Lecture 18

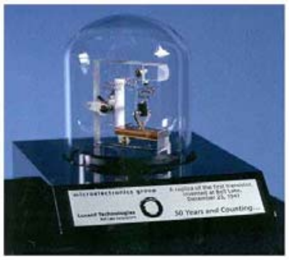
Bipolar Junction Transistors (BJTs)

In this lecture you will learn:

- The operation of bipolar junction transistors
- Forward and reverse active operations, saturation, cutoff
- Ebers-Moll model
- Small signal models

Bipolar Transistors

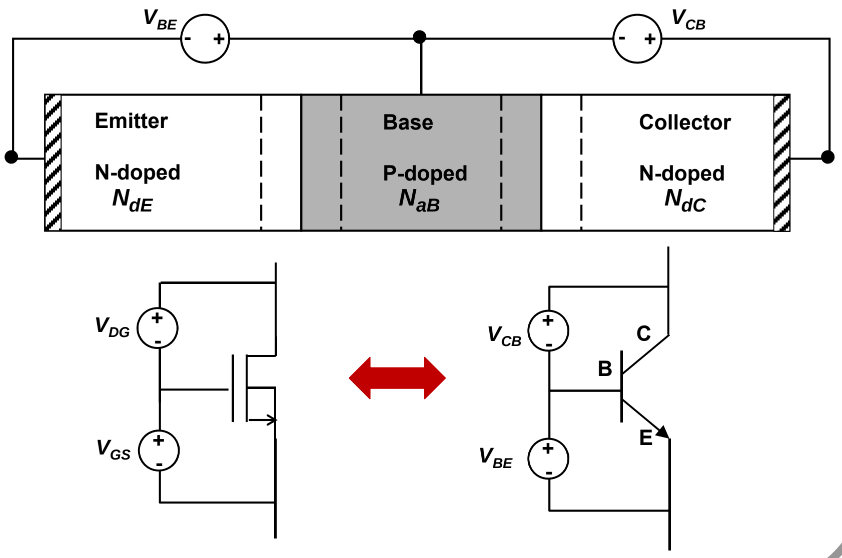
First Bipolar Transistor (AT&T Bell Labs)



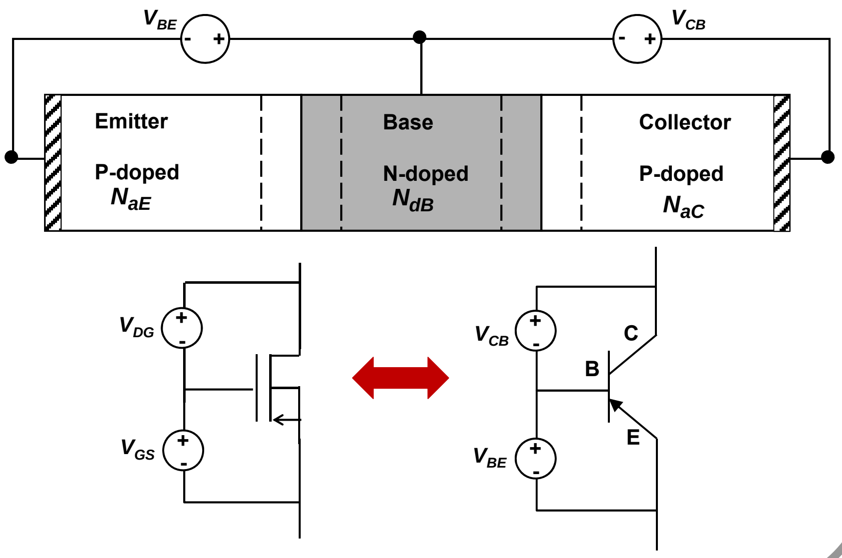
First Bipolar Transistor (AT&T Bell Labs)

William Shockley, John Bardeen, Walter Brattain
(Nobel Prize for the Transistor, AT&T Bell Labs)

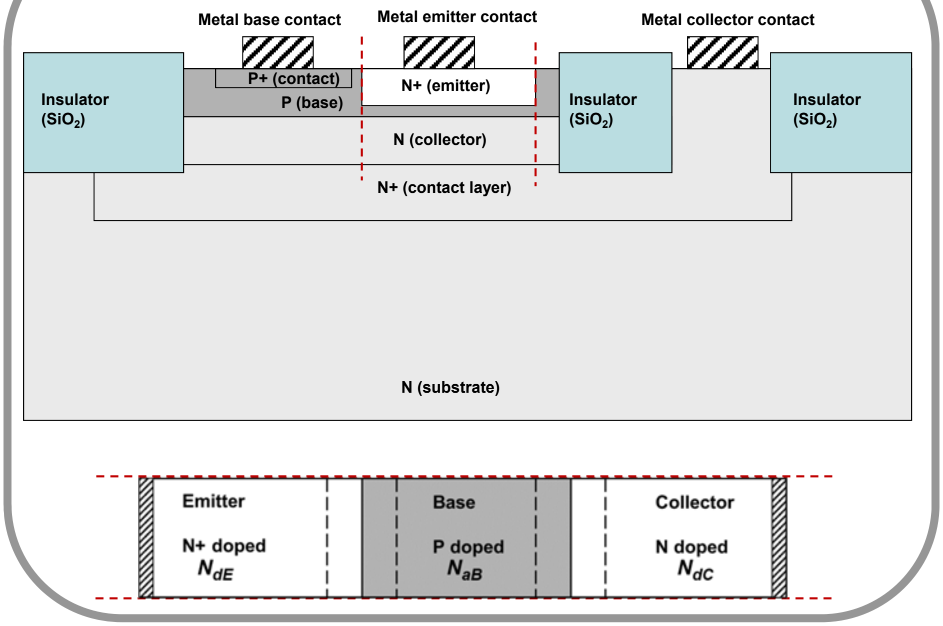
NPN Bipolar Junction Transistor



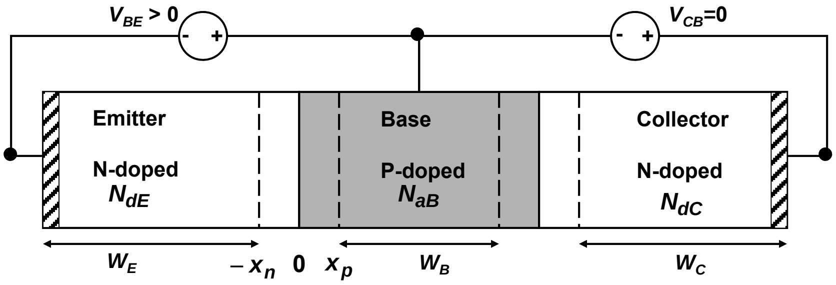
PNP Bipolar Junction Transistor



A Silicon NPN BJT



NPN BJT: Basic Operation



Suppose:

