Lecture 18

PHYC 161 Fall 2016

Announcement from Bibek Pokharel:

NEW SESSION THIS WEEK: There will be an SI/HELP session THIS FRIDAY Oct 7, 2-4 pm just outside room 111

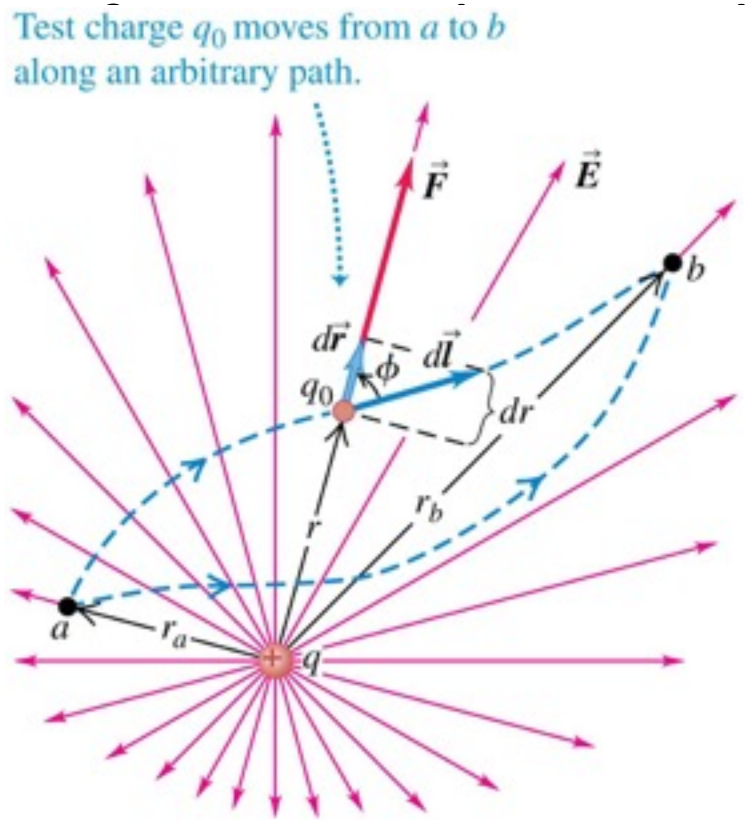
CANCEL NEXT MONDAY: There will be NO SI session on Monday, Oct 10.

Electrical Potential Energy and Work Done by Field on Charges

Change in EPE

was defined as:

$$\Delta U_g = -W_g = -\int_1^2 \vec{F}_g \cdot d\vec{r}$$



If moving from a —> b

positive (test) charge

W>0, ΔU <0

decreases potential energy

imagine a negative charge

W<0, ΔU >0

*inc*reases potential energy

Electric Field from the Potential

- Let's say that somehow we have determined the electric potential everywhere in space from a charge distribution.

$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{r} \Rightarrow$$

$$\int_a^b dV = -\int_a^b \vec{E} \cdot d\vec{r} \Rightarrow$$

$$dV = -\vec{E} \cdot d\vec{r}$$

$$dV = -\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right) \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$dV = -E_x dx - E_y dy - E_z dz$$

Electric Field from the Potential

- If we now hold y and z constant (so that dy and dz are zero) then,

$$dV = -E_x dx - \cancel{E_y dy} - \cancel{E_z dz} \Rightarrow$$

$$dV = -E_x dx \Rightarrow$$

$$E_x = -\left.\frac{dV}{dx}\right|_{\text{y and z constant}} \equiv -\frac{\partial V}{\partial x}$$

- Likewise then, for the other components of the electric field,

$$E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

Potential gradient

- The components of the electric field can be found by taking partial derivatives of the electric potential:

Electric field components found from potential: $E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$

