Physics 2514

Lecture 20

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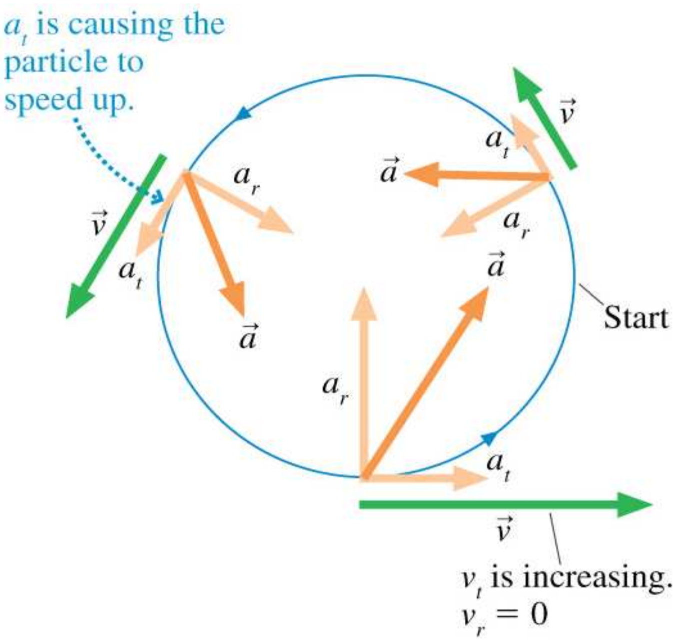
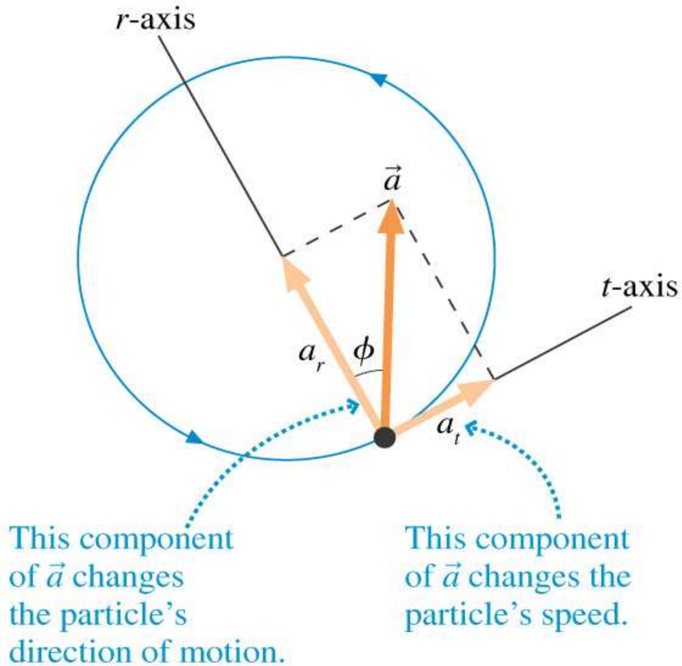
Goals

Discussion of circular motion.

We will apply Newton’s second law to circular motion.

Circular Motion





Review

Kinematic equations and variables for circular motion

Motion along arc is 1-D with tangential acceleration and velocity specifying motion

