Lecture 14 : Electromagnetic induction

Looking forward at …

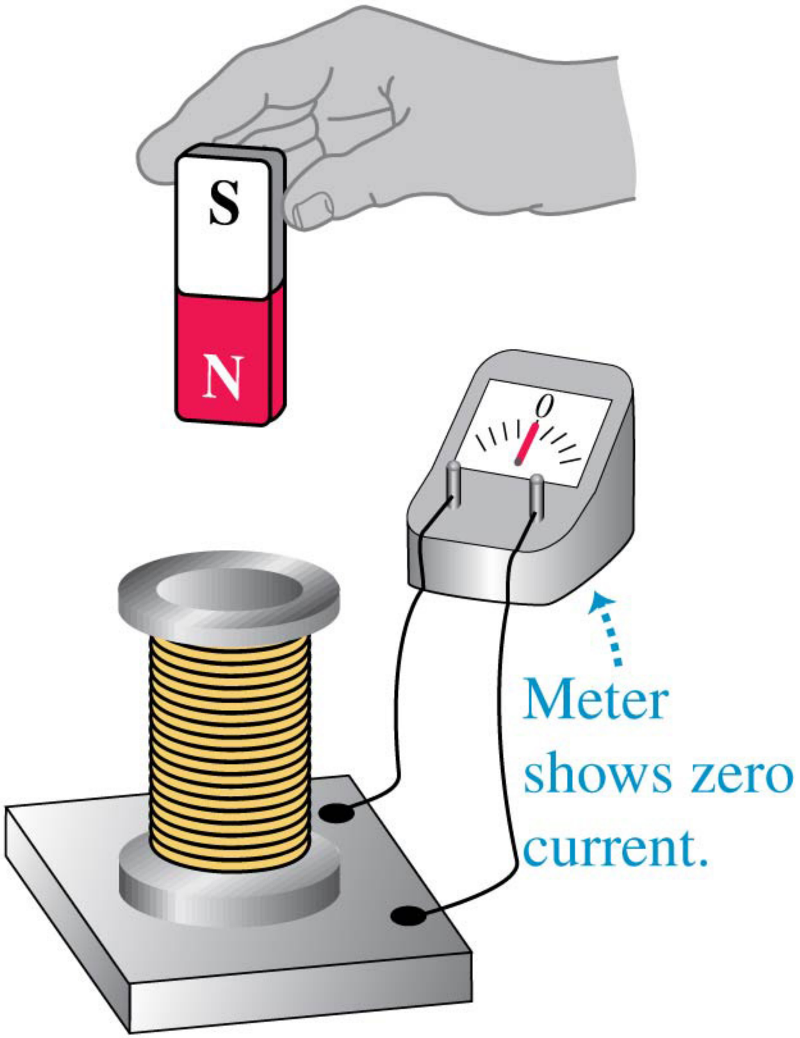
- how Faraday's law relates the induced emf in a loop to the change in magnetic flux through the loop.
- how to determine the direction of an induced emf.
- how a changing magnetic flux generates a circulating electric field.

Introduction

- The card reader at a gas station scans the information that is coded in a magnetic pattern on the back of your card.
- Why must you remove the card quickly rather than hold it motionless in the card reader's slot?
- Energy conversion makes use of electromagnetic induction.
- Faraday's law and Lenz's law tell us about induced currents.
- Maxwell's equations describe the behavior of electric and magnetic fields in *any* situation.

Induction experiment: Slide 1 of 4

- During the 1830s, several pioneering experiments with magnetically induced emf were carried out.
- In the figure shown, a coil of wire is connected to a galvanometer.
- When the nearby magnet is stationary, the meter shows no current.

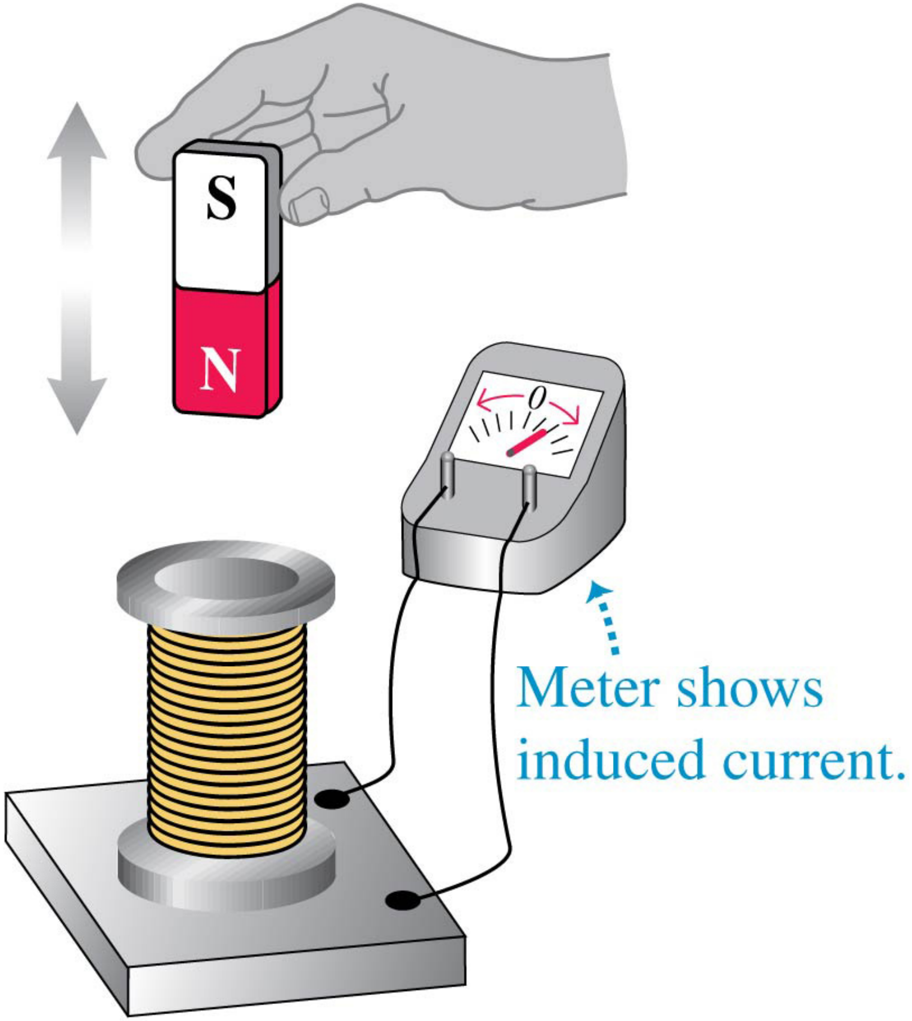


Meter shows zero current.

Induction experiment: Slide 2 of 4

- When we move the magnet either toward or away from the coil, the meter shows current in the circuit, but only while the magnet is moving.

- We call this an **induced current**, and the corresponding emf required to cause this current is called an **induced emf**.

