\text{number free e-trons}}{(n_n)} \approx \underset{\text{number donor atoms}}{(N_D)}}^{\text{they will be equal...}}$$

The key here is that number of free electrons (conductivity) is dependent on doping concentration, not temperature…

Doped Semiconductors

***p*-type semiconductor**

- **Q:** How can one find the concentration?
  - **A:** Use the formula to right, adapted for the *p*-type semiconductor.

action: combine this with equation on previous slide

$$\overbrace{p_p \times n_p = n_i^2}$$

number of holes in *p*-type

number of free electrons in *p*-type

number of free electrons and holes in thermal equil.

$$n_p \approx \frac{n_i^2}{n_A}$$

Doped Semiconductors

***n*-type semiconductor**

- **Q:** How can one find the concentration?
    - **A:** Use the formula to right, adapted for the *n*-type semiconductor.

$$\overbrace{p_n \times n_n = n_i^2}^{\textbf{action}\text{: combine this with equation on previous slide}}$$

p_n	n_n	n_i^2
number of holes in n-type	number of free electrons in n-type	number of free electrons and holes in thermal equil.

$$p_n \approx \frac{n_i^2}{n_D}$$

Doped Semiconductors

- ***p*-type semiconductor**
  - n_p will have the same dependence on temperature as n_i^2
  - the concentration of holes (p_n) will be much larger than free electrons
  - holes are the majority charge carriers
  - free electrons are the minority charge carrier

- ***n*-type semiconductor**
  - p_n will have the same dependence on temperature as n_i^2
  - the concentration of free electrons (n_n) will be much larger than holes
  - electrons are the majority charge carriers
  - holes are the minority charge carrier

Doped Semiconductors

It should be emphasized that a piece of *n*-type or *p*-type silicon is electrically neutral; the charge of the majority free carriers (electrons in the *n*-type and holes in the *p*-type silicon) are neutralized by the bound charges associated with the impurity atoms.

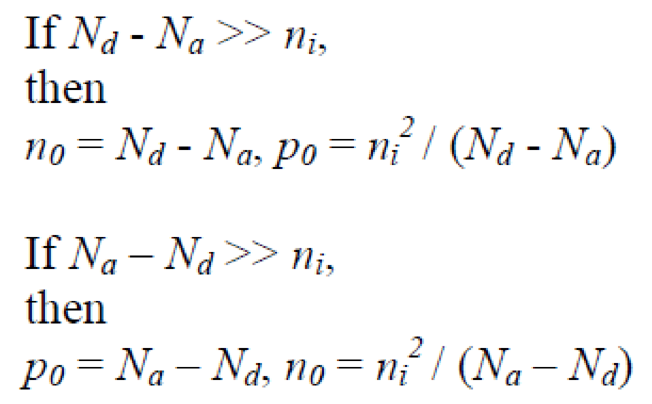
Let
n_o : thermal-equilibrium concentration of electrons
p_o : thermal-equilibrium concentration of holes
N_d : concentration of donor atoms
N_a : concentration of acceptor atoms
N_d+ : concentration of positively charged donors (ionized donors)
N_a- : concentration of negatively charged acceptors (ionized acceptors)

Doped Semiconductors

At equilibrium, the product of the majority and minority carrier concentration is a constant, and this is mathematically expressed by the Law of Mass Action.

$$n_0 p_0 = n_i^2$$

by the charge neutrality condition, $n_0 + N_a^- = p_0 + N_d^+$



If $N_d - N_a >> n_i$,
then
$n_0 = N_d - N_a$, $p_0 = n_i^2 / (N_d - N_a)$

If $N_a – N_d >> n_i$,
then
$p_0 = N_a – N_d$, $n_0 = n_i^2 / (N_a – N_d)$

Example 2: Doped Semiconductor

- Consider an n-type silicon for which the dopant concentration is $N_D = 10^{17}/\text{cm}^3$. Find the electron and hole concentrations at $T = 300\text{K}$.
- **Solution**

The concentration of the majority electrons is $n_n \simeq N_D = 10^{17}\ /\text{cm}^3$

The concentration of the minority holes is $p_n \simeq \frac{{n_i}^2}{N_D}$

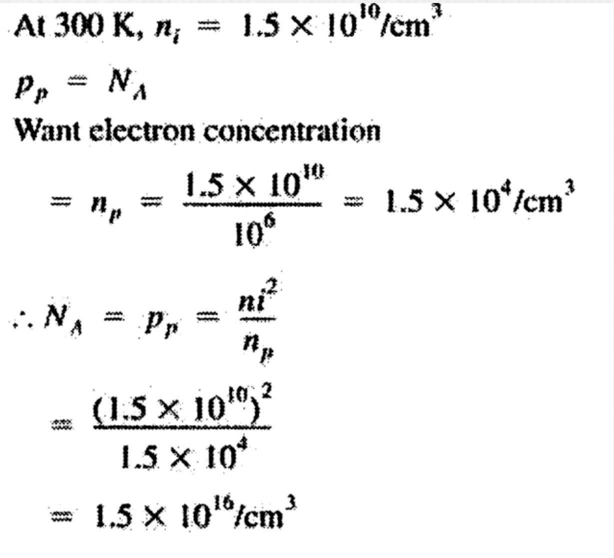
In Example 1, we found that at $T = 300$ K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$. Thus,

$$p_n \simeq \frac{(1.5 \times 10^{10})^2}{10^{17}} = 2.25 \times 10^3/\text{cm}^3$$

Observe that $n_n \gg n_i$ and that n_n is vastly higher than p_n.

Example 3 : Doped Semiconductor

For a silicon crystal doped with boron, what must N_A be if at $T = 300$ K the electron concentration drops below the intrinsic level by a factor of $\mathbf{10^6}$?



At 300 K, $n_i = 1.5 \times 10^{10}/\text{cm}^3$

$p_p = N_A$

Want electron concentration

$$= n_p = \frac{1.5 \times 10^{10}}{10^6} = 1.5 \times 10^4/\text{cm}^3$$

$$\therefore N_A = p_p = \frac{ni^2}{n_p}$$

$$= \frac{(1.5 \times 10^{10})^2}{1.5 \times 10^4}$$

$$= 1.5 \times 10^{16}/\text{cm}^3$$

Current Flow in Semiconductors

1. Drift Current

There are two distinctly different mechanisms for the movement of charge carriers and hence for current flow in semiconductors: **drift** and **diffusion**.

- **Q:** What happens when an electrical field (E) is applied to a semiconductor crystal?
  - **A:** Holes are accelerated in the direction of E, free electrons are attracted.
- **Q:** How is the velocity of these carriers defined?

μ_p =hole mobility
E=electric field

μ_n =electron mobility
E=electric field

$$v_{p-drift} = \mu_p E \qquad\qquad v_{n-drift} = -\mu_n E$$

1. Drift Current

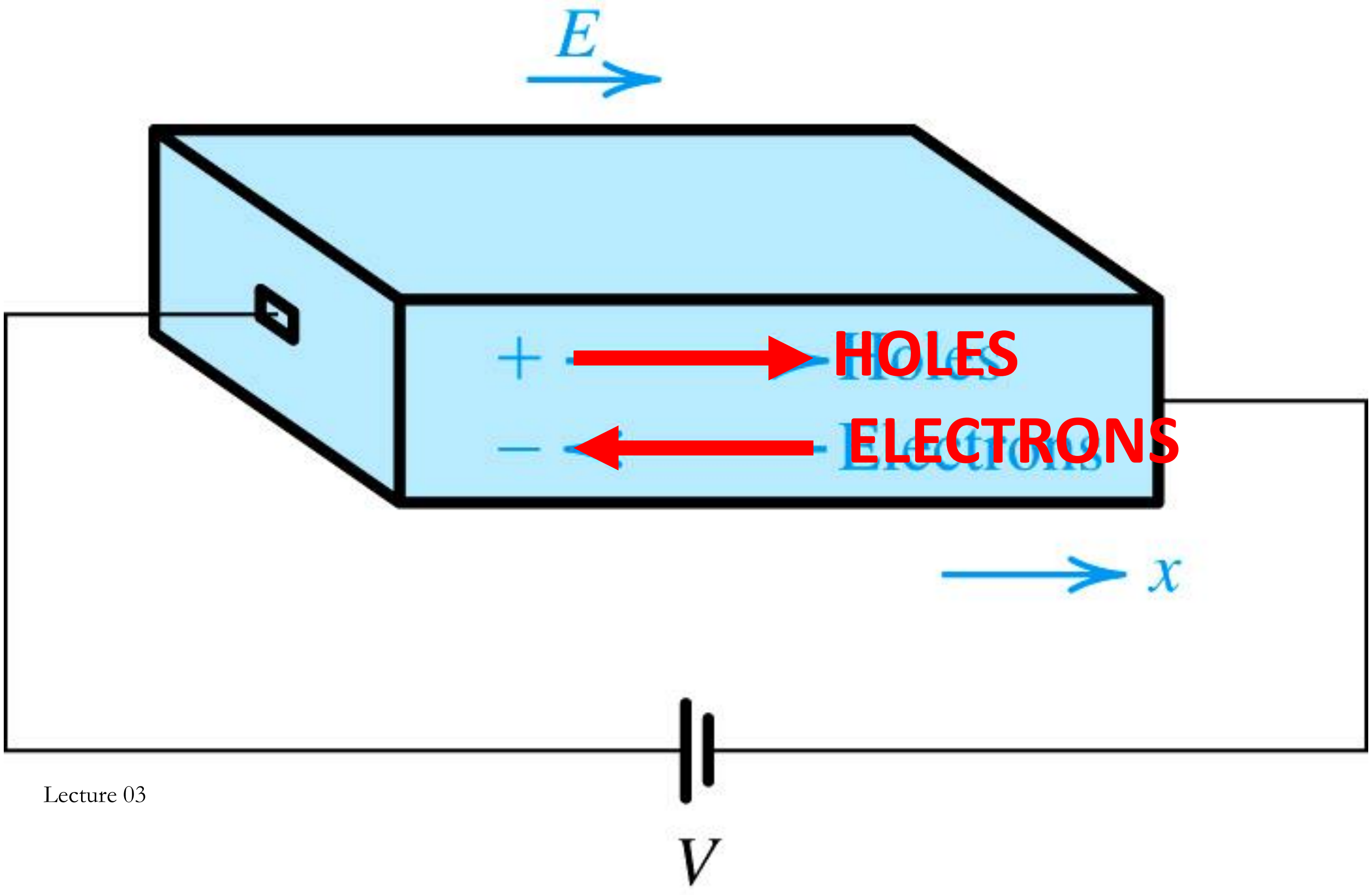
note that electrons move with velocity **2.5 times higher** than holes

E (volts / cm)

μ_p (cm^2/Vs) = 480 for silicon

μ_n (cm^2/Vs) = 1350 for silicon

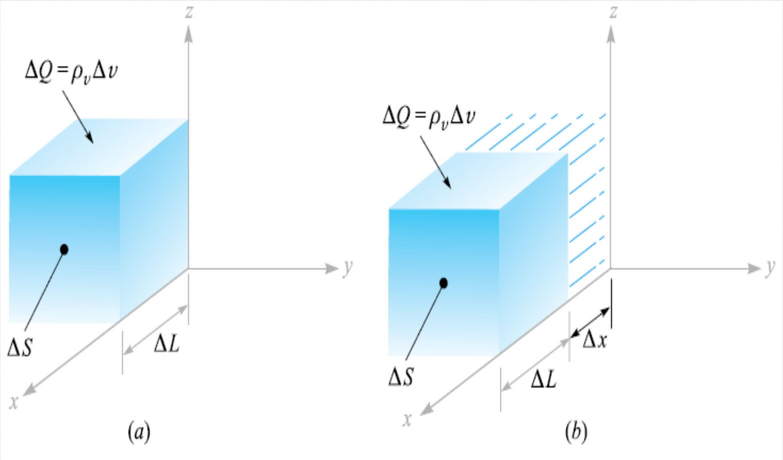
An electric field *E* established in a bar of silicon causes the holes to drift in the direction of *E* and the free electrons to drift in the opposite direction. Both the hole and electron drift currents are in the direction of *E*.



Current and Current Density

Current density may be related to the velocity of volume charge density at a point.

Consider the element of charge $\Delta Q = \rho_v \Delta v = \rho_v \Delta S \Delta L$, as shown in Figure (*a*).



Current and Current Density

- To simplify the explanation, assume that the charge element is oriented to the x-axis and has only an *x* component of velocity $\Delta Q = \rho_v \Delta S \Delta x.$
- If the charge element ΔQ moved a distance Δx in the time interval Δt, as indicated in Figure (*b*), the resulting current will be

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \frac{\Delta x}{\Delta t} \Rightarrow \Delta I = \rho_v \, \Delta S \, v_x$$

Where v_x represents the *x* component of the velocity **v**. In terms of current density,

we find

$$J_x = \rho_v \, v_x$$

and in general

$$\mathbf{J} = \rho_v \mathbf{v}$$

1. Drift Current

- Assume that, for the single-crystal silicon bar on previous slide, the concentration of holes is defined as p and electrons as n.
- **Q:** What is the current component attributed to the flow of holes (not electrons)?

1. Drift Current

- **step #1:** Consider a plane perpendicular to the *x* direction.
- **step #2:** Define the hole charge that crosses this plane.

I_p = current flow attributed to holes
A = cross-sectional area of silicon
q = magnitude of the electron charge
p = concentration of holes
$v_{p-drift}$ = drift velocity of holes

$$\overbrace{I_p = A\underbrace{qp}_{\rho_v}v_{p-drift}}$$

$$\mathbf{J} = \rho_v \mathbf{v}$$

1. Drift Current

PART A: What is the current component attributed to the flow of holes (not electrons)?

- **step #3:** Substitute in $\mu_p E$.
- **step #4:** Define current density as $J_p = I_p / A$.

I_p = current flow attributed to holes
A = cross-sectional area of silicon
q = magnitude of the electron charge
p = concentration of holes
μ_p = hole mobility
E = electric field

$$\overbrace{I_p = Aqp\mu_p E}$$

$$\boxed{J_p = qp\mu_p E}$$

1. Drift Current

- **Q:** What is the current component attributed to the flow of electrons (not holes)?
  - **A:** to the right…
- **Q:** How is total drift current defined?
  - **A:** to the right…

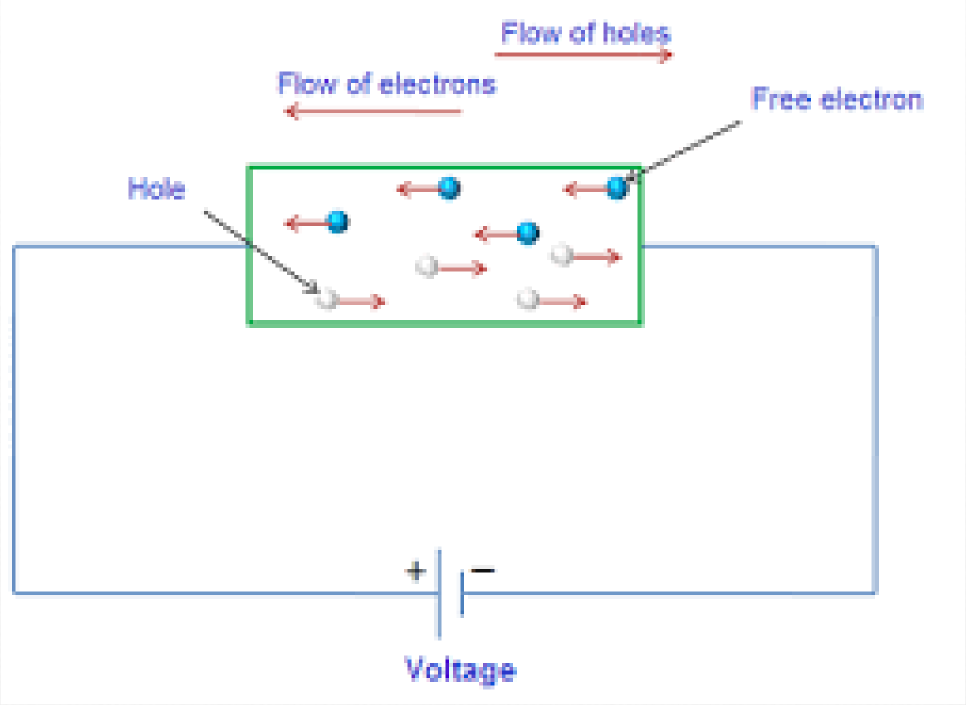
I_n = current flow attributed to electrons
A = cross-sectional area of silicon
q = magnitude of the electron charge
n = concentration of free electrons
μ_n = electron mobility
E = electric field

$$\overbrace{I_n = -Aqv_{n-drift}}$$

$$J_n = qn\mu_n E$$

$$J = J_p + J_n = \underbrace{q(p\mu_p + n\mu_n)}_{\text{this is conductivity } (\sigma)} E$$

