Lecture 1: Bond graph Theory

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Outline

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4. Causality
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   - Constrained causality elements
   - Preferred causality elements
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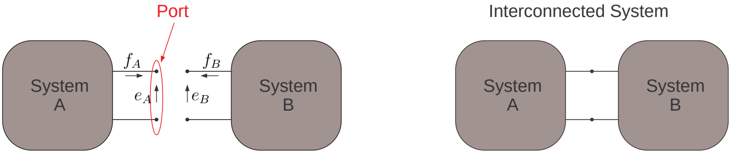
Interconnection systems



- A port is an interface of an element with other.
- A port contains two variables, e (effort) and f (flow).
- Usually, a port exchanges power.

Interconnection example: LC circuit

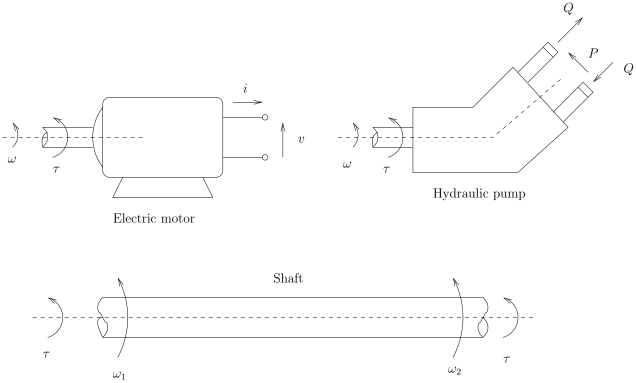
- Interconnection rules

$$v_L = v_C, \quad i_L = -i_C.$$

- effort variables (v_C, v_L), flow variables (i_C, i_L).

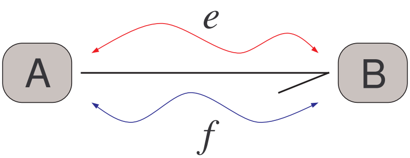
Engineering multiports

Some engineering subsystems:



Interconnection: Electrical motor + Shaft + Hydraulic pump

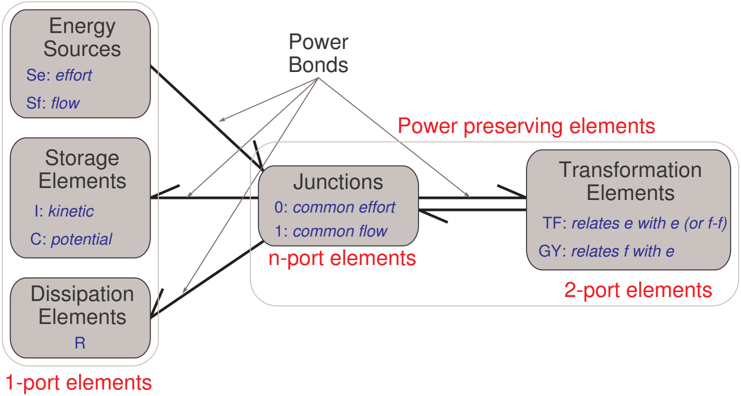
Ports and bonds



- A and B are related by a power bond.
- The half arrow represents the positive power flow convention.
- Pair of signals: flow f and effort e, called power variables.
- Flow: current, speed...
- Effort: voltage, torque...

Ports and bonds

The Bond Graph Theory can be, roughly speaking, summarized in:



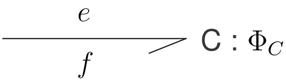
Basic elements of the BG theory

- Each element is represented by a multiport.
- Port are connected by a *bond*.

- 1-port elements: energy storing, dissipation or supplying elements.
  - C-elements and I-elements.
  - R-elements.
  - Se-elements and Sf-elements.
- 2-ports elements: Transformers and gyrators.
- 3-ports elements: 0 and 1 junctions.

- 2-port and 3-port elements describes a network interconnection.
- 2-port and 3-port elements are power preserving.

C-elements



Constitutive relation

$$\dot{q} = f$$

where q is called displacement, and $e = \Phi_C^{-1}(q)$.