The base-emitter junction is forward biased

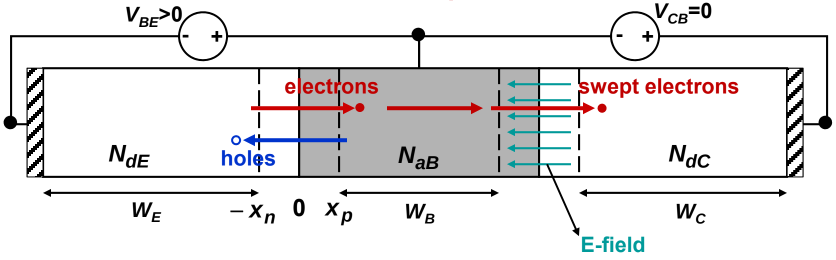
$$V_{BE} > 0$$

The base-collector junction is zero biased

$$V_{CB} = 0$$

This biasing scheme will put the device in the "forward active" operation (to be discussed fully later)

NPN BJT: Basic Operation



Consider the action in the base first ($V_{BE} > 0$ and $V_{CB} = 0$)

- The electrons diffuse from the emitter, cross the depletion region, and enter the base
- In the base, the electrons are the minority carriers
- In the base, the electrons diffuse towards the collector
- As soon as the electrons reach the base-collector depletion region they are immediately swept away into the collector by the strong electric fields in the depletion region

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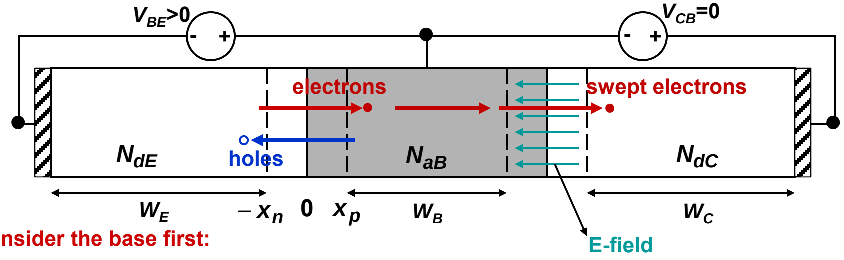
NPN BJT: Electron-Hole Populations



Consider the base first:

In the base, the electron population can be written as:

$$\left.\underbrace{n(x) = n_{po}}_{\text{Equilibrium electron density}} + \underbrace{n'(x)}_{\text{Excess electron density}}\right\} \quad n_{po} = \frac{n_i^2}{N_{aB}}$$

In the base, the excess electron population satisfies the differential equation:

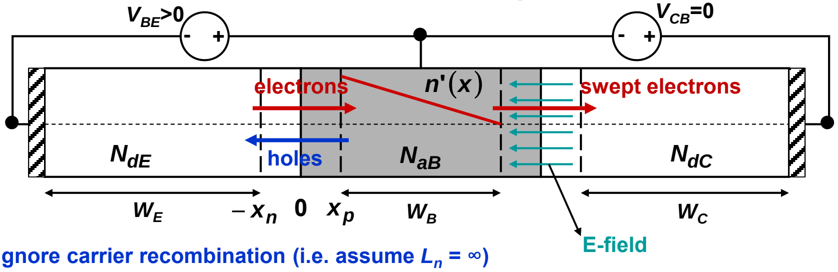
$$\left.\frac{\partial^2 n'(x)}{\partial x^2} - \frac{n'(x)}{L_n^2} = 0\right\}$$

Boundary conditions

$$n'(x_p) = \frac{n_i^2}{N_{aB}}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$n'(x_p + W_B) = \frac{n_i^2}{N_{aB}}\left(e^{\frac{qV_{BC}}{KT}} - 1\right) = 0$$

NPN BJT: Electron-Hole Populations



- Ignore carrier recombination (i.e. assume $L_n = \infty$)

$$\frac{\partial^2 n'(x)}{\partial x^2}=0 \quad \Big\}$$

Boundary conditions

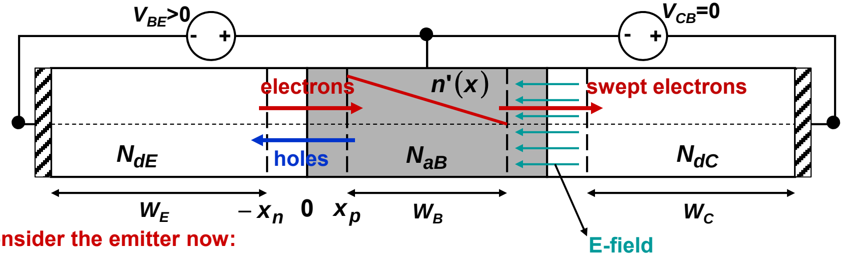
$$n'(x_p)=\frac{n_i^2}{N_{aB}}\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

$$n'(x_p+W_B)=\frac{n_i^2}{N_{aB}}\left(e^{\frac{qV_{BC}}{KT}}-1\right)=0$$

Solution is:

$$n'(x)=n'(x_n)\left(1-\frac{x-x_p}{W_B}\right)=\frac{n_i^2}{N_{aB}}\left(e^{\frac{qV_{BE}}{KT}}-1\right)\left(1-\frac{x-x_p}{W_B}\right)$$

NPN BJT: Electron-Hole Populations



Consider the emitter now:

In the emitter, the hole population can be written as:

$$p(x)=p_{no}+p'(x)$$

Equilibrium hole density

Excess hole density

$$\Big\} \quad p_{no}=\frac{n_i^2}{N_{dE}}$$

In the emitter, the excess hole population satisfies the differential equation:

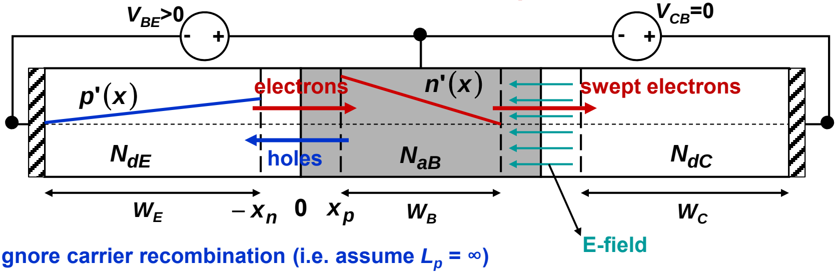
$$\frac{\partial^2 p'(x)}{\partial x^2}-\frac{p'(x)}{L_p^2}=0 \quad \Big\}$$