1. Drift Current



1. Drift Current

- **conductivity** (σ) – relates current density (J) and electrical field (E)
- **resistivity** (ρ) – relates current density (J) and electrical field (E)

$$J = \sigma E$$

$$\boxed{\sigma = q\left(p\mu_p + n\mu_n\right)}$$

$$\boxed{\rho = \frac{1}{q\left(p\mu_p + n\mu_n\right)}}$$

Example 4: Drift current

- **Q(a):** Find the resistivity of intrinsic silicon using following values – $\mu_n = 1350 cm^2/Vs$, $\mu_p = 480 cm^2/Vs$, $n_i = 1.5\mathbf{x10}^{10}/cm^3$.
- **Q(b):** Find the resistivity of *p*-type silicon with $N_A = 10^{16}/cm^2$ and using the following values – $\mu_n = 1110 cm^2/Vs$, $\mu_p = 400 cm^2/Vs$, $n_i = 1.5\mathbf{x10}^{10}/cm^3$

note that doping reduces carrier mobility

Example 5: Drift current

(a) For intrinsic silicon, $p = n = n_i = 1.5 \times 10^{10}/\mathrm{cm}^3$

$$\rho = \frac{1}{q(p\mu_p + n\mu_n)} = \frac{1}{1.6\times10^{-19}(1.5\times10^{10}\times480+1.5\times10^{10}\times1350)}$$

$$= 2.28\times10^5\ \Omega.\mathrm{cm}$$

(b) For the *p*-type silicon $p_p \simeq N_A = 10^{16}/\mathrm{cm}^3$

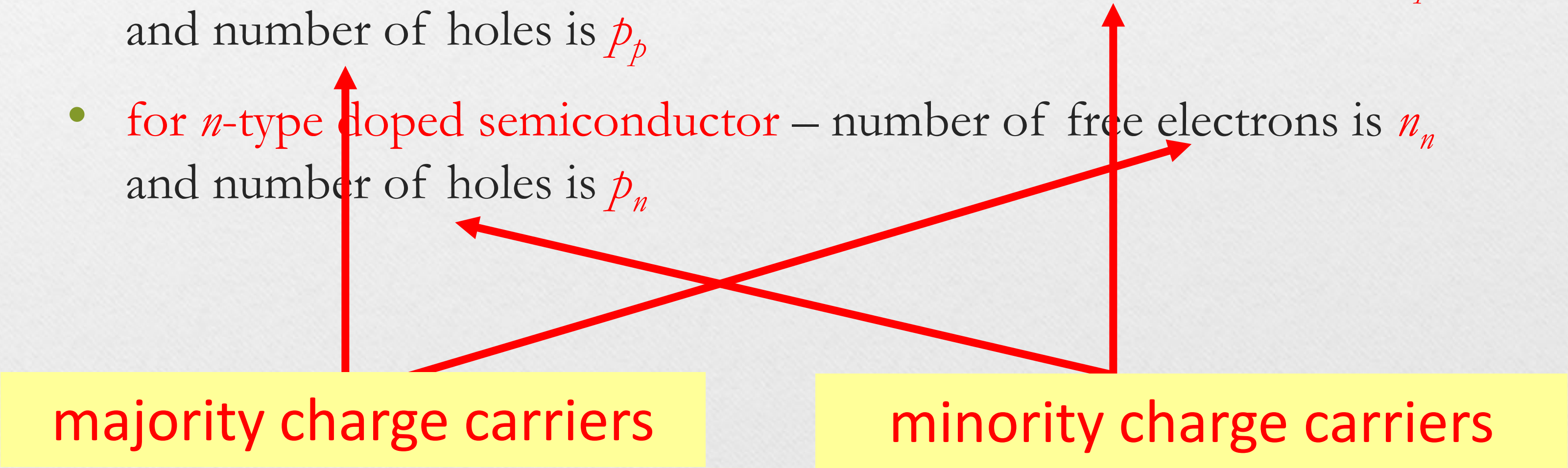
$$n_p \simeq \frac{{n_i}^2}{N_A} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4/\mathrm{cm}^3$$

$$\rho = \frac{1}{q(p\mu_p + n\mu_n)} = \frac{1}{1.6\times10^{-19}(10^{16}\times400+2.25\times10^4\times1110)}$$

$$= 1.56\ \Omega.\mathrm{cm}$$

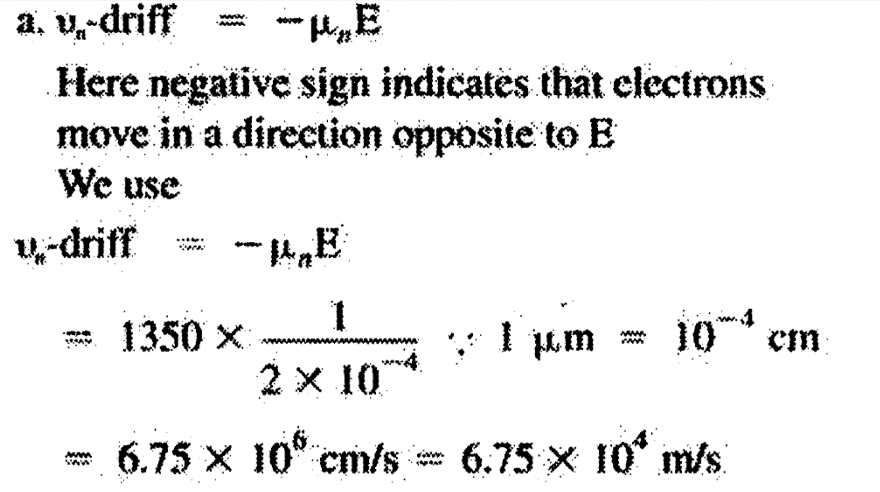
Note…

- for intrinsic semiconductor – number of free electrons is n_i and number of holes is p_i
- for p-type doped semiconductor – number of free electrons is n_p and number of holes is p_p
- for n-type doped semiconductor – number of free electrons is n_n and number of holes is p_n



Example 6: Drift current

A uniform bar of *n*-type silicon of 2 μm length has a voltage of 1 V applied across it. If $N_D = 10^6/\text{cm}^3$ and $\mu_n = 1350\text{cm}^2/\text{V.s}$, find (a) the electron drift velocity, (b) the time it takes an electron to cross the 2-μm length, (c) the drift-current density, and (d) the drift current in the case the silicon bar has a cross sectional area of $0.25\mu\text{m}^2$.



a. $\upsilon_n\text{-drift} = -\mu_n E$

Here negative sign indicates that electrons move in a direction opposite to E

We use

$$\upsilon_n\text{-drift} = -\mu_n E$$

$$= 1350 \times \frac{1}{2 \times 10^{-4}} \quad \because 1\ \mu\text{m} = 10^{-4}\ \text{cm}$$

$$= 6.75 \times 10^6\ \text{cm/s} = 6.75 \times 10^4\ \text{m/s}$$

Example 6: Drift current, contd.

b. Time taken to cross 2μm length

$$= \frac{distance}{v_{drift}} = \frac{2 \times 10^{-6}}{6.75 \times 10^{4}} = 30\text{ps}$$

c. The current density J_n is given by

$$J_n = qn\mu_n E$$

$$= 1.6 \times 10^{-19} \times 10^{16} \times 1350 \times \frac{1}{2 \times 10^{-4}} = 1.08 \times 10^{4}\ \text{A/cm}^2$$

d. Drift current $I_n = J_n A$

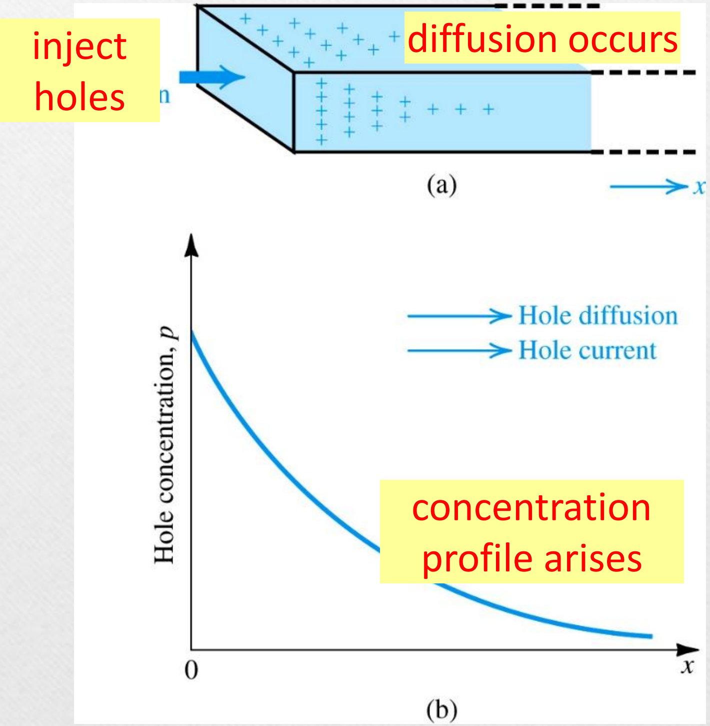
$$I_n = 0.25 \times 10^{-8} \times 1.08 \times 10^{4} = 27\mu\text{A}$$

2. Diffusion Current

- **carrier diffusion** – is the flow of charge carriers from area of high concentration to low concentration.
  - It requires non-uniform distribution of carriers.
- **diffusion current** – is the current flow that results from diffusion.

2. Diffusion Current

- Take the following example…
  - inject holes – By some unspecified process, one injects holes in to the left side of a silicon bar.
  - concentration profile arises – Because of this continuous hole inject, a concentration profile arises.
  - diffusion occurs – Because of this concentration gradient, holes will flow from left to right.



inject holes

diffusion occurs

concentration profile arises

2. Diffusion Current

- **Q:** How is diffusion current defined?

J_p= current flow density attributed to holes
q= magnitude of the electron charge
D_p= diffusion constant of holes (12cm^2/s for silicon)
$\mathbf{p}(x)$= hole concentration at point x
$d\mathbf{p}/dx$= gradient of hole concentration

hole diffusion current density : $J_p = -qD_p \frac{d\mathbf{p}(x)}{dx}$

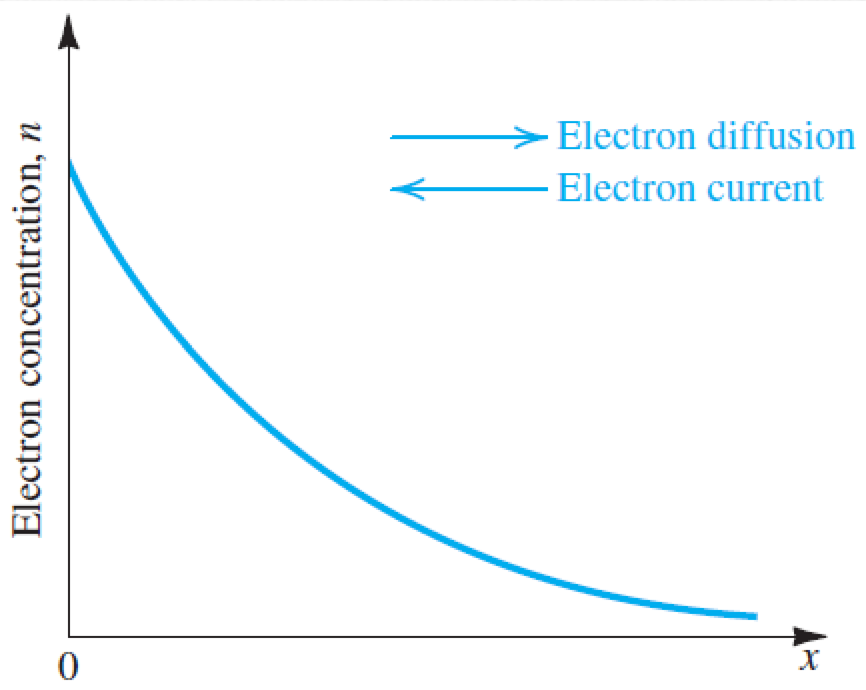
Unit: A/cm^2

electron diffusion current density : $J_n = +qD_n \frac{d\mathbf{n}(x)}{dx}$

J_n= current flow density attributed to free electrons
D_n= diffusion constant of electrons (35cm^2/s for silicon)
$\mathbf{n}(x)$= free electron concentration at point x
$d\mathbf{n}/dx$= gradient of free electron concentration

2. Diffusion Current

Observe that a negative (dn/dx) gives rise to a negative current, a result of the convention that the positive direction of current is taken to be that of the flow of positive charge (and opposite to that of the flow of negative charge).



Example 7: Diffusion current

- Consider a bar of silicon in which a hole concentration **p**(x) described below is established.
- **Q(a):** Find the hole-current density J_p at $x = 0$.
- **Q(b):** Find current I_p.
    - Note the following parameters: $p_0 = 10^{16}/cm^3$, $L_p = 1\mu m$, $A = 100\mu m^2$

$$\mathbf{p}(x) = p_0 e^{-x/L_p}$$

Example 7: Diffusion current, contd.

$$J_p = -qD_p \frac{dp(x)}{dx} = -qD_p \frac{d}{dx}\left[p_o e^{-x/L_p}\right]$$

$$J_p(0) = q\frac{D_p}{L_p} p_0 = 192 \text{ A/cm}^2$$

$$I_p = J_p \times A = 192\ \mu\text{A}$$

3. Relationship Between D and μ?

- **Q:** What is the relationship between diffusion constant (D) and mobility (μ)?
  - **A: thermal voltage** (V_T)
- **Q:** What is this value?
  - **A:** at $T = 300K$, $V_T = 25.9mV$

the relationship between diffusion constant and mobility is defined by thermal voltage

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$$

known as Einstein Relationship

$$V_T = \frac{kT}{q}$$

Summary of Semiconductor currents

- drift current density (J_{drift})
  - affected by – an electric field (E).
- diffusion current density (J_{diff})
  - affected by – concentration gradient in free electrons and holes.

A= cross-sectional area of silicon, q= magnitude of the electron charge,
p= concentration of holes, n= concentration of free electrons,
μ_p = hole mobility, μ_n = electron mobility, E= electric field

$$\overbrace{\textbf{drift current density :}\quad J_{drift} = J_{p-drift} + J_{n-drift} = q(p\mu_p + n\mu_n)E}$$

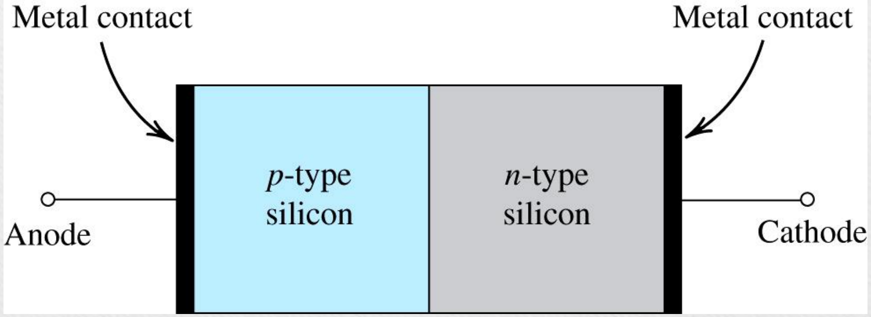
$$\underbrace{\textbf{diffusion current density :}\quad J_{diff} = J_{p-diff} + J_{n-diff} = -qD_p \frac{d\mathbf{p}(x)}{dx} + qD_n \frac{d\mathbf{n}(x)}{dx}}$$

D_p = diffusion constant of holes (12cm^2/s for silicon), D_n = diffusion constant of electrons (35cm^2/s for silicon),
$\mathbf{p}(x)$= hole concentration at point x, $\mathbf{n}(x)$= free electron concentration at point x,
$d\mathbf{p}/dx$= gradient of hole concentration, $d\mathbf{n}/dx$= gradient of free electron concentration

4. The pn Junction with Open-Circuit Terminals
4.1. Physical Structure

[*pn* junction](#) structure

- *p*-type semiconductor
- *n*-type semiconductor
- metal contact for connection



Simplified physical structure of the *pn* junction. As the *pn* junction implements the junction diode, its terminals are labeled anode and cathode.

