Fluid Mechanics: Fundamentals of Fluid Mechanics, 7th Edition,
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Lecture- 05
Fluid Statics (Part B)

Dr. Dhafer Manea Hachim AL-HASNAWI
Assist Proof
Al-Furat Al-Awsat Technical University
Engineering Technical College / Najaf
email:coj.dfr@atu.edu.iq

Learning Objectives

- After completing this Lecture, you should be able to:

1. calculate the hydrostatic pressure force on a plane or curved submerged surface.

Outline

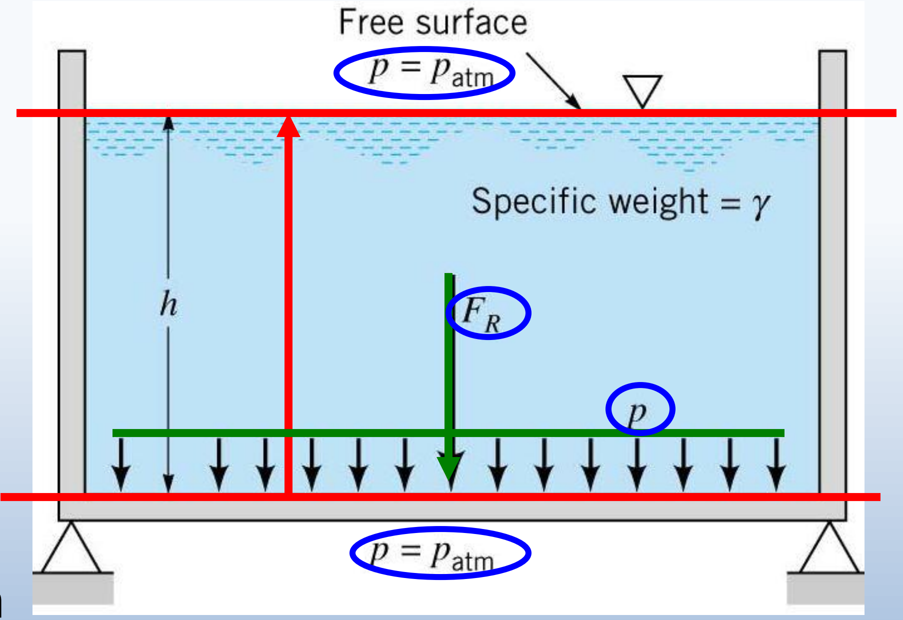
- Hydrostatic Force on a Plane Surface
- Hydrostatic Force on a Curved Surface
- Example Problems

Hydrostatic Force on a Plane Surface: Tank Bottom

Simplest Case: Tank bottom with a uniform pressure distribution

$$p - \gamma h = p_{atm} - p_{atm}$$

$$p = \gamma h$$



Now, the resultant Force:

$$F_R = \text{pA}$$

Acts through the Centroid

A = area of the Tank Bottom

Hydrostatic Force on a Plane Surface: General Case

The origin O is at the Free Surface.

θ is the angle the plane makes with the free surface.

y is directed along the plane surface.

A is the area of the surface.

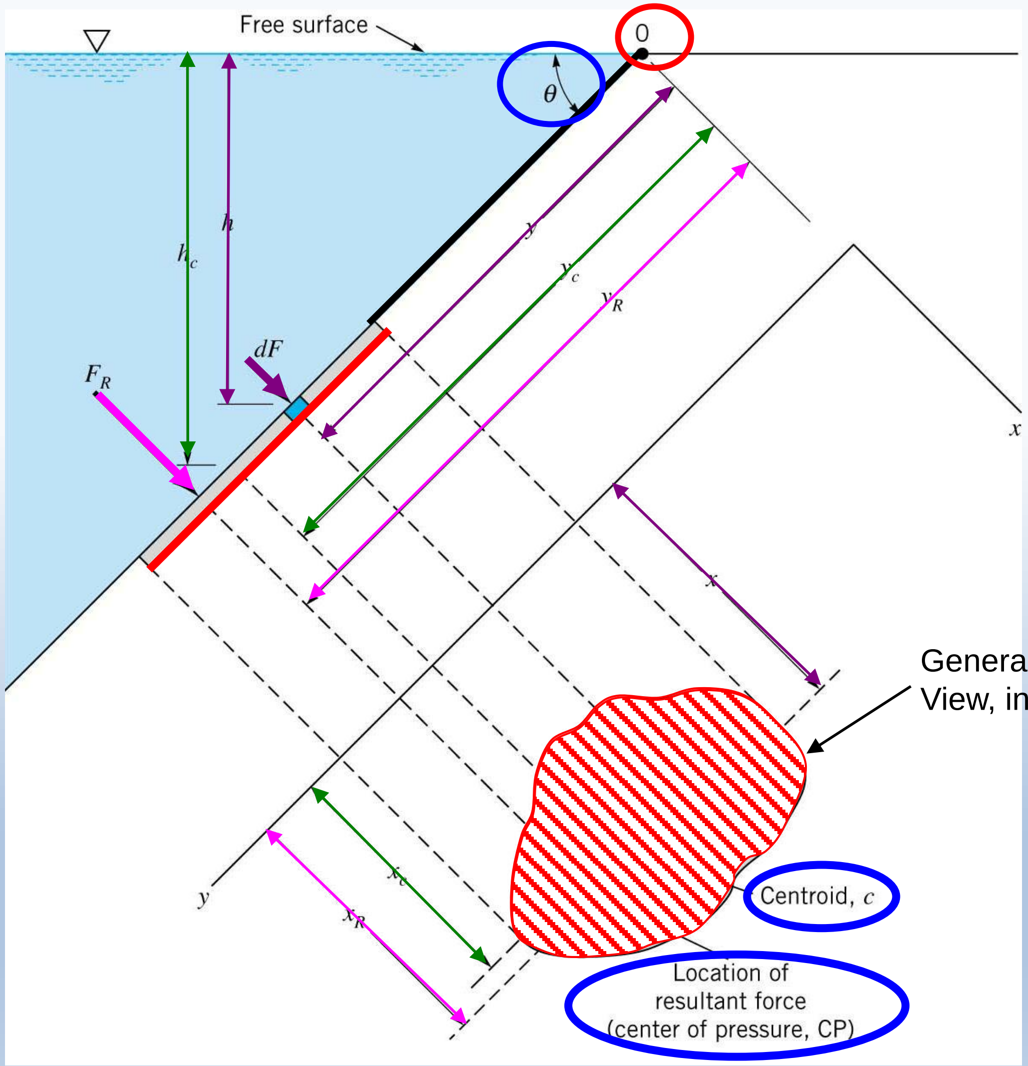
dA is a differential element of the surface.