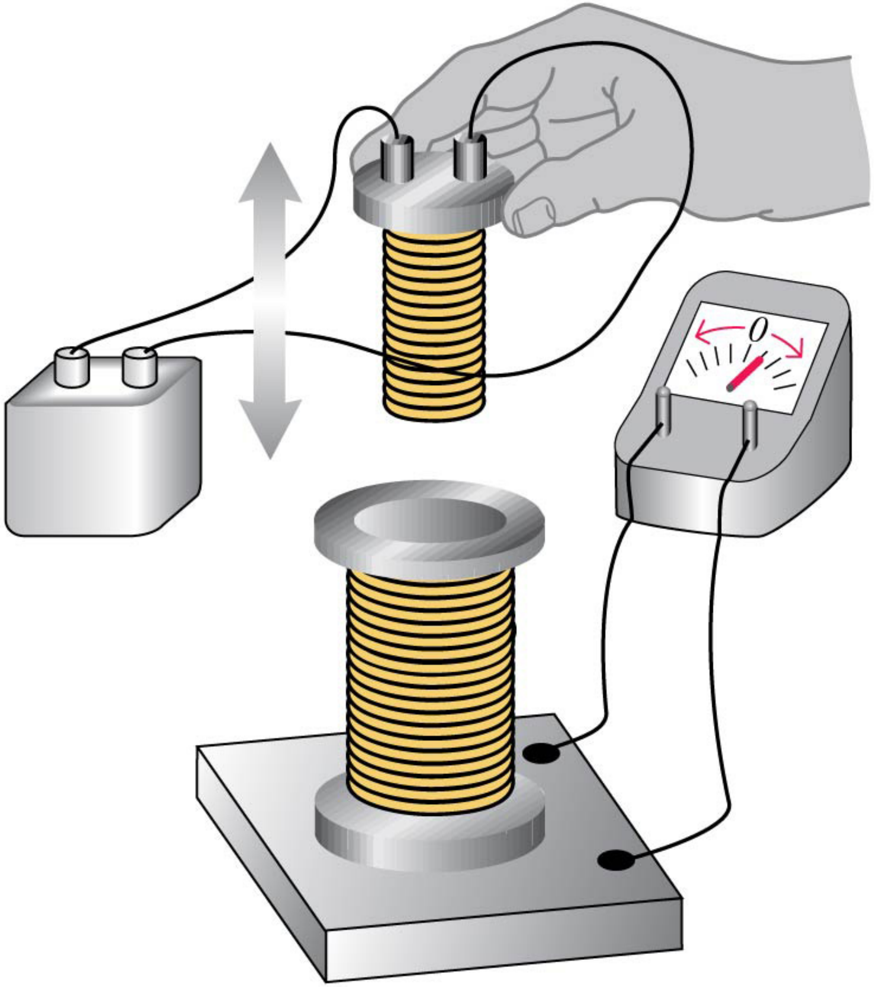


Meter shows induced current.

Induction experiment: Slide 3 of 4

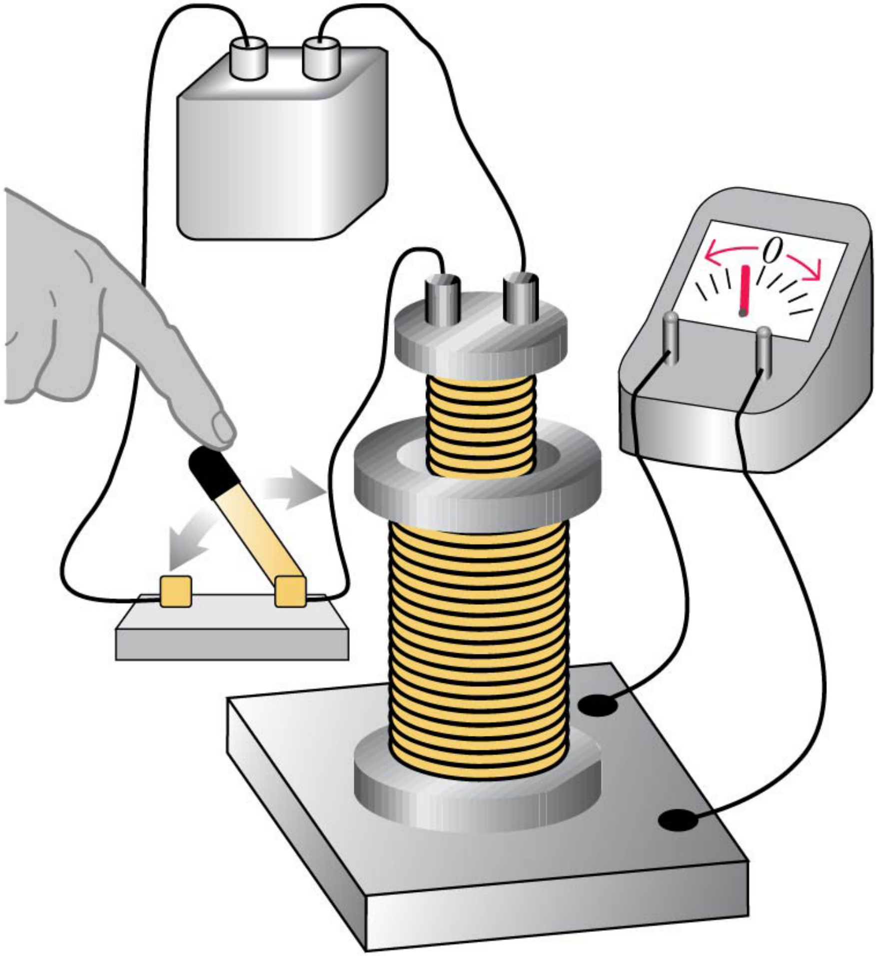
- In this figure we replace the magnet with a second coil connected to a battery.

- When we move the second coil toward or away from the first, there is current in the first coil, but only while one coil is moving relative to the other.



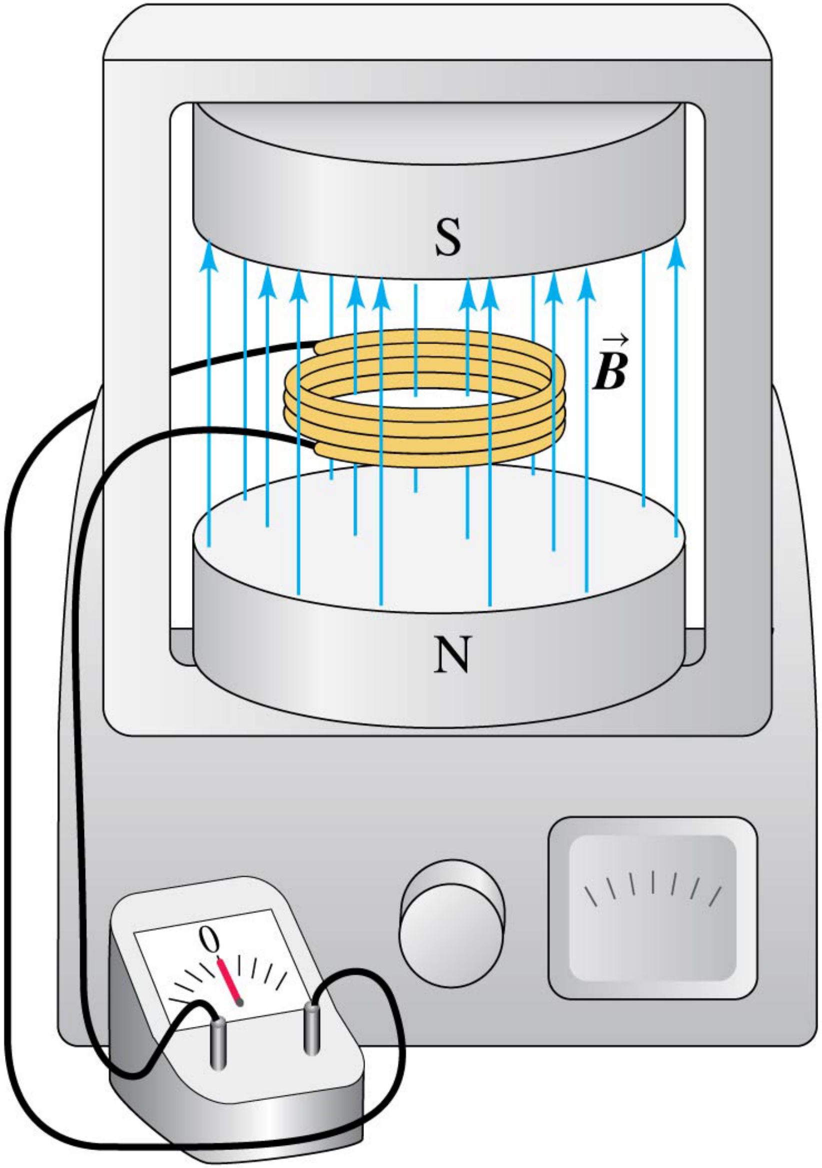
Induction experiment: Slide 4 of 4

- Using the two-coil setup of the previous slide, we keep both coils stationary and vary the current in the second coil by opening and closing the switch.
- The induced current in the first coil is present only while the current in the second coil is *changing*.



