Lecture 2: Non-Ideal Amps and Op-Amps

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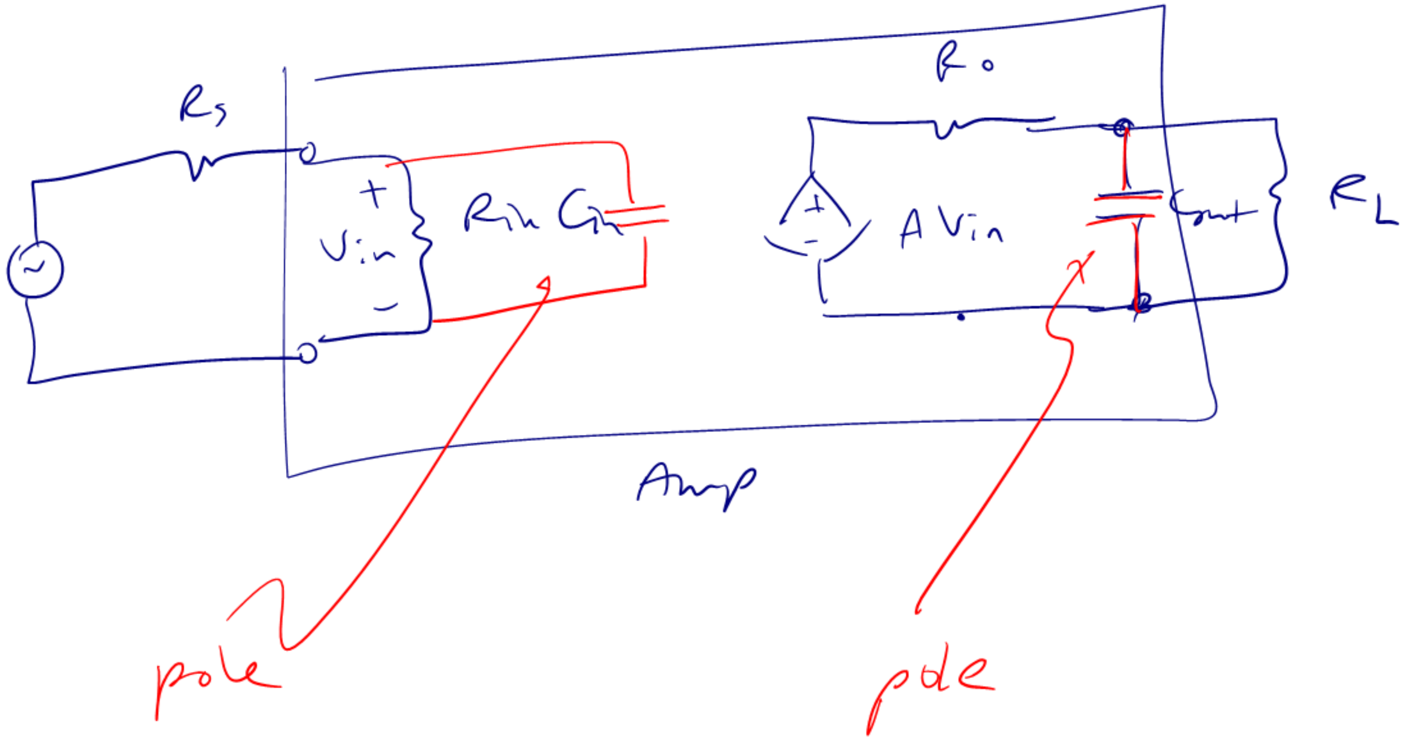
Practical Op-Amps

- Linear Imperfections:
  - Finite open-loop gain ($A_0 < \infty$)
  - Finite input resistance ($R_i < \infty$)
  - Non-zero output resistance ($R_o > 0$)
  - Finite bandwidth / Gain-BW Trade-Off
- Other (non-linear) imperfections:
  - Slew rate limitations
  - Finite swing
  - Offset voltage
  - Input bias and offset currents
  - Noise and distortion

