Lecture 22 Interference, Diffraction

Looking forward at …

- what happens when two waves combine, or interfere, in space.
- how to understand the interference pattern formed by the interference of two coherent light waves.
- What is the pattern of interference from two slits, when the screen at which we project it is very far away: double slit (Young's) experiment.
- what is a single-slit diffraction pattern.
- what happens when coherent light shines on an array of narrow, closely spaced slits.
- how x-ray diffraction reveals the arrangement of atoms in a crystal.
- how diffraction sets limits on the smallest details that can be seen with an optical system.

Introduction

- Why do soap bubbles show vibrant color patterns, even though soapy water is colorless?

- What causes the multicolored reflections from DVDs?
- We will now look at optical effects, such as interference, that depend on the wave nature of light.

Principle of superposition

- The term **interference** refers to any situation in which two or more waves overlap in space.

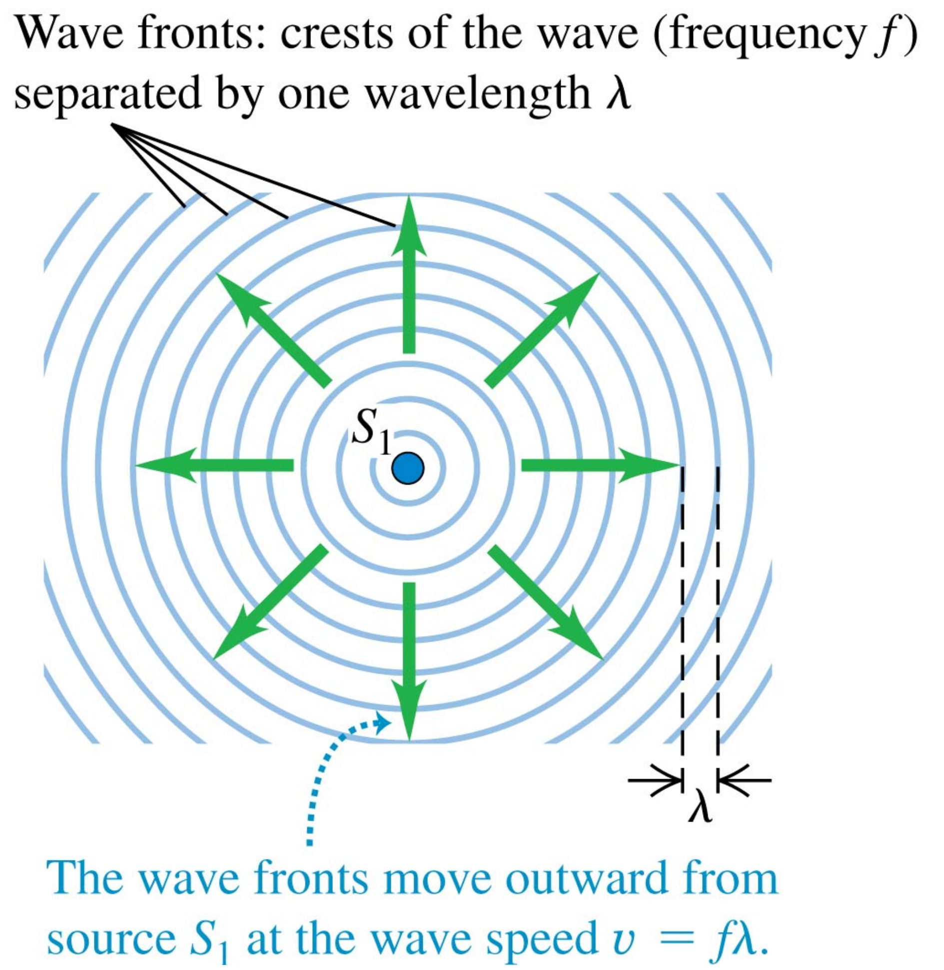
- When this occurs, the total wave at any point at any instant of time is governed by the **principle of superposition**:

When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.