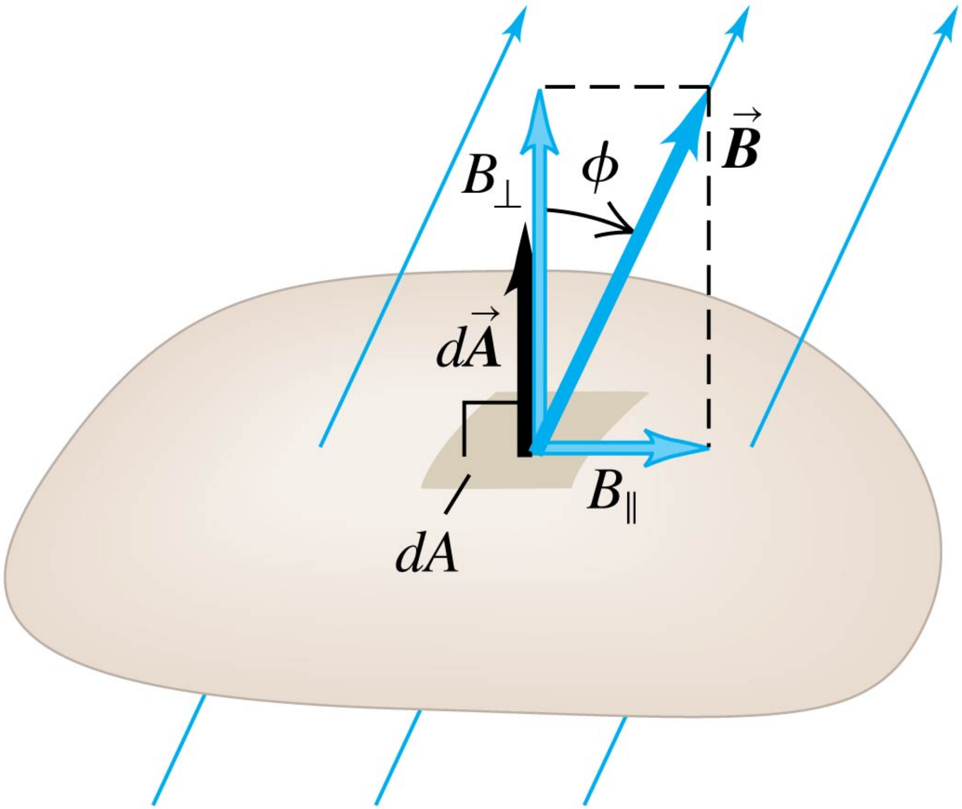
A coil in a magnetic field

- Shown is a coil in a magnetic field.
- When the magnetic field is constant and the shape, location, and orientation of the coil do not change, no current is induced in the coil.
- A current is induced when any of these factors *change*.



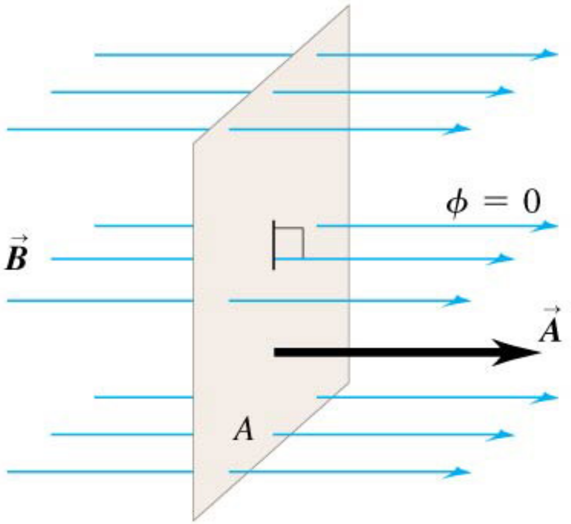
Magnetic flux (Review of Section 27.3)

- To define the *magnetic flux*, we can divide any surface into elements of area dA.
- The magnetic flux through the area element is defined to be $d\Phi_B = B_\perp dA$.



Magnetic flux through a flat area: Orientation 1 of 3

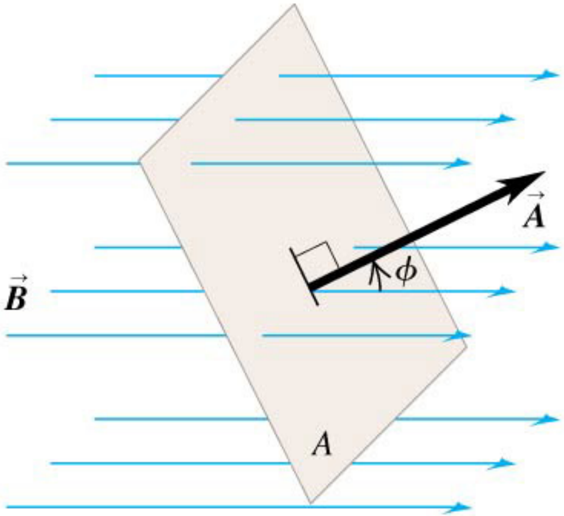
- The maximum magnetic flux through a surface occurs when the surface is face-on to the magnetic field.
- In this case the magnetic flux is simply *BA*.



Surface is face-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.

Magnetic flux through a flat area: Orientation 2 of 3

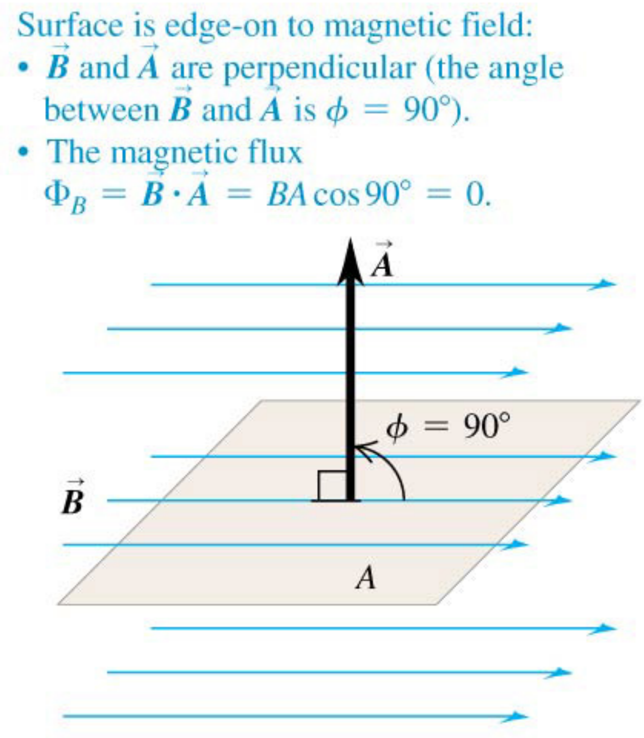
- When the surface is at some angle relative to the magnetic field, the magnetic flux is between 0 and *BA*.



Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{B}$ and $\vec{A}$ is ϕ.
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$.

Magnetic flux through a flat area: Orientation 3 of 3

- When the surface is edge-on to the magnetic field, the magnetic flux through the surface is zero.



Surface is edge-on to magnetic field:

- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 90°$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos 90° = 0$.