Integral relation from f to e

$$e(t) = \Phi_C^{-1} \underbrace{\left(q(t_0) + \int_{t_0}^{t} f(\tau) \mathrm{d}\tau \right)}_{q(t)}$$

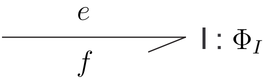
Linear case

For an ideal capacitor:

$$\Phi_C^{-1}(q) = \frac{q}{C} \qquad v = \frac{q}{C}$$

$$v(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) \mathrm{d}\tau$$

I-elements



Constitutive relation

$$\dot{p} = e$$

where p is called momentum, and $f = \Phi_I^{-1}(p)$.

Integral relation from e to f

$$f(t) = \Phi_I^{-1}\underbrace{\left(p(t_0) + \int_{t_0}^{t} e(\tau)\mathrm{d}\tau\right)}_{p(t)}$$

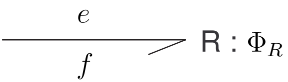
Linear case

For an ideal inductor:

$$\Phi_I^{-1}(p) = \frac{p}{L} \qquad i = \frac{p}{L}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\mathrm{d}\tau$$

R-elements



Constitutive relation

$$e = \Phi_R(f)$$

or $f = \Phi_R^{-1}(e)$, algebraic relation.

Represents a power dissipation.

Linear case

For an ideal resistor:

$$\Phi_R(f) = Rf$$

$$v(t) = Ri(t)$$

Se-elements

$$E : \mathrm{Se} \; \overset{e}{\underset{f}{\xrightharpoonup{\qquad\qquad}}}$$

Constitutive relation

$$e = E(t).$$

- The effort e, does not depend on f!!!.
- Output power flow.

Examples

- Electrical voltage source.
- Torque source.

Sf-elements

$$F : \mathrm{Sf} \ \overset{e}{\underset{f}{\xrightharpoondown{\qquad\qquad}}}$$

Constitutive relation

$$f = F(t).$$

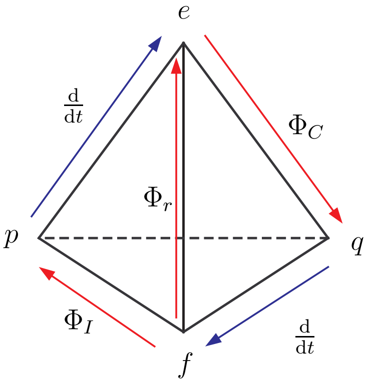
- Flux f, does not depend on e!!!.
- Output power flow.

Examples

- Electrical current source.
- Speed source.

Tetrahedron of state

The R, C and I-type elements can be placed along the tetrahedron of state:



Energy: C-element

The energy variation for an element is describe by:

$$H(t) - H(t_0) = \int_{t_0}^{t} e(\tau) f(\tau) \mathrm{d}\tau.$$

C-element

The constitutive relation is: $e = \Phi_C^{-1}(q), \quad \dot{q} = f.$
For a linear case, $\Phi_C^{-1}(q) = \frac{q}{C}$,

$$H(q) - H(q_0) = \frac{1}{2C}(q^2 - q_0^2).$$

Energy: I-element

The energy variation for an element is describe by:

$$H(t) - H(t_0) = \int_{t_0}^{t} e(\tau) f(\tau) \mathrm{d}\tau.$$

I-element

The constitutive relation is: $f = \Phi_I^{-1}(p), \quad \dot{q} = e.$
For a linear case, $\Phi_I^{-1}(p) = \frac{p}{I}$,

$$H(p) - H(p_0) = \frac{1}{2I}(p^2 - p_0^2).$$

Energy: R-element

The energy variation for an element is describe by:

$$H(t) - H(t_0) = \int_{t_0}^{t} e(\tau) f(\tau) \mathrm{d}\tau.$$

R-element

The constitutive relation is: $f = \Phi_R^{-1}(e)$ or $e = \Phi_R(f)$.
Then,

$$H(t) - H(t_0) = \int_{t_0}^{t} e(\tau) \Phi_R^{-1}(e(\tau)) \mathrm{d}\tau.$$

2-port elements

- 2-port elements are power preserving.
- With the sign convention of the figure, then power conservation means

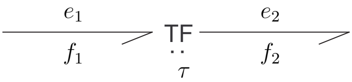
$$e_1 f_1 = e_2 f_2.$$

- Any power flowing into one side of the 2-port is simultaneously flowing out of the other side.