Boundary conditions

$$p'(-x_n)=\frac{n_i^2}{N_{dE}}\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

$$p'(-x_n-W_E)=0$$

NPN BJT: Electron-Hole Populations



- **Ignore carrier recombination (i.e. assume $L_p = \infty$)**

$$\frac{\partial^2 p'(x)}{\partial x^2} = 0 \quad \Big\}$$

Boundary conditions

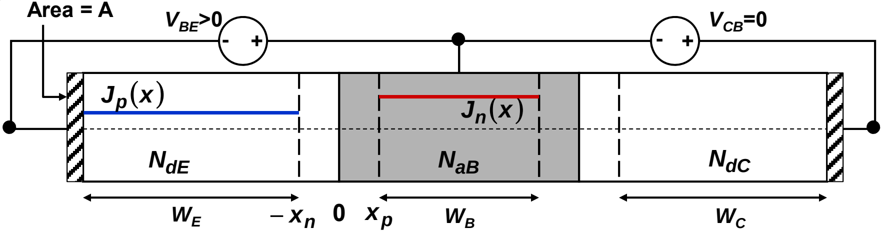
$$p'(-x_n) = \frac{n_i^2}{N_{dE}}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$p'(-x_n - W_E) = 0$$

Solution is:

$$p'(x) = p'(-x_n)\left(1 + \frac{x + x_n}{W_E}\right) = \frac{n_i^2}{N_{dE}}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)\left(1 + \frac{x + x_n}{W_E}\right)$$

NPN BJT: Electron and Hole Current Densities



In the base:

- **The electron current is:**

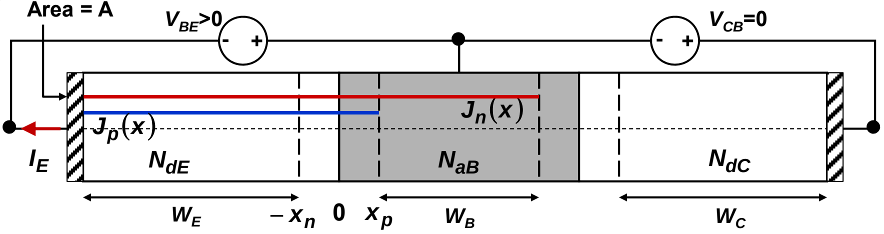
$$J_n(x) \approx q\, D_n \frac{\partial n(x)}{\partial x} = qn_i^2 \frac{D_n}{N_{aB}W_B}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

In the emitter:

- **The hole current is:**

$$J_p(x) \approx -q\, D_p \frac{\partial p(x)}{\partial x} = qn_i^2 \frac{D_p}{N_{dE}W_E}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

NPN BJT: Terminal Currents

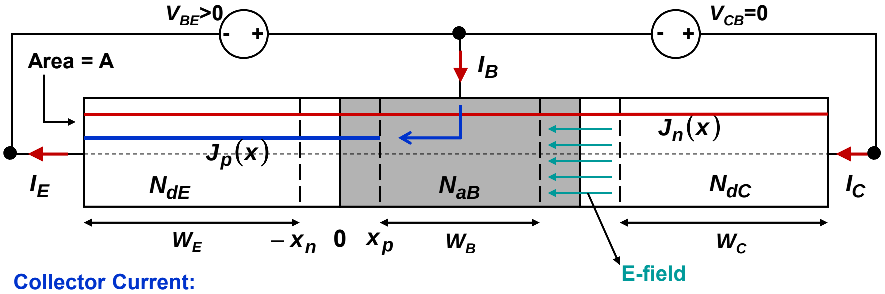


Emitter current:

- The current flowing out of the emitter is the sum of the total electron and total hole currents in the emitter:

$$I_E = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E} + \frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

NPN BJT: Terminal Currents



Collector Current:

- The current going into the collector is due to the electrons that got swept from the Base through the Base-Collector depletion region by the electric-fields:

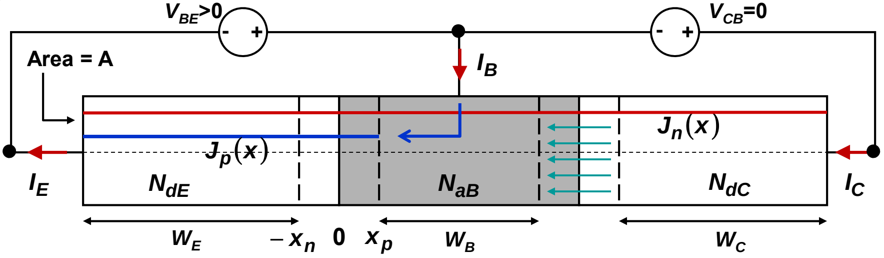
$$I_C = qn_i^2 A\left(\frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

Base Current:

- The current going into the Base is due to the holes that got injected from the base into the emitter:

$$I_B = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E}\right)\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

NPN BJT: Terminal Currents

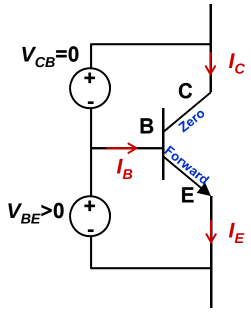


$$I_E = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E}+\frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

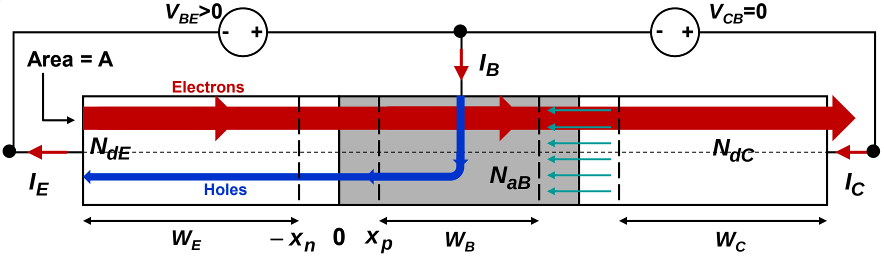
$$I_C = qn_i^2 A\left(\frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

$$I_B = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E}\right)\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

$$I_E = I_B + I_C$$



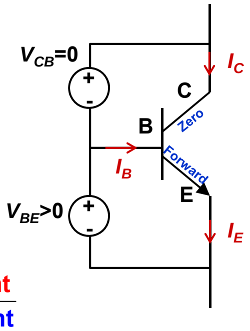
NPN BJT: Terminal Currents



$$I_E = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E}+\frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

$$I_C = qn_i^2 A\left(\frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$

$$I_B = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E}\right)\left(e^{\frac{qV_{BE}}{KT}}-1\right)$$



$$I_E = I_B + I_C \qquad \beta_F = \frac{I_C}{I_B} = \frac{\text{Red current}}{\text{Blue current}}$$