dF is the force acting on the differential element.

C is the centroid.

CP is the center of Pressure

F_R is the resultant force acting through CP



General Shape: Planar View, in the x-y plane

Hydrostatic Force on a Plane Surface: General Case

Then the force acting on the differential element:

$$dF = \gamma h \, dA$$

Then the resultant force acting on the entire surface:

$$F_R = \int_A \gamma h \, dA$$

$$= \int_A \gamma y \sin\theta \, dA$$

We note h = ysinθ

With γ and θ taken as constant:

$$F_R = \gamma \sin\theta \int_A y \, dA$$

We note, the integral part is the first moment of area about the x-axis

$$\int_A y \, dA = y_c A$$

Where y_c is the y coordinate to the centroid of the object.

$$F_R = \gamma A \underline{y_c \sin\theta}$$

h_c

$$F_R = \gamma h_c A$$

Hydrostatic Force on a Plane Surface: Location

Now, we must find the location of the center of Pressure where the Resultant Force Acts:
"The Moments of the Resultant Force must Equal the Moment of the Distributed Pressure Force"

Moments about the x-axis:

$$F_R y_R = \int_A y\, dF = \int_A \gamma \sin\theta\, y^2\, dA$$

$$dF = \gamma h\, dA$$

And, note h = ysinθ

We note, $F_R = \gamma A y_c \sin\theta$

Second moment of Intertia, I_x

Then,

$$y_R = \frac{\int_A y^2\, dA}{y_c A} \quad \Rightarrow \quad y_R = \frac{I_x}{y_c A}$$

Parallel Axis Thereom:

$$I_x = I_{xc} + A y_c^2$$

I_{xc} **is the second moment of inertia through the centroid**

Substituting the parallel Axis thereom, and rearranging:

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

We, note that for a submerged plane, the resultant force always acts below the centroid of the plane.