For EM waves: displacements-> E and B fields

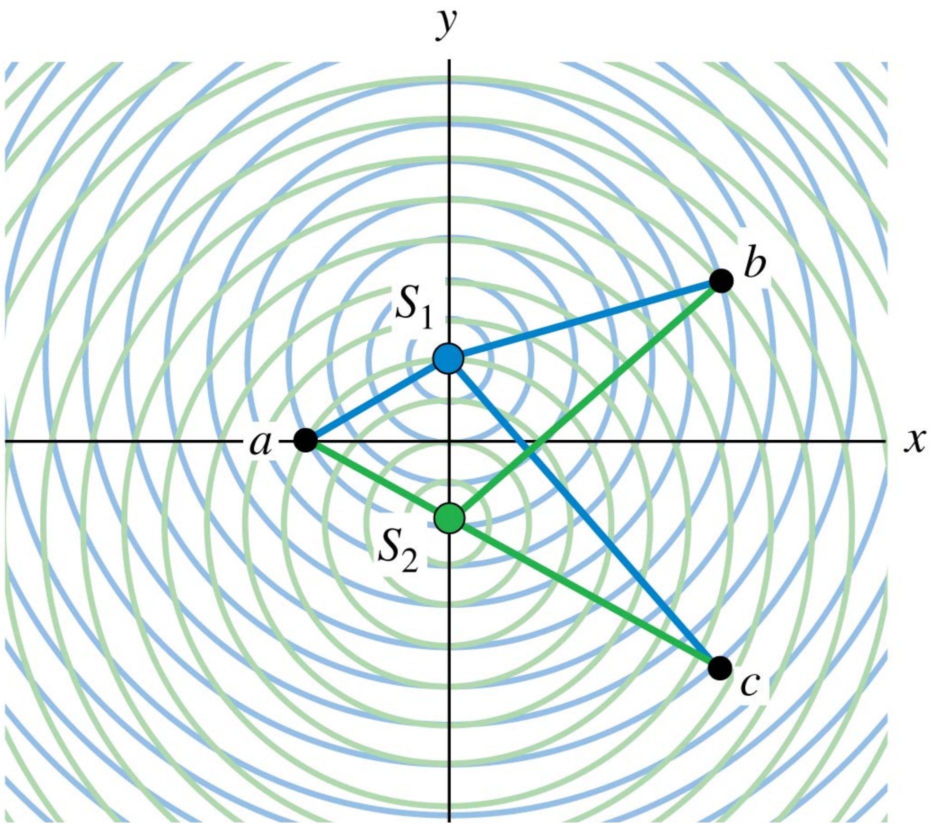
Wave fronts from a single source

- Interference effects are most easily seen when we combine sinusoidal waves with a single frequency and wavelength.
- Shown is a "snapshot" of a single source S_1 of sinusoidal waves and some of the wave fronts produced by this source.



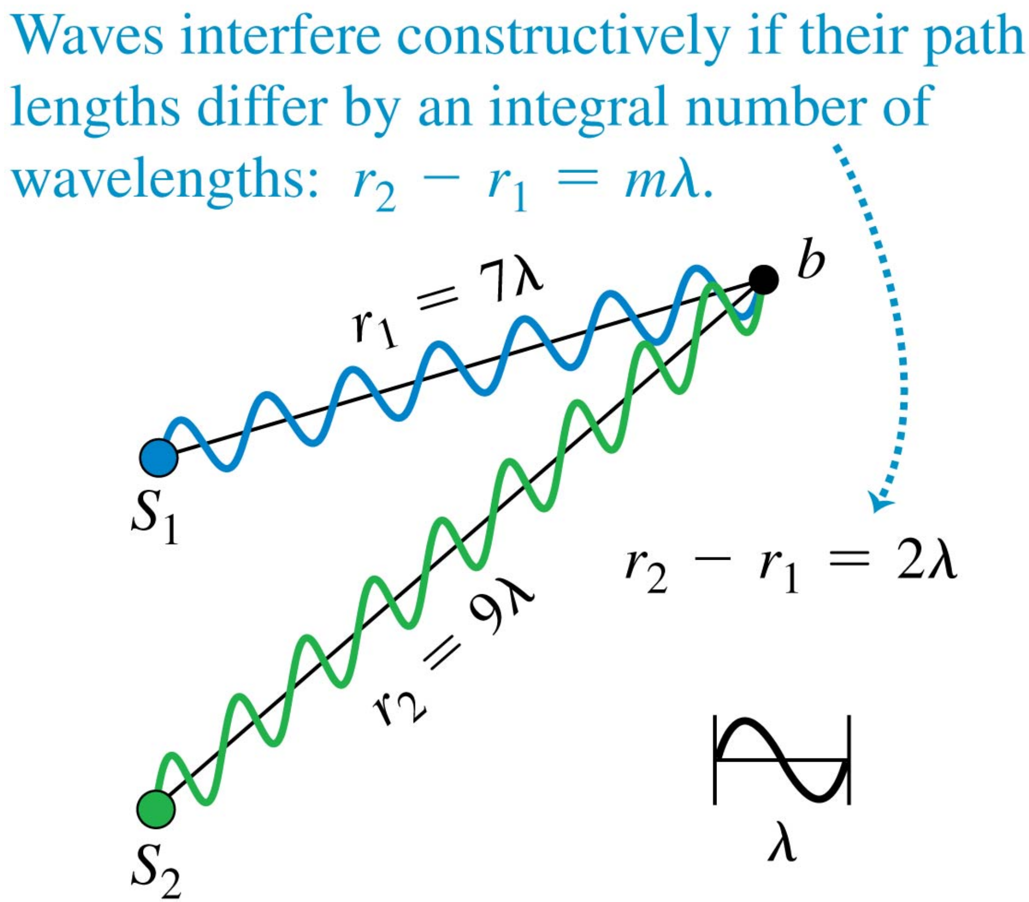
Constructive and destructive interference

- Shown are two identical sources of monochromatic waves, S_1 and S_2.
- The two sources are permanently *in phase*; they vibrate in unison.
- Constructive interference occurs at point *a* (equidistant from the two sources).