$$s = s_0 + v_{ot}t + \frac{1}{2}a_t t^2$$

$$v_t = v_{0t} + a_t t$$

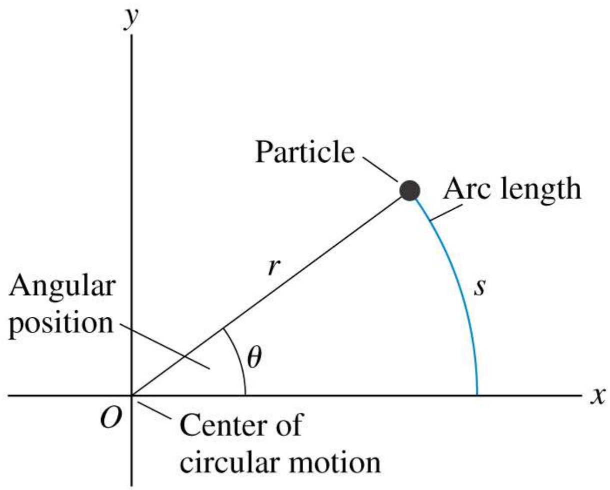
Now in terms of angular variables

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

Centripetal acceleration

$$a_r = \frac{v^2}{r}$$

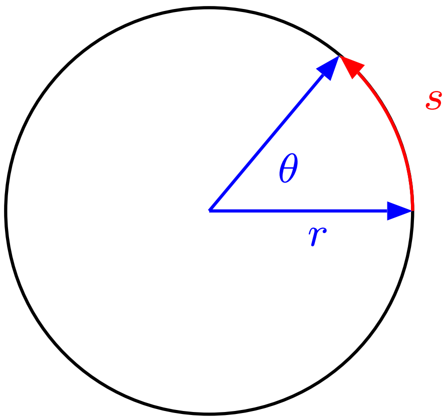


$$\theta = \frac{s}{r} \qquad \omega = \frac{v_t}{r}$$

$$\alpha = \frac{a_t}{r}$$

Example 1

A 5.0 m diameter merry-go-round is initially turning with a 4.0 s period. It slows down and stops in 20 s. How many revolutions does it make as it slows down?



Use angular variables

Known:

$T = 4$ s $\Rightarrow \omega_0 = 2\pi/T = 1.57$ rad/s

$t = 20$ s $\Rightarrow \omega_f = 0$

Unknown:

No. of revolutions

$\alpha =$?

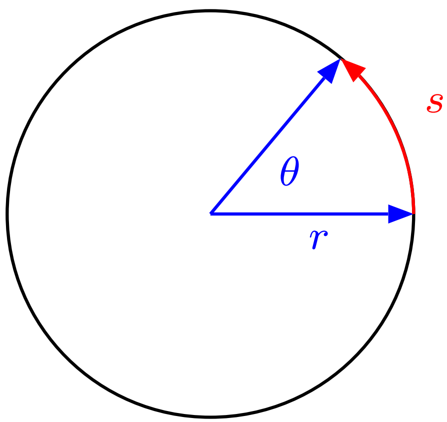
$\omega_f = \omega_0 + \alpha t \Rightarrow \alpha = -0.0785$ rad/s^2

$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \Delta\theta = 15.7$ rad $\Rightarrow \frac{\Delta\theta}{2\pi} = 2.5$ rev

Example 1

A 5.0 m diameter merry-go-round is initially turning with a 4.0 s period. It slows down and stops in 20 s. How many revolutions does it make as it slows down?

Use linear variables (arc-length)



Known:

$T = 4$ s $\Rightarrow v_{t0} = 2\pi r/T = 3.93$ m/s

$t = 20$ s $\Rightarrow v_{tf} = 0$

Unknown:

No. of revolutions

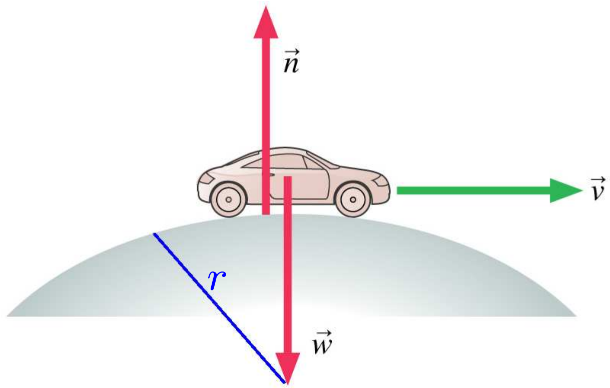
$a_t = ?$

$$v_{tf} = v_{f0} + a_t t \Rightarrow a_t = -0.196 \text{ m/s}^2$$

$$s - s_0 = v_{t0} t + \frac{1}{2} a_t t^2 \Rightarrow \Delta s = 39.4 \text{ m} \Rightarrow \frac{\Delta s}{2\pi r} = 2.5 \text{ rev}$$

Example 2

A car drives over the top of a hill that has a radius of 50 m. What maximum speed can the car have without flying off the road at the top of the hill?



Apply Newton's second law

Known

$r = 50$ cm

$n = 0$

Unknown

v_t

$$mg - n = m\frac{v_t^2}{r} \Rightarrow v_t = \sqrt{gr} \approx 22 \text{ m/s}$$

Clicker

A 75 kg man weights himself at the north pole and at the equator. Which scale reading is greater?

1. north pole;
2. Equator;
3. Both are equal.

Solution

A 75 kg man weights himself at the north pole and at the equator. How much lighter is his apparent weight at the equator?