Simple Model of Amplifier

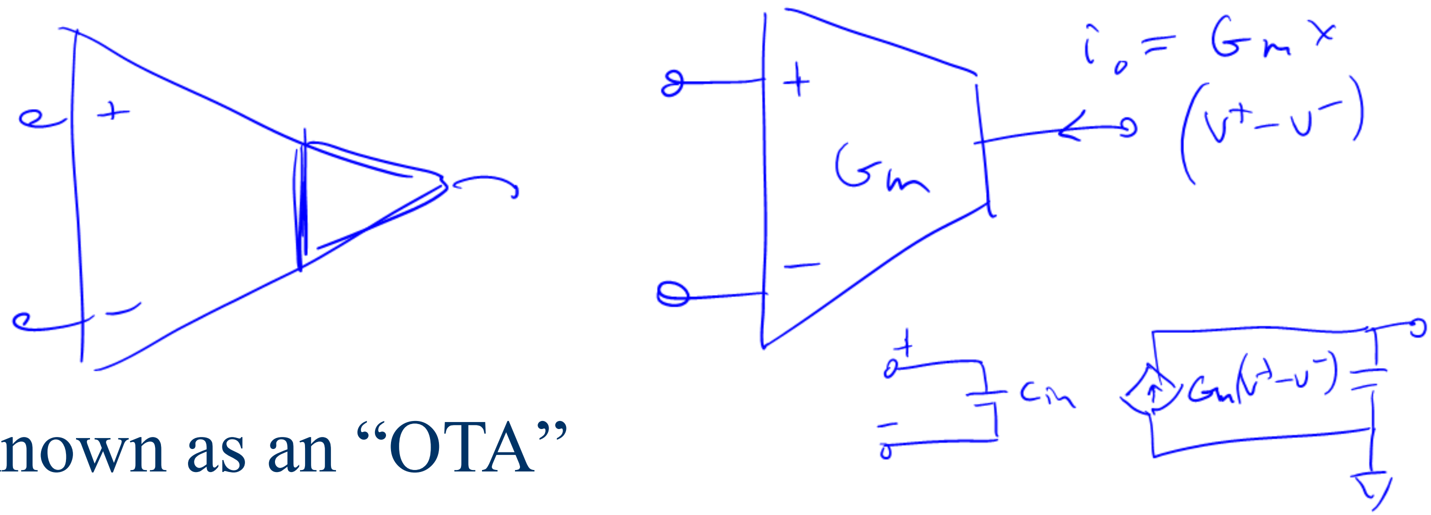
- Input capacitance and output capacitance are added
- Any amplifier has input capacitance due to transistors and packaging / board parasitics
- Output capacitance is usually dominated by the load
  - Driving cables or a board trace
  - Intrinsic capacitance of actuator