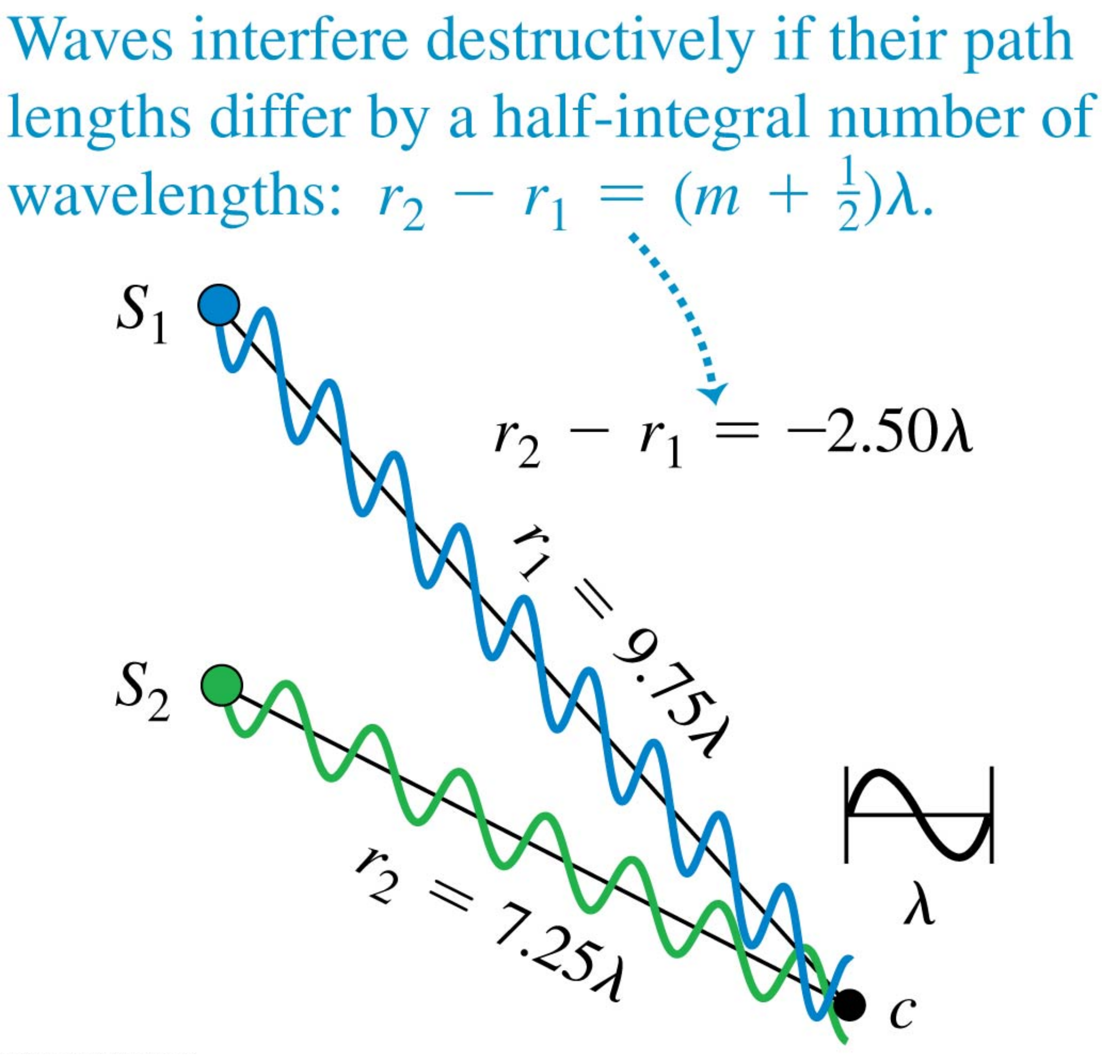
Conditions for constructive interference

- The distance from S_2 to point b is exactly two wavelengths greater than the distance from S_1 to b.
- The two waves arrive *in phase*, and they reinforce each other.
- This is called **constructive interference**.



