Lecture 19

Introduction to ANOVA

STAT 512

Spring 2011

Background Reading

KNNL: 15.1-15.3, 16.1-16.2

Topic Overview

- Categorical Variables
- Analysis of Variance
- Lots of Terminology
- An ANOVA example

Categorical Variables

- To this point, with the exception of the last lecture, all explanatory variables have been quantitative; e.g. comparing X = 3 to X = 5 makes sense numerically

- For ***categorical*** or ***qualitative*** variables there is no ‘numerical’ labeling; or if there is, it isn’t meaningful.

Example

- Five medical treatments – ten subjects on each treatment.
- Goal: Compare the treatments in terms of their effectiveness
  - If there were two treatments, what would we use?

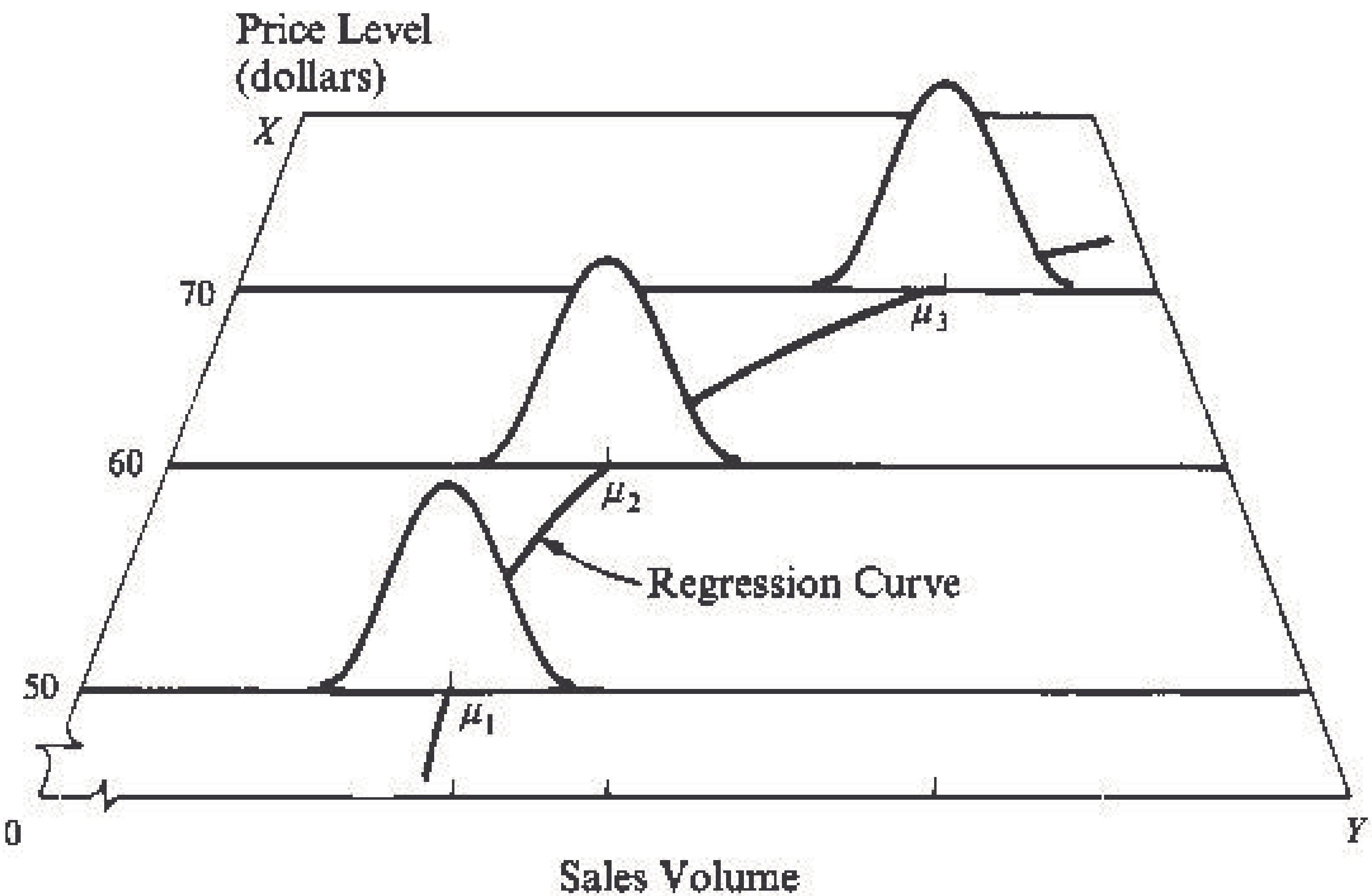
ANOVA

- ***ANOVA = Analysis of Variance***

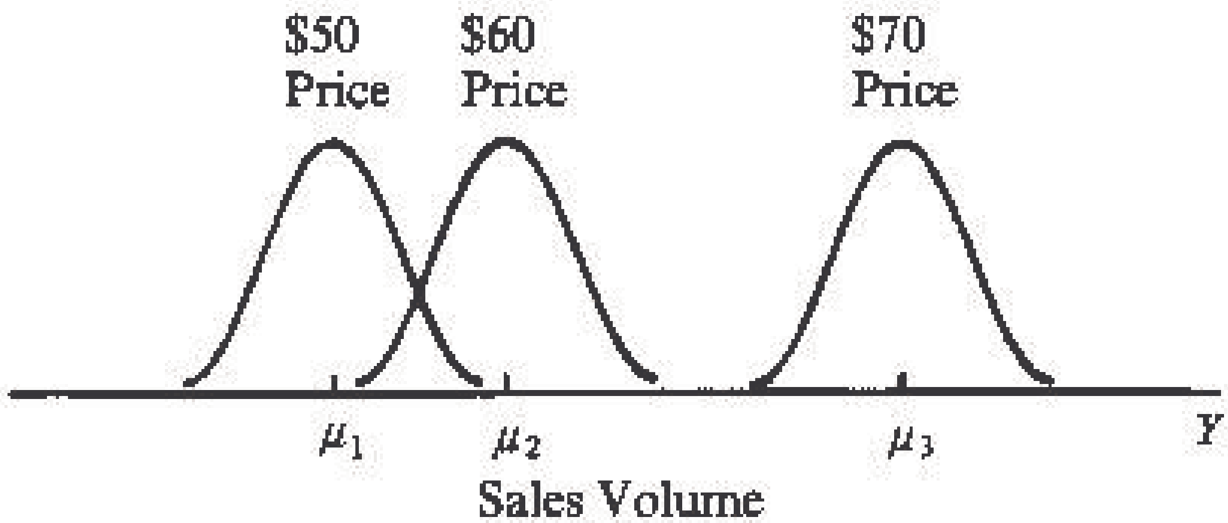
- Compare means among treatment groups, without assuming any parametric relationships (regression does assume such a relationship).

- Example: Price vs. Sales Volume

Regression Model



ANOVA Model



KEY DIFFERENCE: No assumption is made about the manner in which Price and Sales Volume are related.

Similarities to Regression

- Assumptions on errors identical as to regression
- We assume each population is normal and the variances are identical. We also assume independence.
- Can get "predicted values" for each group, as well as CI's.

Differences

- No specific relationship is assumed.
- Goal becomes: look for differences among the groups.

Terminology

- We may refer to any qualitative predictor variable as a ***factor***.
- Each factor has a certain number of ***levels***.
- ***Experimental factors*** are “set” or “assigned” to the experimental units; ***observational factors*** are characteristics of the experimental units that cannot be assigned.

Terminology (2)

- Factors are ***qualitative*** if they represent traits that could not be placed in some logical numerical order.
  - GENDER, BRAND, DRUG
- Factors are ***quantitative*** if levels are described by numerical quantities on an equal interval scale.
  - AGE, TEMPERATURE

Terminology (3)

- A ***Treatment*** is a specific experimental condition (determined by factors and levels of each factor).
- The ***Experimental Unit*** (Basic Unit of Study) is the smallest unit to which a treatment can be assigned.
- A design is called ***balanced*** if each treatment is replicated the same number of times (i.e. same number of EU's per treatment).

Examples

Five medications – each used for 10 subjects

- Medication is an experimental factor; EU is the subject (person) receiving the medication.
- There are five treatments, which may or may not have any logical "ordering"
- Design is balanced (generally) since we are able to assign the treatments.