Transfer Function



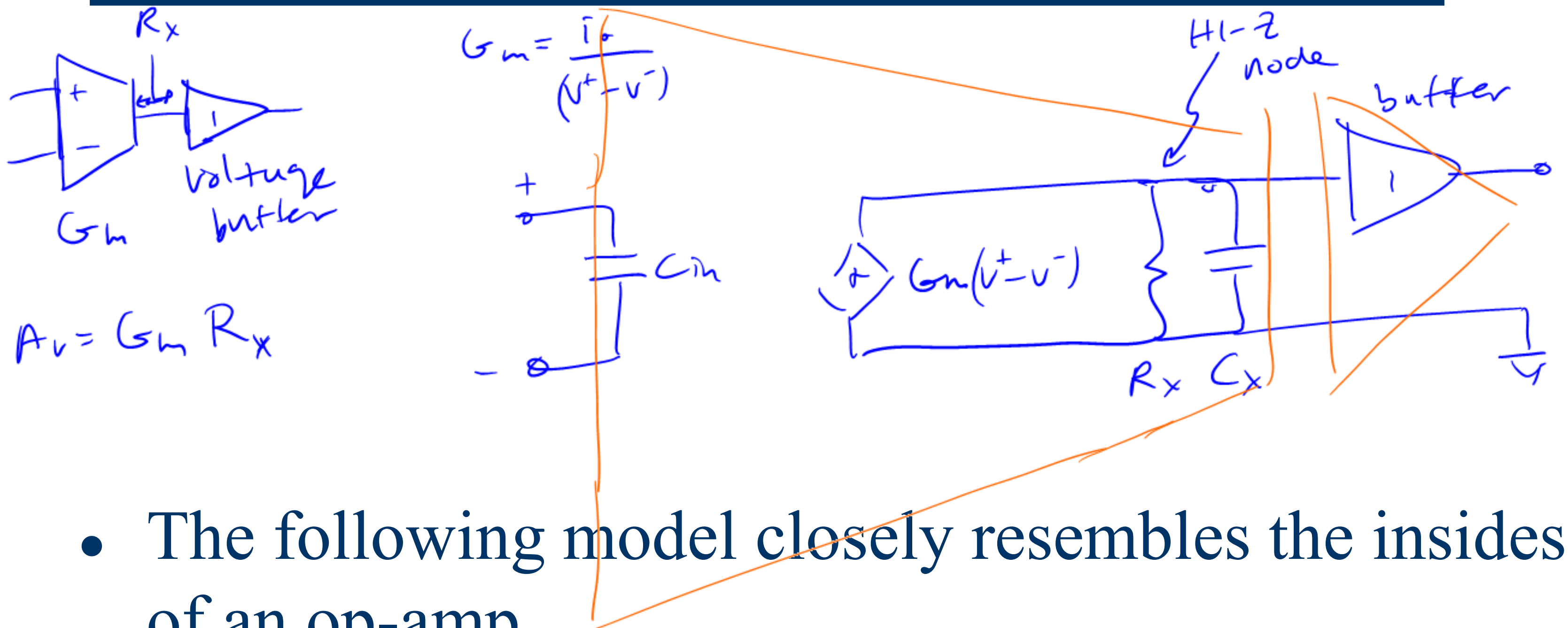
- Using the concept of impedance, it’s easy to derive the transfer function

Operational Transconductance Amp



- Also known as an "OTA"
  - If we "chop off" the output stage of an op-amp, we get an OTA
- An OTA is essentially a G_m amplifier. It has a current output, so if we want to drive a load resistor, we need an output stage (buffer)
- Many op-amps are internally constructed from an OTA + buffer

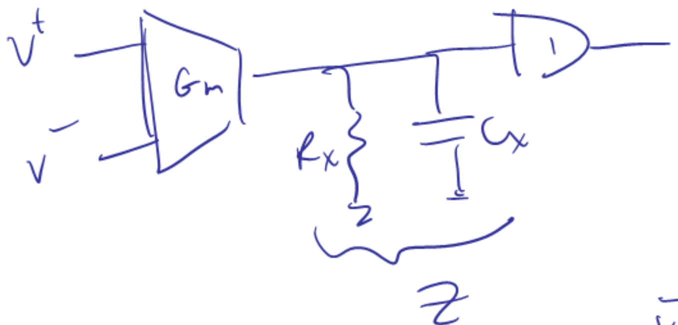
Op-Amp Model



- The following model closely resembles the insides of an op-amp.
- The input OTA stage drives a high Z node to generate a very large voltage gain.
- The output buffer then can drive a low impedance load and preserve the high voltage gain

Op-Amp Gain / Bandwidth

- The dominant frequency response of the op-amp is due to the time constant formed at the high-Z node



$$v_o = (v^+ - v^-) G_m Z$$

$$= (v^+ - v^-) G_m \frac{R_x \frac{1}{j\omega C_x}}{R_x + \frac{1}{j\omega C_x}}$$

$$\frac{v_o}{v^+ - v^-} = G_m R_x / (1 + j\omega R_x C_x)$$

- An interesting observation is that the gain-bandwidth product depends on G_m and C_x only