Faraday's law of induction

- When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop:

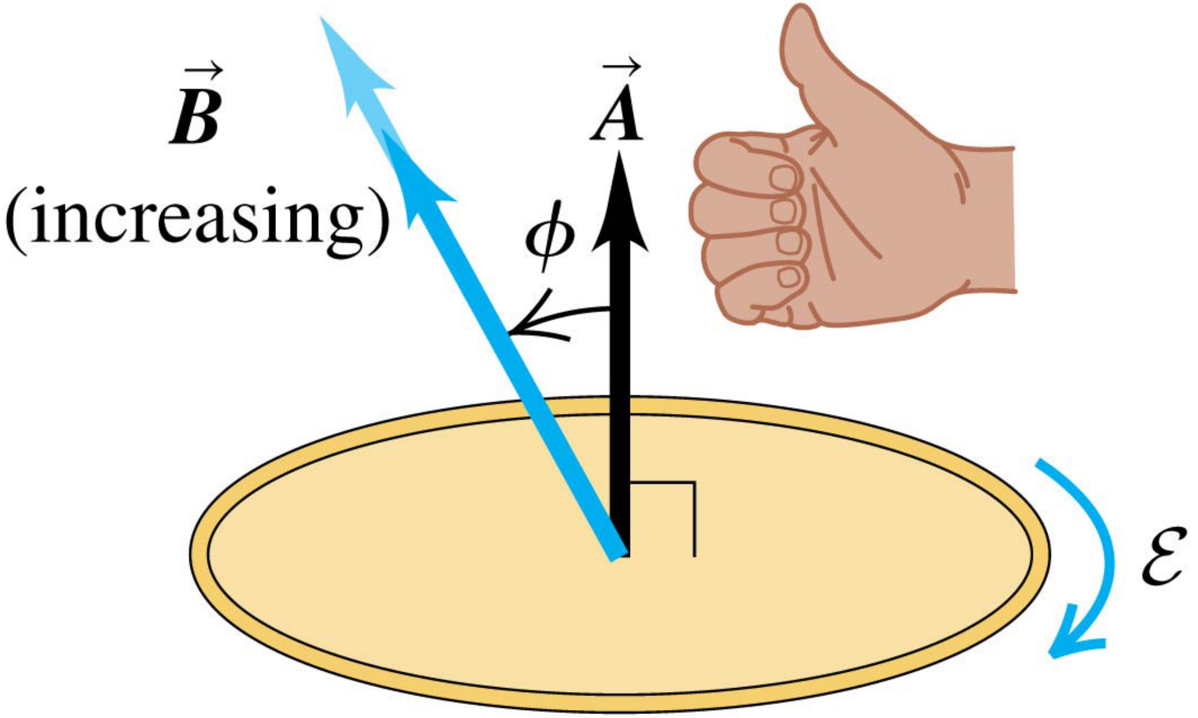
Faraday's law:
The induced emf in a closed loop ...

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

... equals the negative of the time rate of change of magnetic flux through the loop.

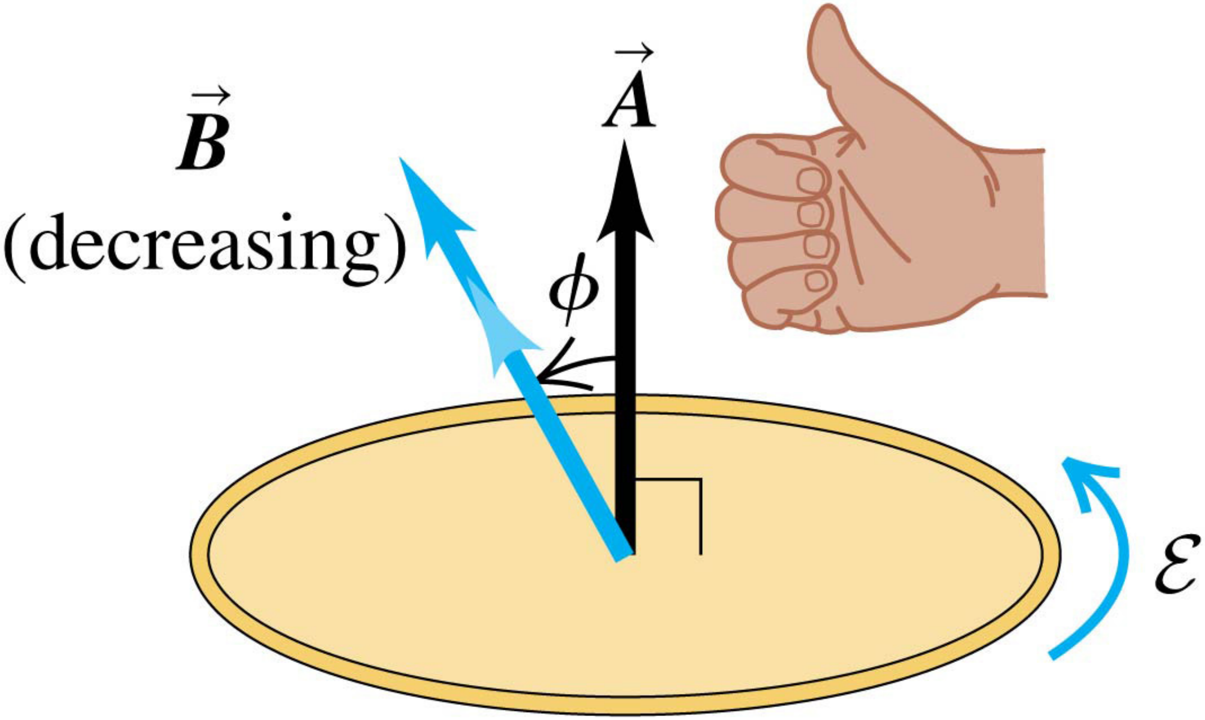
- Recall that the unit of magnetic flux is the weber (Wb).
- $1\ \mathrm{T \cdot m^2} = 1\ \mathrm{Wb}$, so $1\ \mathrm{V} = 1\ \mathrm{Wb/s}$.

Determining the direction of the induced emf: Slide 1 of 4



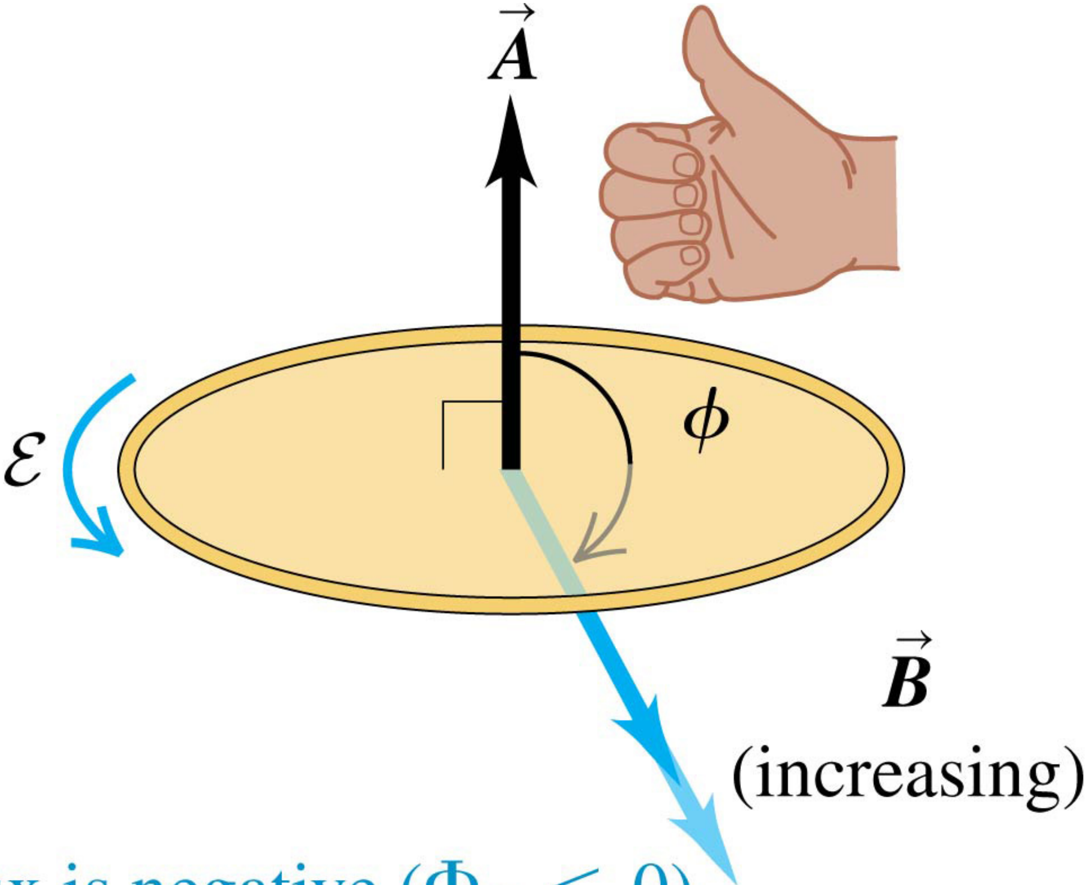
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Determining the direction of the induced emf: Slide 2 of 4