Hydrostatic Force on a Plane Surface: Location

Moments about the y-axis: $F_R x_R = \int_A x dF$

$dF = \gamma h \, dA$

And, note h = ysinθ

$$F_R x_R = \int_A \gamma \sin\theta \, xy \, dA$$

We note, $F_R = \gamma A y_c \sin\theta$

Second moment of Intertia, I_{xy}

Then, $x_R = \frac{\int_A xy \, dA}{y_c A} = \frac{I_{xy}}{y_c A}$

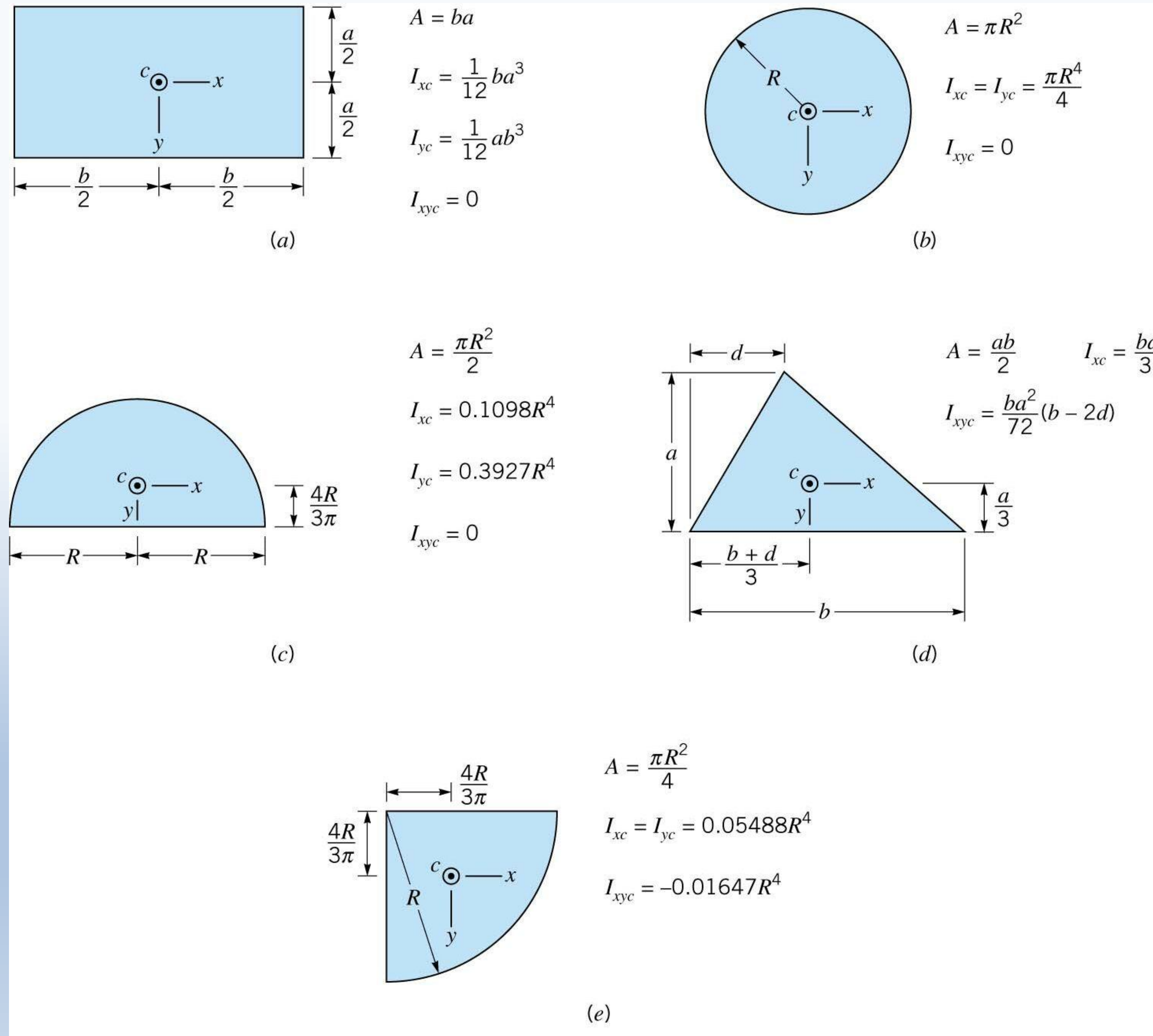
Parallel Axis Thereom:

$I_{xy} = I_{xyc} + A x_c y_c$ **I_{xc} is the second moment of inertia through the centroid**

Substituting the parallel Axis thereom, and rearranging:

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

Hydrostatic Force on a Plane Surface: Geometric Properties



Centroid Coordinates

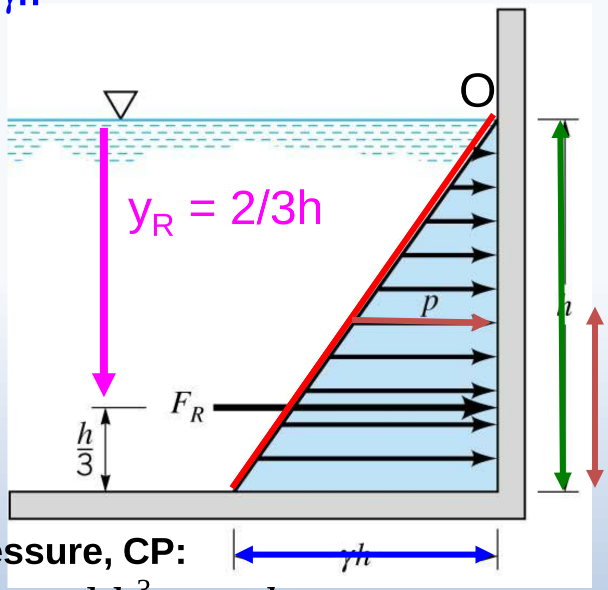
Areas

Moments of Inertia

Hydrostatic Force: Vertical Wall

Find the Pressure on a Vertical Wall using Hydrostatic Force Method

Pressure varies linearly with depth by the hydrostatic equation:
The magnitude of pressure at the bottom is $p = \gamma h$



The depth of the fluid is "h" into the board

The width of the wall is "b" into the board

$$F_R = p_{av} A$$

By inspection, the average pressure occurs at h/2, $p_{av} = \gamma h/2$

$$F_R = \gamma\left(\frac{h}{2}\right)A$$

The resultant force act through the center of pressure, CP:

y-coordinate:

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$I_{xc} = \frac{1}{12}bh^3$$

$$y_c = \frac{h}{2}$$

$$A = bh$$

$$y_R = \frac{bh^3}{12\frac{h}{2}(bh)} + \frac{h}{2}$$

$$y_R = \frac{h}{6} + \frac{h}{2} = \frac{2}{3}h$$

Hydrostatic Force: Vertical Wall

x-coordinate:

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

$$I_{xyc} = 0$$

$$y_c = \frac{b}{2}$$

$$A = bh$$

$$x_R = \frac{0}{\frac{h}{2}(bh)} + \frac{b}{2}$$

$$x_R = \frac{b}{2}$$

Center of Pressure:

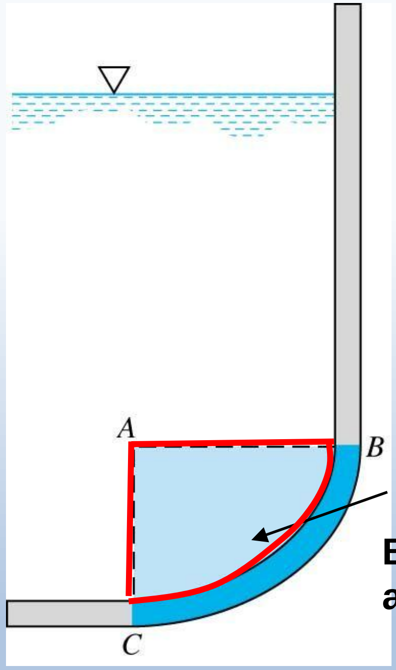
$$\left(\frac{b}{2}, \frac{2h}{3}\right)$$

Now, we have both the resultant force and its location.

The pressure prism is a second way of analyzing the forces on a vertical wall.

Hydrostatic Force on a Curved Surface

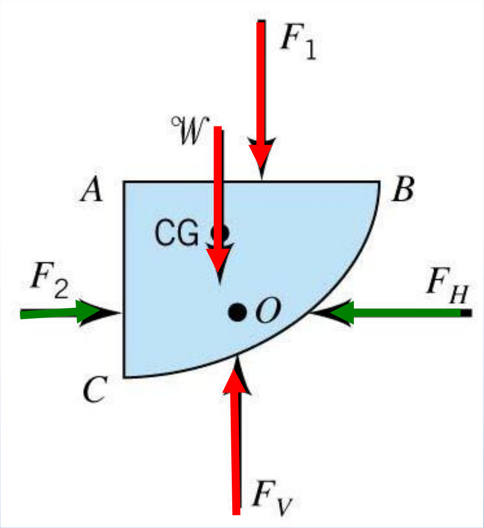
- **General theory of plane surfaces does not apply to curved surfaces**
- **Many surfaces in dams, pumps, pipes or tanks are curved**
- **No simple formulas by integration similar to those for plane surfaces**
- **A new method must be used**



Isolated Volume

Bounded by AB an AC and BC

Then we mark a F.B.D. for the volume:



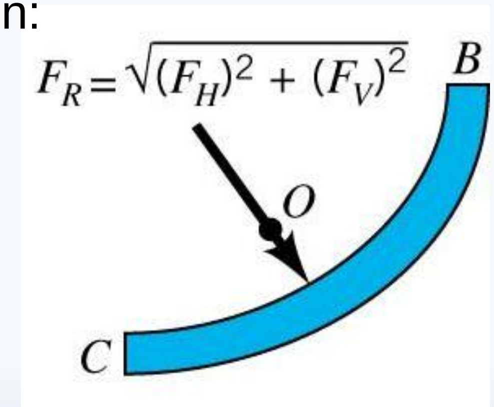
F_1 and F_2 is the hydrostatic force on each planar face

F_H and F_V is the component of the resultant force on the curved surface.

W is the weight of the fluid volume.

Hydrostatic Force on a Curved Surface

Now, balancing the forces for the Equilibrium condition:



Horizontal Force: $F_H = F_2$

Vertical Force: $F_V = F_1 + \mathcal{W}$

Resultant Force: $F_R = \sqrt{(F_H)^2 + (F_V)^2}$

The location of the Resultant Force is through O by sum of Moments:

Y-axis: $$F_1 x_1 + W x_c = F_V x_V$$

X-axis: $$F_2 x_2 = F_H x_H$$

Soda Pop Bottle Curved Surface:

Chapter Summary

Pressure gradient in a stationary fluid	$\frac{dp}{dz} = -\gamma$	(2.4)
Pressure variation in a stationary incompressible fluid	$p_1 = \gamma h + p_2$	(2.7)
Hydrostatic force on a plane surface	$F_R = \gamma h_c A$	(2.18)
Location of hydrostatic force on a plane surface	$y_R = \frac{I_{xc}}{y_c A} + y_c$	(2.19)
	$x_R = \frac{I_{xyc}}{y_c A} + x_c$	(2.20)

Example 2.6 Hydrostatic Force on a Plane Circular Surface

GIVEN The 4-m-diameter circular gate of Fig. E2.6*a* is located in the inclined wall of a large reservoir containing water ($\gamma = 9.80 \text{ kN/m}^3$). The gate is mounted on a shaft along its horizontal diameter, and the water depth is 10 m above the shaft.

FIND Determine

(a) the magnitude and location of the resultant force exerted on the gate by the water and

(b) the moment that would have to be applied to the shaft to open the gate.