4.2. Operation with Open-Circuit Terminals

- **Q:** What is state of *pn* junction with open-circuit terminals?
- **A:** Read the below…
  - *p*-type material contains majority of holes
    - these holes are neutralized by equal amount of bound negative charge
  - *n*-type material contains majority of free electrons
    - these electrons are neutralized by equal amount of bound positive charge

4.2. Operation with Open-Circuit Terminals

bound charge

- charge of opposite polarity to free electrons / holes of a given material
- neutralizes the electrical charge of these majority carriers
- does not affect concentration gradients

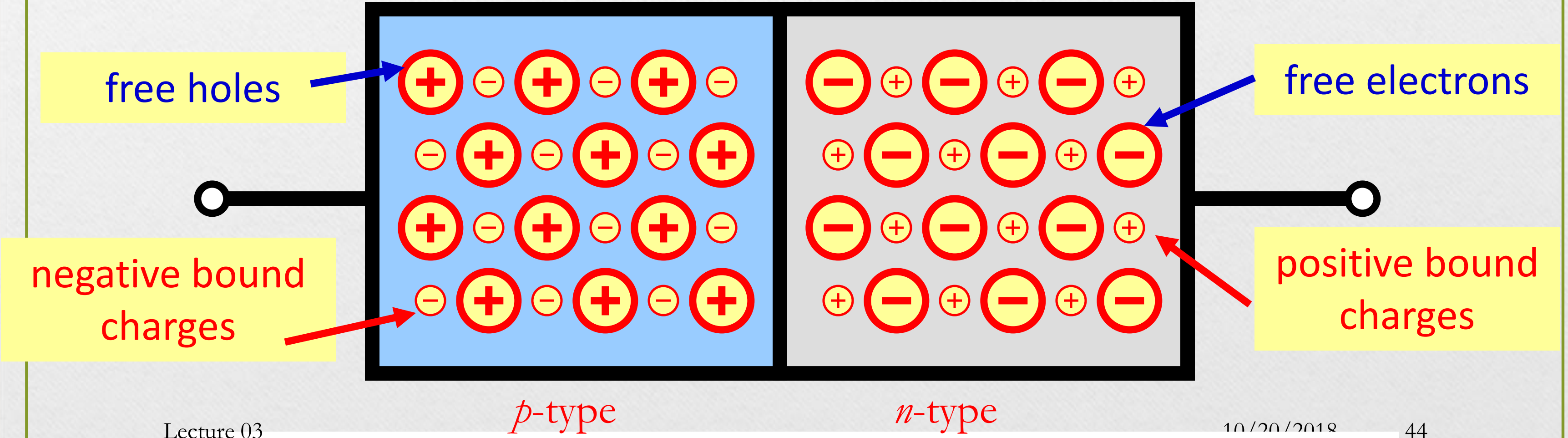
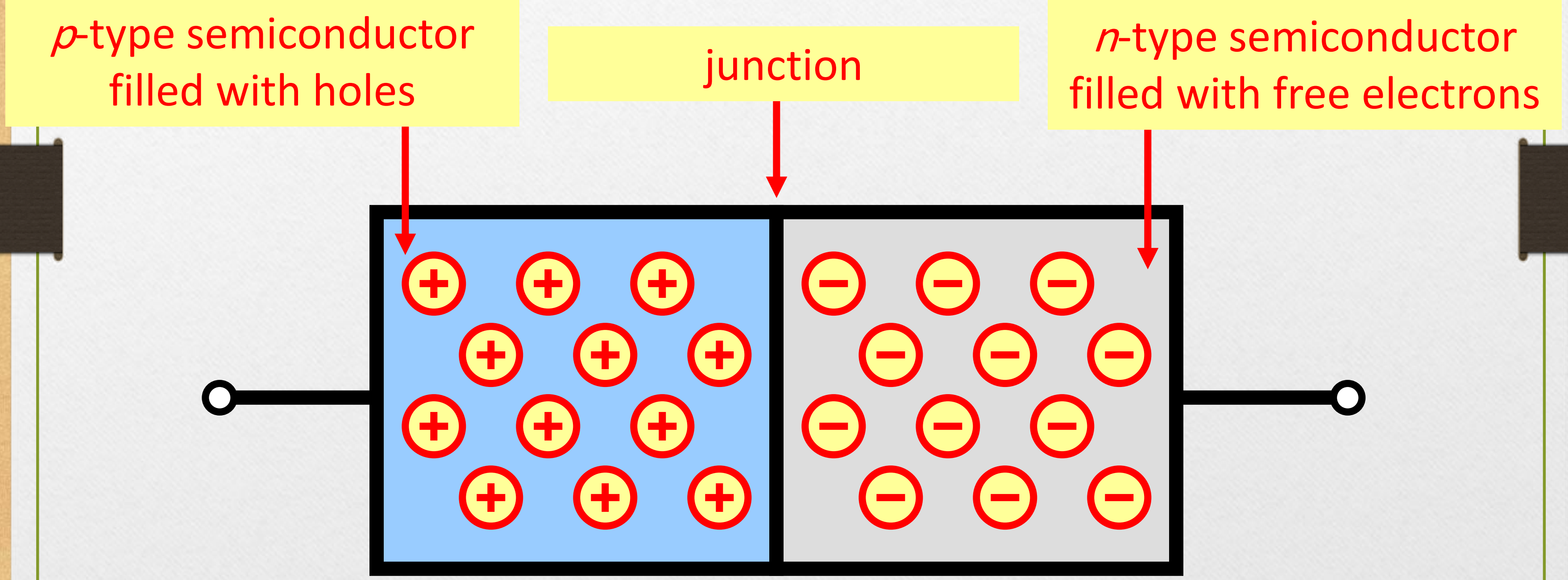


Figure: The *pn* junction with no applied voltage (open-circuited terminals).

4.2. Operation with Open-Circuit Terminals

- **Q:** What happens when a *pn*-junction is newly formed – aka. when the *p*-type and *n*-type semiconductors first touch one another?
  - **A:** See following slides…

Step #1: The *p*-type and *n*-type semiconductors are joined at the junction.



p-type semiconductor filled with holes

junction

n-type semiconductor filled with free electrons

Figure: The *pn* junction with no applied voltage (open-circuited terminals).

Step #2: Diffusion begins. Those free electrons and holes which are closest to the junction will recombine and, essentially, eliminate one another.

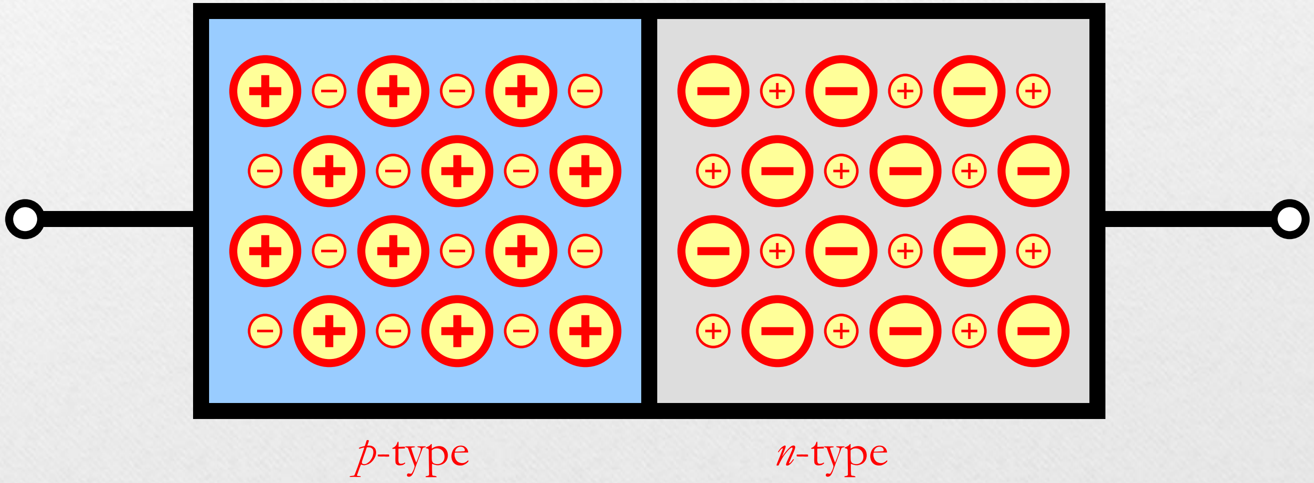


Figure: The *pn* junction with no applied voltage (open-circuited terminals).

Step #3: The depletion region begins to form – as diffusion occurs and free electrons recombine with holes.

The depletion region is filled with "uncovered" bound charges – who have lost the majority carriers to which they were linked.

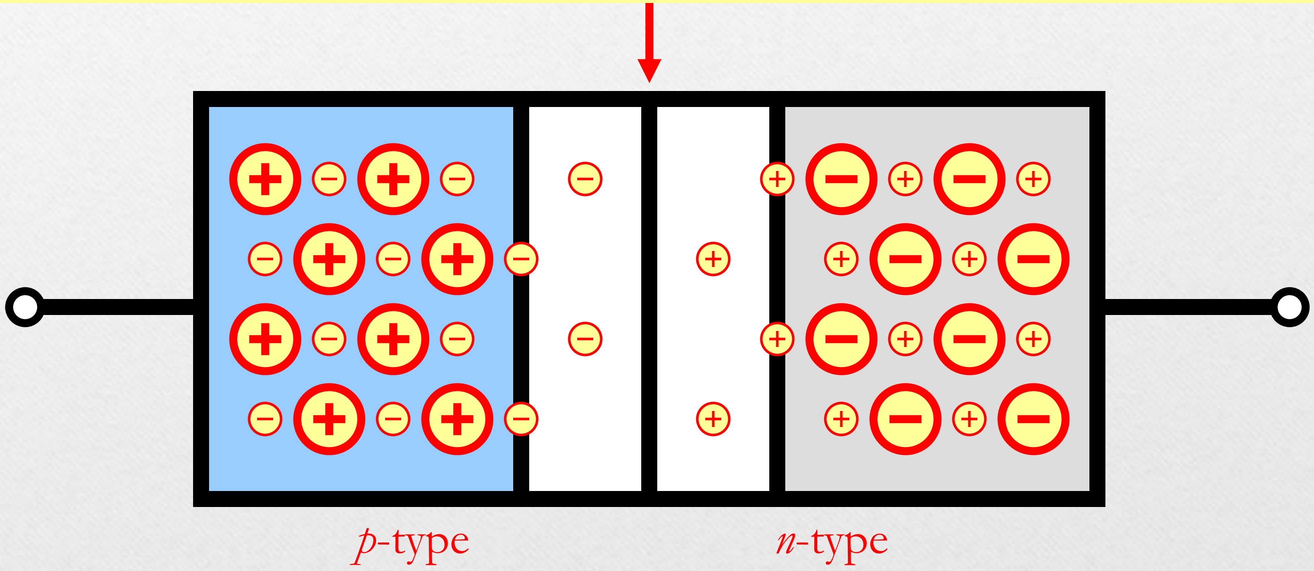
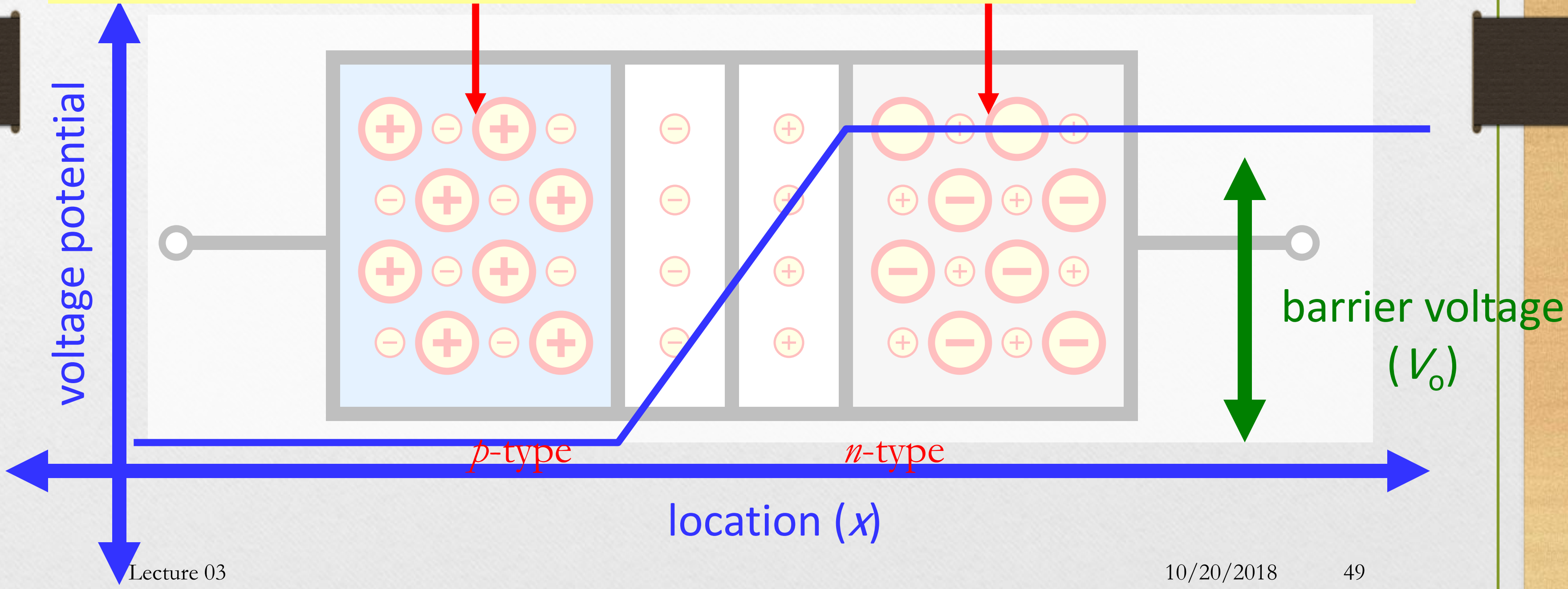


Figure: The *pn* junction with no applied voltage (open-circuited terminals).

Step #4: The "uncovered" bound charges affect a voltage differential across the depletion region. The magnitude of this barrier voltage (V_0) differential grows, as diffusion continues.

No voltage differential exists across regions of the *pn*-junction outside of the depletion region because of the neutralizing effect of positive and negative bound charges.



Step #5: The barrier voltage (V_0) is an electric field whose polarity opposes the direction of diffusion current (I_D). As the magnitude of V_0 increases, the magnitude of I_D decreases.

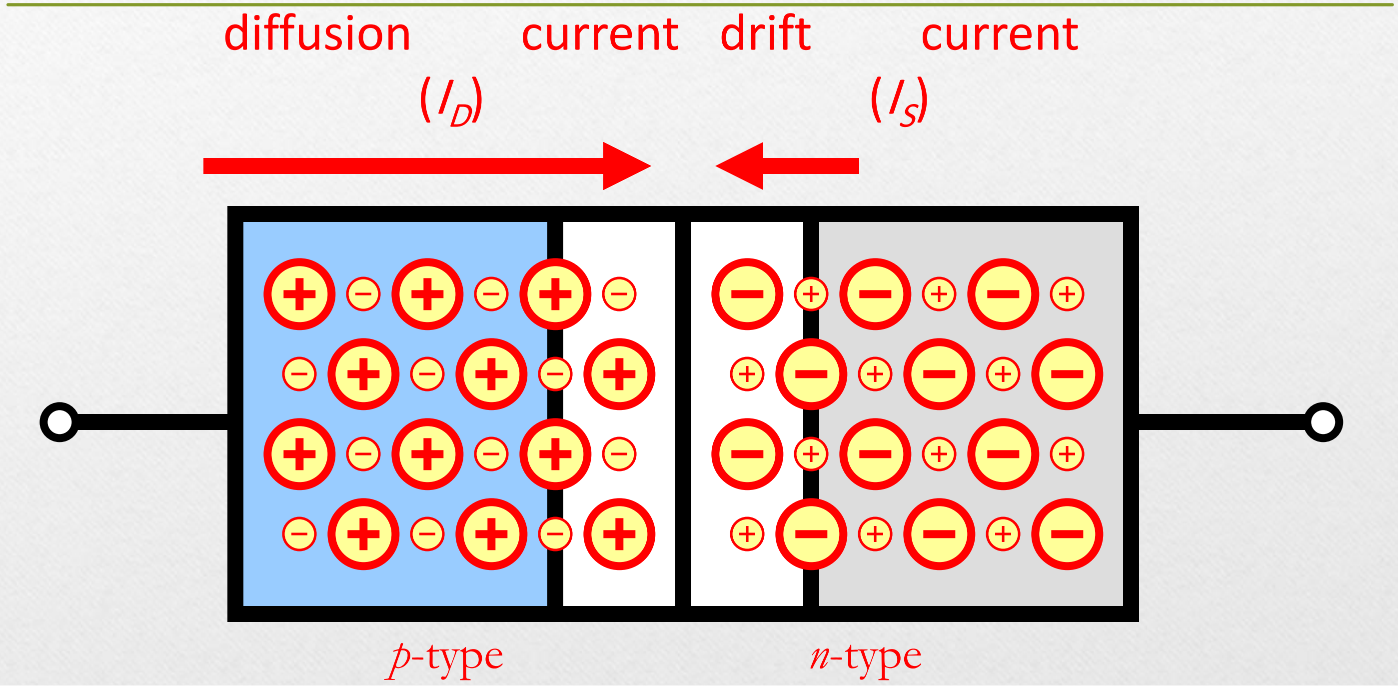
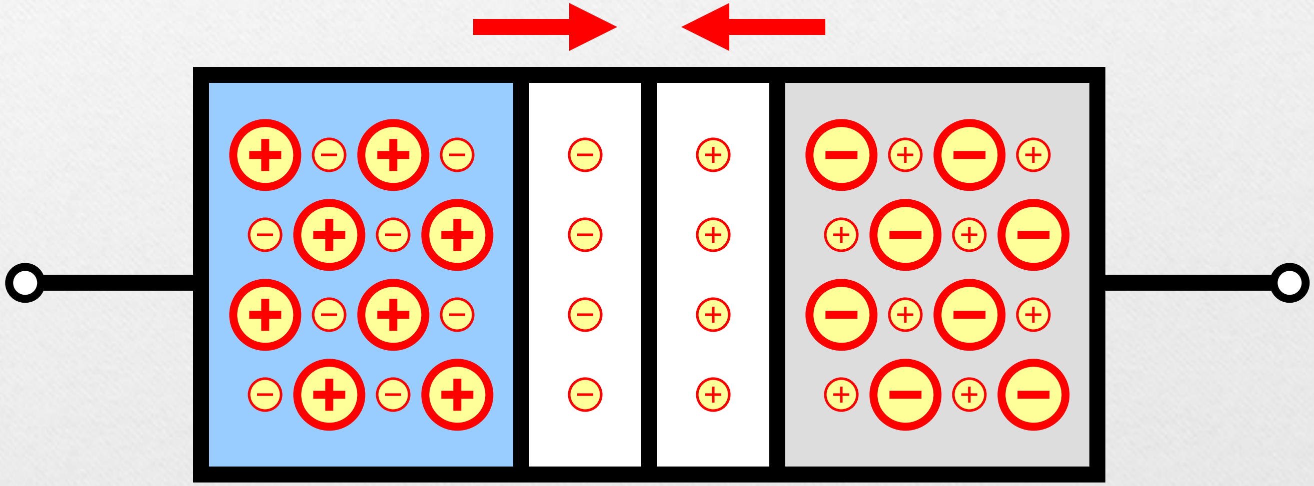


Figure: The *pn* junction with no applied voltage (open-circuited terminals).