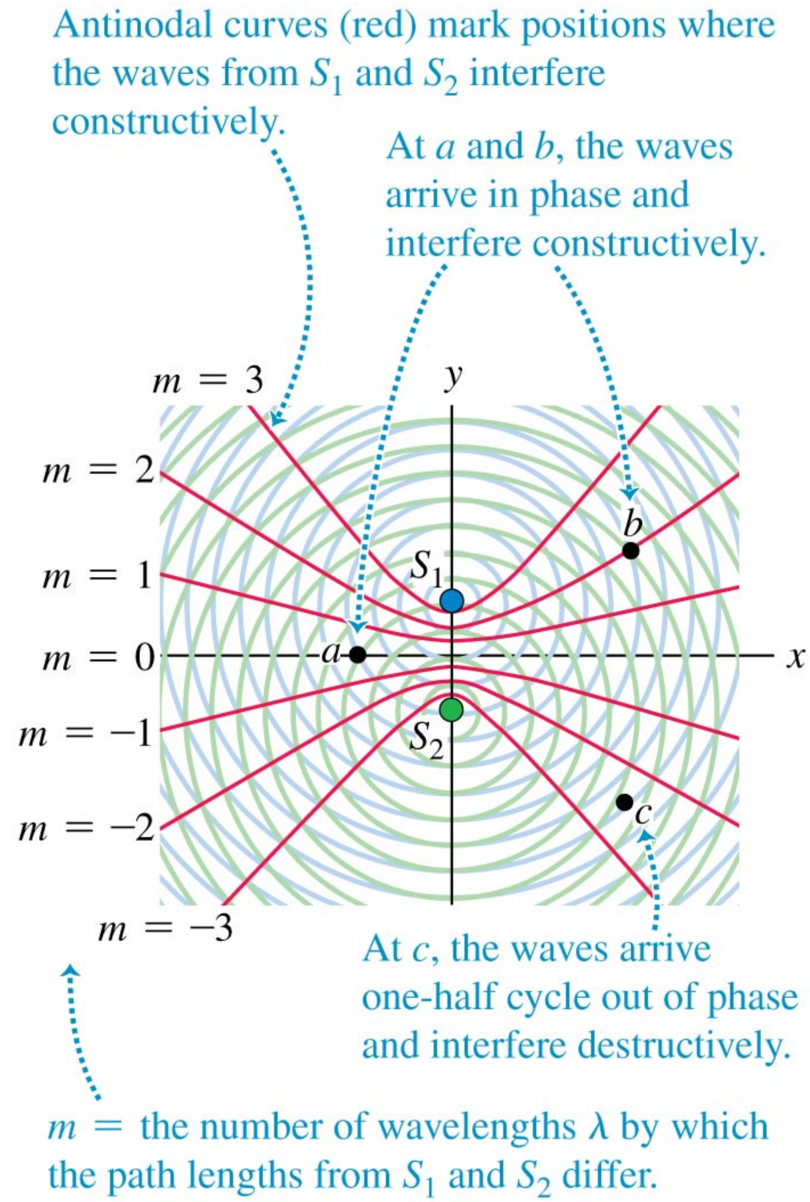
Conditions for destructive interference

- The distance from S_1 to point c is a *half-integral* number of wavelengths greater than the distance from S_2 to c.
- The two waves cancel or partly cancel each other.
- This is called **destructive interference**.



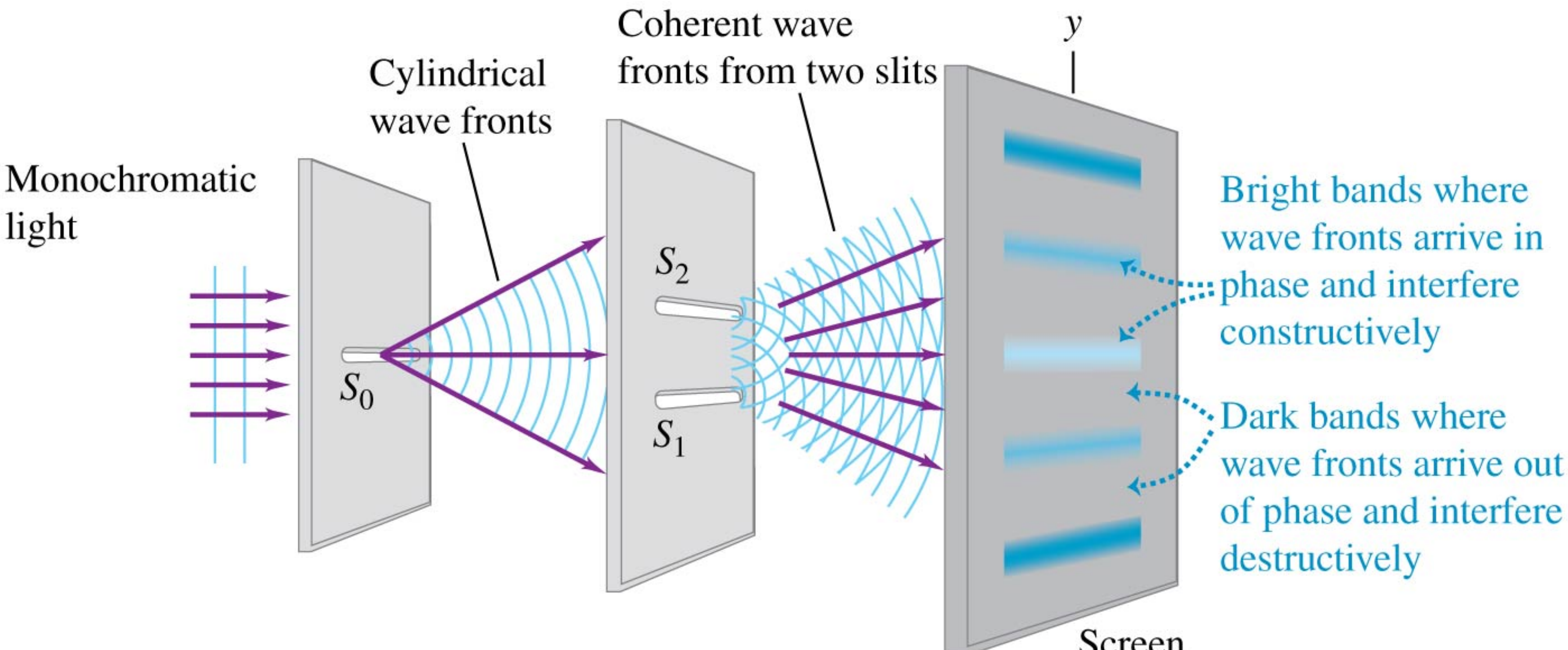
Constructive and destructive interference

- Shown are two identical sources of monochromatic waves, S_1 and S_2, which are in phase and at a distance $m\lambda < d < (m+1)\lambda$
- The red curves show all positions where constructive interference occurs; these curves are called **antinodal curves**.
- Not shown are the **nodal curves**, which are the curves that show where destructive interference occurs.
- The destructive interference is not perfect for $m \neq 0$ and small distances



Two-source interference of light

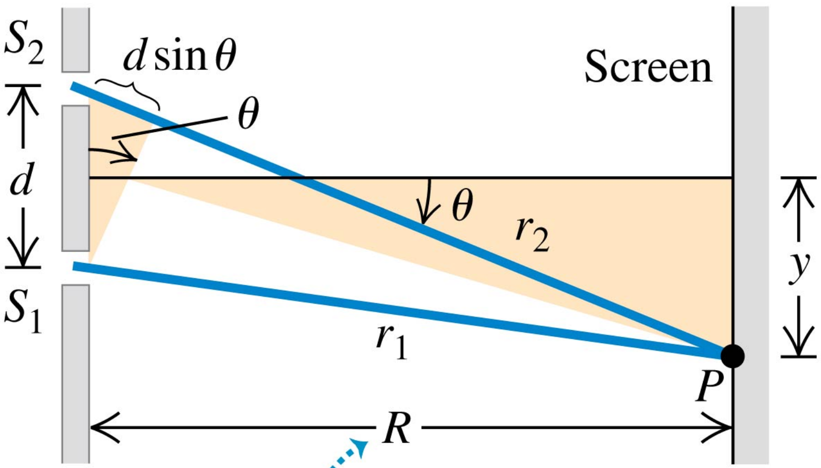
- Shown below is one of the earliest quantitative experiments to reveal the interference of light from two sources, first performed by Thomas Young.
- The interference of waves from slits S_1 and S_2 produces a pattern on the screen.