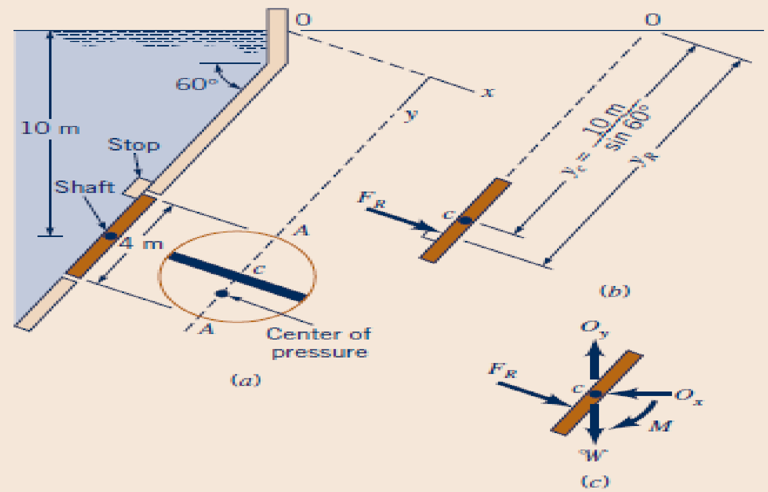


Figure E2.6*a–c*

Solution

(a) To find the magnitude of the force of the water we can apply Eq. 2.18,

$$F_R = \gamma h_c A$$

and since the vertical distance from the fluid surface to the centroid of the area is 10 m, it follows that

$$\begin{aligned} F_R &= (9.80 \times 10^3 \text{ N/m}^3)(10 \text{ m})(4\pi \text{ m}^2) \\ &= 1230 \times 10^3 \text{ N} = 1.23 \text{ MN} \end{aligned} \quad \text{(Ans)}$$

To locate the point (center of pressure) through which F_R acts, we use Eqs. 2.19 and 2.20,

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad y_R = \frac{I_{xc}}{y_c A} + y_c$$

For the coordinate system shown, $x_R = 0$ since the area is symmetrical, and the center of pressure must lie along the diameter *A-A*. To obtain y_R, we have from Fig. 2.18

$$I_{xc} = \frac{\pi R^4}{4}$$

and y_c is shown in Fig. E2.6*b*. Thus,

$$\begin{aligned} y_R &= \frac{(\pi/4)(2 \text{ m})^4}{(10 \text{ m}/\sin 60°)(4\pi \text{ m}^2)} + \frac{10 \text{ m}}{\sin 60°} \\ &= 0.0866 \text{ m} + 11.55 \text{ m} = 11.6 \text{ m} \end{aligned}$$

and the distance (along the gate) below the shaft to the center of pressure is

$$y_R - y_c = 0.0866 \text{ m} \quad \text{(Ans)}$$

We can conclude from this analysis that the force on the gate due to the water has a magnitude of 1.23 MN and acts through a point along its diameter *A-A* at a distance of 0.0866 m (along the gate) below the shaft. The force is perpendicular to the gate surface as shown in Fig. E2.6*b*.

COMMENT By repeating the calculations for various values of the depth to the centroid, h_c, the results shown in Fig. E2.6*d* are obtained. Note that as the depth increases, the distance between the center of pressure and the centroid decreases.

(b) The moment required to open the gate can be obtained with the aid of the free-body diagram of Fig. E2.6*c*. In this diagram 𝒲 is the weight of the gate and O_x and O_y are the horizontal and vertical reactions of the shaft on the gate. We can now sum moments about the shaft

$$\sum M_c = 0$$

and, therefore,

$$\begin{aligned} M &= F_R (y_R - y_c) \\ &= (1230 \times 10^3 \text{ N})(0.0866 \text{ m}) \\ &= 1.07 \times 10^5 \text{ N} \cdot \text{m} \end{aligned} \quad \text{(Ans)}$$

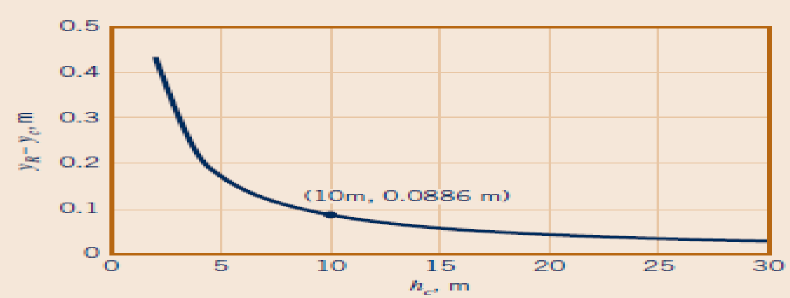


Figure E2.6*d*

EXAMPLE 2.7 Hydrostatic Pressure Force on a Plane Triangular Surface

GIVEN An aquarium contains seawater ($\gamma = 64.0 \text{ lb/ft}^3$) to a depth of 1 ft as shown in Fig. E2.7*a*. To repair some damage to one corner of the tank, a triangular section is replaced with a new section as illustrated in Fig. E2.7*b*.

FIND Determine

(a) the magnitude of the force of the seawater on this triangular area, and

(b) the location of this force.