Ten age groups – 50 subjects

- Age is an observational, quantitative factor; subject is again the EU; Design is probably not balanced

Examples (2)

Blood Type

- Observational factor
- Qualitative factor
- Again design probably not balanced

Brand of Product

- Observational, qualitative factor
- Design likely balanced by arrangement

Multiple Factors

- With two or more factors, each combination of levels is generally called a ***treatment combination***
- Can treat as single variable if desired
- Example: Blood Type * Medication
  - 4 blood types
  - 5 medications
  - 20 treatment combinations

Crossed Factors

- Two factors are ***crossed*** if all factor combinations are represented.
- Example: Blood Type * Medication

	1	2	3	4	5
A	xx	xx	xx	xx	xx
B	xx	xx	xx	xx	xx
AB	xx	xx	xx	xx	xx
O	xx	xx	xx	xx	xx

Note: This type of table is called a ***design chart***.

Nested Factors

- One factor has levels that are unique to a given level of another factor
- Example: Plant * Operator

Plant #1	Plant #2	Plant #3
Op #1 Op #2 Op #3	Op #4 Op #5 Op #6	Op #7 Op #8 Op #9

- We say: Operators are nested within manufacturing plants.

Control Groups

- Often a ***control*** or ***placebo*** treatment is used. This treatment is more of a “standard” than a treatment, as it is the case of no treatment at all.

- Comparing treatments to controls can be a very effective way of showing that a treatment is effective.

Fixed vs. Random Factors

- For the most part, we will consider only ***fixed effect models*** in this class. A factor is called ***fixed*** because the levels are chosen in advance of the experiment and we were interested in differences in response among <u>those specific levels</u>.
- Note: ***Random*** factors will need to be treated differently, since their levels are chosen randomly from a large population of possible levels.

Randomization

- Completely separate concept from random effects.
- In an experimental study, generally want to avoid any potential bias in the design by ***randomizing treatments to experimental units*** whenever possible.
- Randomization may be ***constrained***. Example: Have 100 people, 50 men and 50 women. Randomly assign each of the 5 treatments to 10 men and 10 women.

Experimental Designs

- Completely Randomized Design
- Factorial Experiments
- Randomized Complete Block Designs
- Nested Designs
- Repeated Measures Designs
- Incomplete Block Designs
- We'll discuss some of these. More thorough experimental design course: STAT 514.

