Lecture 1: Introduction

Reading materials: Sections 1.1 – 1.6

1. Introduction

- People became interested in vibration when the first musical instruments, probably whistles or drums, were discovered.

- Most human activities involve vibration in one form or other. For example, we hear because our eardrums vibrate and see because light waves undergo vibration.

- Any motion that repeats itself after an interval of time is called *vibration* or *oscillation*.

- The general terminology of "Vibration" is used to describe oscillatory motion of mechanical and structural systems.

- The Vibration of a system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternately.

- The terminology of "Free Vibration" is used for the study of natural vibration modes in the absence external loading.

- The terminology of "Forced Vibration" is used for the study of motion as a result of loads that vary rapidly with time. Loads that vary rapidly with time are called dynamic loads.

- If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as "undamped vibration". If any energy is lost in this way, however, is called "damped vibration".

- If the system is damped, some energy is dissipated in each cycle of vibration and must be replaced by an external source if a state of steady vibration is to be maintained.

2. Branches of Mechanics

- Rigid bodies
  Statics & Dynamics; Kinematics & Dynamics of Mechanical Systems

- Fluid mechanics

- Deformable bodies
  Structural analysis: assuming loads do not change over time or change very "slowly"
  Vibrations or Dynamic analysis: considering more general case when loads vary with time.
  Finite element analysis: a powerful numerical method for both static and dynamic analysis.

- Vibration analysis procedure

  Step 1: Mathematical modeling
  Step 2: Derivation of governing equations
  Step 3: Solution of the governing equations
  Step 4: Interpretation of the results

3. Other Basic Concepts of Vibration

- A vibratory system, in general, includes a means for storing potential energy (spring or elasticity), a means for storing kinetic energy (mass or inertia), and a means by which energy is gradually lost (damper).

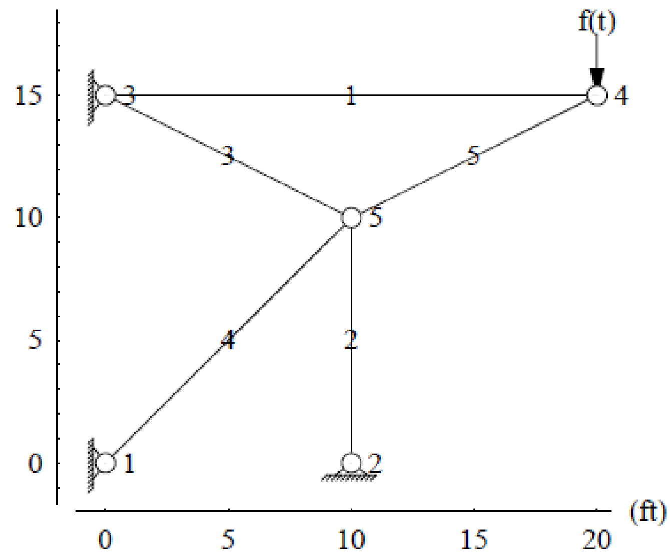
- The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom (DOF) of the system.

  Examples:

- A large number of practical systems can be described using a finite number of DOFs. Systems with a finite number of DOFs are called *discrete* or *lumped parameter systems*.

- Some systems, especially those involving continuous elastic members, have an infinite number of DOFs. Those systems are called *continuous* or *distributed systems*.

4. Plane truss example (Matrix method)



- Element equations

- Global equations

- Boundary conditions

- Stiffness matrix and unknown variables

$$\begin{pmatrix} 379857. & 111803. & -223607. & -111803. \\ 111803. & 55901.7 & -111803. & -55901.7 \\ -223607. & -111803. & 557699. & 110485. \\ -111803. & -55901.7 & 110485. & 534789. \end{pmatrix} \begin{pmatrix} u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix}$$