NPN BJT: Circuit Level Parameters

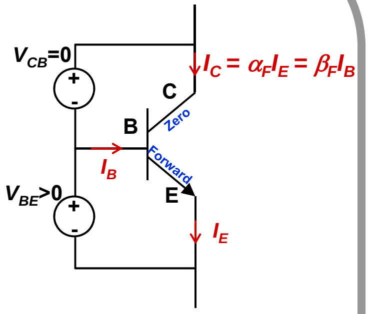
Current gain β_F :

Current gain of the BJT in the forward active operation is defined as the ratio of the collector and base currents:

$$\beta_F = \frac{I_C}{I_B} = \frac{D_n}{N_{aB}W_B}\frac{N_{dE}W_E}{D_p} \Rightarrow I_C = \beta_F I_B$$

Typical values of β_F are between 20-200 and:

$$N_{dE} >> N_{aB} > N_{dC}$$



$$I_E = I_B + I_C$$

α_F :

In the forward active operation α_F is defined as the ratio of the collector and emitter currents:

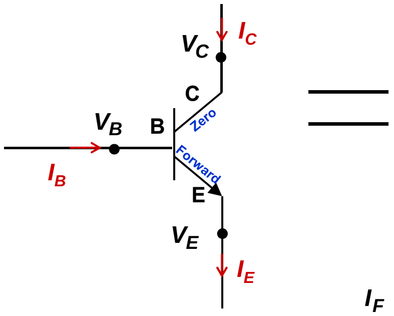
$$\alpha_F = \frac{I_C}{I_E} = \frac{\frac{D_n}{N_{aB}W_B}}{\frac{D_p}{N_{dE}W_E} + \frac{D_n}{N_{aB}W_B}} \Rightarrow I_C = \alpha_F I_E$$

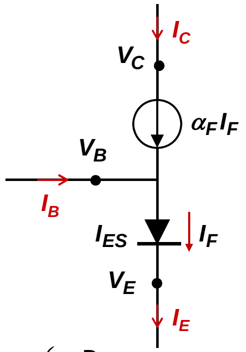
Transistor relation:

α_F and β_F are related: $$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

NPN BJT: Ebers-Moll Model for Forward Active Operation

Suppose: $\begin{cases} V_{BE} > 0 \\ V_{CB} = 0 \end{cases}$





$$I_F = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E} + \frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$= I_{ES}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

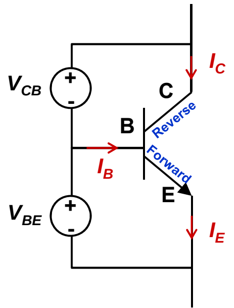
$$I_B = (1-\alpha_F)I_F$$
$$I_C = \alpha_F I_F$$
$$I_E = I_F$$

The circuit level simplified model with an ideal diode and a current-controlled current source models the NPN transistor in the forward active operation

NPN BJT: Forward Active Operation

$V_{BE} > 0$

$V_{CB} \geq 0$



Forward active operation

$$\beta_F = \frac{I_C}{I_B}$$

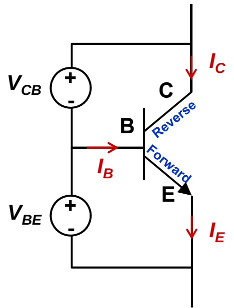
$$\alpha_F = \frac{I_C}{I_E}$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

NPN BJT: Forward and Reverse Active Operations

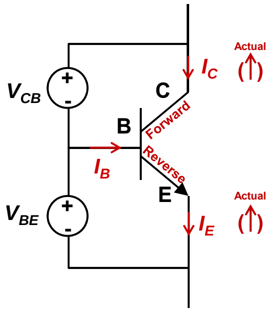
$V_{BE} > 0$

$V_{CB} \geq 0$



$V_{CB} < 0$

$V_{BE} \leq 0$



Forward active operation

$$\beta_F = \frac{I_C}{I_B}$$

$$\alpha_F = \frac{I_C}{I_E}$$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$

Reverse active operation

$$\beta_R = -\frac{I_E}{I_B} = \frac{D_n}{N_{aB}W_B}\frac{N_{dC}W_C}{D_p}$$

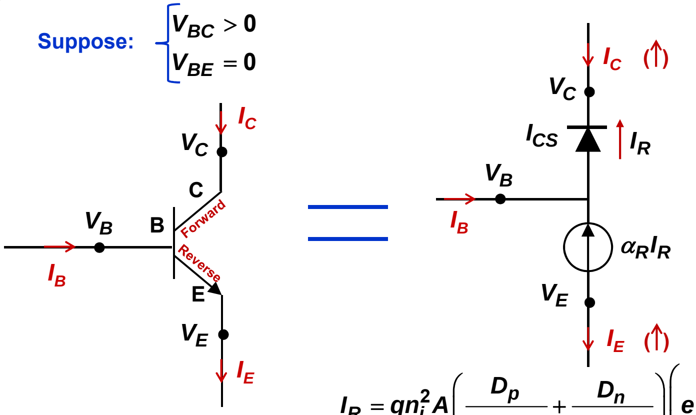
$$\alpha_R = \frac{I_E}{I_C}$$

$$\beta_R = \frac{\alpha_R}{1-\alpha_R}$$

In a well designed transistor: $\beta_F >> \beta_R$

NPN BJT: Ebers-Moll Model for Reverse Active Operation

Suppose: $\begin{cases} V_{BC} > 0 \\ V_{BE} = 0 \end{cases}$



$$I_R = qn_i^2 A\left(\frac{D_p}{N_{dC}W_C} + \frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BC}}{KT}} - 1\right)$$

$$= I_{CS}\left(e^{\frac{qV_{BC}}{KT}} - 1\right)$$

$$\boxed{\begin{aligned} I_B &= (1-\alpha_R)I_R \\ I_C &= -I_R \\ I_E &= -\alpha_R I_R \end{aligned}}$$