$$BW = \frac{1}{R_x C_x}$$

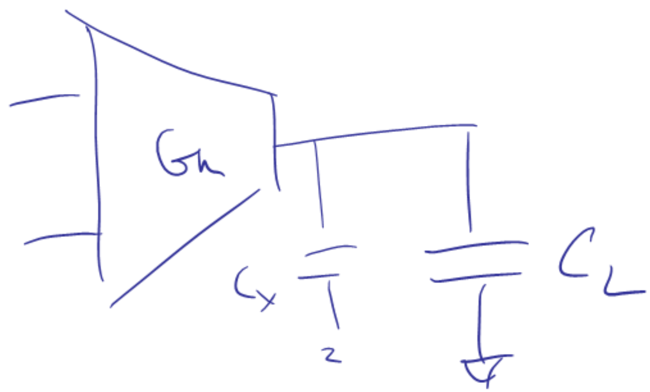
$$G_o = G_m R_x$$

$$G_o \times BW =$$

$$GBW = \frac{G_m}{C_x}$$

Preview: Driving Capacitive Loads

- In many situations, the load is capacitor rather than a resistor
- For such cases, we can directly use an OTA (rather than a full op-amp) and the gain / bandwidth product are now determined by the load capacitance



$$GBW = \frac{G_m}{(C_L + C_x)}$$

$$= \frac{G_m}{C_L}$$

$C_L \gg C_x$

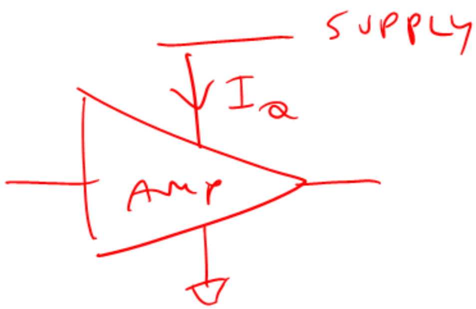
OTA Power Consumption

- For a fixed load, the current consumption of the OTA is fixed by the gain/bandwidth requirement, assuming load dominates