Step #6: Equilibrium is reached, and diffusion ceases, once the magnitudes of diffusion and drift currents equal one another – resulting in no net flow.

Once equilibrium is achieved, no net current flow exists ($I_{net} = I_D - I_S$) within the *pn*-junction while under open-circuit condition.



4.2. Operation with Open-Circuit Terminals

- ***pn*-junction built-in voltage** (V_0) – is the equilibrium value of barrier voltage.
  - It is defined to the right.
  - Generally, it takes on a value between 0.6 and 0.9 V for silicon at room temperature.
  - This voltage is applied across depletion region, not terminals of *pn* junction.
    - Power cannot be drawn from V_0.

V_0 = barrier voltage
V_T = thermal voltage
N_A = acceptor doping concentration
N_D = donor doping concentration
n_i = concentration of free electrons...
...in intrinsic semiconductor

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Report 1

Prove that the pn - junction built - in voltage is given by

$$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

it can be derived from the equality of drift current and diffusion current at equibrium

$\therefore$ electron drift current = electron diffusion current

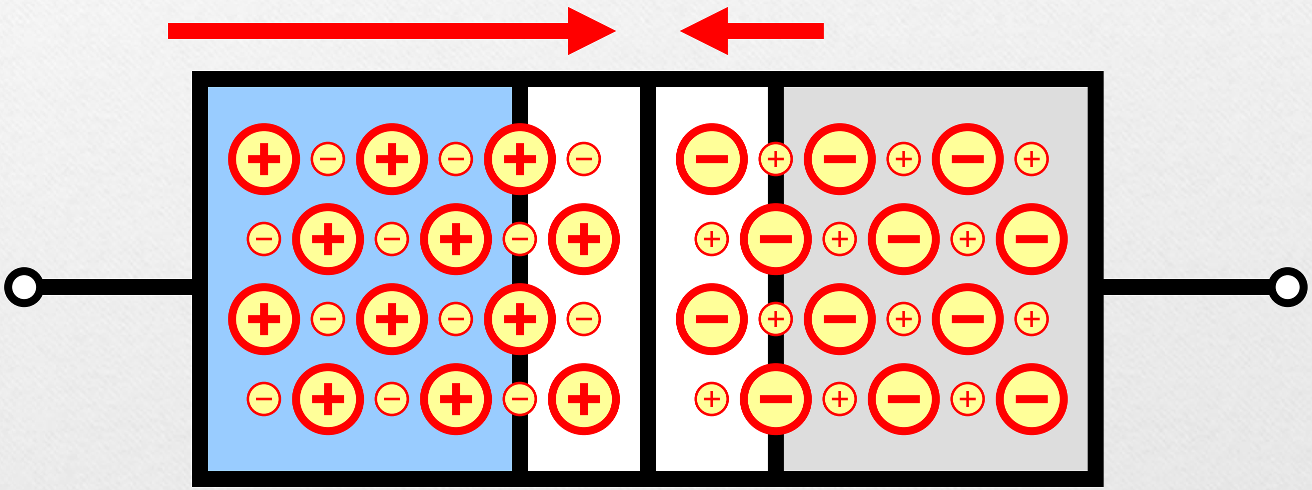
The Drift Current I_S and Equilibrium

- In addition to majority-carrier diffusion current (I_D), a component of current due to minority carrier drift exists (I_S).
- Specifically, some of the thermally generated holes in the *p*-type and *n*-type materials move toward and reach the edge of the depletion region.
- Therefore, they experience the electric field (V_0) in the depletion region and are swept across it.
    - Unlike diffusion current, the polarity of V_0 reinforces this drift current.

4.2. Operation with Open-Circuit Terminals

- Because these holes and free electrons are produced by thermal energy, I_S is heavily dependent on temperature
- Any depletion-layer voltage, regardless of how small, will cause the transition across junction. Therefore I_S is independent of V_0.
- **drift current** (I_S) – is the movement of these minority carriers.
  - aka. electrons from *p*-side to *n*-side of the junction

Note that the magnitude of drift current (I_S) is unaffected by level of diffusion and / or V_0. It will be, however, affected by temperature.



diffusion current (I_D)

drift current (I_S)

Figure: The *pn* junction with no applied voltage (open-circuited terminals).

4.2. Operation with Open-Circuit Terminals

- **Q:** How is the charge stored in both sides of the depletion region defined?
  - **A:** Refer to equations to right. Note that these values should equal one another.

$|Q_+|=$ magnitude of charghe on *n*-side of junction

$q=$ magnitude of electric charge

$A=$ cross-sectional area of junction

$x_n=$ penetration of depletion region into *n*-side

$N_D=$ concentration of donor atoms

$$|Q_+| = qAx_nN_D$$

$$|Q_-| = qAx_pN_A$$

$|Q_-|=$ magnitude of charghe on *n*-side of junction

$q=$ magnitude of electric charge

$A=$ cross-sectional area of junction

$x_p=$ penetration of depletion region into *p*-side

$N_A=$ concentration of acceptor atoms

4.2. Operation with Open-Circuit Terminals

- **Q:** What information can be derived from this equality?
  - **A:** In reality, the depletion region exists almost entirely on one side of the *pn*-junction – due to great disparity between $N_A > N_D$.

$$qAx_pN_A = qAx_nN_D \quad \rightarrow \quad \frac{x_n}{x_p} = \frac{N_A}{N_D}$$

4.2. Operation with Open-Circuit Terminals

W = width of depletion region

ε_S = electrical permiability of silicon $(11.7\varepsilon_0 = 1.04\mathbf{E}{-}12\,F/cm)$

q = magnitude of electron charge

N_A = concentration of acceptor atoms

N_D = concentration of donor atoms

V_0 = barrier / junction built-in voltage

$$W = x_n + x_p = \overbrace{\sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}}$$

- Note that both x_p and x_n may be defined in terms of the depletion region width (W).

$$x_n = W\frac{N_A}{N_A + N_D}$$

$$x_p = W\frac{N_D}{N_A + N_D}$$

4.2. Operation with Open-Circuit Terminals

- **Q:** What has been learned about the *pn*-junction?
  - **A:** composition
    - The *pn* junction is composed of two silicon-based semiconductors, one doped to be *p*-type and the other *n*-type.
  - **A:** majority carriers
    - Are generated by doping.
    - Holes are present on *p*-side, free electrons are present on *n*-side.

3.4.2. Operation with Open-Circuit Terminals

- **Q:** What has been learned about the *pn*-junction?
  - **A:** bound charges
    - Charge of majority carriers are neutralized electrically by bound charges.
  - **A:** diffusion current I_D
    - Those majority carriers close to the junction will diffuse across, resulting in their elimination.

4.2. Operation with Open-Circuit Terminals

- **Q:** What has been learned about the pn-junction?
  - **A:** depletion region
    - As these carriers disappear, they release bound charges and effect a voltage differential V_0.
  - **A:** depletion-layer voltage
    - As diffusion continues, the depletion layer voltage (V_0) grows, making diffusion more difficult and eventually bringing it to halt.

4.2. Operation with Open-Circuit Terminals

- **Q:** What has been learned about the pn-junction?
  - **A:** minority carriers
    - Are generated thermally.
    - Free electrons are present on *p*-side, holes are present on *n*-side.
  - **A:** drift current I_S
    - The depletion-layer voltage (V_0) facilitates the flow of minority carriers to opposite side.
  - **A:** open circuit equilibrium $I_D = I_S$

5. The *pn* Junction with an Applied Voltage

5.1. Qualitative Description of Junction Operation

- Figure to right shows *pn*-junction under three conditions:
  - (a) open-circuit – where a barrier voltage V_0 exists.
  - (b) reverse bias – where a dc voltage V_R is applied.
  - (c) forward bias – where a dc voltage V_F is applied.

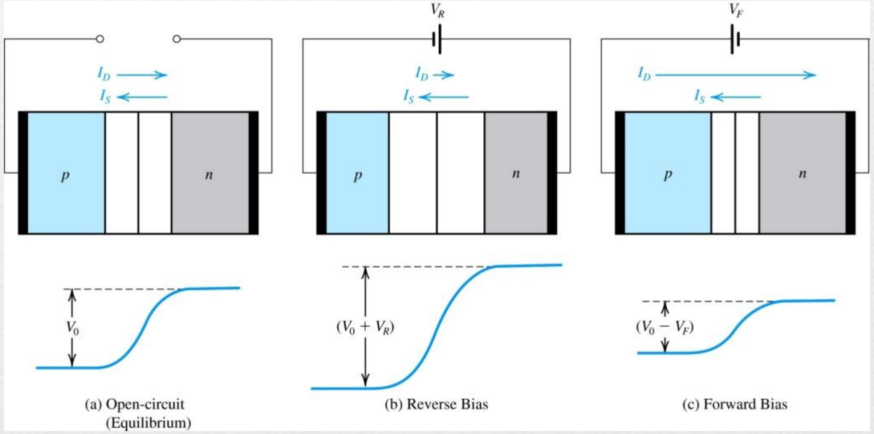


Figure 11: The *pn* junction in: **(a)** equilibrium; **(b)** reverse bias; **(c)** forward bias.

1) no voltage applied

2) voltage differential across depletion zone is V_0

3) $I_D = I_S$

1) negative voltage applied

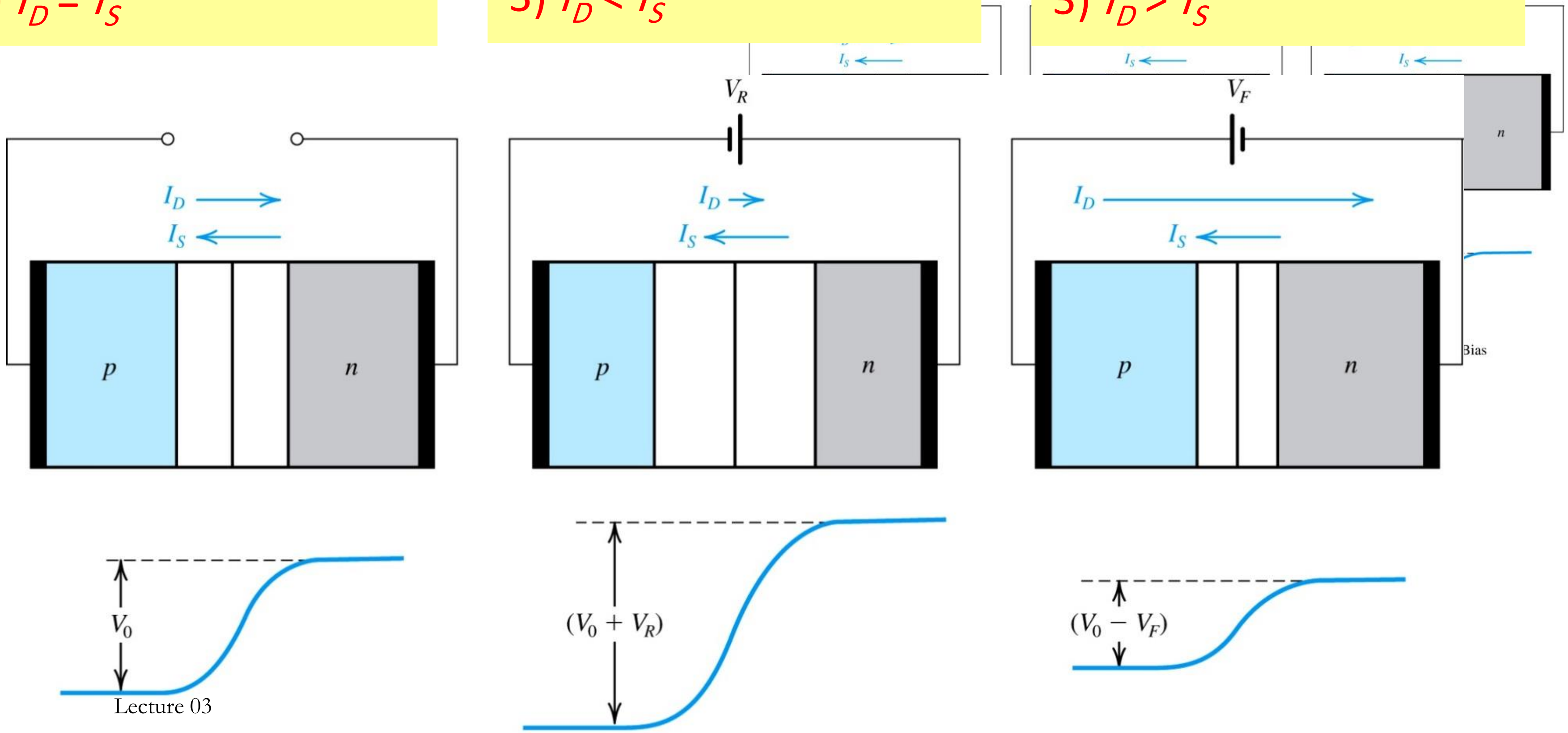
2) voltage differential across depletion zone is $V_0 + V_R$

3) $I_D < I_S$

1) positive voltage applied

2) voltage differential across depletion zone is V_0 - V_F

3) $I_D > I_S$

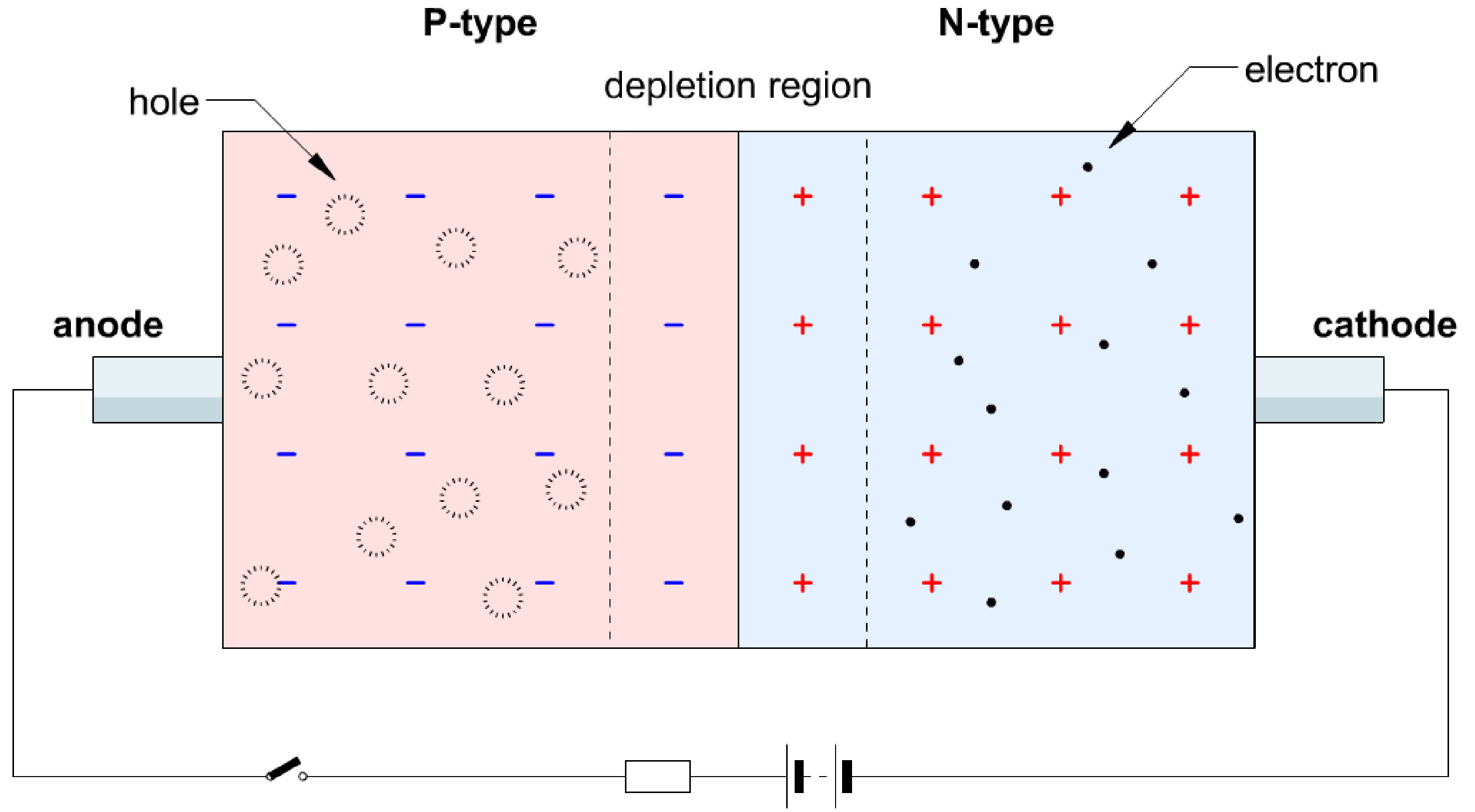


(a) Open-circuit (Equilibrium)

(b) Reverse Bias

(c) Forward Bias

5.1. Qualitative Description of Junction Operation



5.1. Qualitative Description of Junction Operation

- **reverse bias case**
  - the externally applied voltage V_R adds to the barrier voltage V_0
    - …increase effective barrier
  - this reduces rate of diffusion, reducing I_D
    - if $V_R > 1V$, I_D will fall to $0A$
  - the drift current I_S is unaffected, but dependent on temperature
  - result is that *pn* junction will conduct **small** drift current I_S

minimal current flows in reverse-bias case

- **forward bias case**
  - the externally applied voltage V_F subtracts from the barrier voltage V_0
    - …decrease effective barrier
  - this increases rate of diffusion, increasing I_D
  - the drift current I_S is unaffected, but dependent on temperature
  - result is that *pn* junction will conduct **significant** current $I_D - I_S$

significant current flows in forward-bias case

Forward-Bias Case

- Observe that decreased barrier voltage will be accompanied by…
    - (1) decrease in stored uncovered charge on both sides of junction
    - (2) smaller depletion region
- Width of depletion region shown to right.