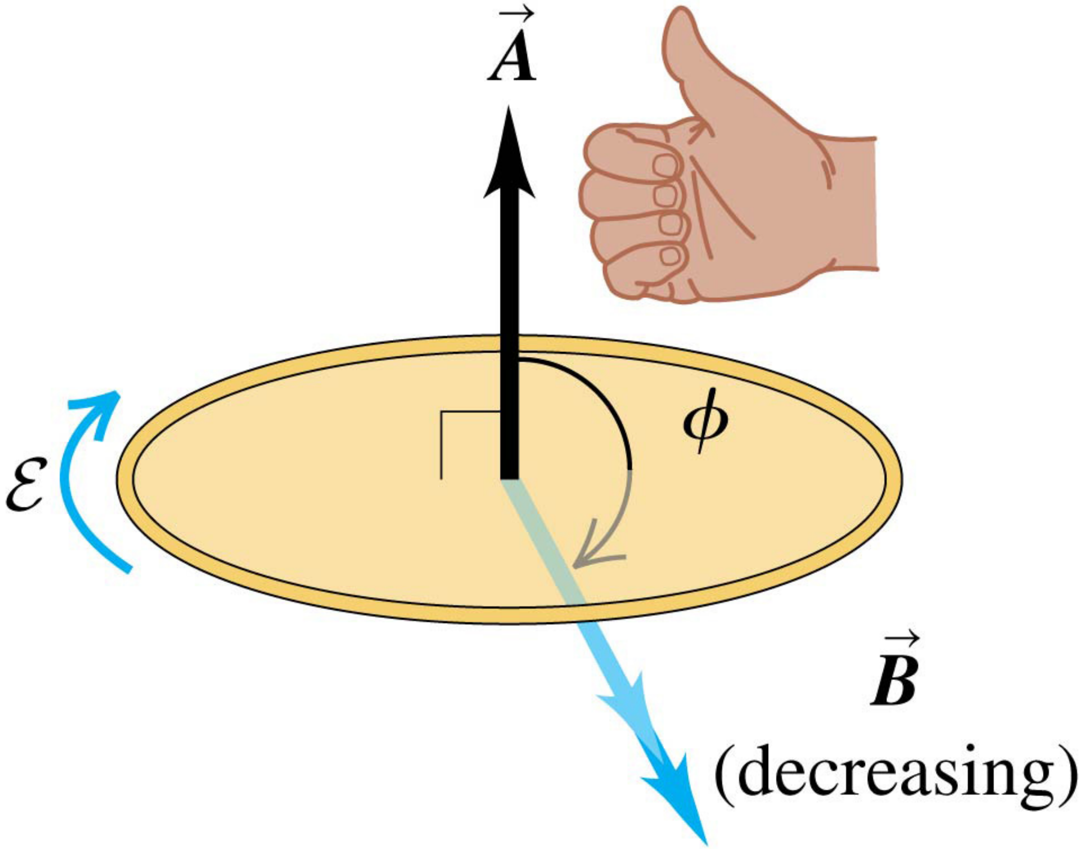
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

Determining the direction of the induced emf: Slide 3 of 4



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

Determining the direction of the induced emf: Slide 4 of 4



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Faraday’s law for a coil

- A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature.
- The resulting induced emf is far larger than would be possible with a single loop of wire.
- If a coil has N identical turns and if the flux varies at the same rate through each turn, total emf is:

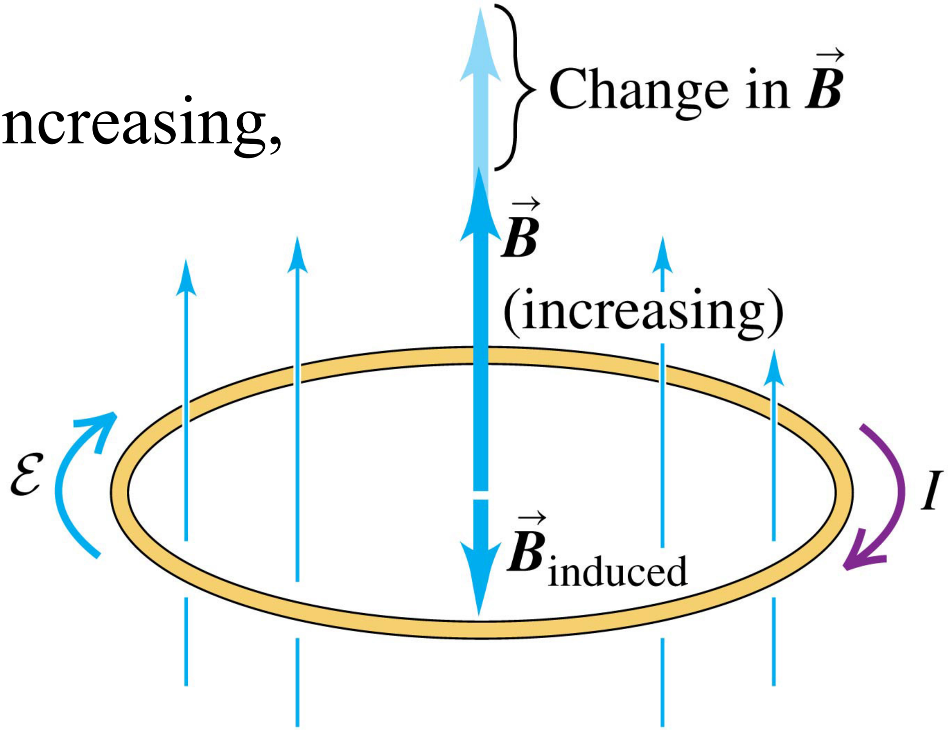
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Lenz's law

- Lenz's law is a convenient method for determining the direction of an induced current or emf:

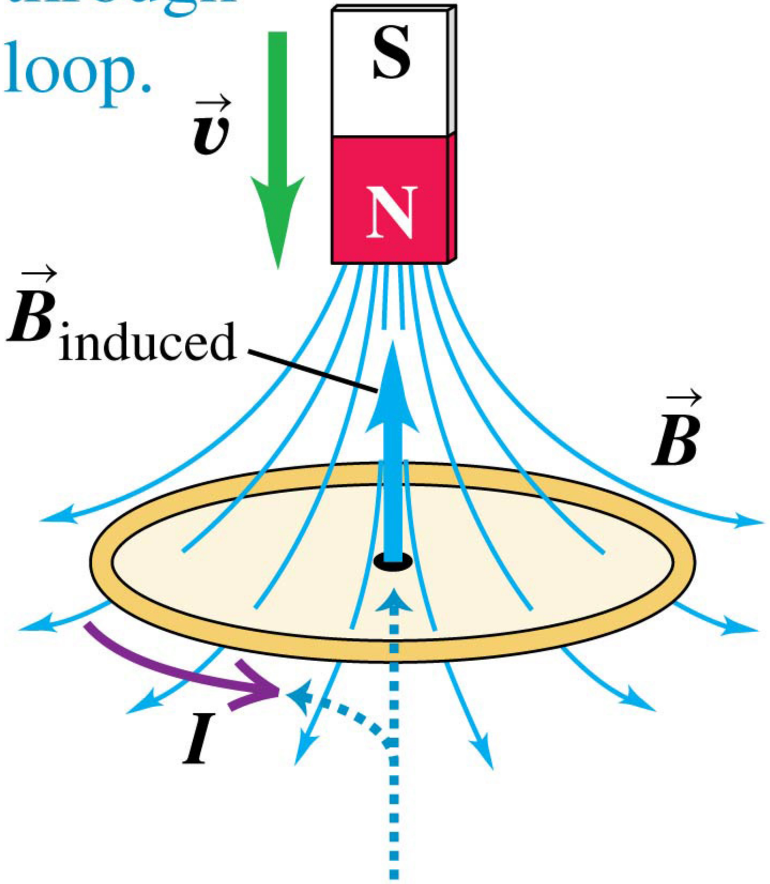
> **The direction of any magnetic induction effect is such as to oppose the cause of the effect.**

- For example, in the figure there is a uniform magnetic field through the coil.
- The magnitude of the field is increasing, so there is an induced emf driving a current, as shown.