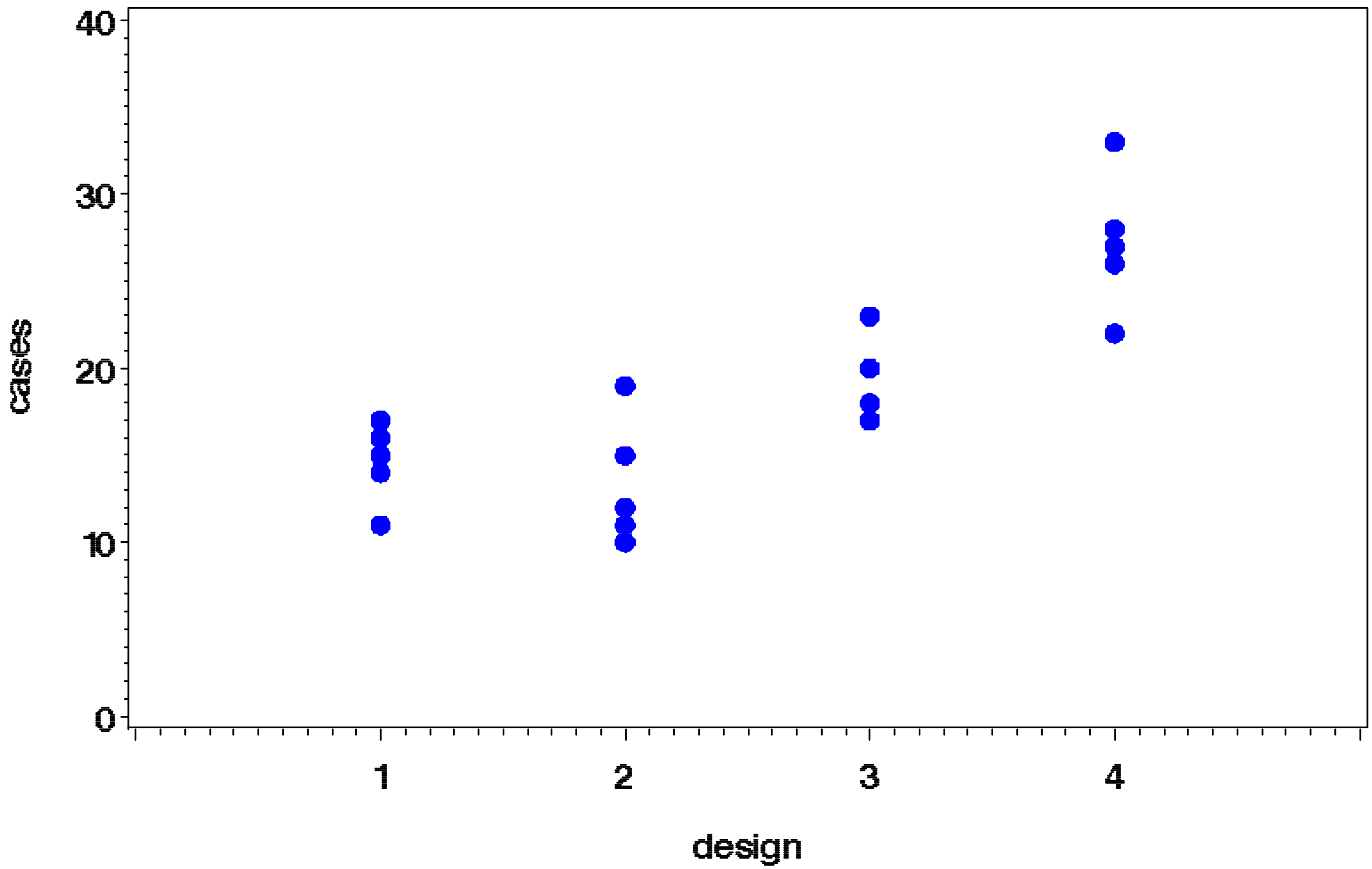
Example

- Kenton Food Company Example (p685)
- Compare four different package designs (numbered 1, 2, 3, 4 in no particular order)
- Response: # of cases sold
- 20 stores, but one was destroyed by fire during the study; 19 observations
- SAS file: kenton.sas

Data

Design 1	Design 2	Design 3	Design 4
11	12	23	27
17	10	20	33
16	15	18	22
14	19	17	26
15	11		28

Scatter Plot



ANOVA Code (SAS)

```
proc glm data=kenton;
   class design;
   model cases=design;
   means design /bon lines cldiff;
```

- Class statement identifies ALL categorical variables (separate by spaces as in model)
- Means statement requests comparisons of the group means (lots of options)

Output

Source	DF	SS	MS	F Value	Pr > F
Model	3	588	196	18.59	<.0001
Error	15	158	10.5		
Total	18	746			

R-Square	Coeff Var	Root MSE	cases Mean
0.788055	17.43042	3.247563	18.63158

Output (2)

```
Bonferroni (Dunn) t Tests for cases
NOTE: This test controls the Type I
experimentwise error rate, but it generally
has a higher Type II error rate than Tukey's
for all pairwise comparisons.
  Alpha                              0.05
  Error Degrees of Freedom             15
  Error Mean Square              10.54667
  Critical Value of t             3.03628
Comparisons significant at the 0.05 level
are indicated by ***.
```

Output (3)

design Comparison	Difference Means	Simultaneous 95% Confidence Limits		
4 - 3	7.700	1.085	14.315	***
4 - 1	12.600	6.364	18.836	***
4 - 2	13.800	7.564	20.036	***
3 - 4	-7.700	-14.315	-1.085	***
3 - 1	4.900	-1.715	11.515	
3 - 2	6.100	-0.515	12.715	
1 - 4	-12.600	-18.836	-6.364	***
1 - 3	-4.900	-11.515	1.715	
1 - 2	1.200	-5.036	7.436	
2 - 4	-13.800	-20.036	-7.564	***
2 - 3	-6.100	-12.715	0.515	
2 - 1	-1.200	-7.436	5.036	