Two-source interference of light

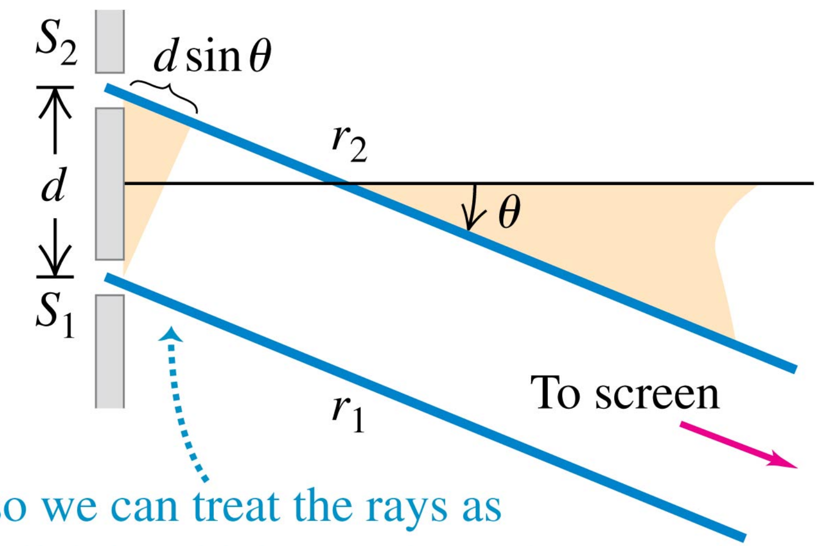
- (b) shows the actual geometry of Young's experiment.
- If the distance R to the screen is much greater than the distance d between the slits, we can use the approximate geometry shown in (c).

(b) Actual geometry (seen from the side)



In real situations, the distance R to the screen is usually very much greater than the distance d between the slits ...

(c) Approximate geometry



... so we can treat the rays as parallel, in which case the path difference is simply $r_2 - r_1 = d \sin\theta$.

Interference from two slits

- Constructive interference (reinforcement) occurs at points where the path difference is an integral number of wavelengths, $m\lambda$. **Here λ is the wavelength in**

- So the bright regions on the screen occur at angles θ for which

Constructive interference, two slits:

Distance between slits — Wavelength

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Angle of line from slits to *m*th bright region on screen

- Similarly, destructive interference (cancellation) occurs, forming dark regions on the screen, at points for which the path difference is a half-integral number of wavelengths.

Destructive interference, two slits:

Distance between slits — Wavelength

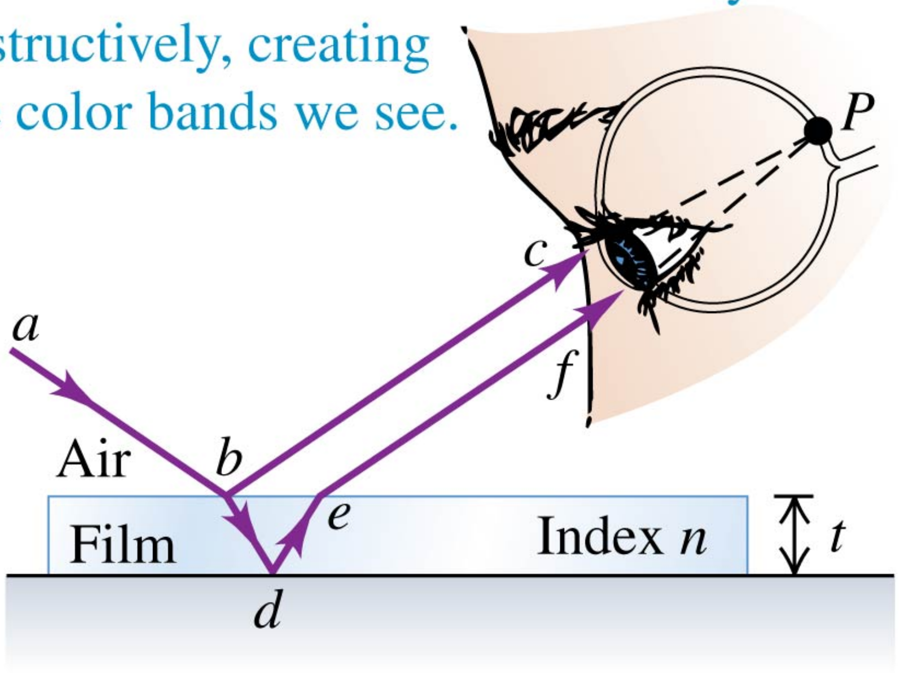
$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