W = width of depletion region

ε_S = electrical permiability of silicon ($11.7\varepsilon_0 = 1.04\mathbf{E}{-12}\,F/cm$)

q = magnitude of electron charge

N_A = concentration of acceptor atoms

N_D = concentration of donor atoms

V_0 = barrier / junction built-in voltage

V_F = externally applied forward-bias voltage

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\underbrace{(V_0 - V_F)}_{\substack{\textbf{action:}\\ \text{replace } V_0\\ \text{with } V_0 - V_F}}}$$

$$Q_J = A\sqrt{2\varepsilon_S q\left(\frac{N_A N_D}{N_A + N_D}\right)\underbrace{(V_0 - V_F)}_{\substack{\textbf{action:}\\ \text{replace } V_0\\ \text{with } V_0 - V_F}}}$$

Q_J = magnitude of charge stored on either side of depletion region

Reverse-Bias Case

- Observe that increased barrier voltage will be accompanied by…
  - (1) increase in stored uncovered charge on both sides of junction
  - (2) wider depletion region
- Width of depletion region shown to right.

$W =$ width of depletion region

$\varepsilon_S =$ electrical permiability of silicon ($11.7\varepsilon_0 = 1.04\mathbf{E}{-}12\,F/cm$)

$q =$ magnitude of electron charge

$N_A =$ concentration of acceptor atoms

$N_D =$ concentration of donor atoms

$V_0 =$ barrier / junction built-in voltage

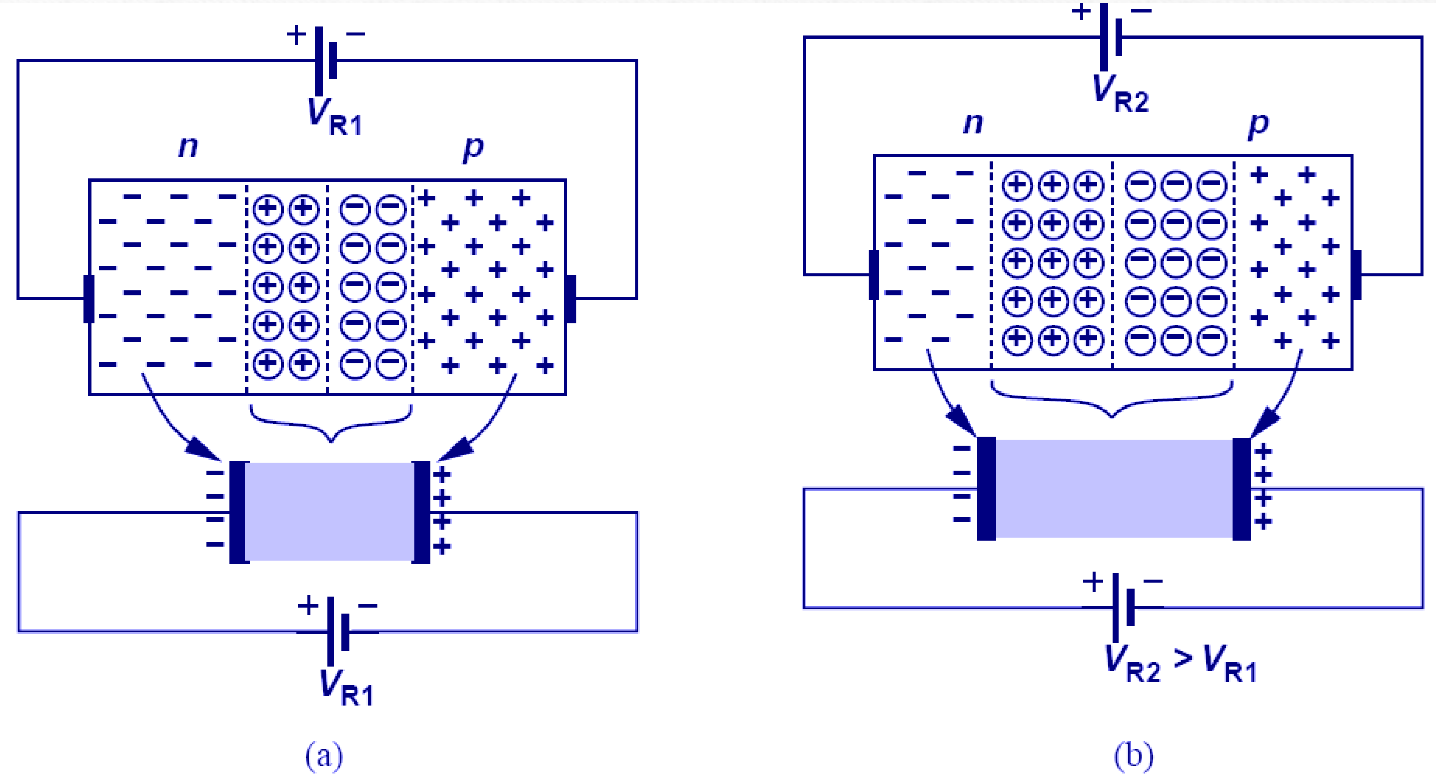
$V_R =$ externally applied reverse-bias voltage

$$W = x_n + x_p = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\underbrace{(V_0+V_R)}_{\substack{\textbf{action:}\\ \text{replace } V_0\\ \text{with } V_0+V_R}}}$$

$$Q_J = A\sqrt{2\varepsilon_S q\left(\frac{N_A N_D}{N_A+N_D}\right)\underbrace{(V_0+V_R)}_{\substack{\textbf{action:}\\ \text{replace } V_0\\ \text{with } V_0+V_R}}}$$

$Q_J =$ magnitude of charge stored on either side of depletion region

Reverse Biased Diode's Application: Voltage-Dependent Capacitor



(a)

(b)

The PN junction can be viewed as a capacitor. By varying V_R, the depletion width changes, changing its capacitance value; therefore, the PN junction is actually a voltage-dependent capacitor.

Example 7: PN junctions

Consider a *pn* junction in equilibrium at room temperature ($T = 300$ K) for which the doping concentrations are $N_A = 10^{18}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$ and the cross-sectional area $A = 10^{-4}\ \text{cm}^2$. Calculate p_p, n_{p0}, n_n, p_{n0}, V_0, W, x_n, x_p, and Q_J. Use $n_i = 1.5 \times 10^{10}/\text{cm}^3$.

Solution

$$p_p \simeq N_A = 10^{18}\ \text{cm}^{-3}$$

$$n_{p0} = \frac{n_i^2}{p_p} \simeq \frac{n_i^2}{N_A} = \frac{(1.5 \times 10^{10})^2}{10^{18}} = 2.25 \times 10^2\ \text{cm}^{-3}$$

$$n_n \simeq N_D = 10^{16}\ \text{cm}^{-3}$$

$$p_{n0} = \frac{n_i^2}{n_n} \simeq \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\ \text{cm}^{-3}$$

$$V_O = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

where

$$V_T = \frac{kT}{q} = \frac{8.62 \times 10^{-5} \times 300\ (\text{eV})}{q\qquad (\text{e})}$$
$$= 25.9 \times 10^{-3}\ \text{V}$$

Thus,

$$V_0 = 25.9 \times 10^{-3} \ln\left(\frac{10^{18} \times 10^{16}}{2.25 \times 10^{20}}\right)$$
$$= 0.814\ \text{V}$$

To determine W we use Eq. (3.26): $W = x_n + x_p = \sqrt{\dfrac{2\varepsilon_s}{q}\left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right)V_0}$

$$W = \sqrt{\frac{2 \times 1.04 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{18}} + \frac{1}{10^{16}}\right) \times 0.814}$$

$$= 3.27 \times 10^{-5} \text{ cm} = 0.327 \ \mu\text{m}$$

To determine x_n and x_p we use Eq. (3.27) and (3.28), respectively:

$$x_n = W\frac{N_A}{N_A + N_D}$$

$$= 0.327\frac{10^{18}}{10^{18} + 10^{16}} = 0.324 \ \mu\text{m}$$

$$x_p = W\frac{N_D}{N_A + N_D}$$

$$= 0.327\frac{10^{16}}{10^{18} + 10^{16}} = 0.003 \ \mu\text{m}$$

$$Q_J = |Q_\pm| = Aq\left(\frac{N_A N_D}{N_A + N_D}\right)W$$

Finally, to determine the charge stored on either side of the depletion region, we use Eq. (3.29)

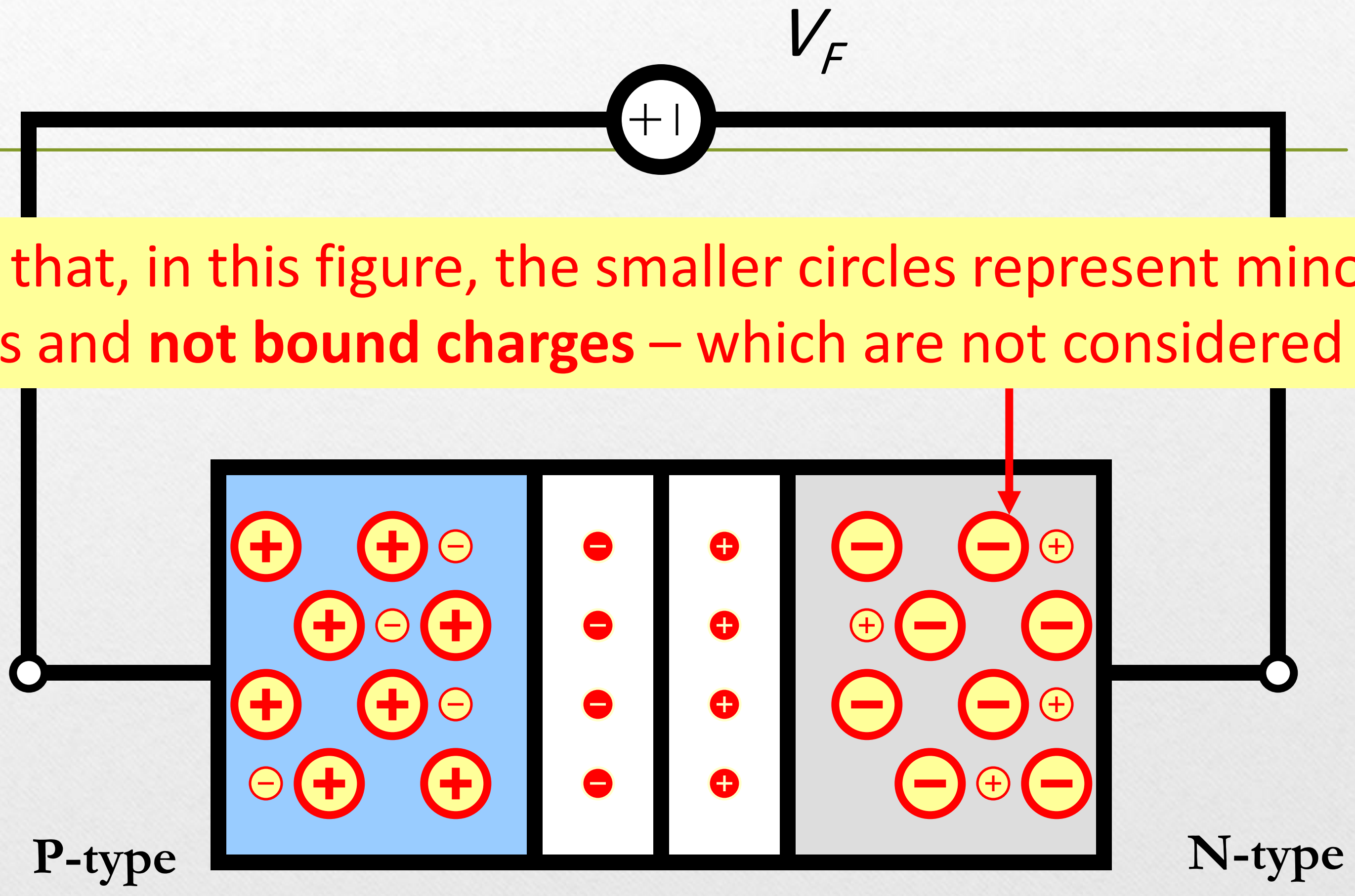
$$Q_J = 10^{-4} \times 1.6 \times 10^{-19}\left(\frac{10^{18} \times 10^{16}}{10^{18} + 10^{16}}\right) \times 0.327 \times 10^{-4}$$

$$= 5.18 \times 10^{-12} \text{ C} = 5.18 \text{ pC}$$

5.2. The Current-Voltage Relationship of the Junction

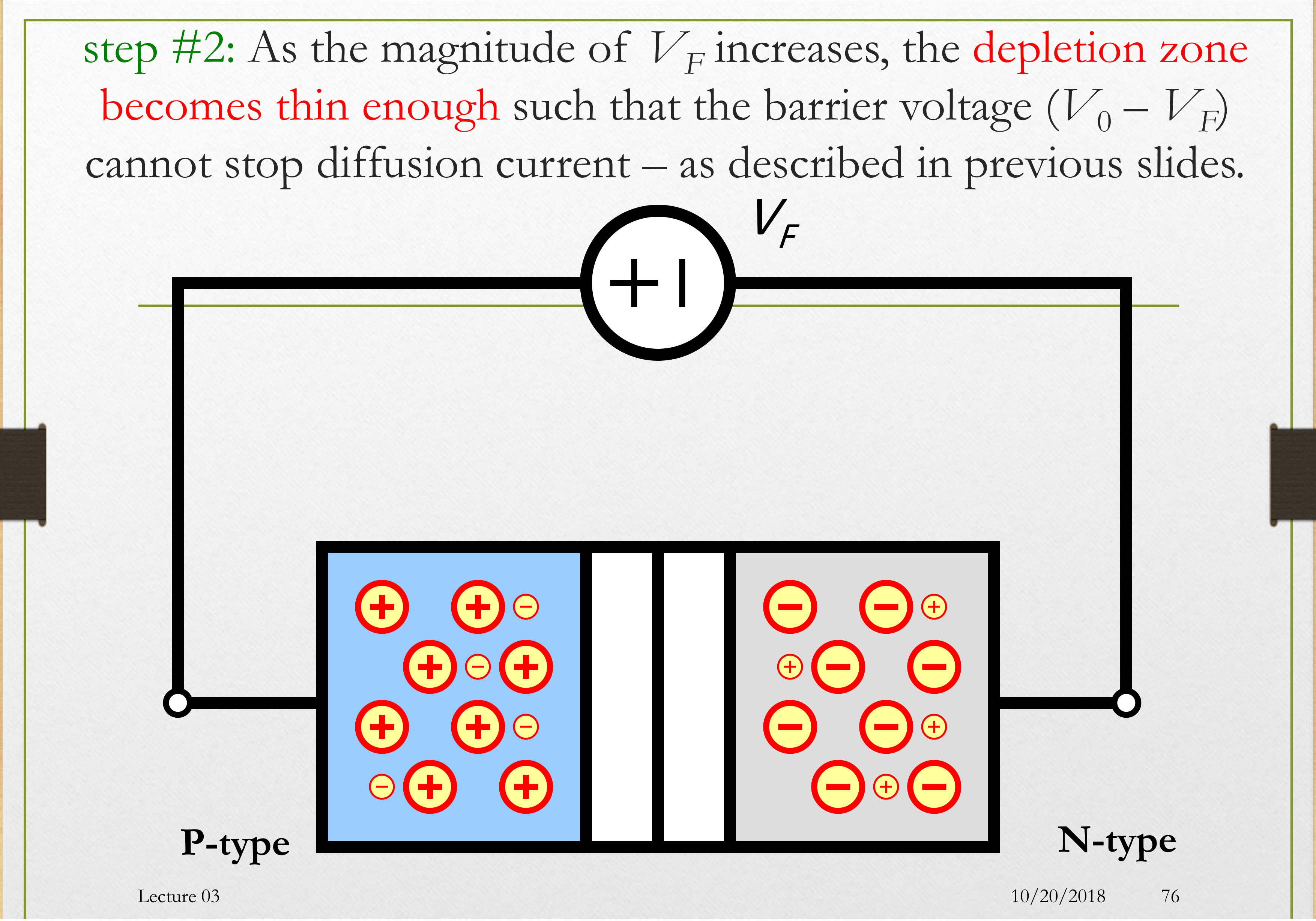
- **Q:** What happens, exactly, when a forward-bias voltage (V_F) is applied to the *pn*-junction?
  - **step #1:** Initially, a small forward-bias voltage (V_F) is applied. It, because of its polarity, pushes majority carriers (holes in *p*-region and electrons in *n*-region) toward the junction and reduces width of the depletion zone.
    - Note, however, that this force is opposed by the built-in voltage (V_0).

step #1: Initially, a small forward-bias voltage (V_F) is applied. It, because of its polarity, pushes majority (holes in *p*-region and electrons in *n*-region) toward the junction and reduces width of the depletion zone.