- North pole $v = 0$ ($v = R\omega$, $R = 0$)

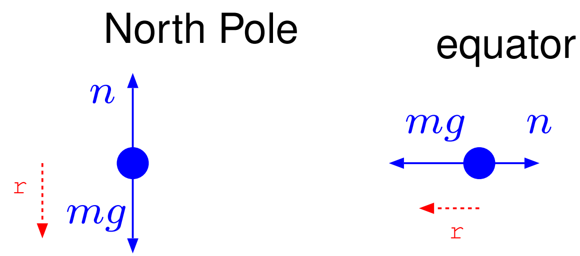
$$mg - n = 0$$

$$\Rightarrow \quad \text{apparent weight} = n = mg$$

- Equator $\omega = \frac{2\pi}{T} = 7.3 \times 10^{-5}$ rad/s

$$mg - n = m\frac{v^2}{R} = mR\omega^2$$

$$\Rightarrow \quad \text{apparent weight} = n = mg - mR\omega^2$$



- Difference is $mR\omega^2 = 2.5$ N or $\approx 1/2$ lb lighter at the equator. $R = 6.37 \times 10^6$ m

Clicker

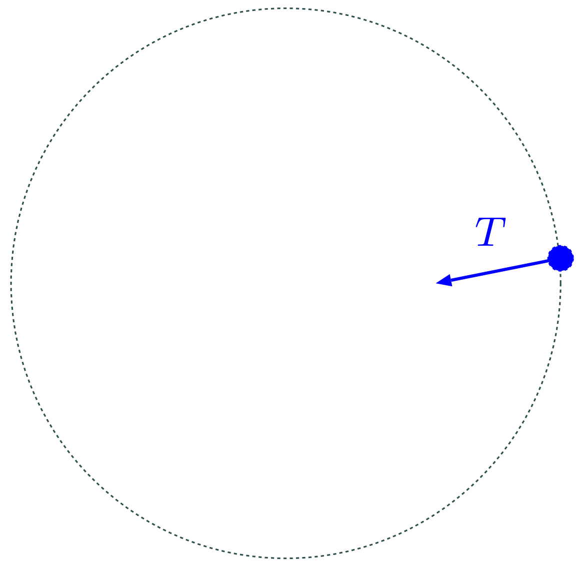
An object attached to a string is whirled in a circle of radius r in the horizontal plane. In addition, the speed of the object is constant. What is the cause of the outward force acting on the object that keeps the string stretched?

1. Centrifugal force;
2. Inertial force;
3. There is no outward force;
4. Gravity;
5. My hand whirling it.

Solution

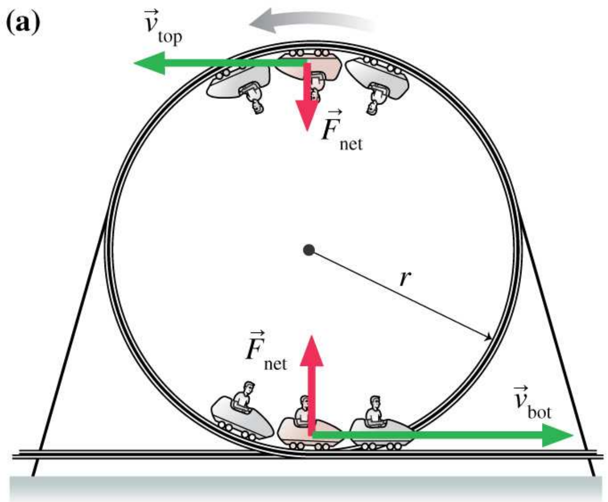
There is no outward force. According to Newton's first law the object wants to continue in a straight line, therefore the tension in the string is pulling it inward from the path it wants to take.

$$T = m\frac{v^2}{r}$$

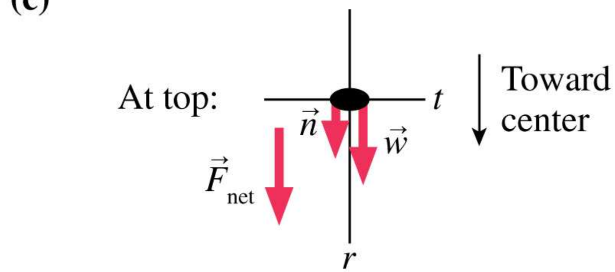


Example 3

Consider a roller coaster, what is the apparent weight of a passenger of mass m at the top and bottom of the of the path? The speed at the top is v_t, at the bottom it is v_b.