Angle of line from slits to *m*th dark region on screen

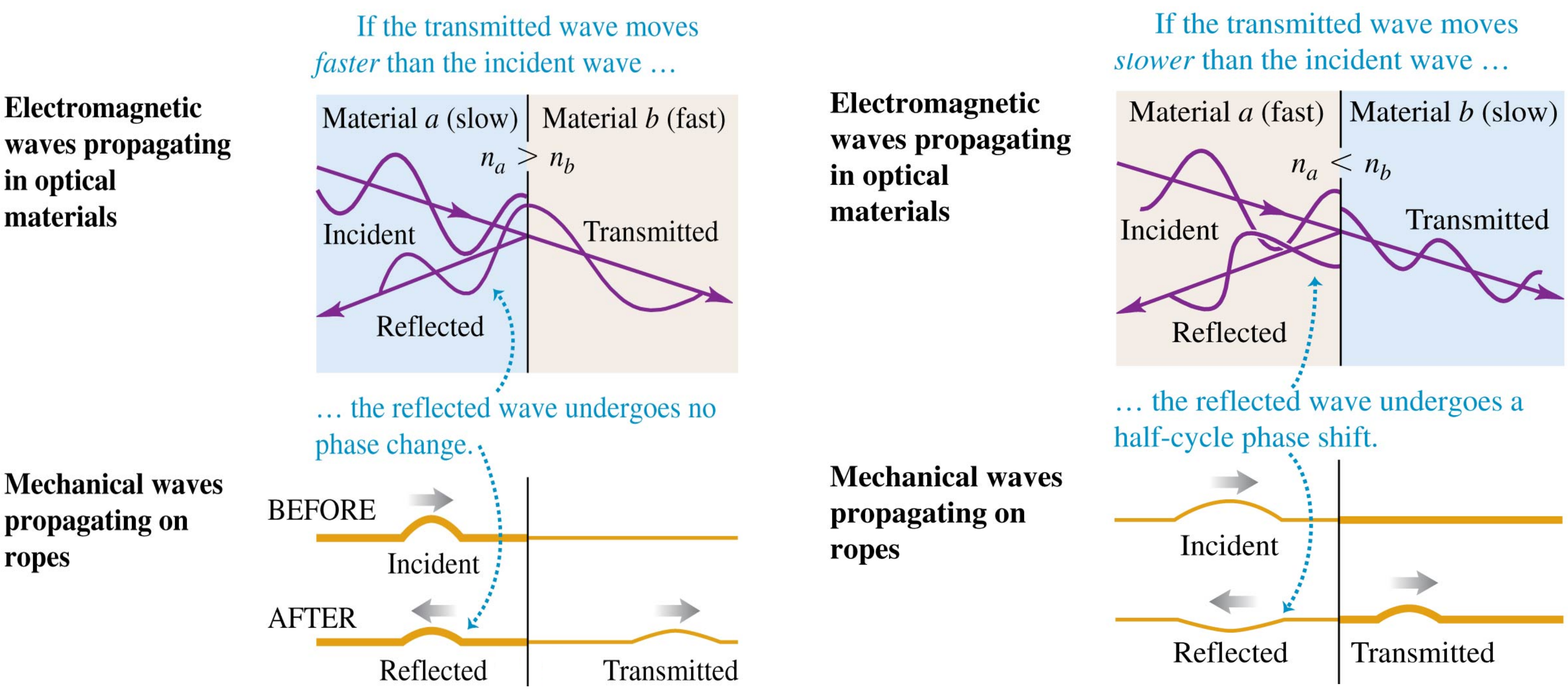
Interference in thin films

Light reflected from the upper and lower surfaces of the film comes together in the eye at *P* and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see.



Phase shifts during reflection



Interference in thin films

- For light of normal incidence on a thin film with wavelength λ in the film, in which neither or both of the reflected waves have a half-cycle phase shift:

Constructive reflection: $2t = m\lambda \quad (m = 0, 1, 2, \ldots)$

(From thin film, no relative phase shift) Thickness of film (t), Wavelength (λ)

Destructive reflection: $2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$

- When only one of the reflected waves has a half-cycle phase shift :

- $\Delta\phi - \pi = \frac{2\pi}{\lambda} 2t - \pi = 2\pi m,$ Dividing by $\frac{2\pi}{\lambda}$ we get:

Constructive reflection: $2t = \left(m + \frac{1}{2}\right)\lambda \quad (m = 0, 1, 2, \ldots)$

(From thin film, half-cycle phase shift) Thickness of film (t), Wavelength (λ)