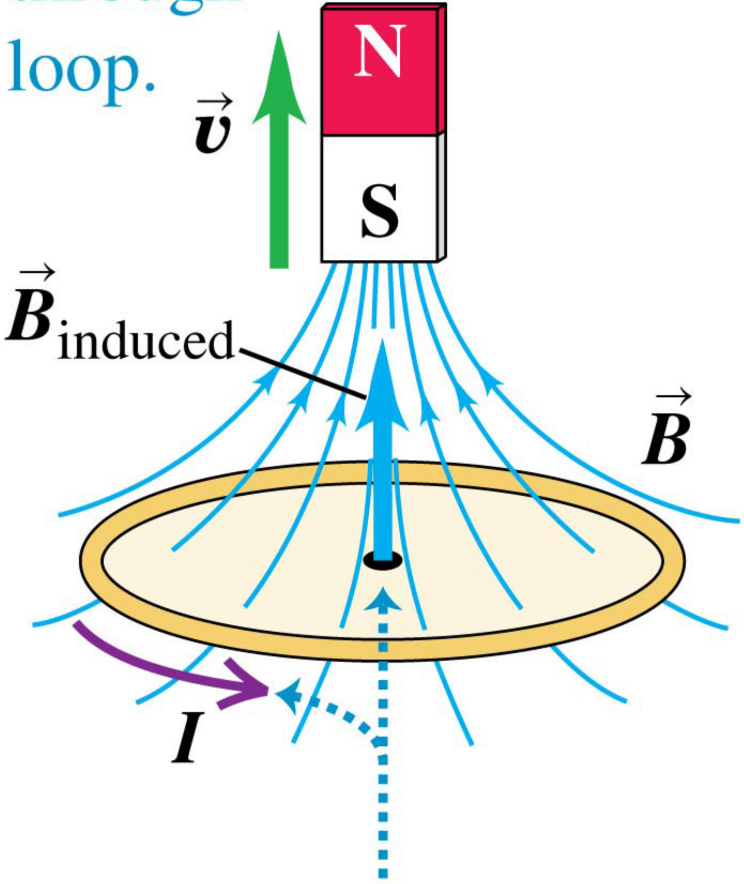


Lenz's law and the direction of induced current

Motion of magnet causes *increasing downward* flux through loop.



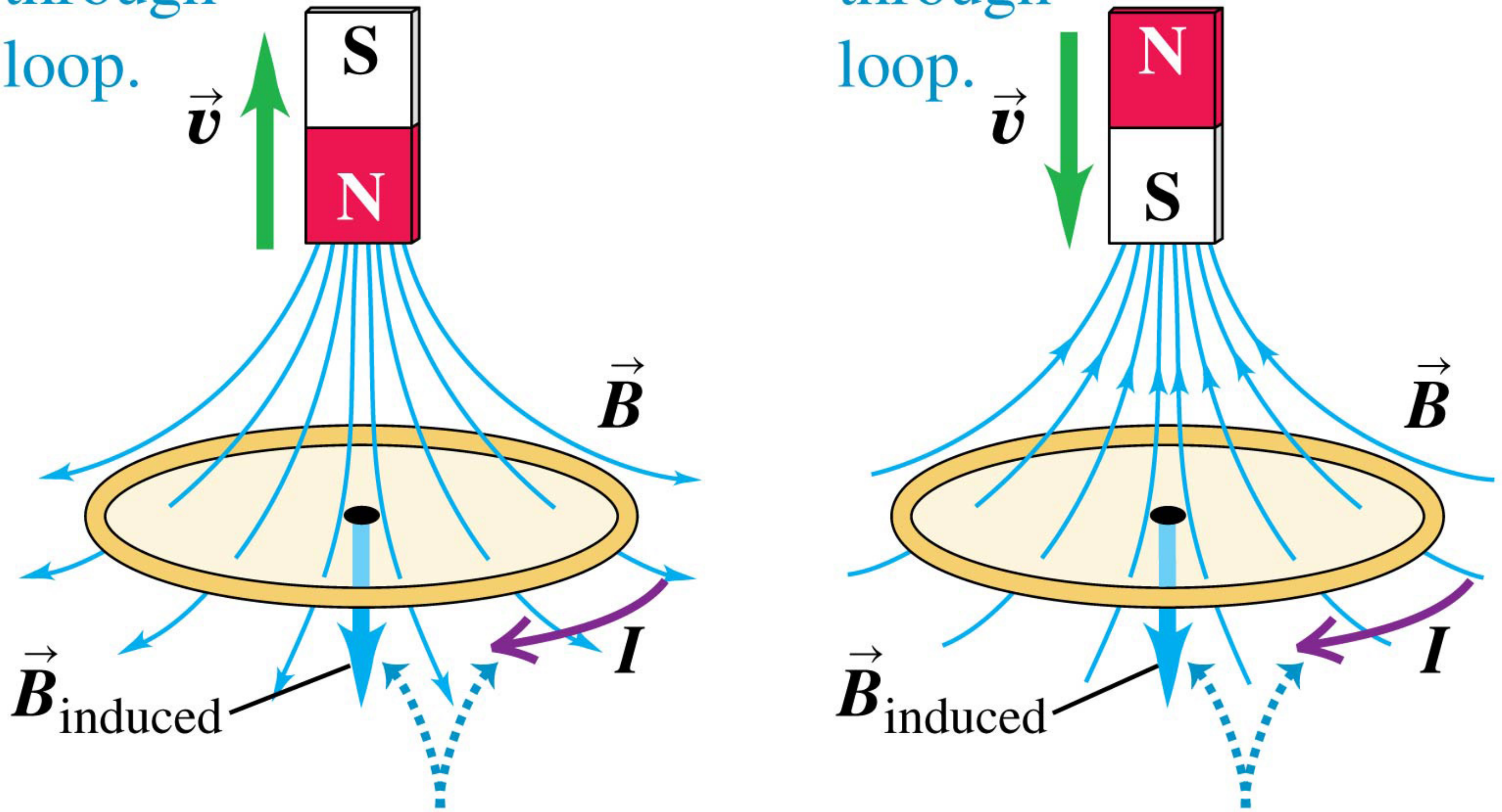
Motion of magnet causes *decreasing upward* flux through loop.



Lenz’s law and the direction of induced current

Motion of magnet causes *decreasing downward* flux through loop.

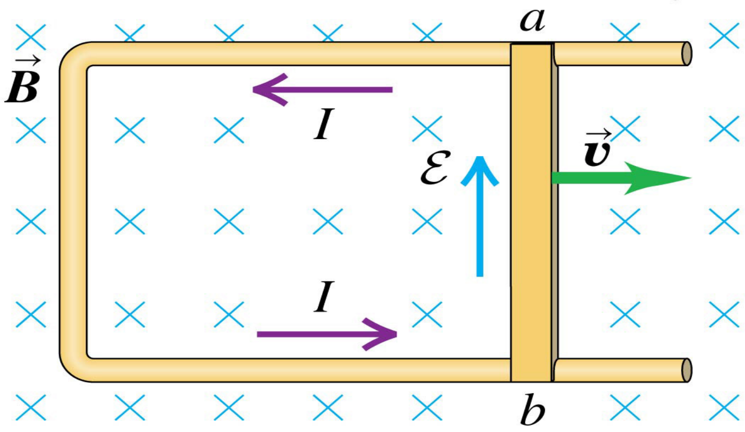
Motion of magnet causes *increasing upward* flux through loop.