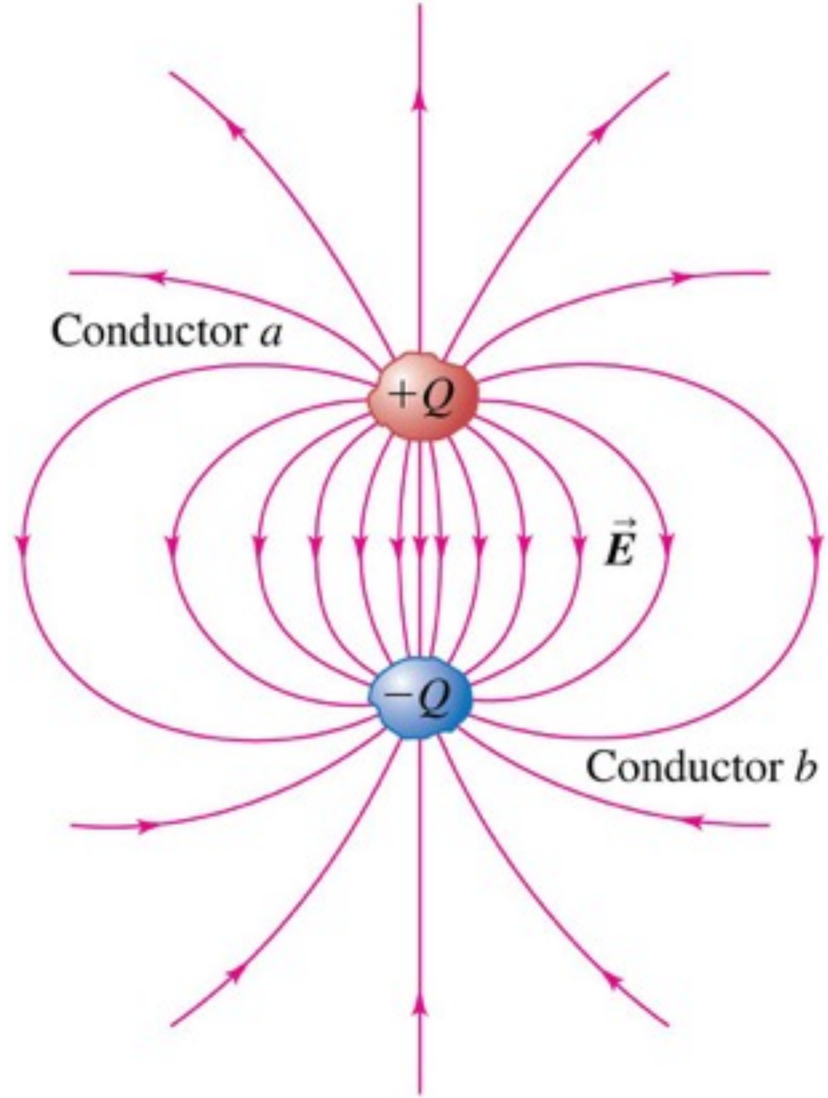
Each electric field component ... equals the negative of the corresponding partial derivative of electric potential function V.

- The electric field is the negative gradient of the potential:

$$\vec{E} = -\vec{\nabla} V$$

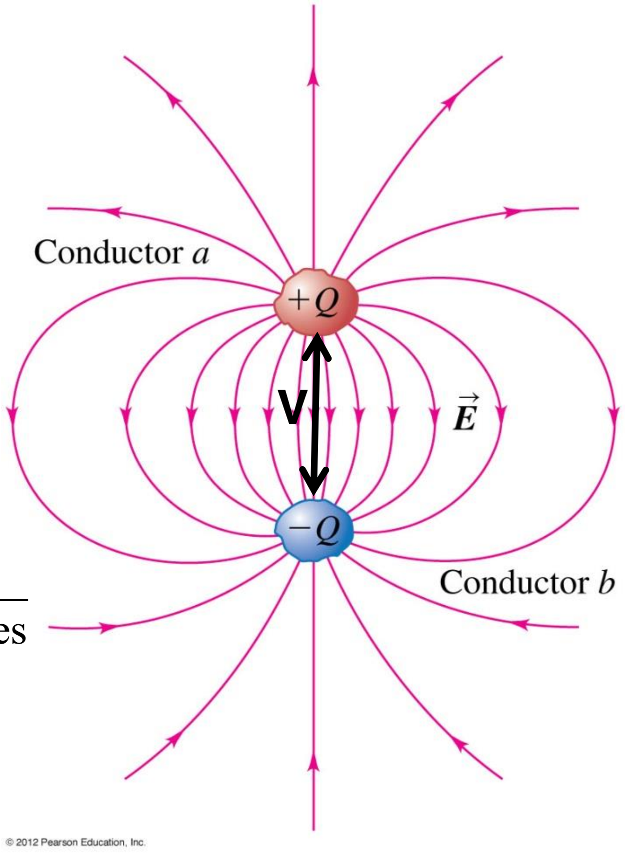
Capacitors

- Any two conductors separated by an insulator (or a vacuum) form a **capacitor**.
- When the capacitor is *charged*, it means the two conductors have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole is zero.