Destructive reflection: $2t = m\lambda \quad (m = 0, 1, 2, \ldots)$

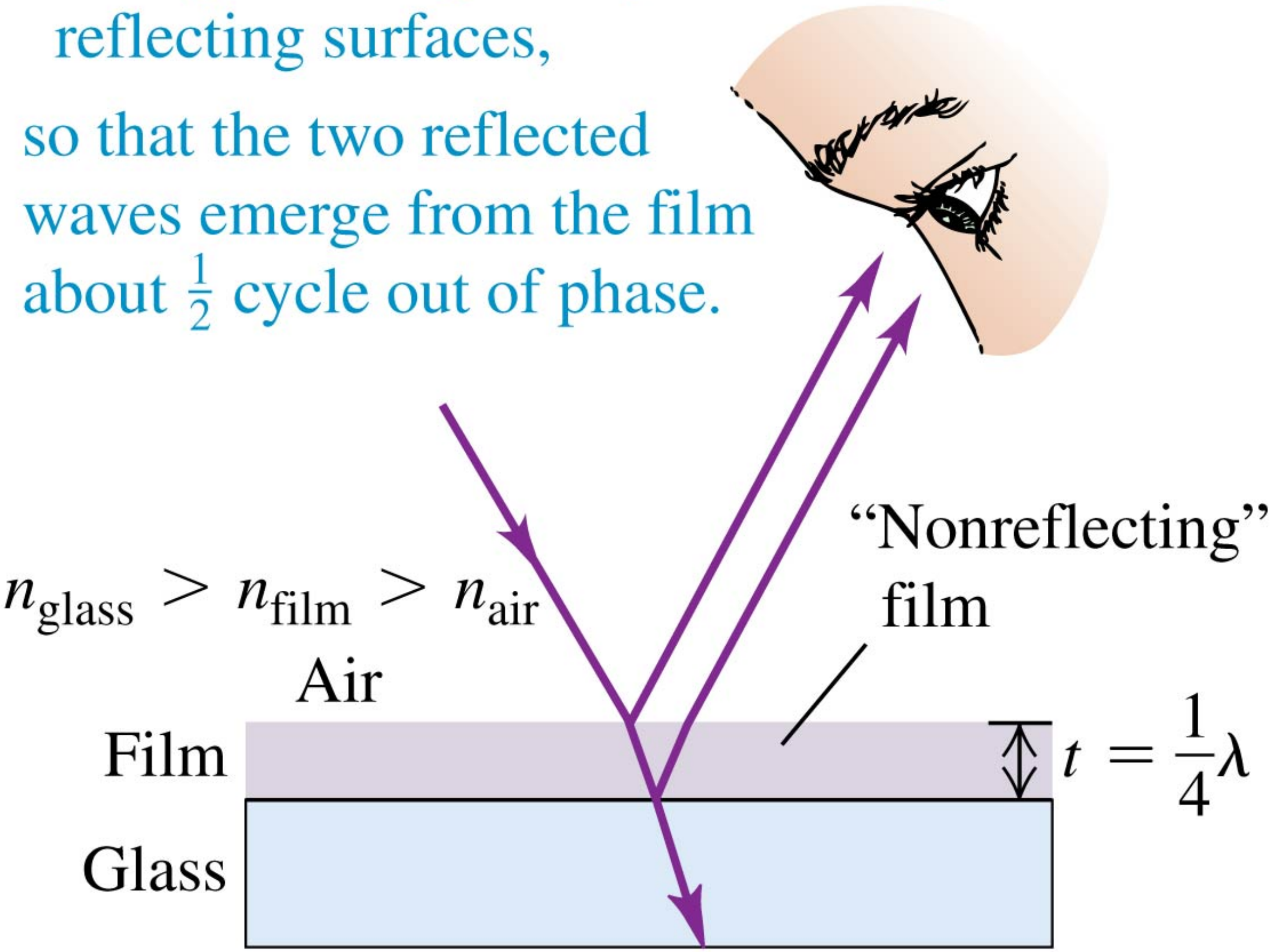
- **Here λ is the wavelength in the film!** $\lambda = \lambda_0/n$ *, where n is the index of refraction of the film.*