SOLUTION

(a) The various distances needed to solve this problem are shown in Fig. E2.7*c*. Since the surface of interest lies in a vertical plane, $y_c = h_c = 0.9$ ft, and from Eq. 2.18 the magnitude of the force is

$$F_R = \gamma h_c A$$
$$= (64.0 \text{ lb/ft}^3)(0.9 \text{ ft})[(0.3 \text{ ft})^2/2] = 2.59 \text{ lb} \qquad \text{(Ans)}$$

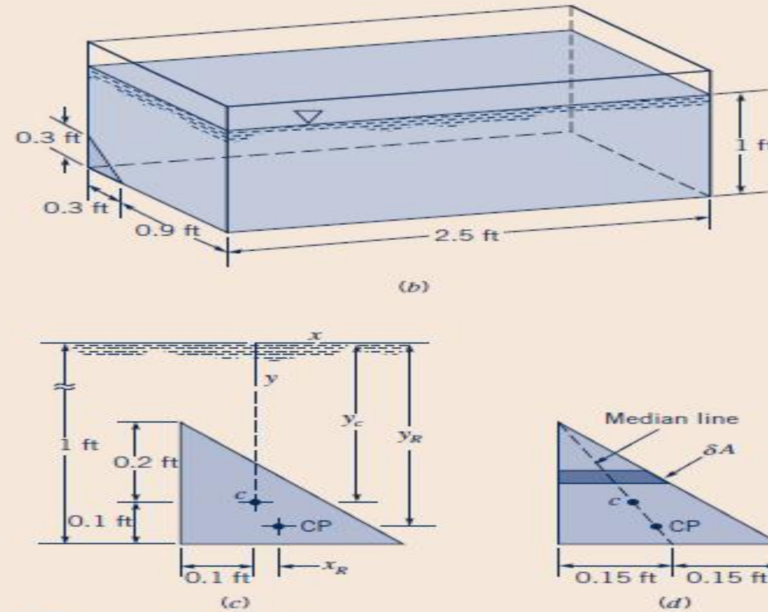
COMMENT Note that this force is independent of the tank length. The result is the same if the tank is 0.25 ft, 25 ft, or 25 miles long.

(b) The *y* coordinate of the center of pressure (CP) is found from Eq. 2.19,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

and from Fig. 2.18

■ Figure E2.7*a* (Photograph courtesy of Tenecor Tanks, Inc.)



■ Figure E2.7*b–d*

$$I_{xc} = \frac{(0.3 \text{ ft})(0.3 \text{ ft})^3}{36} = \frac{0.0081}{36} \text{ ft}^4$$

so that

$$y_R = \frac{0.0081/36 \text{ ft}^4}{(0.9 \text{ ft})(0.09/2 \text{ ft}^2)} + 0.9 \text{ ft}$$
$$= 0.00556 \text{ ft} + 0.9 \text{ ft} = 0.906 \text{ ft} \qquad \text{(Ans)}$$

Similarly, from Eq. 2.20

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

and from Fig. 2.18

$$I_{xyc} = \frac{(0.3 \text{ ft})(0.3 \text{ ft})^2}{72}(0.3 \text{ ft}) = \frac{0.0081}{72} \text{ ft}^4$$

so that

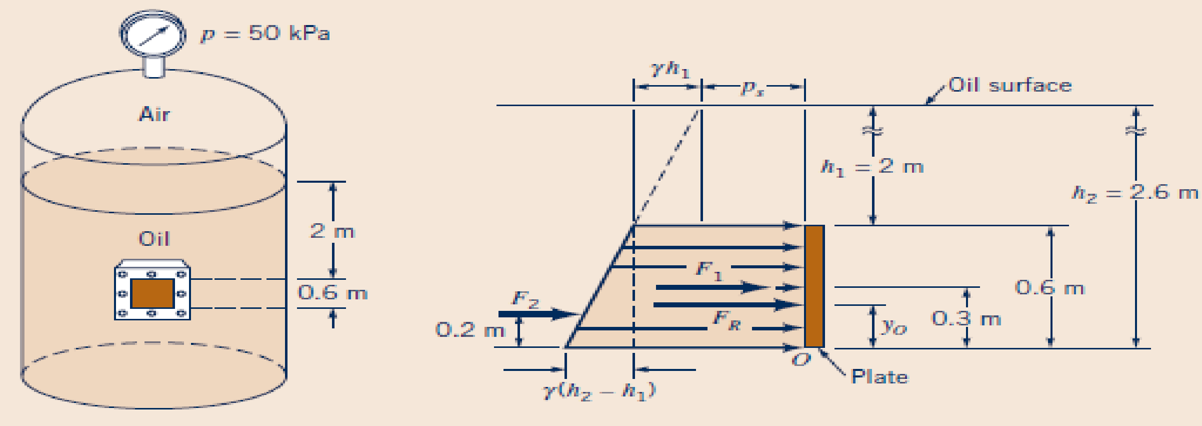
$$x_R = \frac{0.0081/72 \text{ ft}^4}{(0.9 \text{ ft})(0.09/2 \text{ ft}^2)} + 0 = 0.00278 \text{ ft} \qquad \text{(Ans)}$$

COMMENT Thus, we conclude that the center of pressure is 0.00278 ft to the right of and 0.00556 ft below the centroid of the area. If this point is plotted, we find that it lies on the median line for the area as illustrated in Fig. E2.7*d*. Since we can think of the total area as consisting of a number of small rectangular strips of area δA (and the fluid force on each of these small areas acts through its center), it follows that the resultant of all these parallel forces must lie along the median.