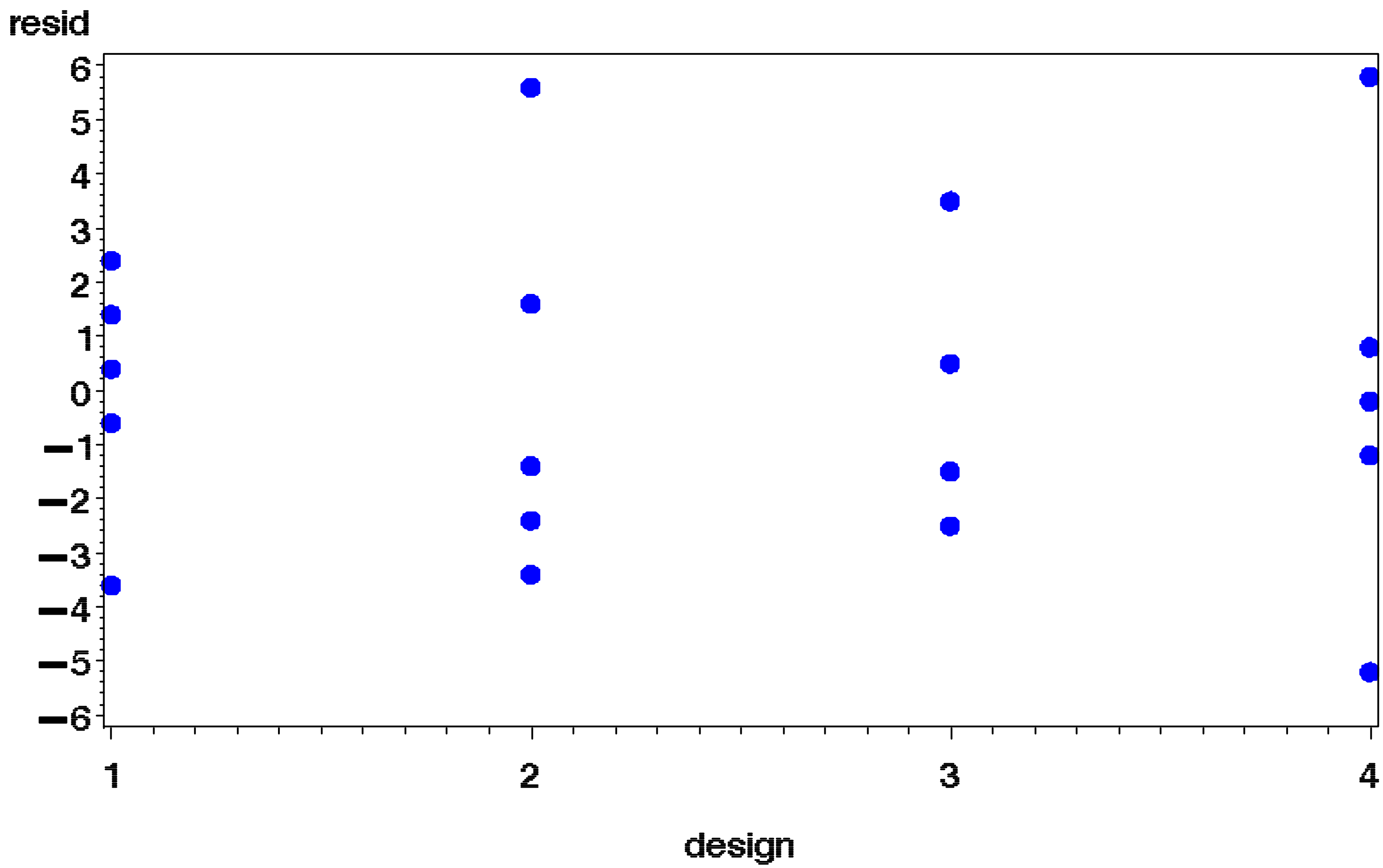
Output (4)

Group	Mean	N	design
A	27.200	5	4
B	19.500	4	3
B			
B	14.600	5	1
B			
B	13.400	5	2