$$C_L >> C_x$$

- G_m scales with current, so driving a larger capacitance requires more power

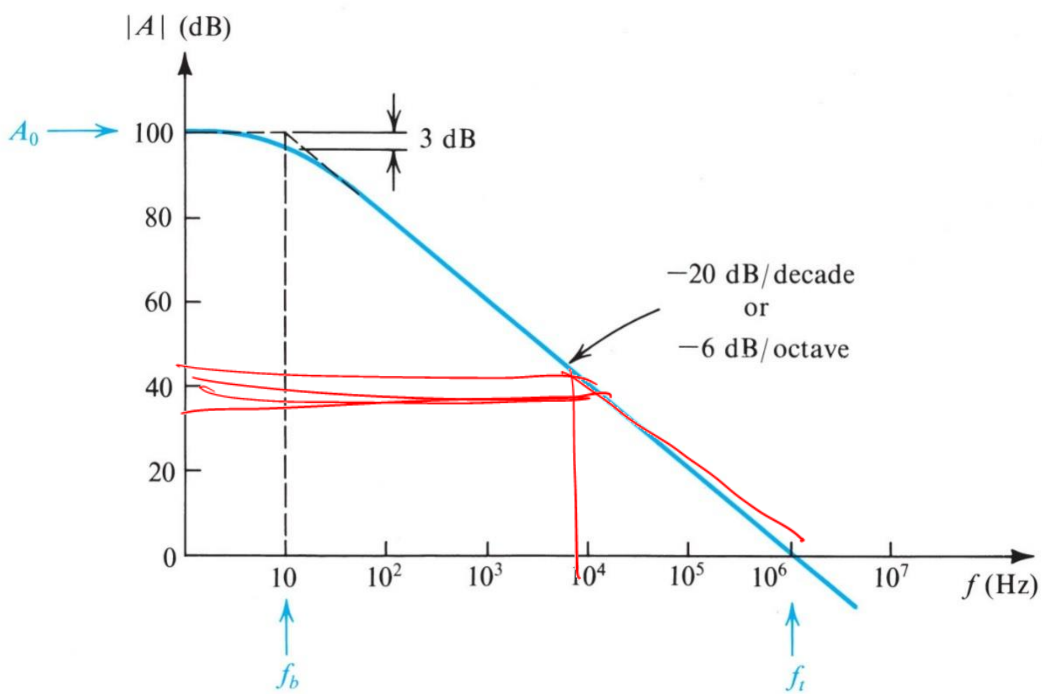
$$G_m \propto f(I_Q)$$



BIAS CURRENT INTO AMP

Gain/Bandwidth Trade-off

Open-Loop Frequency Response



$$A(jw) = \frac{A_0}{1 + jw / w_b}$$

A_0 : dc gain

w_b : 3dB frequency

$w_t = A_0 w_b$: unity-gain bandwidth (or "gain-bandwidth product")

For high frequency, $w >> w_b$

$$A(jw) = \frac{w_t}{jw}$$

$\omega >> \omega_b$ $\left| j \frac{\omega}{\omega_b} \right| >> 1$ GBW

$$A(j\omega) \approx \frac{A_0}{j \frac{\omega}{\omega_b}} = \frac{A_0 \omega_b}{j\omega} = GBW / j\omega$$

Single pole response with a dominant pole at ω_b

$$|A(j\omega)| = 1 \simeq \frac{GBW}{\omega_t}$$

UNITY GAIN FREQ → $\omega_t = GBW$

Bandwidth Extension

- Suppose the core amplifier is single pole with bandwidth:

DC GAIN

$$G(\omega) = \frac{G_0}{1 + j\frac{\omega}{\omega_b}}$$

BW

- When used feedback, the overall transfer function is given by

$$G_{CL} = \frac{G}{1+Gf} = \frac{\frac{G_0}{1+j\omega/\omega_b}}{1+\frac{G_0}{1+j\frac{\omega}{\omega_b}}f}$$

$$G_{CL} = \frac{G_0}{1+j\frac{\omega}{\omega_b} + G_0 f}$$

BW EXPANSION

$$= \frac{G_0/(1+G_0 f)}{1 + j\frac{\omega}{\omega_b(1+G_0 f)}} = \frac{G_{L0}(DC)}{1+j\frac{\omega}{\omega_b'}}$$

Gain / Bandwidth Product in Feedback

- Even though the bandwidth expanded by $(1+T)$, the gain drops by the same factor. So overall the gain-bandwidth (GBW) product is constant
- The GBW product depends only the the G_m of the op-amp and the C_x internal capacitance (or load in the case of an OTA)

$$\omega_b = (1+T)\,\omega_b$$

$$G_{CL} = \frac{G}{1+T}$$

$$\omega_b\, G_{CL} = \cancel{(1+T)}\,\omega_b \frac{G}{\cancel{(1+T)}} = G\,\omega_b = \underline{\underline{GBW}}$$

Unity Gain Frequency

- The GBW product is also known as the unity gain frequency.

$\uparrow \omega_t$

- To see this, consider the frequency at which the gain drops to unity

$$|G(\omega_t)| = \left| \frac{G_0}{1 + j\frac{\omega_t}{\omega_b}} \right| = 1$$

$$G_0 = \sqrt{1 + \left(\frac{\omega_t}{\omega_b}\right)^2}$$

$$G_0^2 - 1 = \left(\frac{\omega}{\omega_b}\right)^2$$

$$\omega = \omega_b \sqrt{G_0^2 - 1} \approx \underbrace{\omega_b G_0}_{GBW}$$

Unity Gain Feedback Amplifier

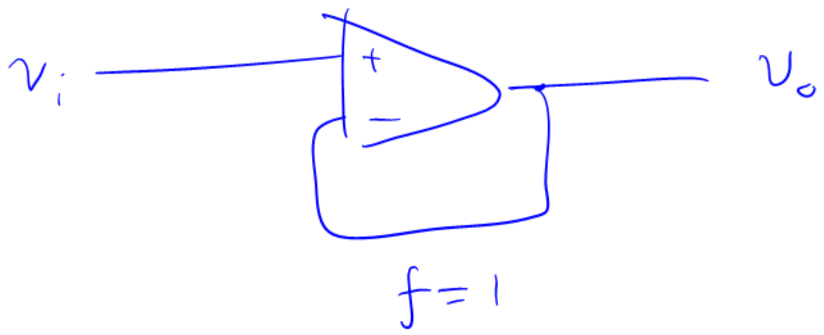
- An amplifier that has a feedback factor f=1, such as a unity gain buffer, has the full GBW product frequency range