2-port elements

- Transformers.
- Gyrators.

TF-elements



One way to satisfy the power conservation is by means of

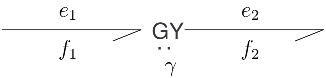
$$e_1 = \tau e_2 \quad \tau f_1 = f_2.$$

This define a 2-port element known as transformer, and denoted by TF. The parameter τ is called the transformer modulus.

Examples

Ideal rigid level, gear pair, electrical transformer, hydraulic ram...

GY-elements



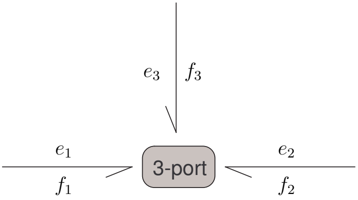
A second way to satisfy the power conservation equation is

$$e_1 = \gamma f_2 \quad \gamma f_1 = e_2.$$

This define a 2-port element known as gyrator, and denoted by GY. The parameter γ is called the gyrator modulus.

Gyrators are somehow more mysterious than transformers. In the early days of bond graph theory, before the importance of the gyrator was recognized, systems which were in fact gyrators were approximated using transformers.
Gyrators are needed, at fundamental level, to model the Hall effect and some electromagnetic phenomena at microwave frequencies, among others.

3-port elements

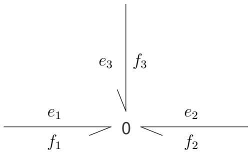


- 3-port elements are power preserving.
- Following the figure power convention,

$$e_1 f_1 + e_2 f_2 + e_3 f_3 = 0.$$

- They are often called junctions.
- They are, in fact, n-port elements.

0-junction



0-junction or, common effort junction.

$$e_1 = e_2 = e_3.$$

- With the power sign convention shown, power conservation is

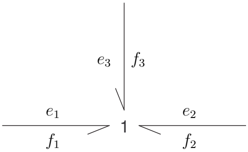
$$e_1 f_1 + e_2 f_2 + e_3 f_3 = 0.$$

- Then using the common effort relation, we get

$$f_1 + f_2 + f_3 = 0$$

which justifies the flow junction name.

1-junction



1-junction or, common flow junction.

$$f_1 = f_2 = f_3.$$

- With the power sign convention shown and using the common flow relation, we get

$$e_1 + e_2 + e_3 = 0$$

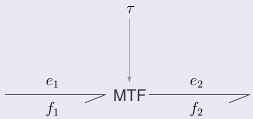
which justifies the effort junction name.

Modulated elements

Some elements can be modulated.

- Some parameters (or constitutive relations) depends on an external signal, which does not has energy contribution.
- In Bond graph Theory, is represented by an activated bond.

Example: a modulated transformer



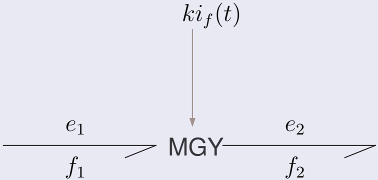
$$e_1 = \tau(t) e_2$$

$$\tau(t) f_1 = f_2$$

In this case the transformer modulus is given by a variable $\tau(t)$.

Other examples

A DC motor



Equations:

$$v = ki_f(t)\omega \quad \tau = ki_f(t)i_a.$$

A variable resistor

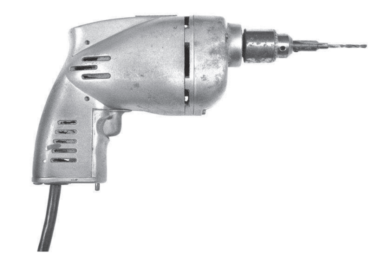
The resistance value is high dependent on the temperature, or in a piezoelectric systems on the position...