At the top



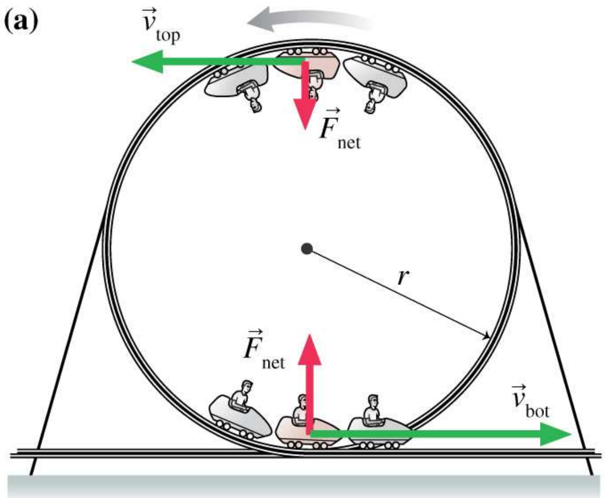
Forces in radial direction

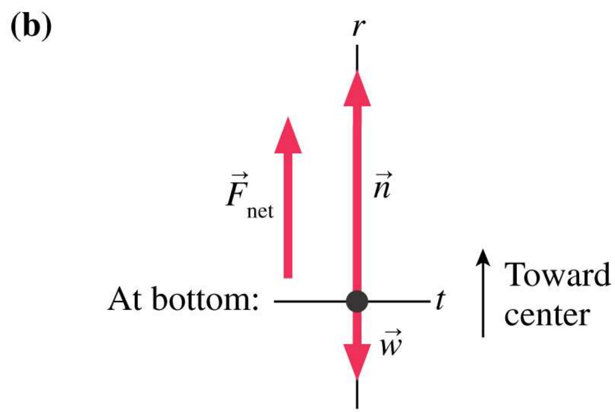
$$n + mg = m\frac{v_t^2}{r} \quad \Rightarrow \quad n = m\frac{v_t^2}{r} - mg$$

Example 3

Consider a roller coaster, what is the apparent weight of a passenger of mass m at the top and bottom of the of the path? The speed at the top is v_t, at the bottom it is v_b.

At the bottom



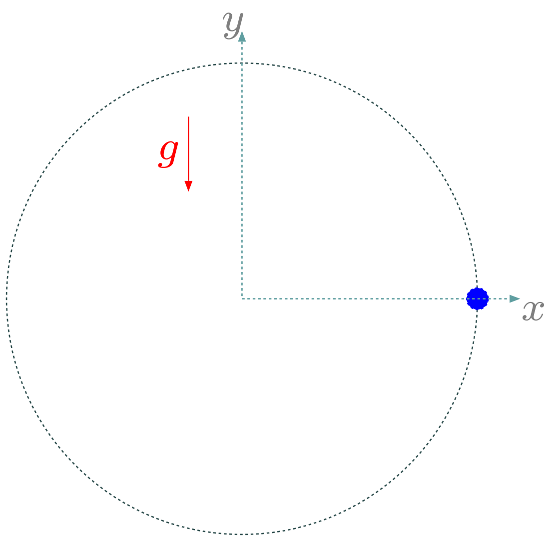


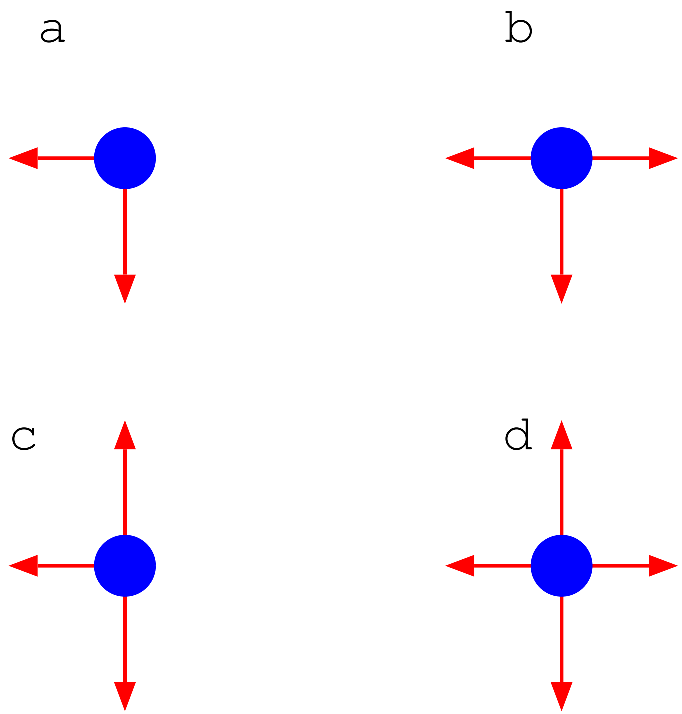
Forces in radial direction

$$n - mg = m\frac{v_b^2}{r} \quad \Rightarrow \quad n = m\frac{v_b^2}{r} + mg$$

Clicker

An object follows a circular trajectory at a constant speed. At the location shown on the figure, which force diagram (free-body diagram) describes the system. *Assume that there is no friction*





Assignment

Start reading Chapter 8