Capacitance

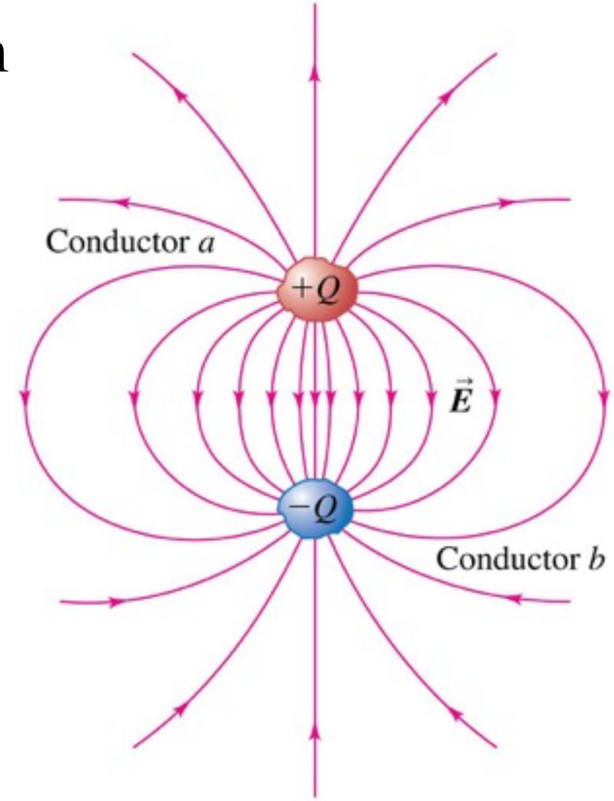
- The capacitance is a property of the ***system*** (not the charge that is put on the system) that relates the amount of charge to the potential difference between the objects.



$$C \equiv \frac{\text{Charge on object}}{\text{Potential of object relative to place where opposite charge resides}}$$

Q24.1

The two conductors *a* and *b* are insulated from each other, forming a capacitor. You increase the charge on *a* to $+2Q$ and increase the charge on *b* to $-2Q$, while keeping the conductors in the same positions. As a result of this change, the capacitance *C* of the two conductors



A. becomes four times as great.

B. becomes twice as great.

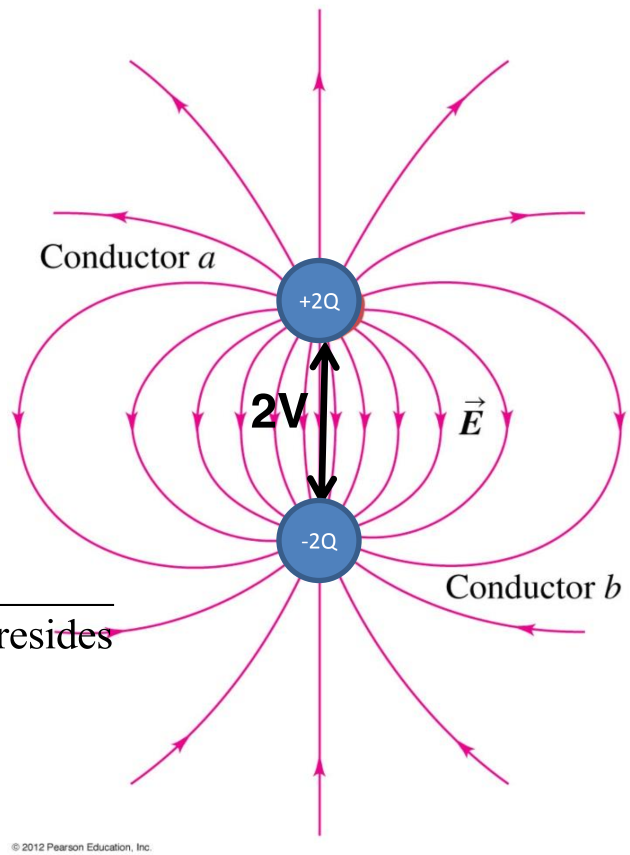
C. remains the same.