$$\omega_b' = \omega_b (1+T)$$

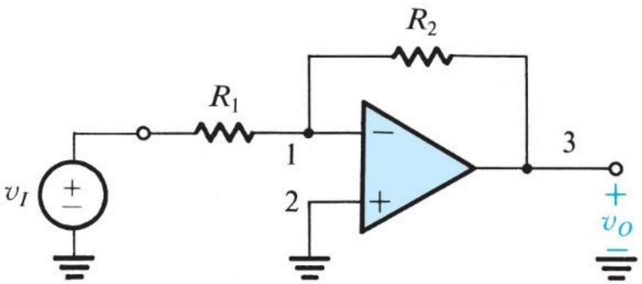
$$T = G f = G \qquad f = 1$$

$$\omega_b' = \omega_b \cdot (1+G) = GBW!$$

FULL BANDWIDTH



Closed-Loop Op Amp



$$v_o = G(v^+ - v^-)$$

$$= G(0 - v^-) = -Gv^-$$

say $i^- = i^+ = 0\text{ A}$

$$\frac{v_i - v^-}{R_1} = \frac{v^- - v_o}{R_2}$$

$$v_i \frac{1}{R_1} = v^-\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{v_o}{R_2}$$

$$v_i = v^-\left(1 + \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} v_o$$

$$= -\frac{v_o}{G}\left(1 + \frac{R_1}{R_2}\right) - \frac{R_1}{R_2} v_o = -v_o\left(\frac{R_1}{R_2} + \frac{1}{G} + \frac{R_1}{R_2}\frac{1}{G}\right)$$

$$\frac{v_o}{v_i} = \frac{-R_2/R_1}{\frac{1}{G}\frac{R_2}{R_1} + \left(\frac{1}{G} + 1\right)} = \frac{-R_2/R_1}{\left(\frac{1 + j\omega/\omega_b}{G_0} + 1\right) + \frac{R_2}{R_1}\frac{\left(1 + j\frac{\omega}{\omega_b}\right)}{G_0}}$$

$$\frac{V_o}{V_i} = \frac{-R_2/R_1}{\frac{1}{G}\frac{R_2}{R_1} + \left(\frac{1}{G}+1\right)} = \frac{-R_2/R_1}{\left(\frac{1+j\omega/\omega_b}{G_0}+1\right) + \frac{R_2}{R_1}\frac{\left(1+j\frac{\omega}{\omega_b}\right)}{G_0}}$$

$$= \frac{-R_2/R_1 \cdot G_0}{1+j\frac{\omega}{\omega_b} + G_0 + \frac{R_2}{R_1}\left(1+j\frac{\omega}{\omega_b}\right)} = \frac{-\frac{R_2}{R_1}G_0/\left(1+G_0+\frac{R_2}{R_1}\right)}{1+j\frac{\omega}{\omega_b}\frac{\left(1+\frac{R_2}{R_1}\right)}{1+G_0+\frac{R_2}{R_1}}}$$

DC GAIN: $-\frac{R_2}{R_1}\frac{G_0}{1+G_0+\frac{R_2}{R_1}} \simeq -\frac{R_2}{R_1}$