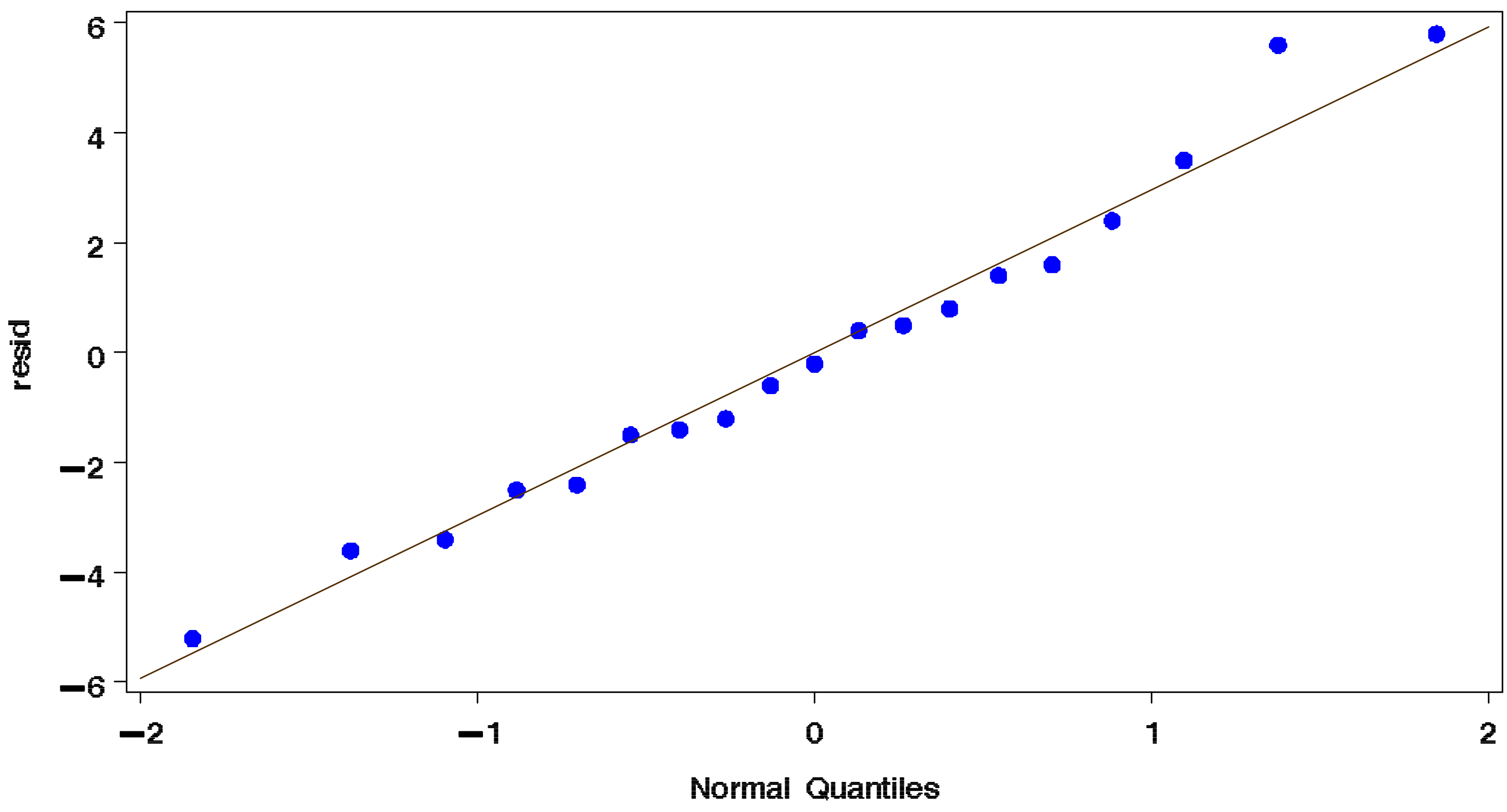
Assumptions

- Should always check normality, constancy of variance assumptions
- Plots to check these are as before
- No obvious problems for this dataset

Residual Plot



Normal QQ Plot



Upcoming in Lecture 20...

- ANOVA Model I (Cell Means)
- Sections 16.3 – 16.6