D. becomes half as great.

E. becomes one-quarter as great.

Capacitance

- Remember, the capacitance is a property of the ***system*** (not the charge that is put on the system) that relates the amount of charge to the potential difference between the objects.
- So, if you double the charge, the potential will also double.
- How is the capacitance affected?

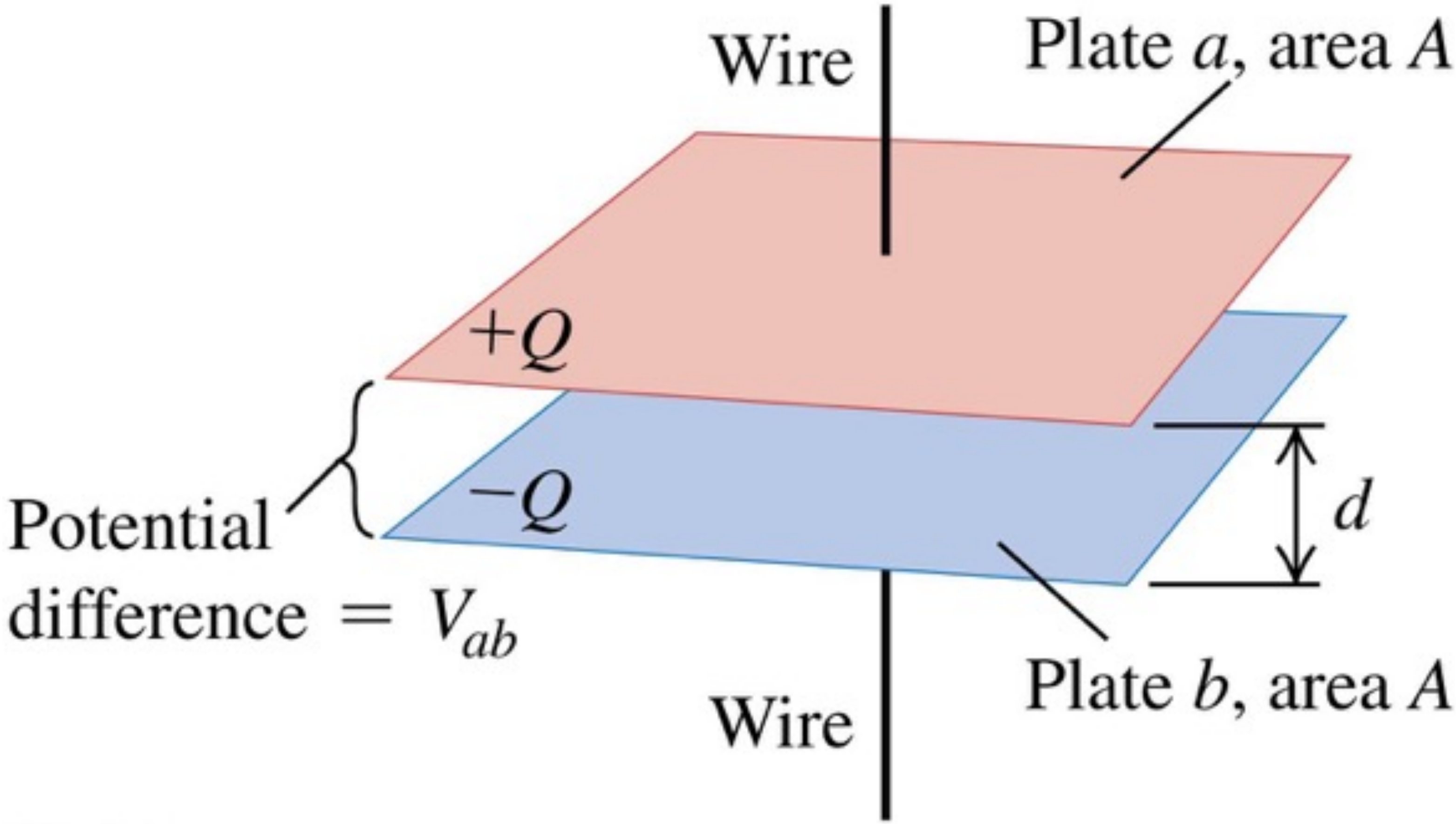


$$C \equiv \frac{\text{Charge on object}}{\text{Potential of object relative to place where opposite charge resides}}$$