The circuit level simplified model with an ideal diode and a current-controlled current source models the NPN transistor in the reverse active operation

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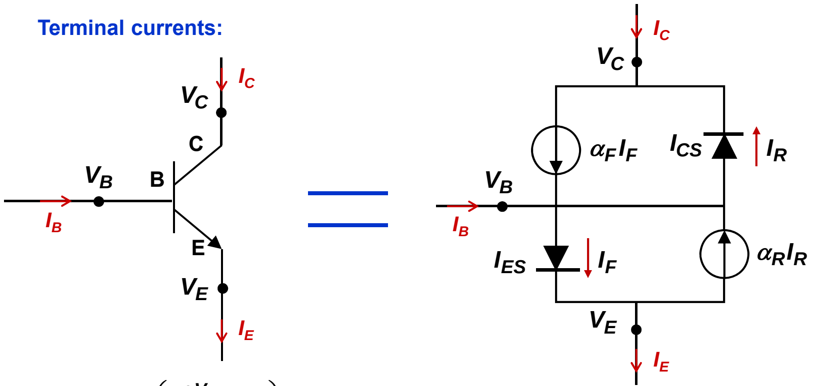
NPN BJT: Ebers-Moll Model and Terminal Currents

Terminal currents:



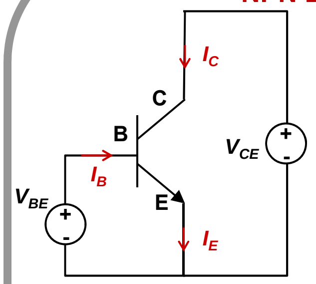
$$I_R = I_{CS}\left(e^{\frac{qV_{BC}}{KT}} - 1\right)$$

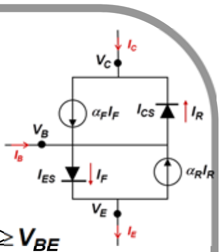
$$I_F = I_{ES}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

And

$$\boxed{\begin{aligned} I_B &= (1-\alpha_F)I_F + (1-\alpha_R)I_R \\ I_C &= \alpha_F I_F - I_R \\ I_E &= I_F - \alpha_R I_R \end{aligned}}$$

NPN BJT: Regimes of Operation - I





In forward active operation:

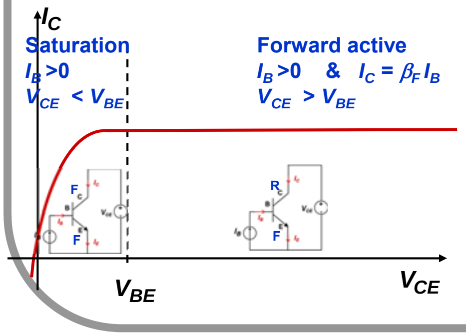
$$I_B > 0 \quad V_{BE} > 0 \quad V_{CB} \geq 0$$

Since: $V_{CE} = V_{CB} + V_{BE}$

$\Rightarrow$ In forward active operation: $V_{CE} \geq V_{BE}$

$$I_C = qn_i^2 A\left(\frac{D_n}{N_{aB}W_B}\right)\left(e^{\frac{qV_{BE}}{KT}} - 1\right) = \beta_F I_B$$

$\Rightarrow$ Independent of V_{CE}



Saturation
$I_B > 0$
$V_{CE} < V_{BE}$

Forward active
$I_B > 0$ & $I_C = \beta_F I_B$
$V_{CE} > V_{BE}$

Forward active:
Base-emitter junction forward biased
Base-collector junction reversed biased

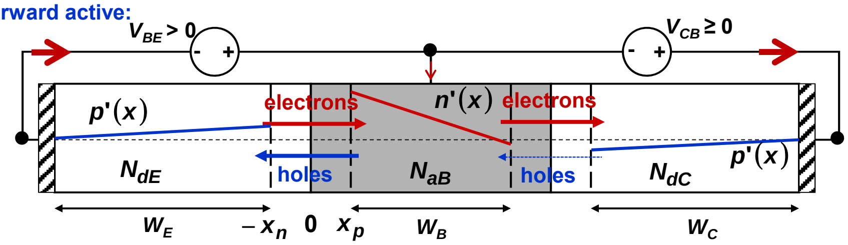
$$I_B > 0 \quad V_{BE} > 0 \quad V_{CB} \geq 0$$

Saturation:
Base-emitter junction forward biased
Base-collector junction forward biased

$$I_B > 0 \quad V_{BE} > 0 \quad V_{CB} < 0$$

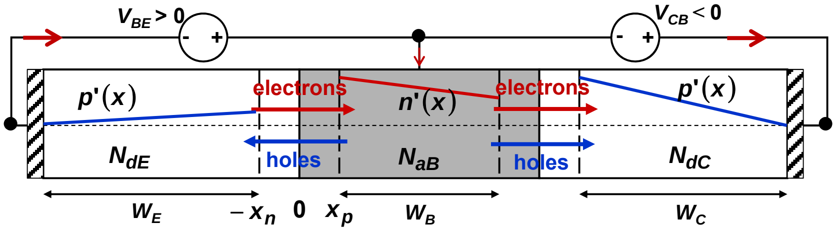
Carrier Densities in Different Regimes of Operation

Forward active:



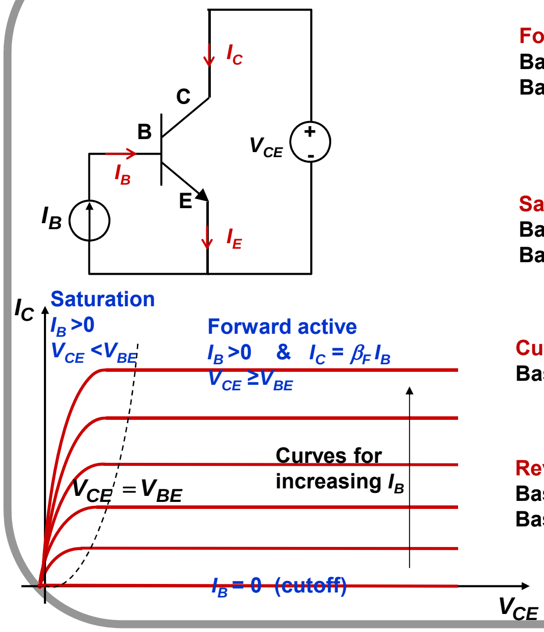
$N_{dE} >> N_{aB} > N_{dC}$

Saturation:



The forward biased base-collector junction reduces the collector current!