They are also MSe, MSf, MI, MC...

Causality

A bond relates two elements, one of them states the effort and the other one the flow.

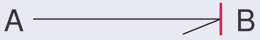
Example: electric drill



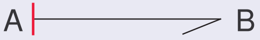
Electric drill fix the effort (torque), and it turns depending on the material.

Causality

The causal stroke determines which element fix the effort.



A fix e and B returns f.



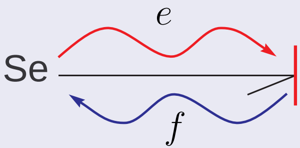
B fix e and A returns f.

- Causality does not depend on the power direction.
- Causality is required to obtain the dynamical equations.
- Causality assignation depends on the interconnected elements.

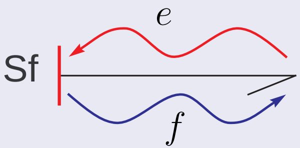
Fixed causality elements

This occurs at sources:

Effort source, Se



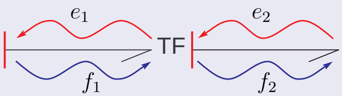
Flow source, Sf



Constrained causality elements

This occurs at:

Transformer, TF



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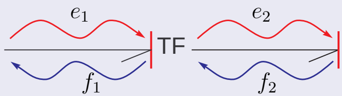
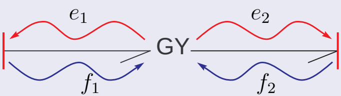


Gyrator, GY



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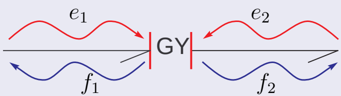
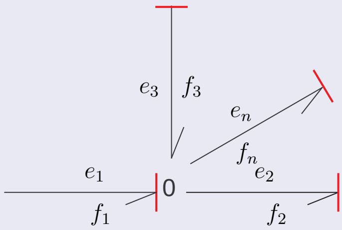


Constrained causality elements

This occurs at:

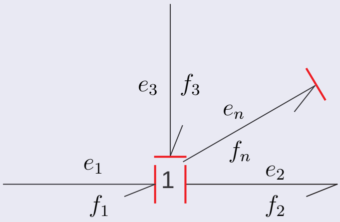
0-junctions

$e_1 = e_2 = \ldots = e_n$



1-junctions

$f_1 = f_2 = \ldots = f_n$



Preferred causality elements

The storage elements have a preferred causality: integral causality.

- Integration is a process which can be realised physically.
- Numerical differentiation is not physically realisable, since information at future points is needed.

Example

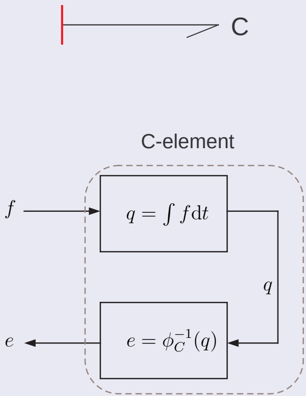
When a voltage v is imposed on an electrical capacitor, the current is:

$$i = C\frac{\mathrm{d}v}{\mathrm{d}t} \quad \text{Differentiation!!!}$$

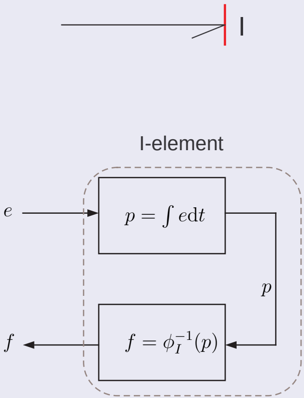
When the current is imposed $v = v_0 + \int i\mathrm{d}t$.

Blocs diagram equivalence

C-element

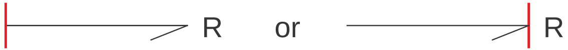


I-element



Indifferent causality elements

When there are no causal constraints.
For a linear R-element, it does not matter which of the port variables is the output.



$$e = Rf$$

$$f = \frac{1}{R}e$$