$$C \equiv \frac{Q}{V} = \frac{2Q}{2V}$$

Parallel-plate capacitor

- A **parallel-plate** capacitor consists of two parallel conducting plates separated by a distance that is small compared to their dimensions.

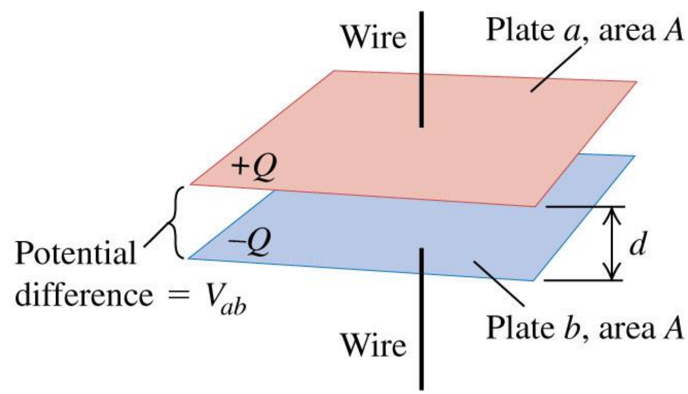


Parallel Plate Capacitor

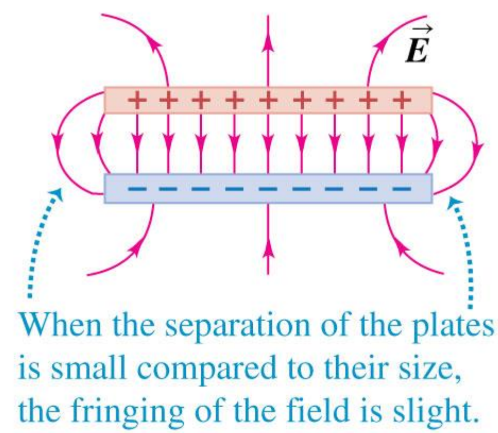
- Steps to find capacitance:
  - Find the potential given a certain amount of charge.
    - Do this by first finding the electric field, then integrating the field to find the potential.
  - Then just divide the charge by the potential difference

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}$$

(a) Arrangement of the capacitor plates



(b) Side view of the electric field $\vec{E}$



Parallel-plate capacitor

- The field between the plates of a parallel-plate capacitor is essentially *uniform*, and the charges on the plates are uniformly distributed over their opposing surfaces.
- When the region between the plates is empty, the capacitance is:

Capacitance of a parallel-plate capacitor in vacuum:

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

- Q: Magnitude of charge on each plate
- V_{ab}: Potential difference between plates
- A: Area of each plate
- d: Distance between plates
- ϵ_0: Electric constant

- The capacitance depends on only the geometry of the capacitor.
- The quantities *A* and *d* are constants for a given capacitor, and ε_0 is a universal constant.

Q24.2

You reposition the two plates of a capacitor so that the capacitance doubles. There is vacuum between the plates. If the charges $+Q$ and $-Q$ on the two plates are kept constant in this process, what happens to the potential difference V_{ab} between the two plates?