or **Kd**

- $f(t) = f_0$

$$\mathbf{Kd} = \mathbf{f}$$

- f(t) is a harmonic force, i.e. $f(t) = -100\cos(7\pi t)$

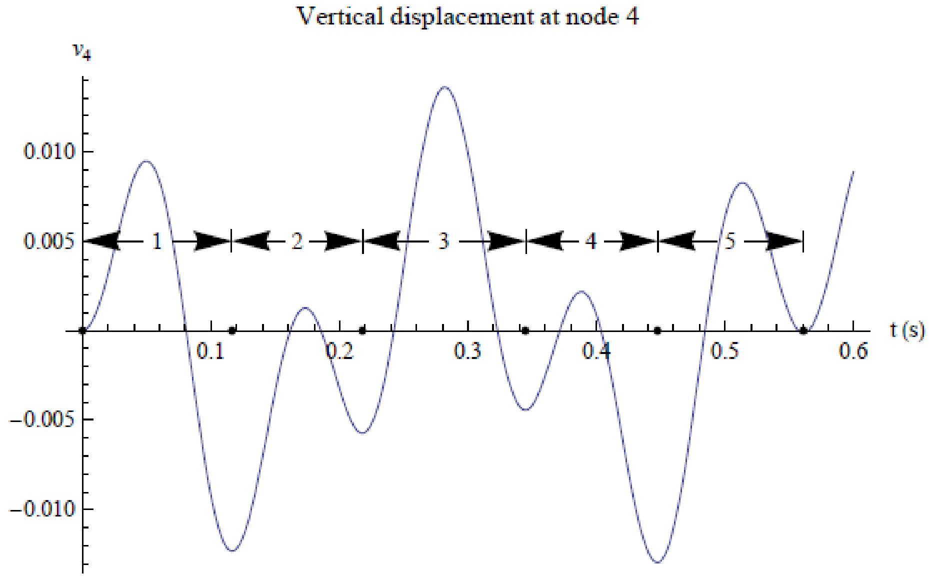
$$\begin{pmatrix} 5.29039 & 0 & 0.0205121 & 0 \\ 0 & 5.29039 & 0 & 0.0205121 \\ 0.0205121 & 0 & 0.170634 & 0 \\ 0 & 0.0205121 & 0 & 0.170634 \end{pmatrix}\begin{pmatrix} \ddot{u}_4 \\ \ddot{v}_4 \\ \ddot{u}_5 \\ \ddot{v}_5 \end{pmatrix} +$$

$$\begin{pmatrix} 379\,857. & 111\,803. & -223\,607. & -111\,803. \\ 111\,803. & 55\,901.7 & -111\,803. & -55\,901.7 \\ -223\,607. & -111\,803. & 557\,699. & 110\,485. \\ -111\,803. & -55\,901.7 & 110\,485. & 534\,789. \end{pmatrix}\begin{pmatrix} u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 100\cos(7\pi t) \\ 0 \\ 0 \end{pmatrix}$$