NEW BW: $\omega_b' = \omega_b\frac{\left(1+G_0+\frac{R_2}{R_1}\right)}{1+\frac{R_2}{R_1}}$

$$\simeq \frac{\omega_b G_0}{1+\frac{R_2}{R_1}}$$

EXPANSION FACTOR

Frequency Response of Closed-Loop Inverting Amplifier Example

GAIN DROPS $\frac{A_0}{\left(\frac{R_2}{R_1}\right)}$

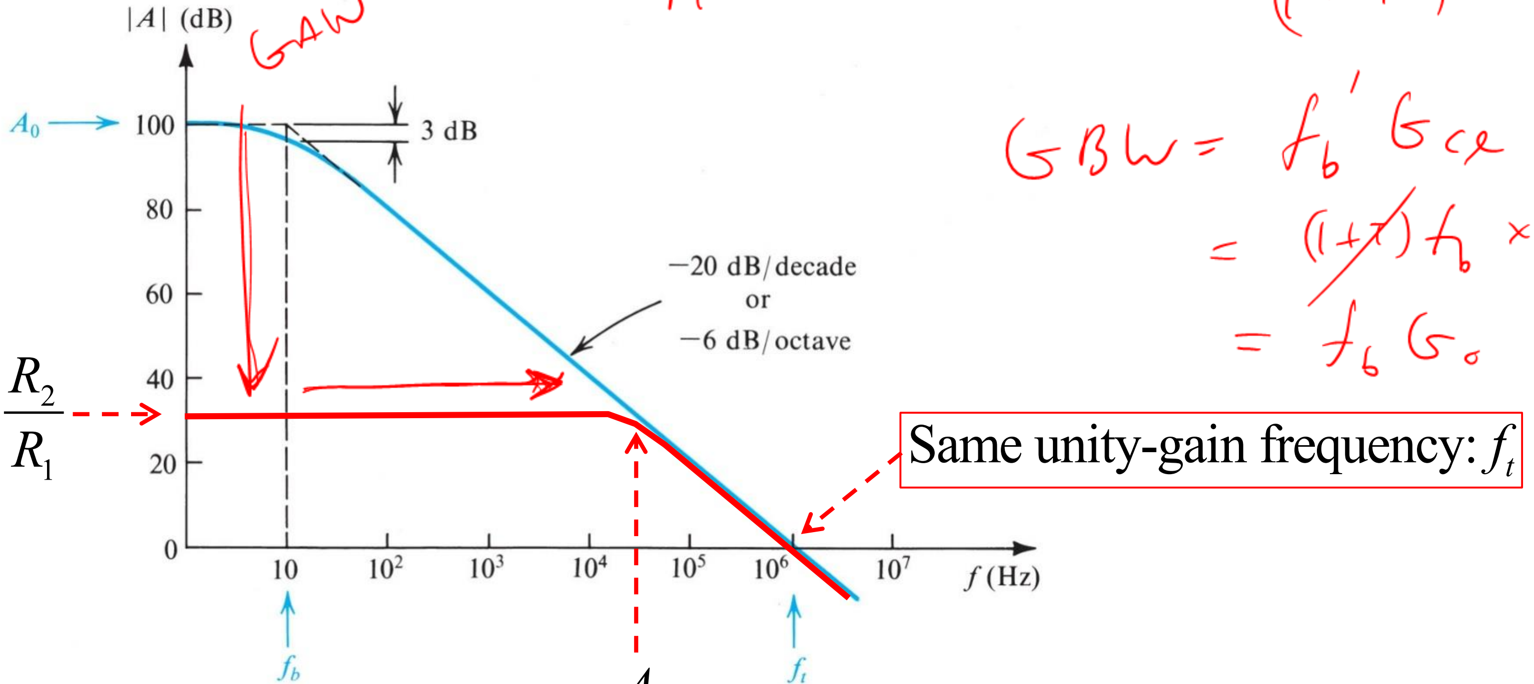
$$f_b' = (1+T) f_b$$

$$G_{cl} = \frac{G_0}{(1+T)}$$

$$GBW = f_b' G_{cl}$$

$$= \cancel{(1+T)} f_b \times \frac{G_0}{\cancel{(1+T)}}$$

$$= f_b G_0$$



$$f_{3dB} \approx \frac{A_0}{R_2/R_1} f_b$$

Non-Dominant Poles

- As we have seen, poles in the system tend to make an amplifier less stable. A single pole cannot do harm since it has a maximum phase shift of 90°
- A second pole in the system is not affected by feedback (prove this) and it will add phase shift as the frequency approaches this second pole
- For this reason, non-dominant poles should be at a much higher frequency than the unity-gain frequency