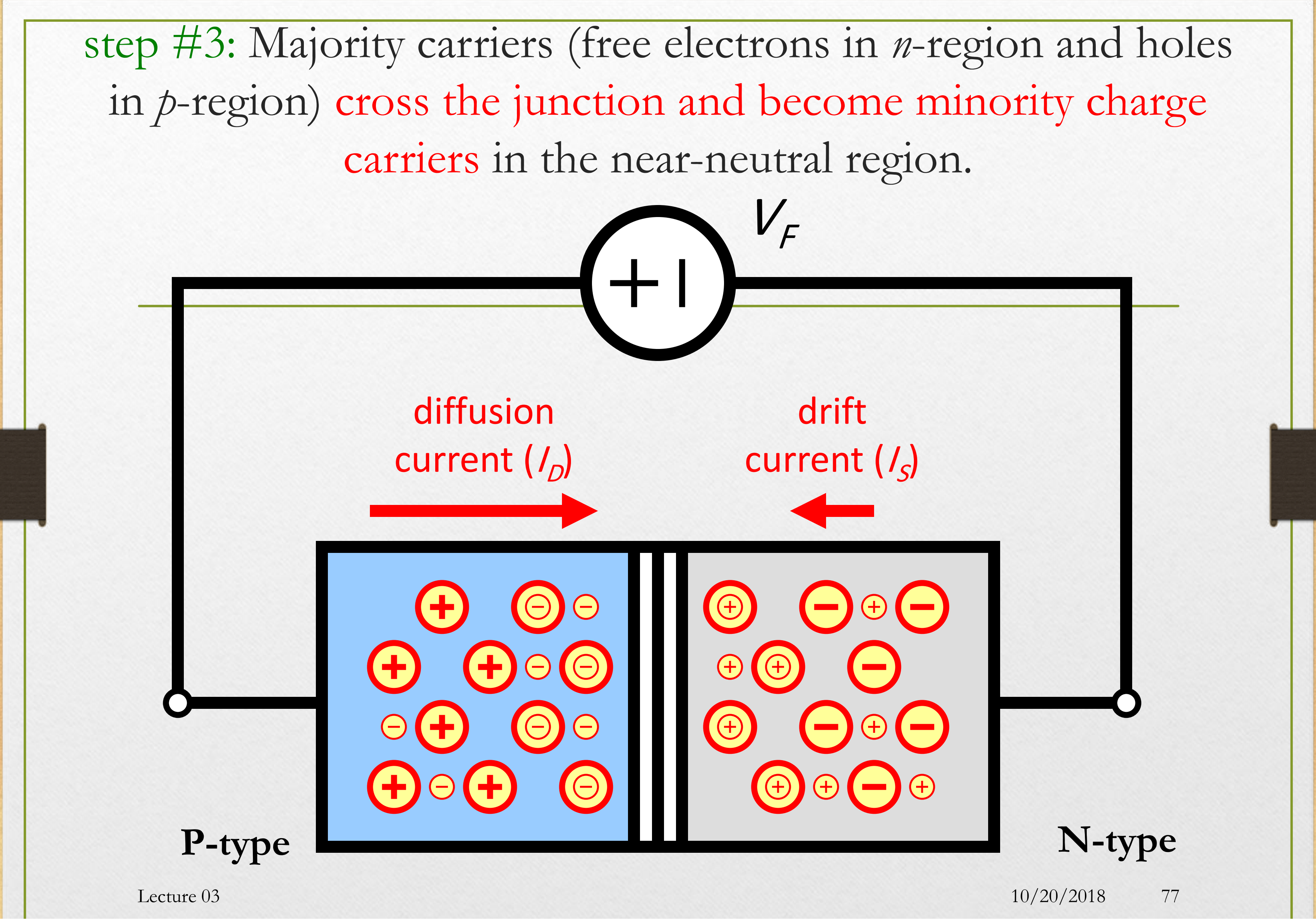
Note that, in this figure, the smaller circles represent minority carriers and **not bound charges** – which are not considered here.

Figure: The *pn* junction with applied voltage.



step #2: As the magnitude of V_F increases, the depletion zone becomes thin enough such that the barrier voltage $(V_0 - V_F)$ cannot stop diffusion current – as described in previous slides.

Figure: The *pn* junction with applied voltage.



step #3: Majority carriers (free electrons in *n*-region and holes in *p*-region) cross the junction and become minority charge carriers in the near-neutral region.

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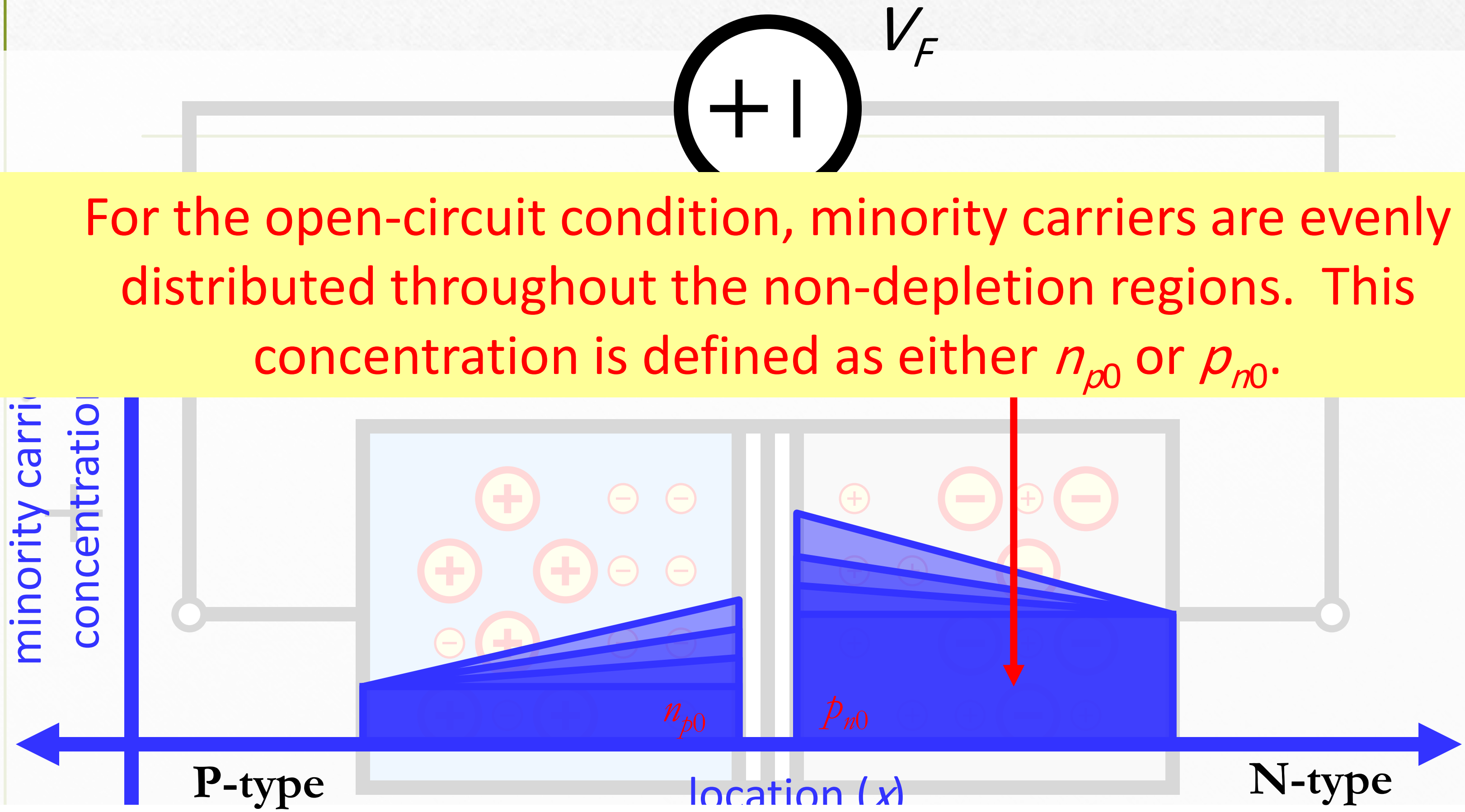
Figure: The *pn* junction with applied voltage.

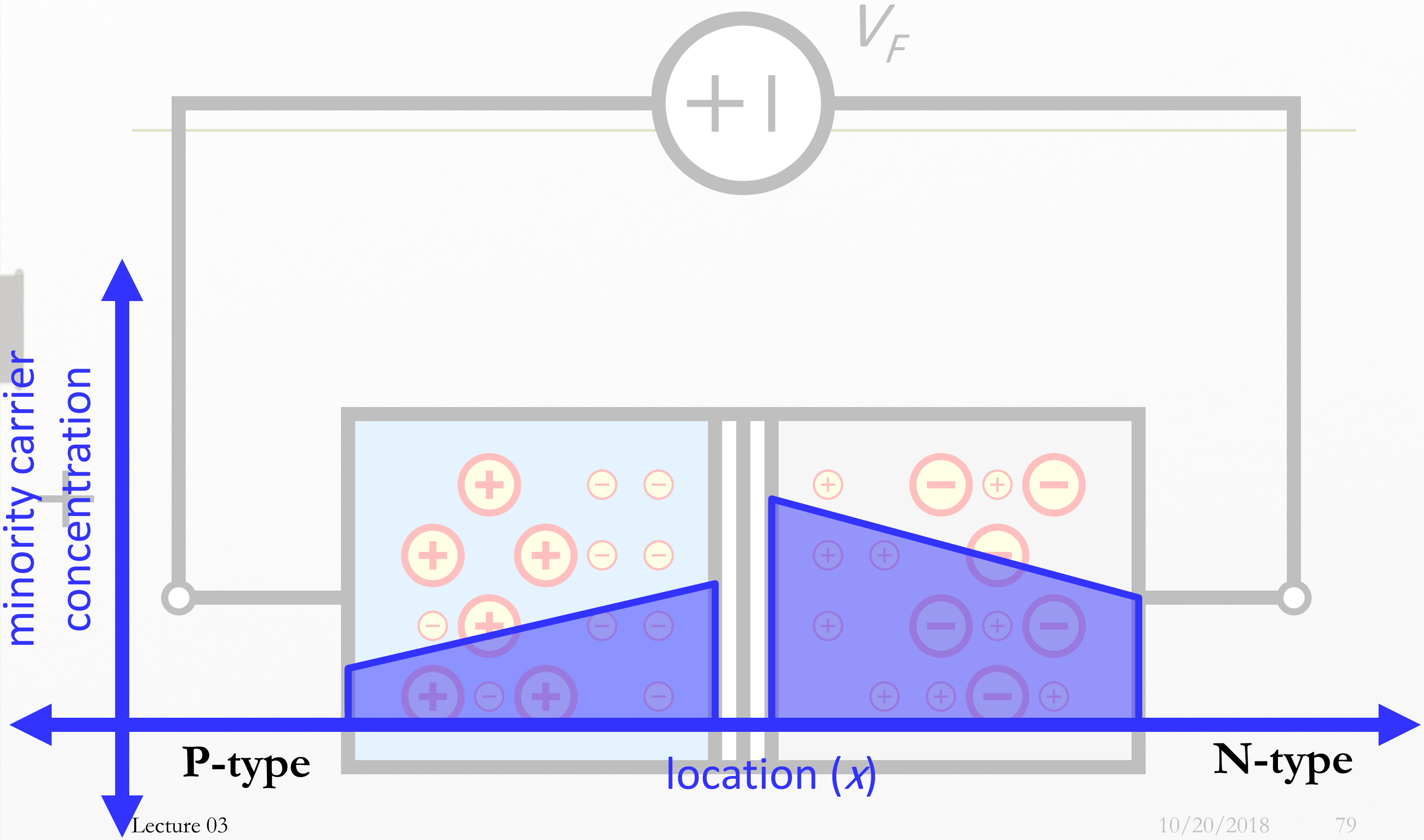
step #4: The concentration of minority charge carriers increases on either side of the junction. A steady-state gradient is reached as rate of majority carriers crossing the junction equals that of recombination.



For the open-circuit condition, minority carriers are evenly distributed throughout the non-depletion regions. This concentration is defined as either n_{p0} or p_{n0}.

Figure: The *pn* junction with no applied voltage (open-circuited terminals).

step #4: The concentration of minority charge carriers increases on either side of the junction. A steady-state gradient is reached as rate of majority carriers crossing the junction equals that of recombination.



step #5+: Diffusion current is maintained – in spite low diffusion lengths (e.g. microns) and recombination – by constant flow of both free electrons and holes towards the ju

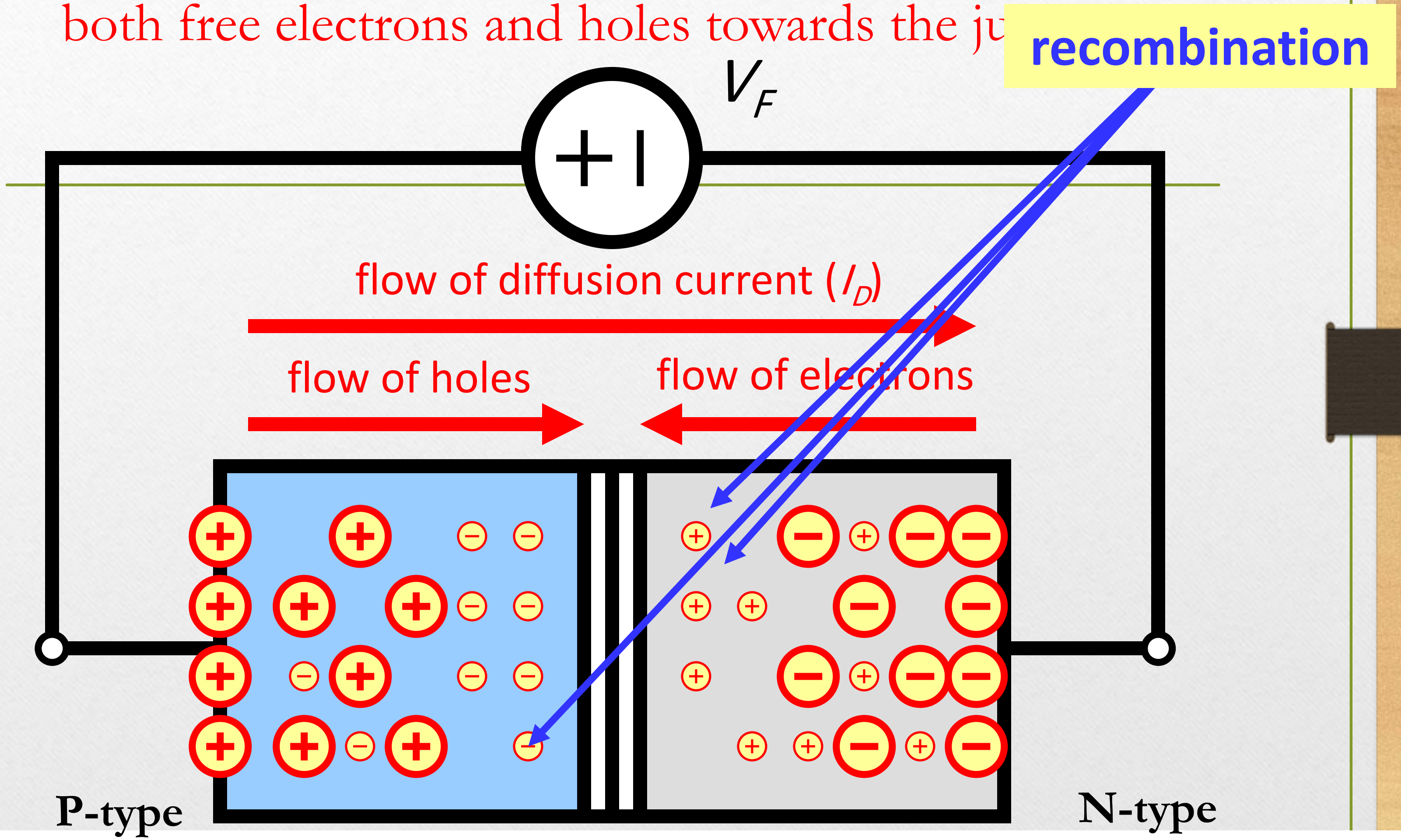


Figure: The *pn* junction with applied voltage.