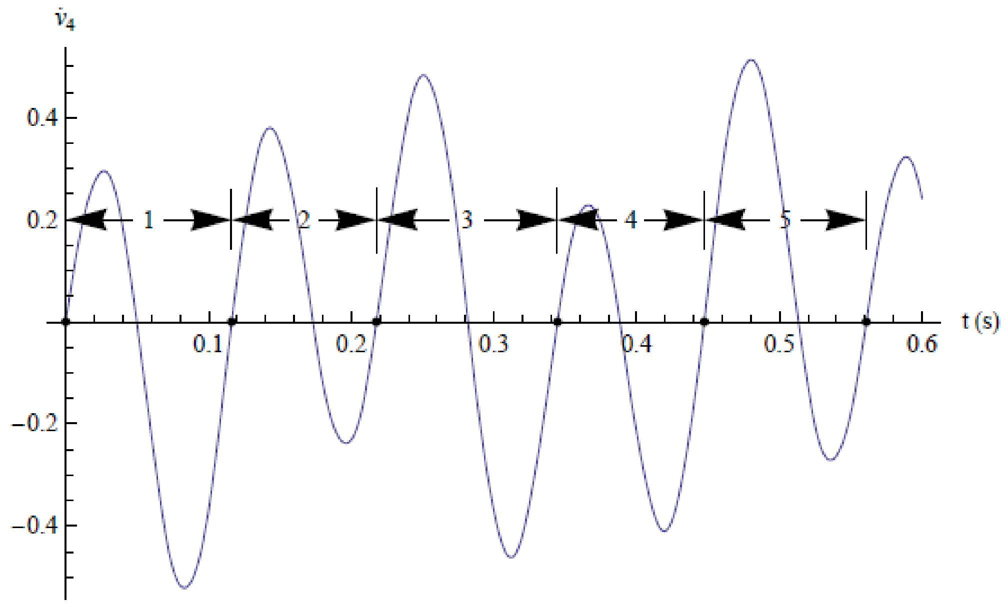
or

$$\mathbf{M\ddot{d}} + \mathbf{Kd} = \mathbf{f}$$

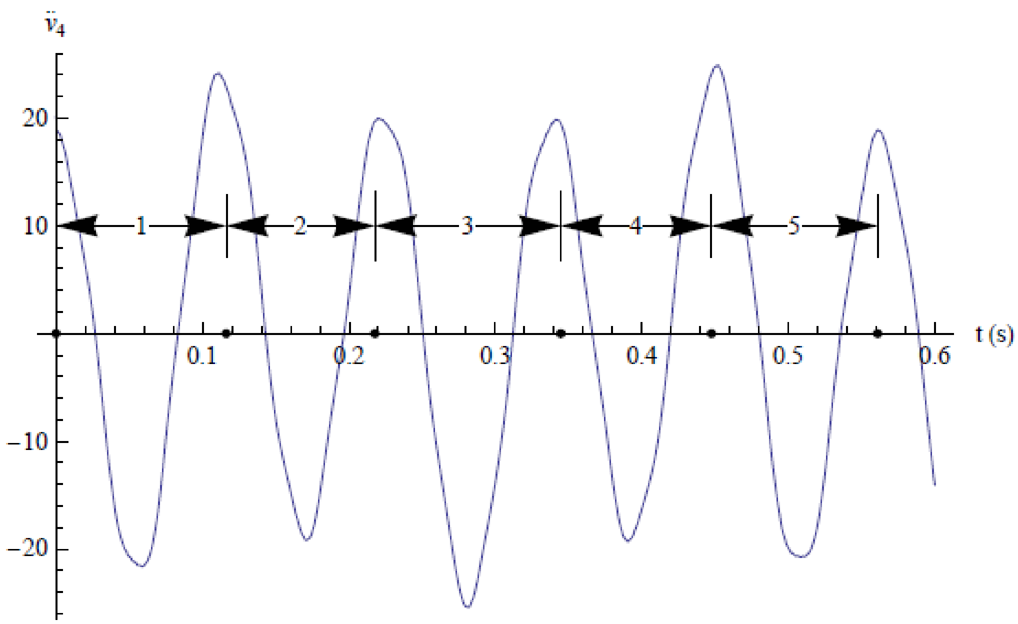
- Solutions

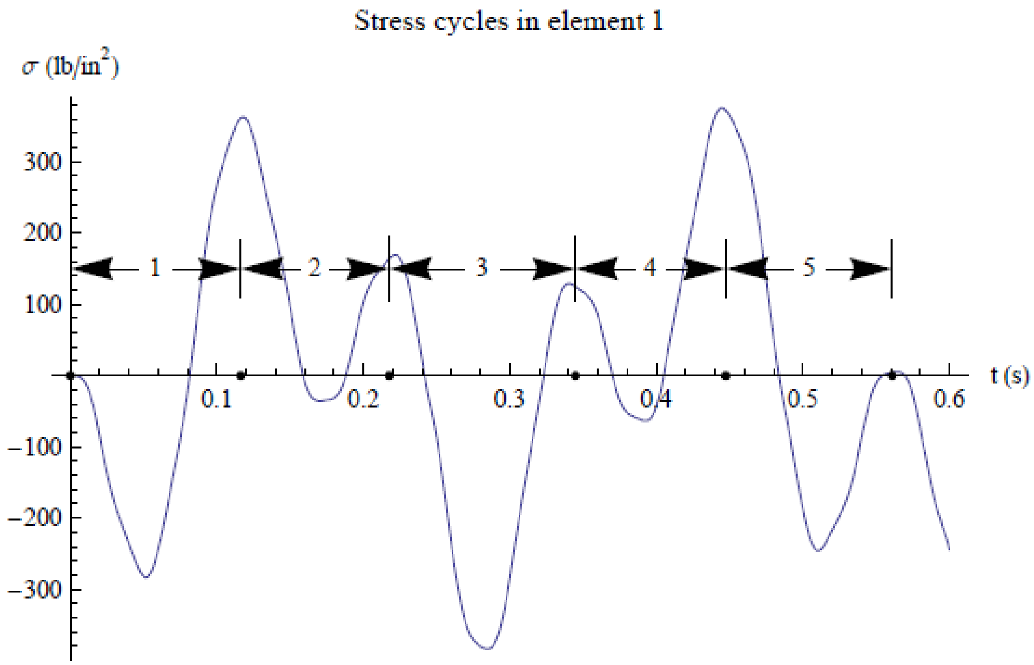


Vertical velocity at node 4



Vertical acceleration at node 4





5. Period and Frequency

- "Period of vibration" is the time that it takes to complete one cycle. It is measured in seconds.