A. V_{ab} becomes four times as great.

B. V_{ab} becomes twice as great.

C. V_{ab} remains the same.

D. V_{ab} becomes half as great.

E. V_{ab} becomes one-quarter as great.

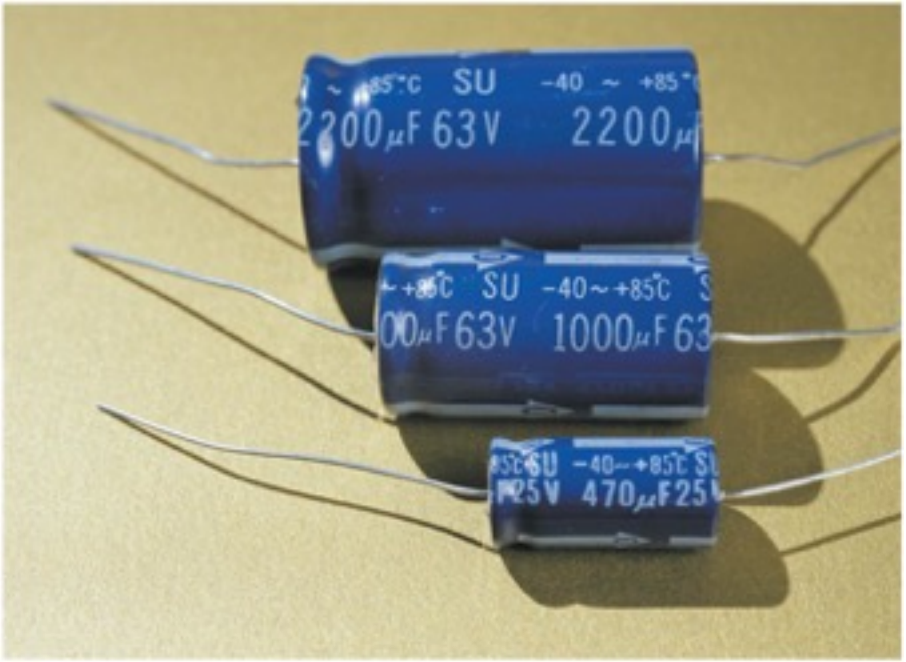
Units of capacitance

- The SI unit of capacitance is the farad, F.

$$1\ \text{F} = 1\ \text{C/V} = 1\ \text{C}^2/\text{N}\cdot\text{m} = 1\ \text{C}^2/\text{J}$$

- One farad is a very large capacitance.
- For the commercial capacitors shown in the photograph, *C* is measured in microfarads

$(1\ \mu\text{F} = 10^{-6}\ \text{F})$



Capacitors in Series

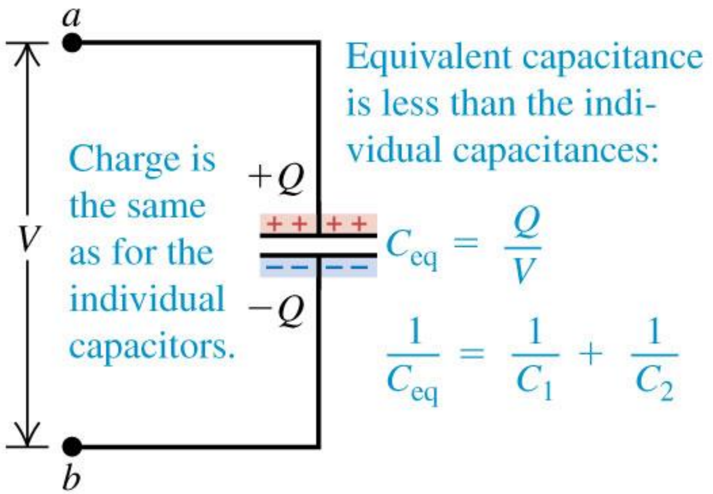
- If you wanted to replace these capacitors with just one equivalent capacitor:

$$C_{\text{eq}} = \frac{Q}{V} = \frac{Q}{V_1 + V_2} \Rightarrow$$

$$\frac{1}{C_{\text{eq}}} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow$$

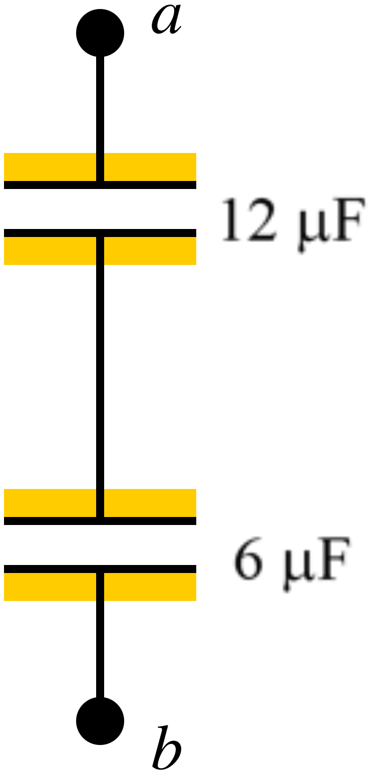
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

(b) The equivalent single capacitor



Q24.3

A 12-μF capacitor and a 6-μF capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?