EXAMPLE 2.8 Use of the Pressure Prism Concept

GIVEN A pressurized tank contains oil ($SG = 0.90$) and has a square, 0.6-m by 0.6-m plate bolted to its side, as is illustrated in Fig. E2.8*a*. The pressure gage on the top of the tank reads 50 kPa, and the outside of the tank is at atmospheric pressure.

FIND What is the magnitude and location of the resultant force on the attached plate?



(*a*) (*b*)

■ **Figure E2.8**

SOLUTION

The pressure distribution acting on the inside surface of the plate is shown in Fig. E2.8*b*. The pressure at a given point on the plate is due to the air pressure, p_s, at the oil surface and the pressure due to the oil, which varies linearly with depth as is shown in the figure. The resultant force on the plate (having an area A) is due to the components, F_1 and F_2, where F_1 and F_2 are due to the rectangular and triangular portions of the pressure distribution, respectively. Thus,

$$\begin{aligned} F_1 &= (p_s + \gamma h_1)A \\ &= [50 \times 10^3\ \text{N/m}^2 \\ &\quad + (0.90)(9.81 \times 10^3\ \text{N/m}^3)(2\ \text{m})](0.36\ \text{m}^2) \\ &= 24.4 \times 10^3\ \text{N} \end{aligned}$$

and

$$\begin{aligned} F_2 &= \gamma\left(\frac{h_2 - h_1}{2}\right)A \\ &= (0.90)(9.81 \times 10^3\ \text{N/m}^3)\left(\frac{0.6\ \text{m}}{2}\right)(0.36\ \text{m}^2) \\ &= 0.954 \times 10^3\ \text{N} \end{aligned}$$

The magnitude of the resultant force, F_R, is therefore

$$F_R = F_1 + F_2 = 25.4 \times 10^3\ \text{N} = 25.4\ \text{kN} \qquad \text{(Ans)}$$

The vertical location of F_R can be obtained by summing moments around an axis through point O so that

$$F_R y_O = F_1(0.3\ \text{m}) + F_2(0.2\ \text{m})$$

or

$$\begin{aligned} y_O &= \frac{(24.4 \times 10^3\ \text{N})(0.3\ \text{m}) + (0.954 \times 10^3\ \text{N})(0.2\ \text{m})}{25.4 \times 10^3\ \text{N}} \\ &= 0.296\ \text{m} \qquad \text{(Ans)} \end{aligned}$$

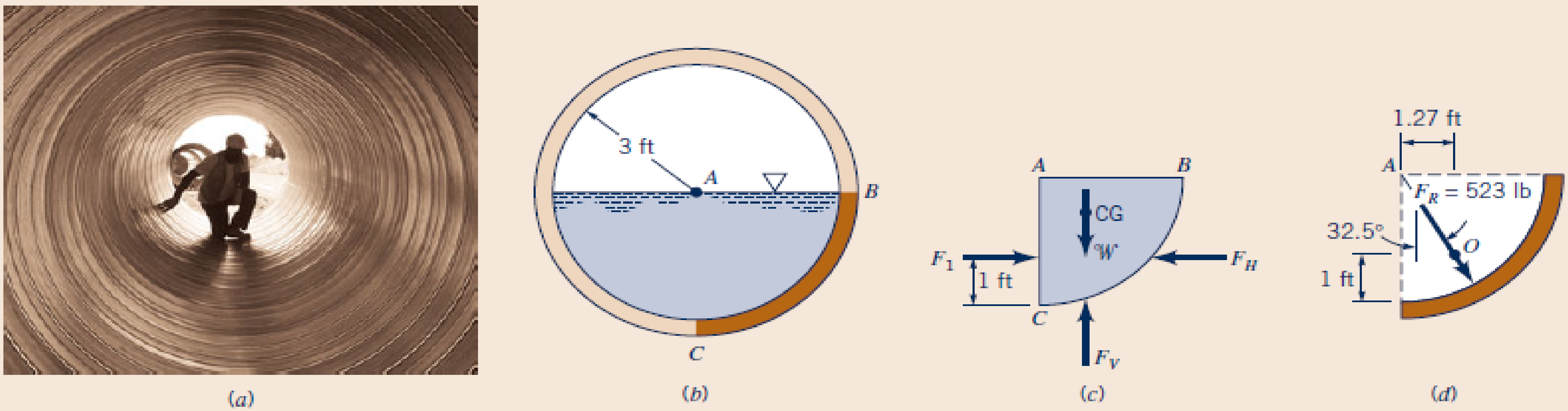
Thus, the force acts at a distance of 0.296 m above the bottom of the plate along the vertical axis of symmetry.

COMMENT Note that the air pressure used in the calculation of the force was gage pressure. Atmospheric pressure does not affect the resultant force (magnitude or location), since it acts on both sides of the plate, thereby canceling its effect.

EXAMPLE 2.9 Hydrostatic Pressure Force on a Curved Surface

GIVEN A 6-ft-diameter drainage conduit of the type shown in Fig. E2.9*a* is half full of water at rest, as shown in Fig. E2.9*b*.

FIND Determine the magnitude and line of action of the resultant force that the water exerts on a 1-ft length of the curved section *BC* of the conduit wall.



■ **Figure E2.9** (Photograph courtesy of CONTECH Construction Products, Inc.)