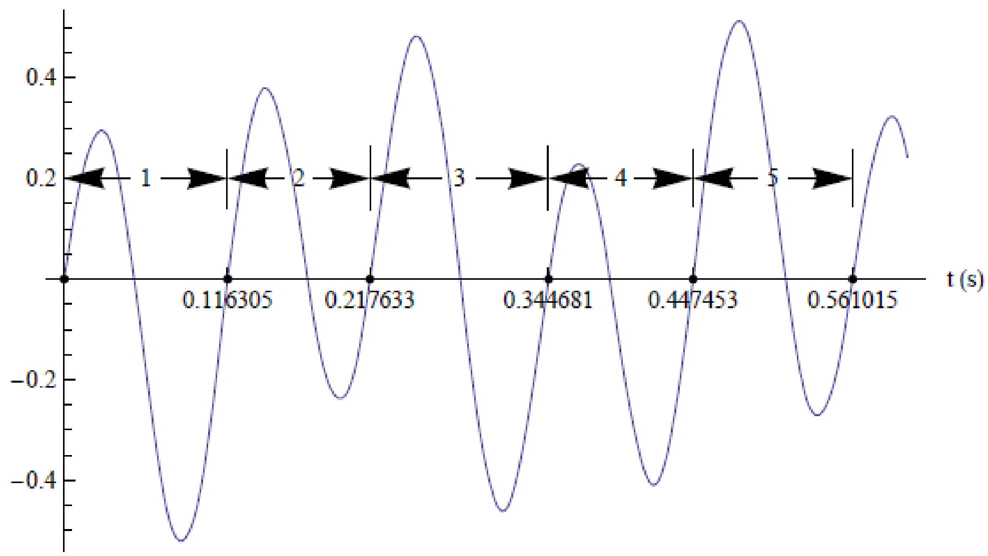
- "Frequency" is the number of cycles per second. It is measured in Hz (1 cycle/second). It could be also measured in radians/second.

Period of vibration: T

Frequency of vibration: $f = (1/T)$ Hz or $\omega = (2\pi/T)$ radians/s

$T=(2\ \pi/\omega) = (1/T)$

Example:



Cycle	Time(s)
1	0.116305
2	0.101328
3	0.127048
4	0.102773
5	0.113562

6. Dynamic loads on structures

- Wind loads
- Blast pressure
- Earthquakes

Etc.

7. Importance of dynamic analysis

- Load magnification & Fatigue effects

- A static load is constant and is applied to the structure for a considerable part of its life. For example, the self weight of building. Loads that are repeatedly exerted, but are applied and removed very slowly, are also considered static loads.

- Fatigue phenomenon can be caused by repeated application of the load. The number of cycles is usually low, and hence this type of loading may cause what is known as low-cycle fatigue.

- Quasi-static loads are actually due to dynamic phenomena but remain constant for relatively long periods.

- Most mechanical and structural systems are subjected to loads that actually vary over time. Each system has a characteristic time to determine whether the load can be considered static, quasi-static, or dynamic. This characteristic time is *the fundamental period of free vibration* of the system.

- Dynamic Load Magnification factor (DLF) is the ratio of the maximum dynamic force experienced by the system and the maximum applied load.

- The small period of vibration results in a small DLF.

- Fatigue phenomenon can be caused by repeated application of the load. The number of cycles and the stress range are important factors in determining the fatigue life.

8. Types of Vibratory Motion

- Oscillatory motion may repeat itself regularly, as in the case of a simple pendulum, or it may display considerable irregularity, as in the case of ground motion during an earthquake.

- If the motion is repeated after equal intervals of time, it is called *periodic motion*. The simplest type of periodic motion is *harmonic motion*.