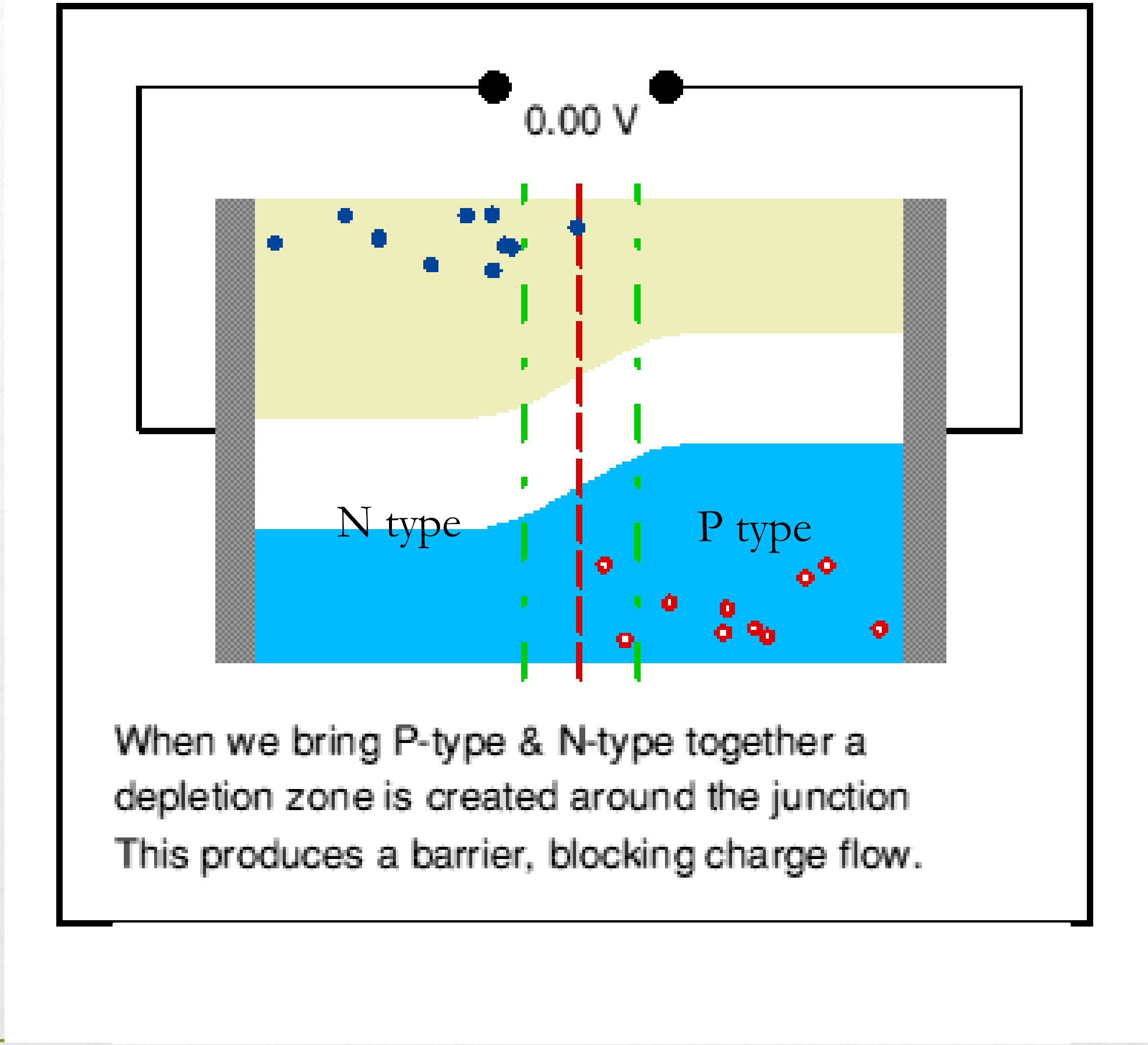
PN junction Characteristics

Conduction band

🟨

Valence band

🟦



When we bring P-type & N-type together a depletion zone is created around the junction This produces a barrier, blocking charge flow.

5.2. The Current-Voltage Relationship of the Junction

- **Q:** For forward-biased case, how is diffusion current (I_D) defined?

$$I = \underbrace{\left(Aqn_i^2 \left[\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right] \right)}_{I_S} (e^{V/V_T} - 1) = I_S (e^{V/V_T} - 1)$$

5.2. The Current-Voltage Relationship of the Junction

$$I = I_S(e^{V/V_T} - 1)$$

- **saturation current** (I_S) – is the maximum reverse current which will flow through *pn*-junction.
  - It is proportional to cross-section of junction (A).
  - Typical value is $10^{-18} A$.

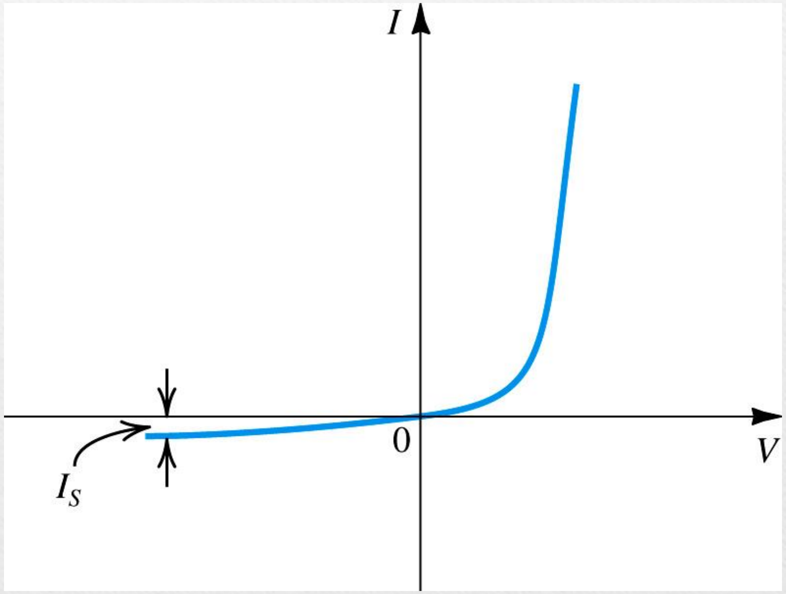


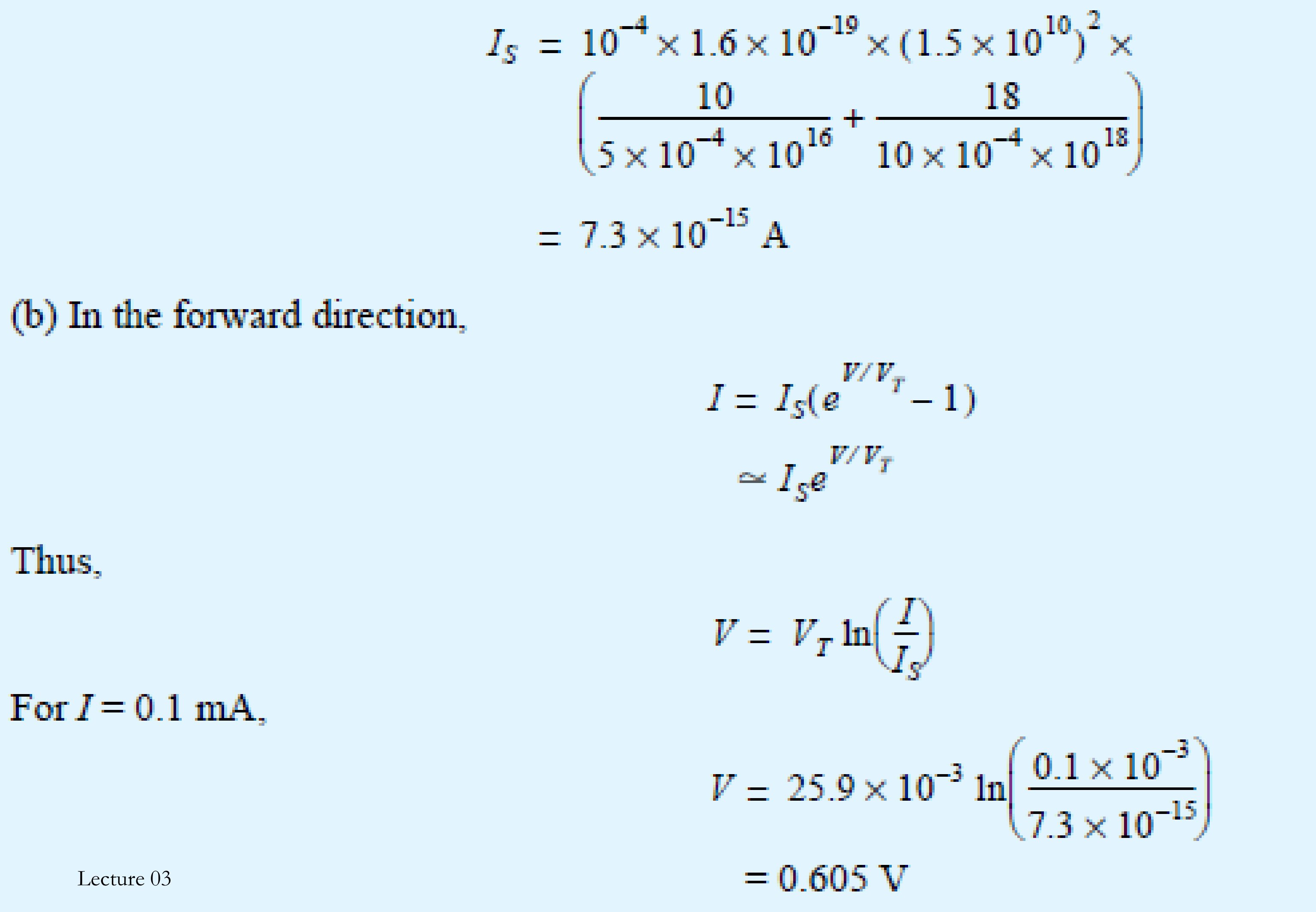
Figure 13: The *pn* junction *I*–*V* characteristic.

Example 6: *pn*-Junction

- Consider a forward-biased *pn* junction conducting a current of $I = 0.1mA$ with following parameters:
  - $N_A = 10^{18}/cm^3$, $N_D = 10^{16}/cm^3$, $A = 10^{-4}cm^2$, $n_i = 1.5\mathbf{e}^{10}/cm^3$, $L_p = 5um$, $L_n = 10um$, D_p (*n*-region) $= 10cm^2/s$, D_n (*p*-region) $= 18cm^2/s$
- **Q(a):** Calculate I_S.
- **Q(b):** Calculate the forward bias voltage (V).
- **Q(c):** Component of current I due to hole injection and electron injection across the junction.

(a)

$$I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$



$$I_S = 10^{-4} \times 1.6 \times 10^{-19} \times (1.5 \times 10^{10})^2 \times \left(\frac{10}{5 \times 10^{-4} \times 10^{16}} + \frac{18}{10 \times 10^{-4} \times 10^{18}}\right)$$

$$= 7.3 \times 10^{-15}\ \mathrm{A}$$

(b) In the forward direction,

$$I = I_S(e^{V/V_T} - 1)$$
$$\simeq I_S e^{V/V_T}$$

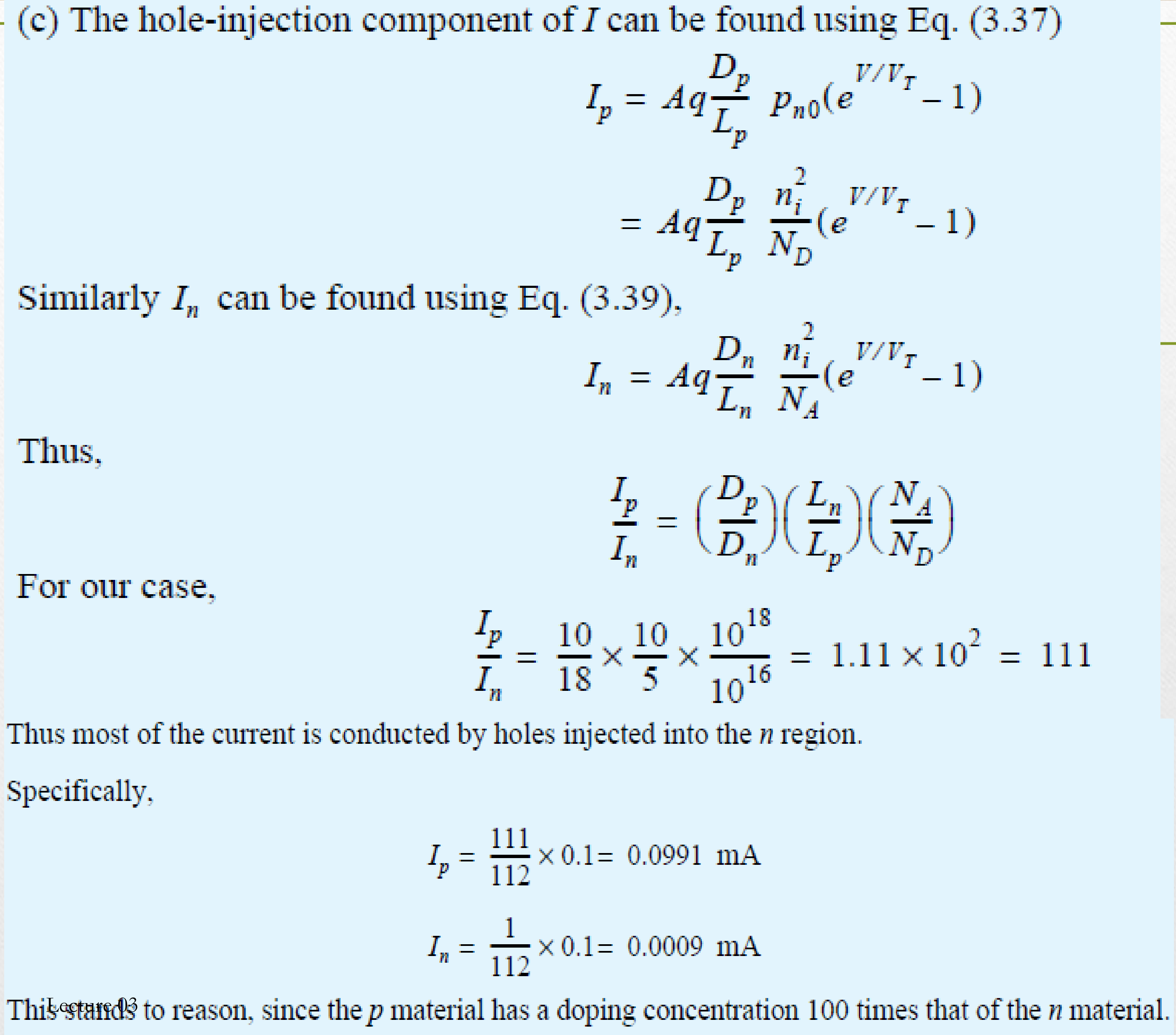
Thus,

$$V = V_T \ln\left(\frac{I}{I_S}\right)$$

For $I = 0.1$ mA,

$$V = 25.9 \times 10^{-3} \ln\left(\frac{0.1 \times 10^{-3}}{7.3 \times 10^{-15}}\right)$$

$$= 0.605\ \mathrm{V}$$



(c) The hole-injection component of I can be found using Eq. (3.37)

$$I_p = Aq\frac{D_p}{L_p}\ p_{n0}(e^{V/V_T} - 1)$$

$$= Aq\frac{D_p}{L_p}\ \frac{n_i^2}{N_D}(e^{V/V_T} - 1)$$

Similarly I_n can be found using Eq. (3.39),

$$I_n = Aq\frac{D_n}{L_n}\ \frac{n_i^2}{N_A}(e^{V/V_T} - 1)$$

Thus,

$$\frac{I_p}{I_n} = \left(\frac{D_p}{D_n}\right)\left(\frac{L_n}{L_p}\right)\left(\frac{N_A}{N_D}\right)$$

For our case,

$$\frac{I_p}{I_n} = \frac{10}{18} \times \frac{10}{5} \times \frac{10^{18}}{10^{16}} = 1.11 \times 10^2 = 111$$

Thus most of the current is conducted by holes injected into the n region.