Motional electromotive force

- When a conducting rod moves perpendicular to a uniform magnetic field, there is a **motional emf** induced.
- A moving charge feels $\vec{F}_m = q\vec{v}\times\vec{B}$,
  $W = F_m L, \mathcal{E} = \frac{W}{q} = vBL$
  $\vec{F}_m$ never does work! Cheating!

Rod connected to stationary conductor



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

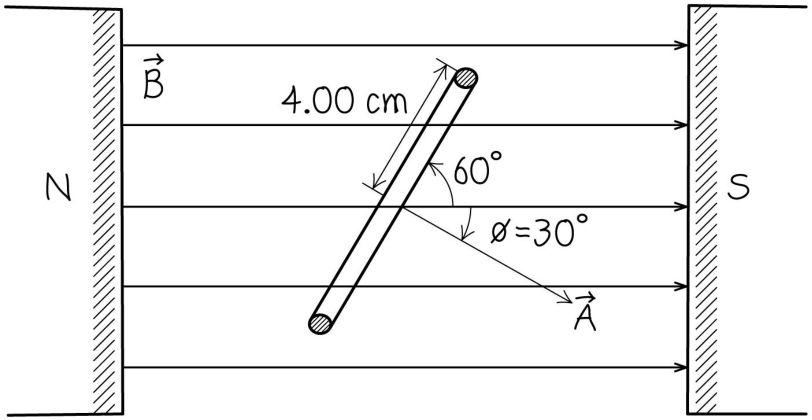
- But… $\vec{F}_m$ is not the only force on q, it rearranges the charges, so that there is also non-zero electrostatic force. Macroscopically, the work is done by the external force acting on the bar!

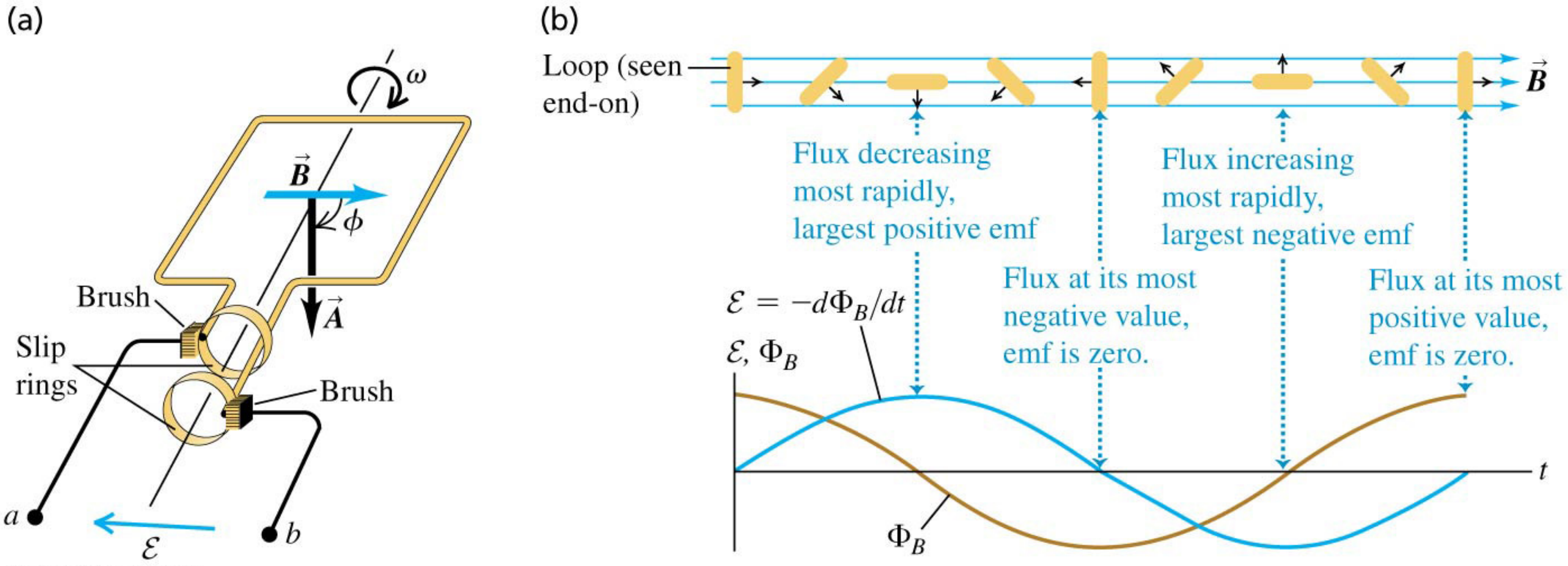
Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$ → $\mathcal{E} = vBL$

Conductor speed → v
Conductor length → L
Magnitude of uniform magnetic field → B

Rotating frame – electric generator

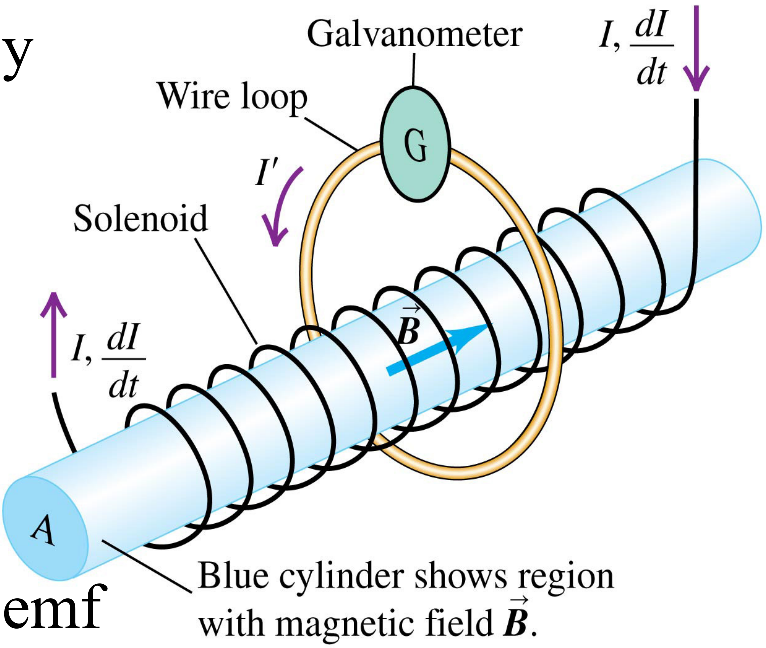
- $\Phi_B(t) = BA\cos\big(\phi(t)\big), \phi(t) = \omega t$
- $\dfrac{d\Phi_B(t)}{dt} = -BA\dfrac{d\cos(\omega t)}{dt} = BA\omega\sin(\omega t)$





Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current *I* changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:



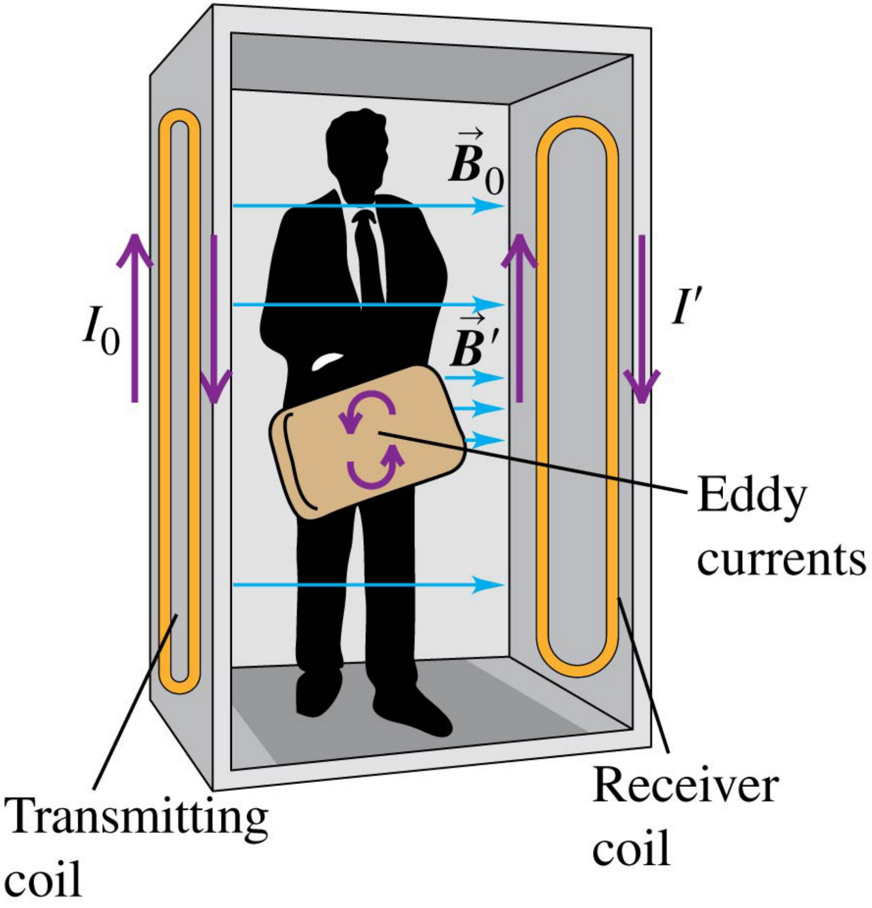
Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E} \cdot d\vec{l}}^{\text{Line integral of electric field around path}} = -\underbrace{\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

Eddy currents

- When a piece of metal moves through a magnetic field or is located in a changing magnetic field, **eddy currents** of electric current are induced.

- The metal detectors used at airport security checkpoints operate by detecting eddy currents induced in metallic objects.



Paramagnetism and diamagnetism

- $\vec{M} = \frac{\vec{\mu}}{V}, \vec{B} = \overrightarrow{B_0} + \mu_0\vec{M} = K_m\,\overrightarrow{B_0},$ where $\overrightarrow{B_0}$ is the external magnetic field. And we assume that $M \sim B_0$
- Indeed, if we have pre-existing magnetic dipoles, they orient along the magnetic field lines. This is called a **paramagnetic** material, the result is that the magnetic field at any point is *greater* than B_0 by a dimensionless factor K_m, called the relative permeability.
- Unlike dielectrics, we have also the opposite case. If an external magnetic field permeates a **diamagnetic** material, the result is a magnetic field that is slightly *less* than B_0 . $K_m<1$. In this case, we don't have pre-existing magnetic dipoles. They are *induced* and are opposite to B_0 , *because of Faraday's Law.*
- The amount by which the relative permeability differs from unity is called the magnetic susceptibility, denoted by χ_m:

$$\chi_m = K_m - 1$$

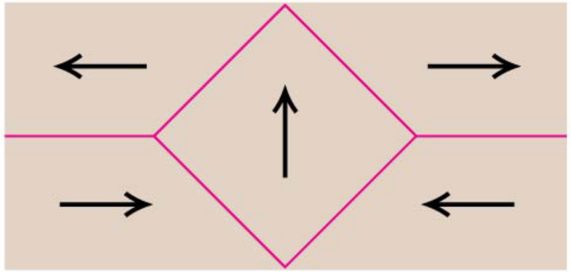
Magnetic susceptibilities of certain materials

	Material	χ_m ($\times 10^{-5}$)
Paramagnetic	Iron ammonium alum	66
	Aluminum	2.2
	Oxygen gas	0.19
Diamagnetic	Bismuth	−16.6
	Silver	−2.6
	Carbon (diamond)	−2.1
	Copper	−1.0

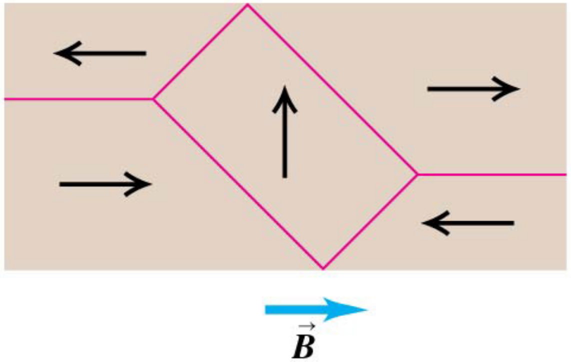
Ferromagnetism

- In **ferromagnetic materials** (such as iron), atomic magnetic moments tend to line up parallel to each other in regions called magnetic domains.
- When there is no externally applied field, the domain magnetizations are randomly oriented.
- When an external magnetic field is present, the domain boundaries shift; the domains that are magnetized in the field direction grow, and those that are magnetized in other directions shrink.

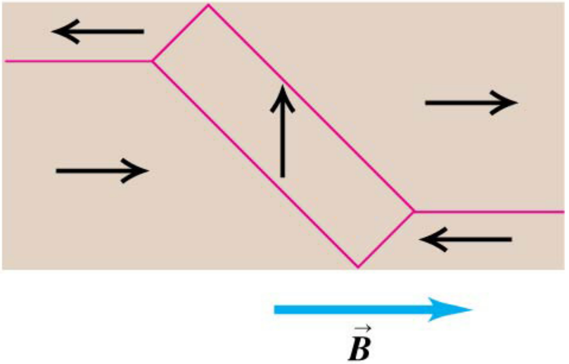
(a) No field

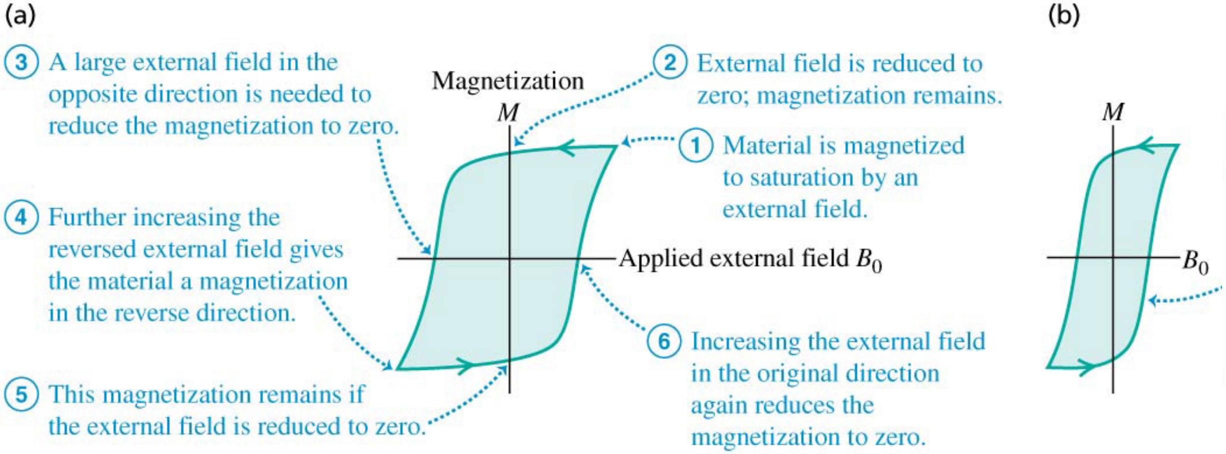


(b) Weak field



(c) Stronger field





③ A large external field in the opposite direction is needed to reduce the magnetization to zero.

④ Further increasing the reversed external field gives the material a magnetization in the reverse direction.

⑤ This magnetization remains if the external field is reduced to zero.

② External field is reduced to zero; magnetization remains.

① Material is magnetized to saturation by an external field.

⑥ Increasing the external field in the original direction again reduces the magnetization to zero.