- Harmonic motion

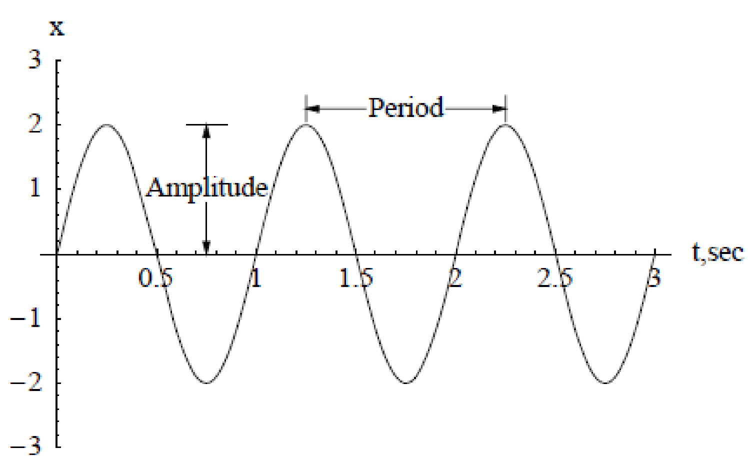
It is described by sine or cosine functions.

$$x(t) = A\sin(\omega\, t)$$

A is the amplitude while ω is the frequency (radians/sec)

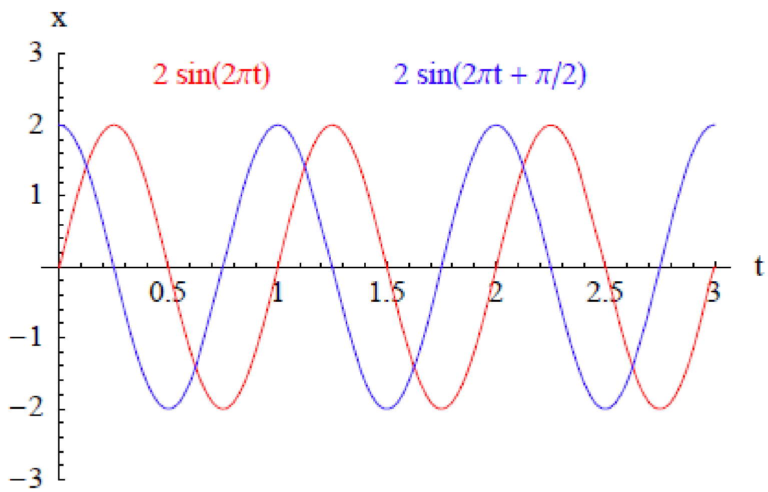
$$\dot{x}(t) = \omega\, A\cos(\omega\, t)$$

$$\ddot{x}(t) = -\omega^2 A\sin(\omega\, t) = -\omega^2 x(t)$$



Plot of $x(t) = 2\sin(2\pi t)$

Two harmonic motions having the same period and/or amplitude could have different phase angle.



Plot of two harmonic functions $2\sin(2\pi t)$ and $2\sin(2\pi t+\pi/2)$

A harmonic motion can be written in terms of exponential functions.

$$\sin\omega t=\frac{e^{i\omega t}-e^{-i\omega t}}{2i};\quad \cos\omega t=\frac{e^{i\omega t}+e^{-i\omega t}}{2}$$

so that

$$e^{i\omega t}=\cos\omega t+i\sin\omega t$$

A harmonic motion could be written as

$$x(t)=a\,e^{i\omega t}$$

Alternative forms for harmonic motion

Generally, a harmonic motion can be expressed as a combination of sine and cosine waves.

$$y(t)=A\cos\omega t+B\sin\omega t \quad\Longleftrightarrow\quad y(t)=Y\sin(\omega t+\theta)$$

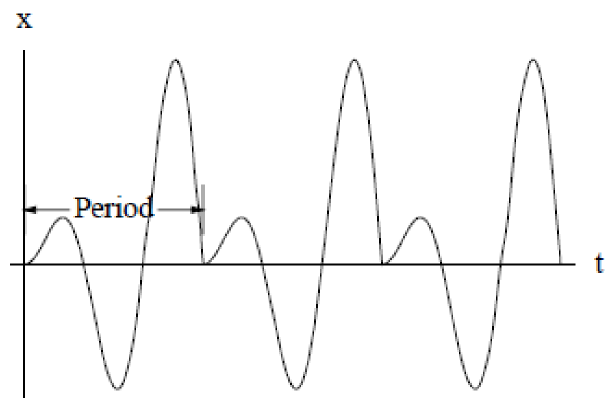
$$Y = \sqrt{A^2 + B^2} \qquad \theta = \tan^{-1}(A / B)$$

or

$$y(t) = A\cos\omega t - B\sin\omega t \iff y(t) = -Y\sin(\omega t - \theta) = Y\cos(\omega t - \theta)$$

Periodic motion

The motion repeats itself exactly.



A general vibratory motion doesn't have a repeating pattern.