NPN-BJT: Regimes of Operation - II



Forward active:

Base-emitter junction forward biased

Base-collector junction reversed biased

$$I_B > 0 \quad V_{BE} > 0 \quad V_{CB} \geq 0$$

$$I_C = \beta_F I_B$$

Saturation:

Base-emitter junction forward biased

Base-collector junction forward biased

$$I_B > 0 \quad V_{BE} > 0 \quad V_{CB} < 0$$

Cutoff:

Base current zero

$$I_B = 0$$

Reverse active:

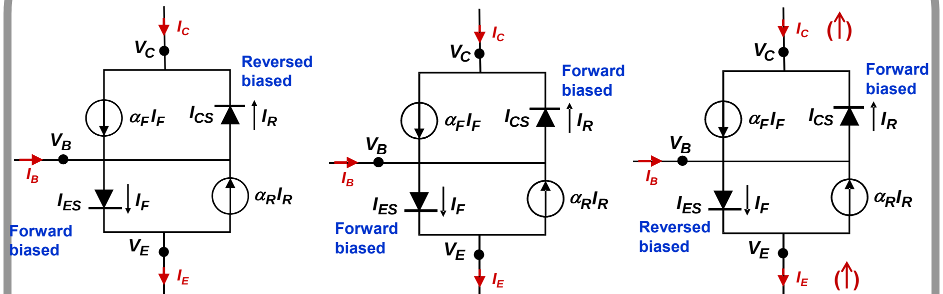
Base-emitter junction reverse biased

Base-collector junction forward biased

$$I_B > 0 \quad V_{BE} \leq 0 \quad V_{CB} < 0$$

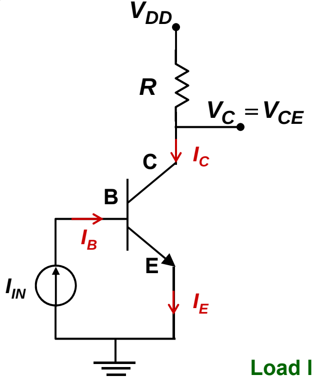
$$I_E = -\beta_R I_B$$

NPN BJT: Different Regimes of Operation



$V_{BE} > 0$	$V_{CB} < 0$	$V_{CB} < 0$
$V_{CB} \geq 0$	$V_{BE} > 0$	$V_{BE} \leq 0$
Forward Active	**Saturation**	**Reverse Active**

NPN BJT: A Simple Amplifier Circuit



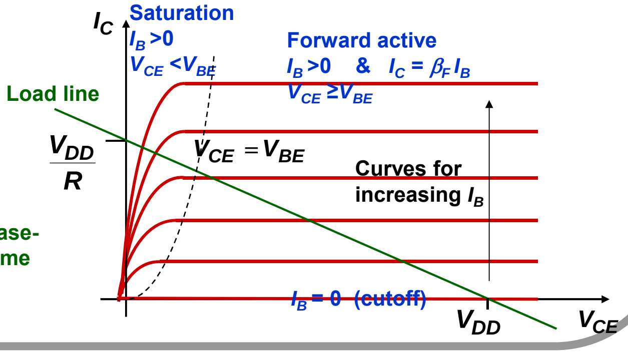
Current gain (in forward active regime):

$$\frac{I_{OUT}}{I_{IN}} = \frac{I_C}{I_B} = \beta_F$$

Load line equation:

$$V_{CE} = V_{DD} - I_C R$$

$$\Rightarrow \quad I_C = \frac{V_{DD} - V_{CE}}{R}$$



Lesson: Don't let the base-collector junction become forward biased

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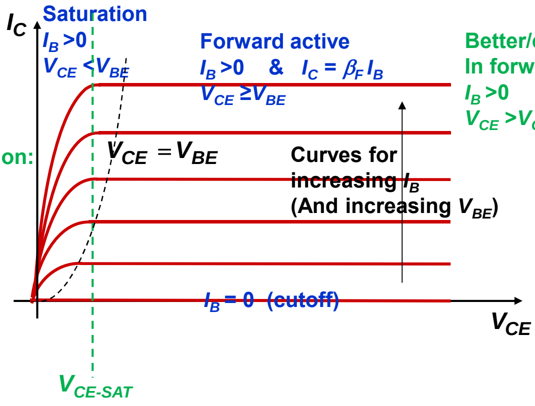
NPN BJT

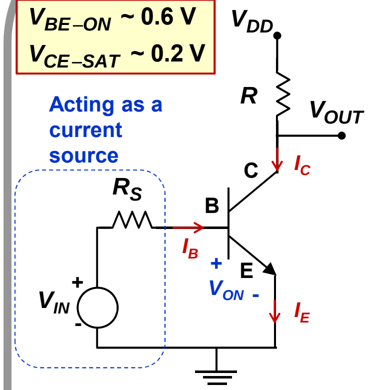
Better/easier definition:
In saturation
$I_B > 0$
$V_{CE} < V_{CE\text{-}SAT}$



Better/easier definition:
In forward active
$I_B > 0$
$V_{CE} > V_{CE\text{-}SAT}$

NPN BJT: Voltage Biasing of a Simple Amplifier Circuit

V_{BE-ON} ~ 0.6 V
V_{CE-SAT} ~ 0.2 V



Approximate analysis of transistor DC biasing:

If: $V_{IN} < V_{BE-ON} \Rightarrow I_B \approx 0 \Rightarrow$ Transistor in cut-off

If: $V_{IN} \ge V_{BE-ON} \Rightarrow$

$$V_{IN} = I_B R_S + V_{BE-ON}$$

$$I_B = \frac{V_{IN} - V_{BE-ON}}{R_S}$$

Assume forward active operation (V_{CE} > $V_{CE\text{-}SAT}$):

$$I_C = \beta_F I_B$$

$$V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta_F \left(\frac{V_{IN} - V_{BE-ON}}{R_s} \right) R$$

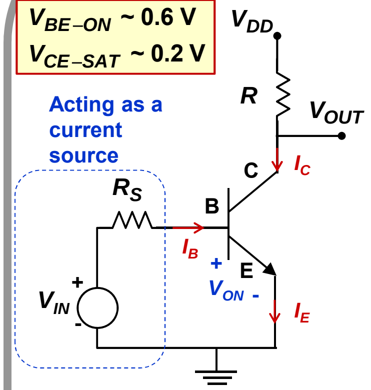
Final Step - confirm if the assumption of forward active operation was valid:

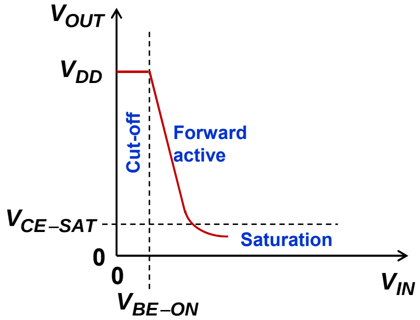
$$V_{CE} \ge V_{CE-SAT}$$

$$\Rightarrow V_{CE} = V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta_F \left(\frac{V_{IN} - V_{BE-ON}}{R_s} \right) R \ge V_{CE-SAT}$$

NPN BJT Amplifier Circuit: Transfer Curve

V_{BE-ON} ~ 0.6 V
V_{CE-SAT} ~ 0.2 V





If: $V_{IN} < V_{BE-ON} \Rightarrow I_B \approx 0 \Rightarrow V_{OUT} = V_{DD}$ $\{$ Transistor in cut-off

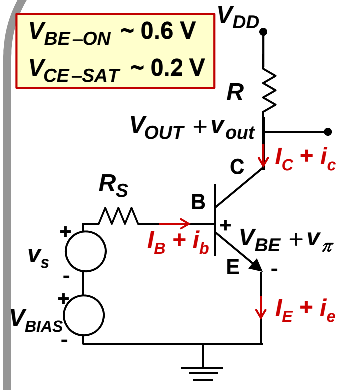
If: $V_{IN} \ge V_{BE-ON} \Rightarrow I_B = \dfrac{V_{IN} - V_{BE-ON}}{R_S}$

$$V_{OUT} = V_{DD} - I_C R = V_{DD} - \beta_F \frac{R}{R_S} (V_{IN} - V_{BE-ON})$$

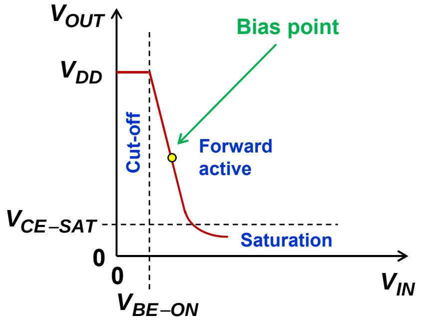
$\}$ Transistor in forward active

If: $I_B > 0$ & $V_{CE} = V_{OUT} \le V_{CE-SAT} \Rightarrow$ $\{$ Transistor in saturation

NPN BJT Common Emitter (CE) Voltage Amplifier


V_{BE-ON} ~ 0.6 V

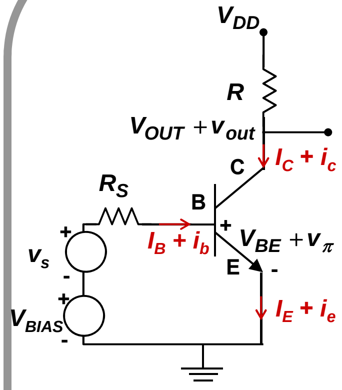
V_{CE-SAT} ~ 0.2 V



We need better techniques to calculate the voltage gain of such amplifier circuits

We need small signal models of the BJTs!

NPN BJT: Small Signal Circuit Model



Base current:

$$I_B = qn_i^2 A\left(\frac{D_p}{N_{dE}W_E}\right)\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$= I_{BS}\left(e^{\frac{qV_{BE}}{KT}} - 1\right)$$

$$\Rightarrow I_B + i_b = I_{BS}\left(e^{\frac{q(V_{BE}+v_\pi)}{KT}} - 1\right)$$

$$\Rightarrow i_b = \frac{\partial I_B}{\partial V_{BE}} v_\pi = \frac{q(I_B + I_{BS})}{KT} v_\pi \approx \frac{qI_B}{KT} v_\pi = g_\pi v_\pi$$

$$\Rightarrow g_\pi = \frac{qI_B}{KT}$$

Collector current:

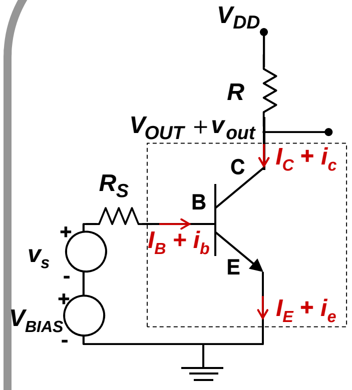
$$I_C + i_c = \beta_F (I_B + i_b)$$

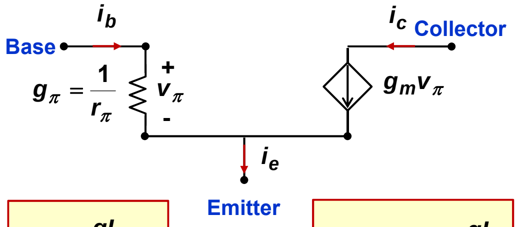
$$\Rightarrow i_c = \beta_F i_b = \beta_F g_\pi v_\pi = g_m v_\pi$$

$$g_m = \beta_F g_\pi = \beta_F \frac{qI_B}{KT} = \frac{qI_C}{KT} \quad \longleftarrow$$