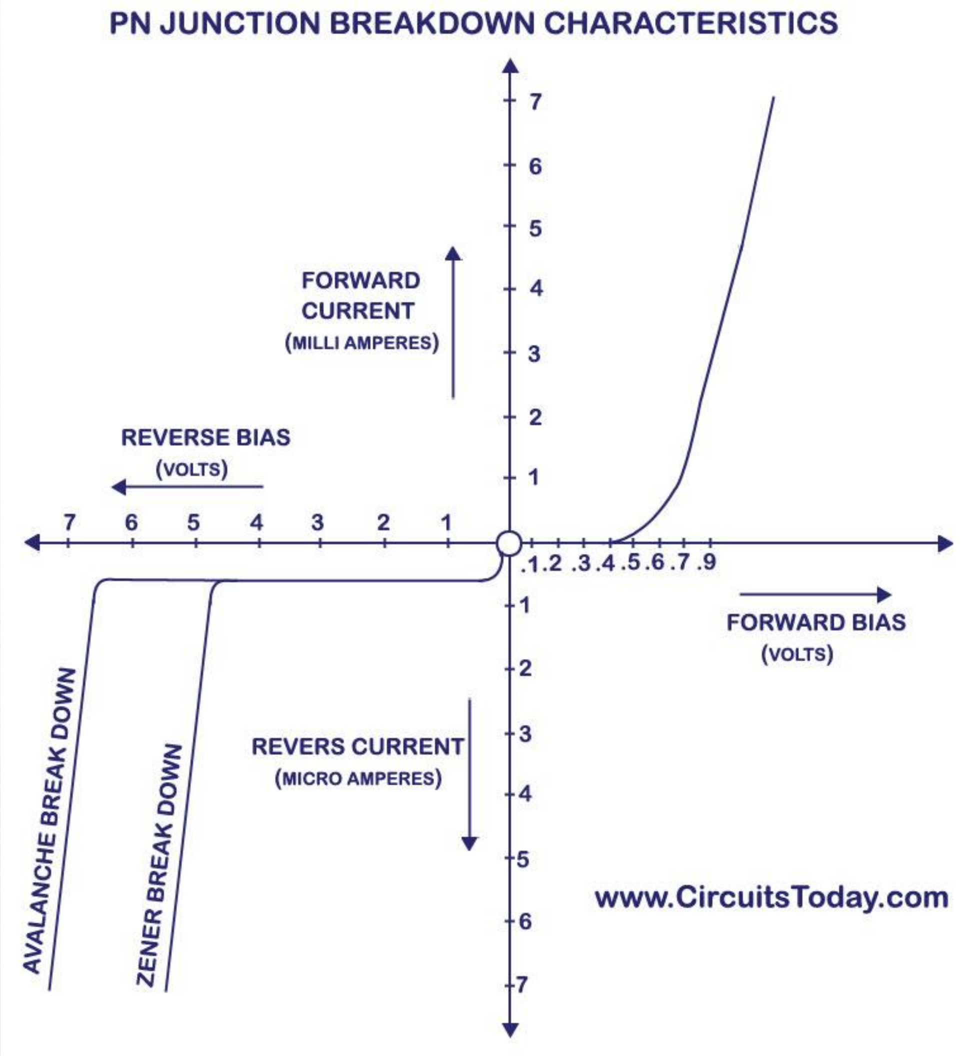
Specifically,

$$I_p = \frac{111}{112} \times 0.1 = 0.0991 \text{ mA}$$

$$I_n = \frac{1}{112} \times 0.1 = 0.0009 \text{ mA}$$

This stands to reason, since the p material has a doping concentration 100 times that of the n material.

5.3 Reverse Breakdown



Zener breakdown

- The electric field in the depletion layer increases to cause breaking covalent bonds and generating electron-hole pairs.
- The electrons generated in this way will be swept by the electric field into the n side and the holes into the p side. Thus these electrons and holes constitute a reverse current across the junction.
- Once the zener effect starts (V_R=5V), a large number of carriers can be generated, with a negligible increase in the junction voltage. Thus the reverse current in the breakdown region will be large and its value must be determined by the external circuit.
- the reverse voltage appearing between the diode terminals will remain close to the specified breakdown voltage V_Z.

Avalanche breakdown

- The minority carriers that cross the depletion region under the influence of the electric field gain sufficient kinetic energy to be able to break covalent bonds in atoms with which they collide.
- The carriers liberated by this process may have sufficiently high energy to be able to cause other carriers to be liberated in another ionizing collision.
- This process keeps repeating in the fashion of an avalanche, with the result that many carriers are created that are able to support any value of reverse current, as determined by the external circuit, with a negligible change in the voltage drop across the junction.

5.3 Reverse Breakdown

- The maximum reverse-bias potential that can be applied before entering the breakdown region is called the peak inverse voltage (referred to simply as the PIV rating) or the peak reverse voltage (denoted the PRV rating).

6. Capacitive Effects in the *pn* Junction

1. Depletion or Junction Capacitance

When a *pn* junction is reverse biased

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}}$$

Where

$$C_{j0} = A\sqrt{\left(\frac{\varepsilon_s q}{2}\right)\left(\frac{N_A N_D}{N_A + N_D}\right)\left(\frac{1}{V_0}\right)}$$

2. Diffusion Capacitance

When a *pn* junction is forward biased

$$C_d = \left(\frac{\tau_T}{V_T}\right) I$$

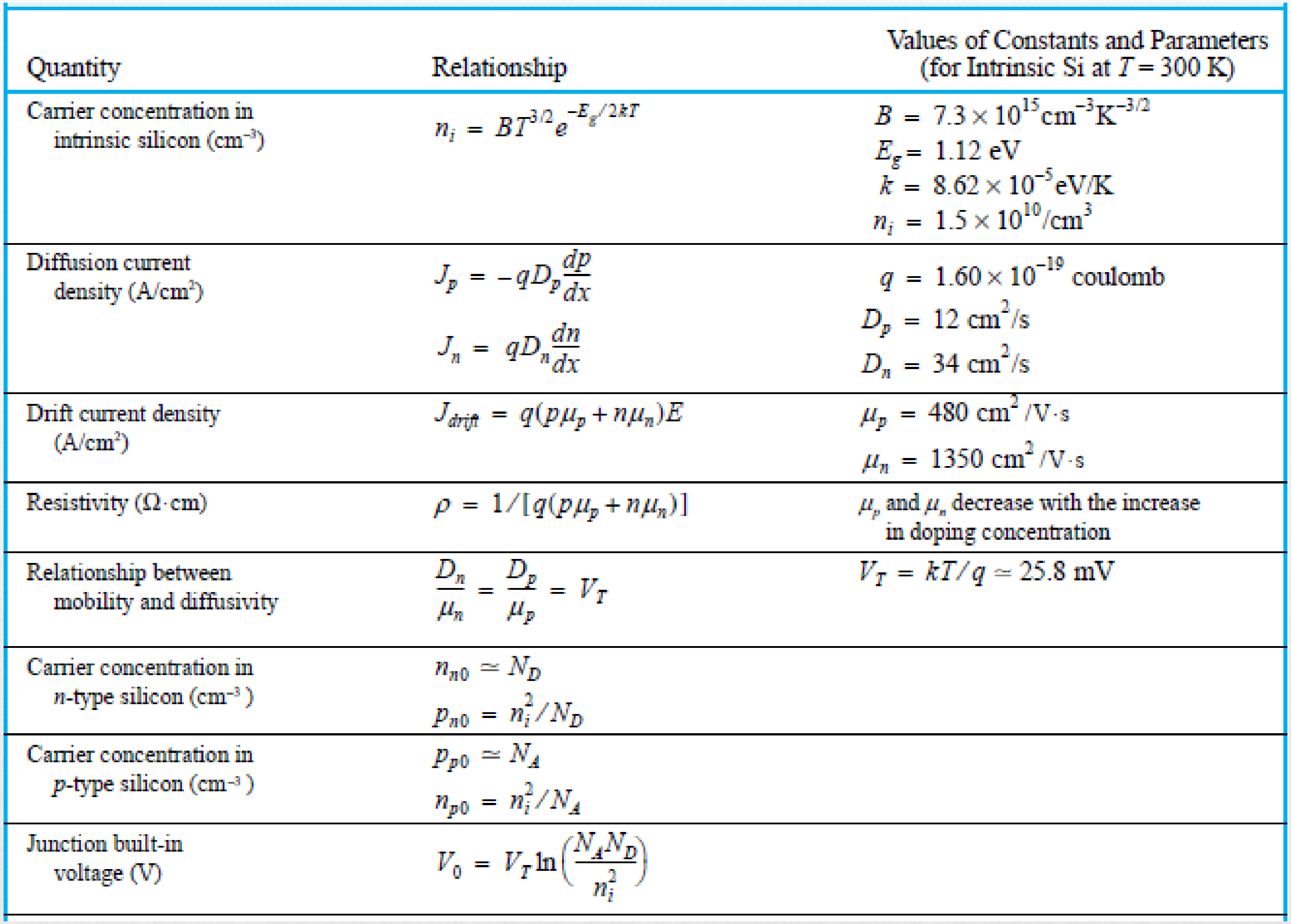
τ_T is the **mean transit time** of the junction.

I is the forward-bias current.

6. Capacitive Effects in the *pn* Junction

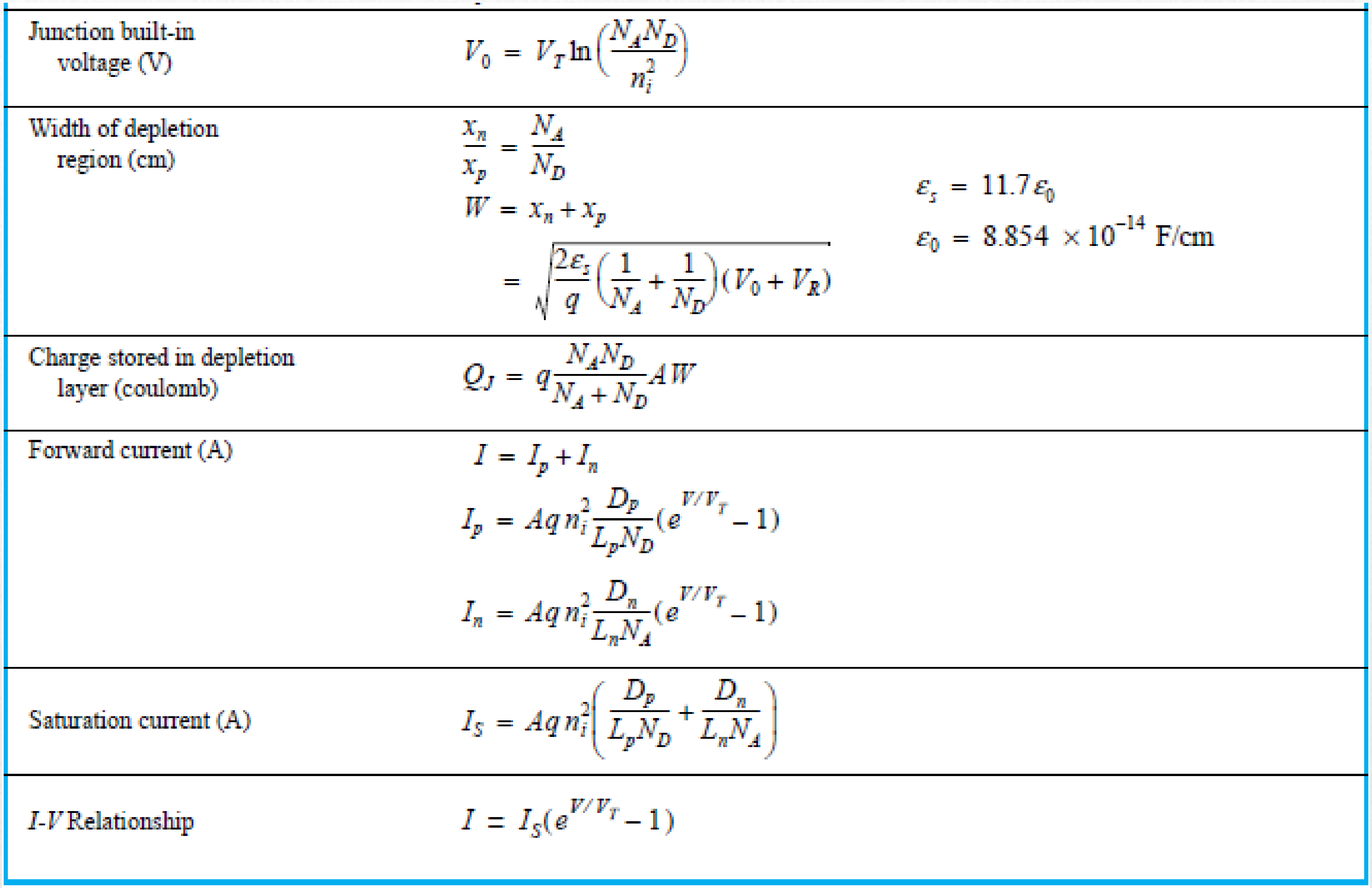
- **junction capacitance:**
  - ✓ due to the dipole in the transition region (associated with the charge stored in the depletion region).
  - ✓ Also called transition region capacitance or depletion layer capacitance.
  - ✓ Dominates under reverse bias conditions.
- **Charge storage (Diffusion) capacitance:**
  - ✓ associated with the minority carrier charge stored in the n and p materials as a result of the concentration profiles established by carrier injection.
  - ✓ Also referred to as diffusion capacitance.
  - ✓ Dominant when the junction is forward biased.

Summary of Important Equations



Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Carrier concentration in intrinsic silicon (cm^{-3})	$n_i = BT^{3/2}e^{-E_g/2kT}$	$B = 7.3 \times 10^{15} \text{cm}^{-3}\text{K}^{-3/2}$ $E_g = 1.12$ eV $k = 8.62 \times 10^{-5}$ eV/K $n_i = 1.5 \times 10^{10}/\text{cm}^3$
Diffusion current density (A/cm^2)	$J_p = -qD_p\frac{dp}{dx}$ $J_n = qD_n\frac{dn}{dx}$	$q = 1.60 \times 10^{-19}$ coulomb $D_p = 12\ \text{cm}^2/\text{s}$ $D_n = 34\ \text{cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{drift} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$ $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$
Resistivity ($\Omega \cdot$cm)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q \simeq 25.8$ mV
Carrier concentration in *n*-type silicon (cm^{-3})	$n_{n0} \simeq N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in *p*-type silicon (cm^{-3})	$p_{p0} \simeq N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	

Summary of Important Equations



Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W = x_n + x_p$ $= \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$	$\varepsilon_s = 11.7\,\varepsilon_0$ $\varepsilon_0 = 8.854 \times 10^{-14}$ F/cm
Charge stored in depletion layer (coulomb)	$Q_J = q\frac{N_A N_D}{N_A + N_D} A W$	
Forward current (A)	$I = I_p + I_n$ $I_p = A q n_i^2 \frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$ $I_n = A q n_i^2 \frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$	
Saturation current (A)	$I_S = A q n_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
I-*V* Relationship	$I = I_S(e^{V/V_T} - 1)$	

Summary (1)

- Today's microelectronics technology is almost entirely based on the semiconductor silicon. If a circuit is to be fabricated as a monolithic integrated circuit (IC), it is made using a single silicon crystal, no matter how large the circuit is.
- In a crystal of intrinsic or pure silicon, the atoms are held in position by covalent bonds. At very low temperatures, all the bonds are intact; No charge carriers are available to conduct current. As such, at these low temperatures, silicone acts as an insulator.

Summary (2)

- At room temperature, thermal energy causes some of the covalent bonds to break, thus generating free electrons and holes that become available to conduct electricity.
- Current in semiconductors is carried by free electrons and holes. Their numbers are equal and relatively small in intrinsic silicon.
- The conductivity of silicon may be increased drastically by introducing small amounts of appropriate impurity materials into the silicon crystal – via process called doping.

Summary (3)

- There are two kinds of doped semiconductor: *n*-type in which electrons are abundant, *p*-type in which holes are abundant.
- There are two mechanisms for the transport of charge carriers in a semiconductor: drift and diffusion.
- Carrier drift results when an electric field (*E*) is applied across a piece of silicon. The electric field accelerates the holes in the direction of *E* and electrons oppositely. These two currents sum to produce drift current in the direction of *E*.

Summary (4)

- Carrier diffusion occurs when the concentration of charge carriers is made higher in one part of a silicon crystal than others. To establish a steady-state diffusion current, a carrier concentration must be maintained in the silicon crystal.
- A basic semiconductor structure is the *pn*-junction. It is fabricated in a silicon crystal by creating a *p*-region in proximity to an *n*-region. The *pn*-junction is a diode and plays a dominant role in the structure and operation of transistors.

Summary (5)

- When the terminals of the *pn*-junction are left open, no current flows externally. However, two equal and opposite currents (I_D and I_S) flow across the junction. Equilibrium is maintained by a built-in voltage (V_0). Note, however, that the voltage across an open junction is $0V$, since V_0 is cancelled by potentials appearing at the metal-to-semiconductor connection interfaces.
- The voltage V_0 appears across the depletion region, which extends on both sides of the junction.

Summary (6)

- The drift current I_S is carried by thermally generated minority electrons in the *p*-material that are swept across the depletion region into the n-side. The opposite occurs in the *n*-material. I_S flows from *n* to *p*, in the reverse direction of the junction. Its value is a strong function of temperature, but independent of V_0.
- Forward biasing of the *pn*-junction, that is applying an external voltage that makes *p* more positive than *n*, reduces the barrier voltage to $V_0 - V$ and results in an exponential increase in I_D (while I_S remains unchanged).