Positive Feedback

- Positive Feedback is also useful
- We can create a comparator circuit with *hysteresis*
- Also, as long as $T < 1$, we can get stable gain … instead of reducing the gain (negative feedback), positive feedback enhances the gain.
- In theory we can boost the gain to any desired level simply by making T close to unity:

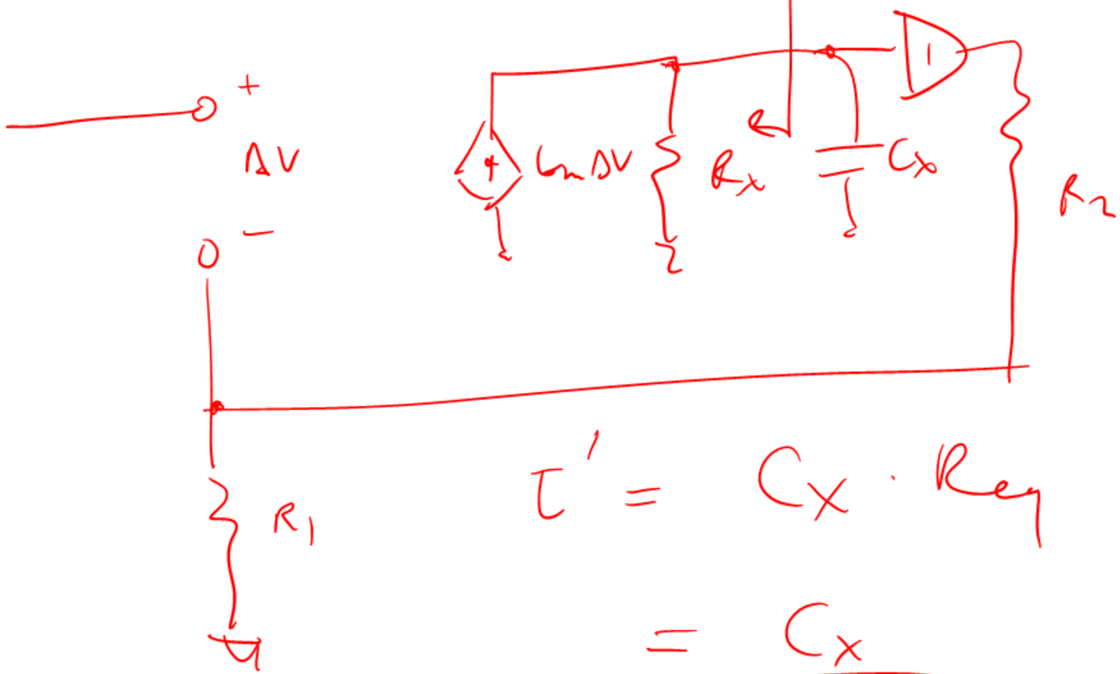
$$T = 1 - \epsilon$$

- ε is a very small number
  - In practice if the gain varies over process / temperature / voltage, then the circuit can go stable and oscillate
  - Positive feedback also has a narrow-banding effect

Back to Circuit Model

- Here's the equivalent circuit for an amplifier with feedback

$R_{eq} = R_x \,||\, \frac{1}{G_m f} \approx \frac{1}{G_m f}$



$\tau' = C_x \cdot R_{eq}$

$= \frac{C_x}{G_m f}$

$\omega_b = \frac{1}{\tau'} = \left(\frac{G_m}{C_x}\right) f \qquad \left(\frac{1}{f}\right)$

unity gain

Approx closed loop gain

$\omega_b = \omega_t \times f$
$= \omega_t / \text{GAIN}$

Circuit Interpretation

- Here we see the action of the feedback is to lower the impedance seen by the G_m by the loop gain, which expands the bandwidth by the same factor

Comment: This is advanced stuff so don't worry if the circuit interpretation is not 100% clear!