Increases linearly with the collector current

NPN BJT: Small Signal Circuit Model

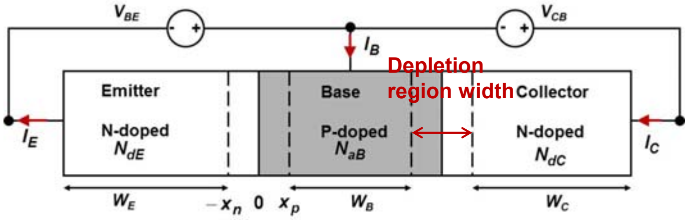


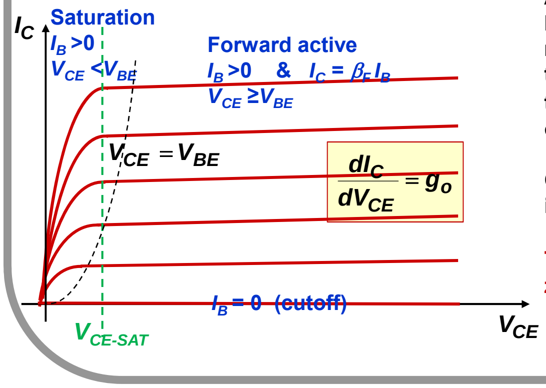


$$g_\pi = \frac{qI_B}{KT}$$

$$g_m = \beta_F g_\pi = \frac{qI_C}{KT}$$

NPN BJT: Forward Active Current vs V_{CE}



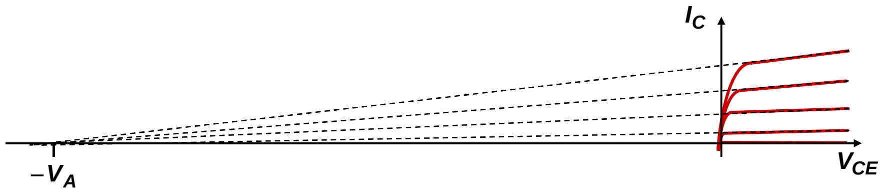


As V_{CE} becomes more positive, the base-collector junction becomes more reverse biased, and the thickness of the depletion increases thereby reducing the thickness W_B of the base

Consequently, the collector current increases

The output conductance g_o is not zero!

NPN BJT: Output Conductance and the Early Voltage

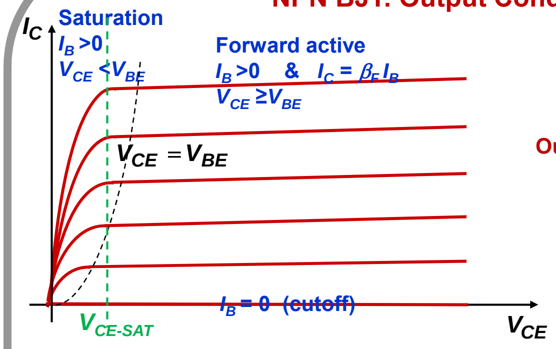


The slope of the I_C vs V_{CE} curves are modeled using the early voltage V_A:

$$\frac{dI_C}{dV_{CE}} = g_o = \frac{I_C}{V_A} = \lambda_n I_C$$

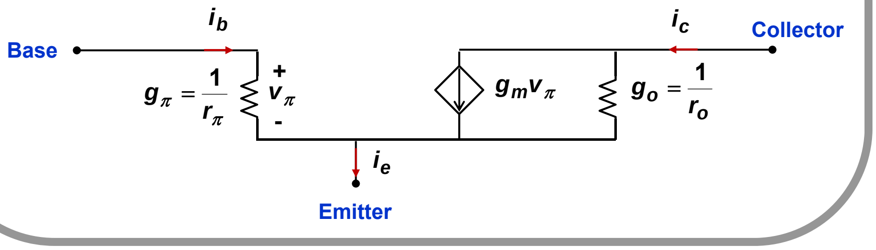
The early voltage is usually in the 50-200 V range

NPN BJT: Output Conductance

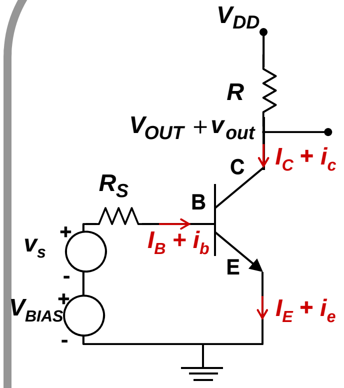


Output conductance:

$$g_o = \frac{1}{r_o} = \frac{\partial I_C}{\partial V_{CE}}$$



NPN BJT Common Emitter (CE) Voltage Amplifier



$$v_\pi = v_s \frac{r_\pi}{r_\pi + R_S}$$

$$i_c = g_m v_\pi + \frac{v_{out}}{r_o}$$

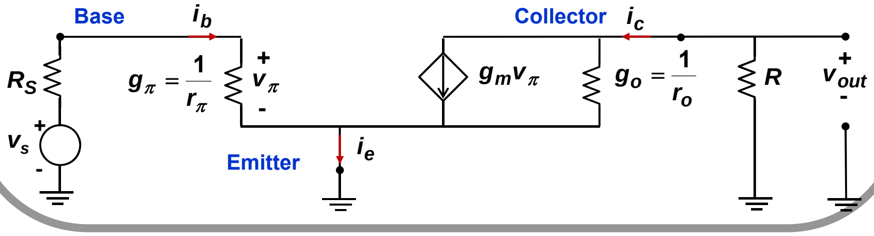
$$v_{out} = -i_c R$$

$$\Rightarrow v_{out} = -g_m (r_o \,||\, R) v_\pi$$

$$\Rightarrow v_{out} = -g_m (r_o \,||\, R) \frac{r_\pi}{r_\pi + R_S} v_s$$

Voltage gain:

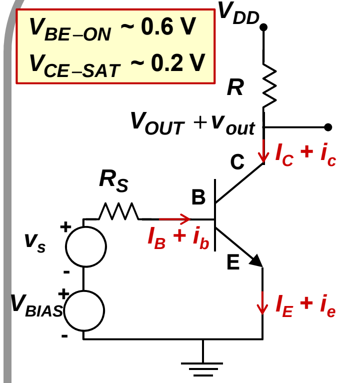
$$A_v = \frac{v_{out}}{v_s} = -g_m (r_o \,||\, R) \frac{r_\pi}{r_\pi + R_S}$$

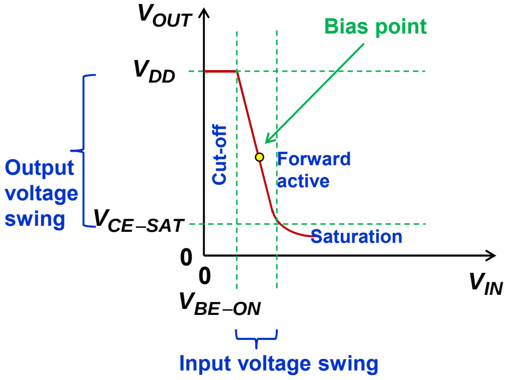


NPN BJT CE Amplifier: Limits of Output Voltage Swing

V_{BE-ON} ~ 0.6 V

V_{CE-SAT} ~ 0.2 V





Minimum output voltage and maximum input voltage:
If the output voltage becomes too small (happens when the input voltage becomes too large), the BJT will go into the saturation region (in the saturation region the gain is small)

Maximum output voltage and minimum input voltage:
If the input voltage becomes too small the BJT will go into cut-off