SOLUTION

We first isolate a volume of fluid bounded by the curved section *BC*, the horizontal surface *AB*, and the vertical surface *AC*, as shown in Fig. E2.9*c*. The volume has a length of 1 ft. The forces acting on the volume are the horizontal force, F_1, which acts on the vertical surface *AC*, the weight, $\mathcal{W}$, of the fluid contained within the volume, and the horizontal and vertical components of the force of the conduit wall on the fluid, F_H and F_V, respectively.

The magnitude of F_1 is found from the equation

$$F_1 = \gamma h_c A = (62.4 \text{ lb/ft}^3)(\tfrac{3}{2} \text{ ft})(3 \text{ ft}^2) = 281 \text{ lb}$$

and this force acts 1 ft above *C* as shown. The weight $\mathcal{W} = \gamma \forall$, where $\forall$ is the fluid volume, is

$$\mathcal{W} = \gamma \forall = (62.4 \text{ lb/ft}^3)(9\pi/4 \text{ ft}^2)(1 \text{ ft}) = 441 \text{ lb}$$

and acts through the center of gravity of the mass of fluid, which according to Fig. 2.18 is located 1.27 ft to the right of *AC* as shown. Therefore, to satisfy equilibrium

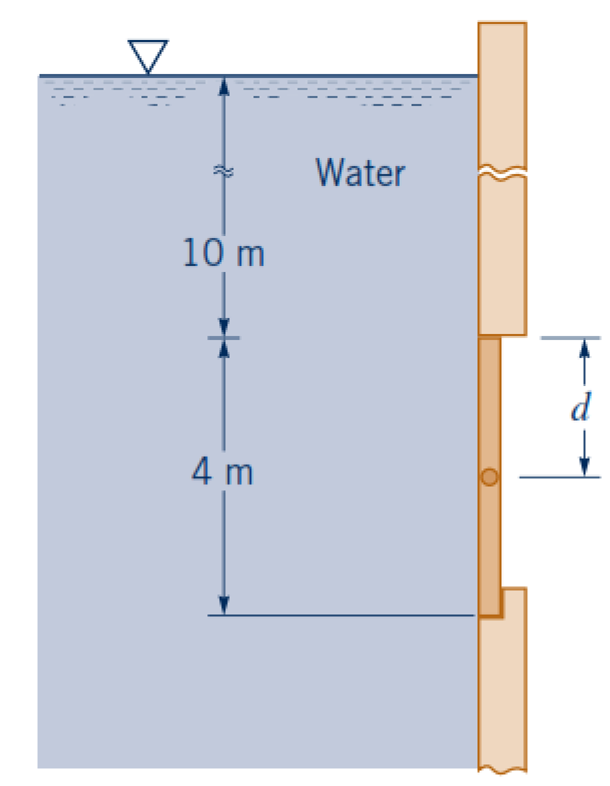
$$F_H = F_1 = 281 \text{ lb} \qquad F_V = \mathcal{W} = 441 \text{ lb}$$

and the magnitude of the resultant force is

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$
$$= \sqrt{(281 \text{ lb})^2 + (441 \text{ lb})^2} = 523 \text{ lb} \qquad \text{(Ans)}$$

The force the water exerts *on* the conduit wall is equal, but *opposite in direction*, to the forces F_H and F_V shown in Fig. E2.9*c*. Thus, the resultant force *on the conduit wall* is shown in Fig. E2.9*d*. This force acts through the point *O* at the angle shown.

2.102 A rectangular gate that is 2 m wide is located in the vertical wall of a tank containing water as shown in Fig. P2.102. It is desired to have the gate open automatically when the depth of water above the top of the gate reaches 10 m. (**a**) At what distance, *d*, should the frictionless horizontal shaft be located? (**b**) What is the magnitude of the force on the gate when it opens?



2.116 A 4-m-long curved gate is located in the side of a reservoir containing water as shown in Fig. P2.116. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point *A*? Explain.

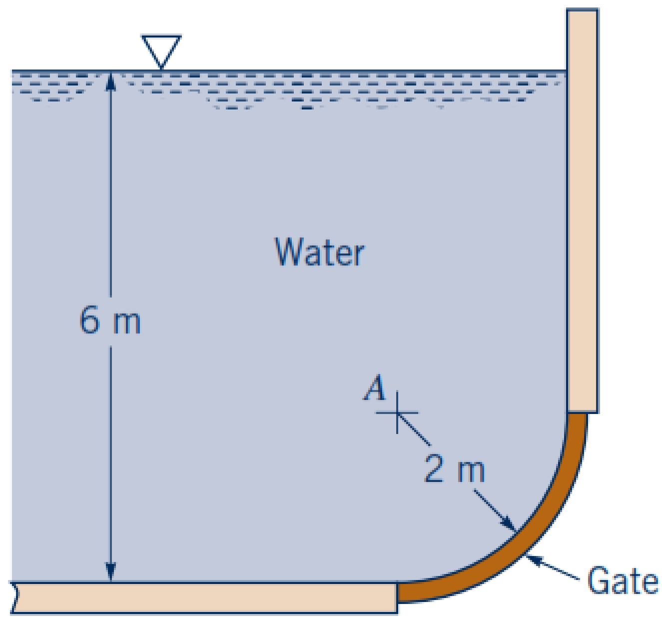


Figure P2.116