A. $C_{eq} = 18\ \mu F$

B. $C_{eq} = 9\ \mu F$

C. $C_{eq} = 6\ \mu F$

D. $C_{eq} = 4\ \mu F$

E. $C_{eq} = 2\ \mu F$

Capacitors in Series

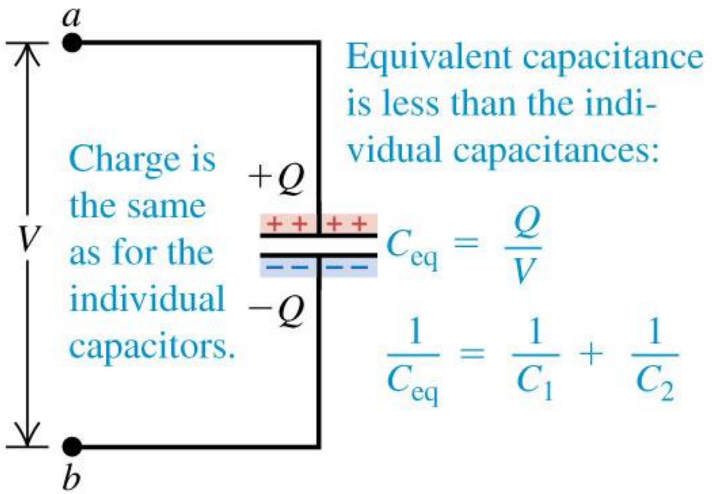
- If you wanted to replace these capacitors with just one equivalent capacitor:

$$C_{eq} = \frac{Q}{V} = \frac{Q}{V_1 + V_2} \Rightarrow$$

$$\frac{1}{C_{eq}} = \frac{V_1 + V_2}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow$$

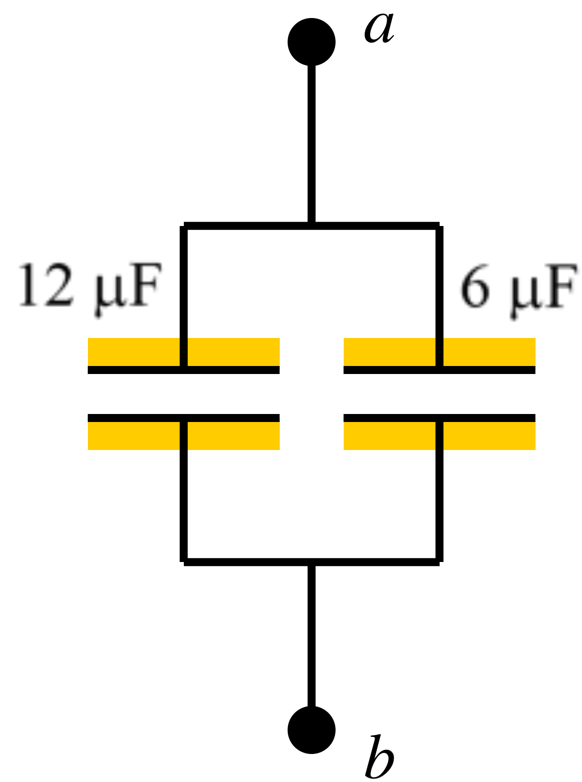
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

(b) The equivalent single capacitor



Q24.5

A 12-μF capacitor and a 6-μF capacitor are connected together as shown. What is the equivalent capacitance of the two capacitors as a unit?



A. $C_{eq} = 18\ \mu F$

B. $C_{eq} = 9\ \mu F$

C. $C_{eq} = 6\ \mu F$

D. $C_{eq} = 4\ \mu F$

E. $C_{eq} = 2\ \mu F$