Nonreflective coatings

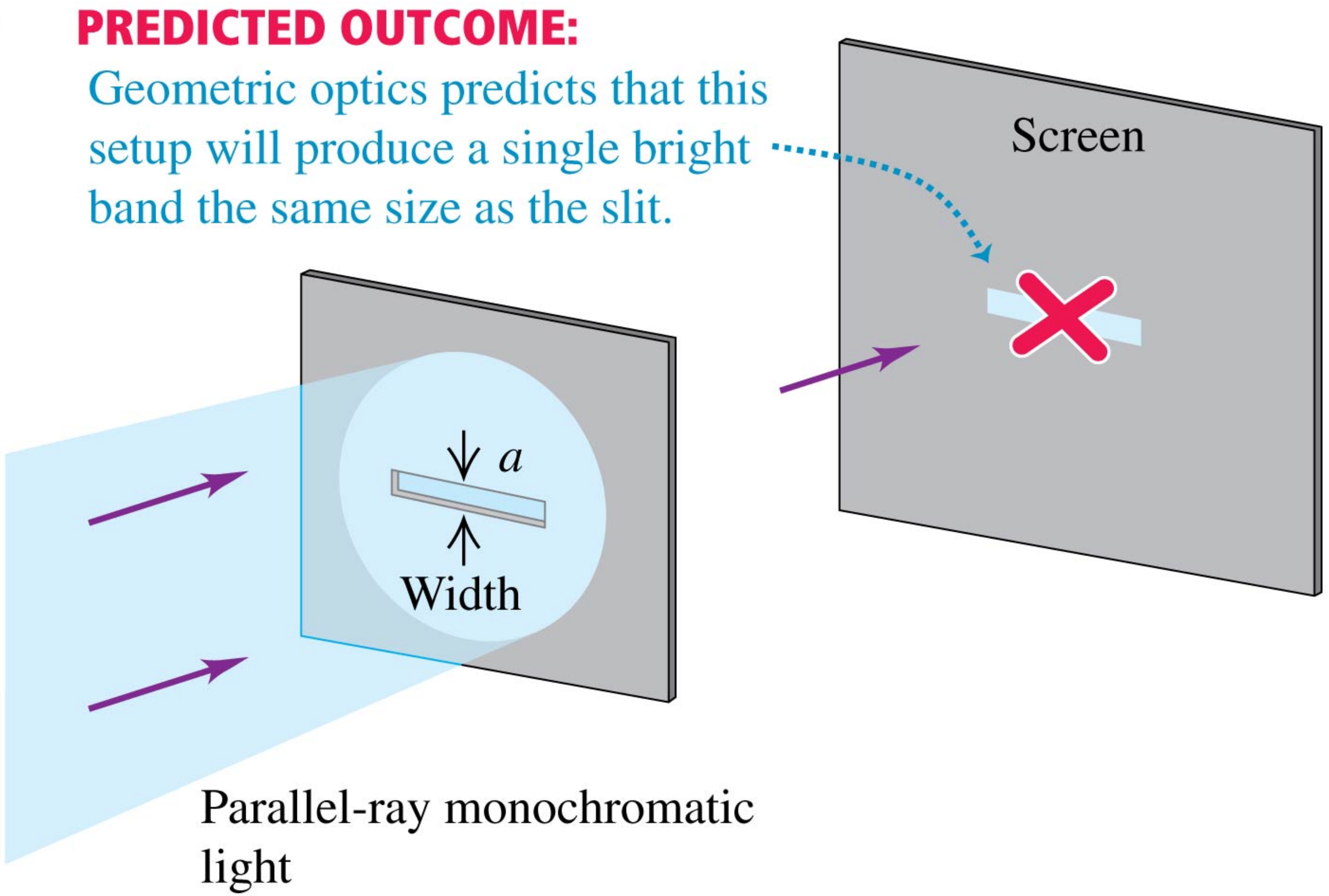
Destructive interference occurs when

- the film is about $\frac{1}{4}\lambda$ thick and
- the light undergoes a phase change at both reflecting surfaces,

so that the two reflected waves emerge from the film about $\frac{1}{2}$ cycle out of phase.



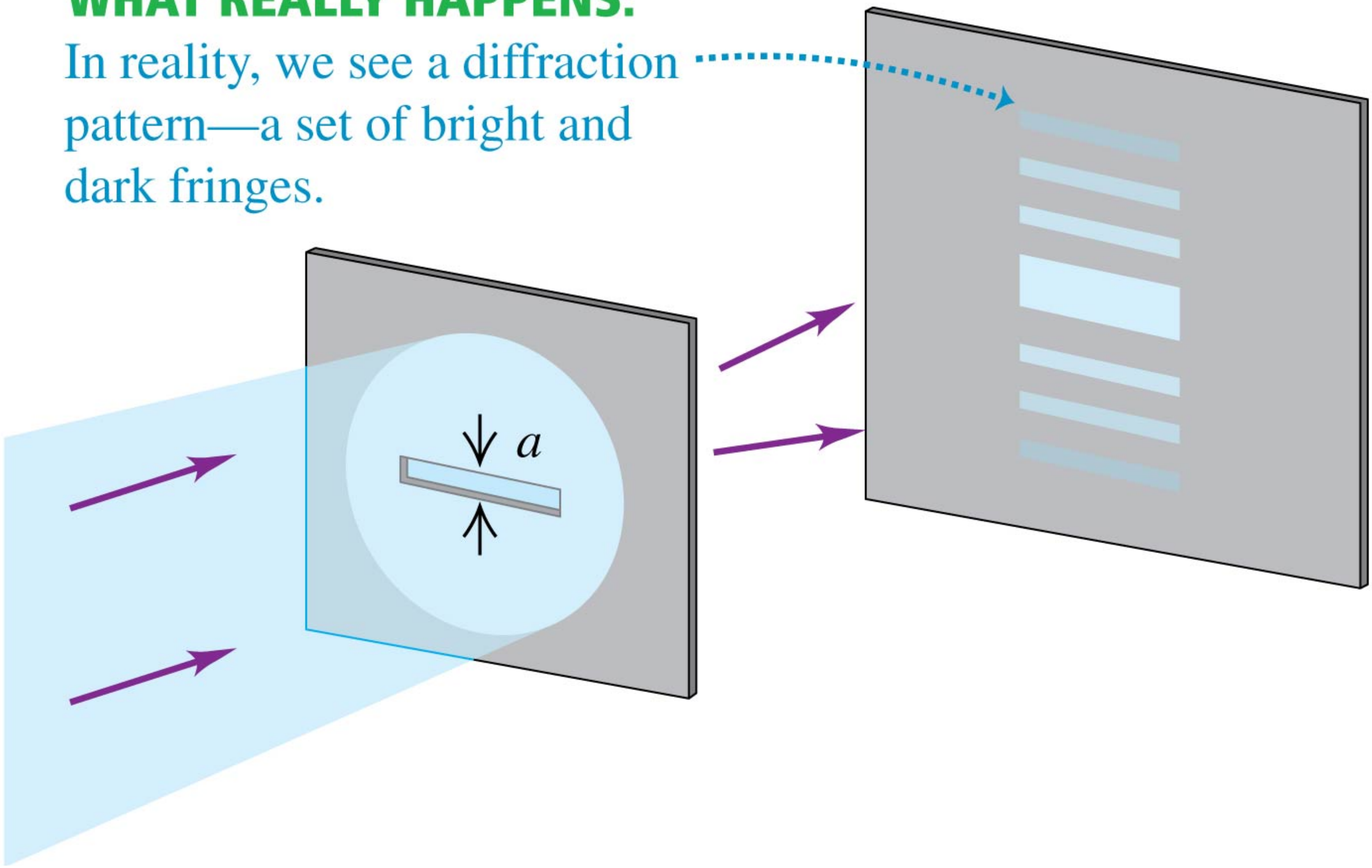
Diffraction from a single slit



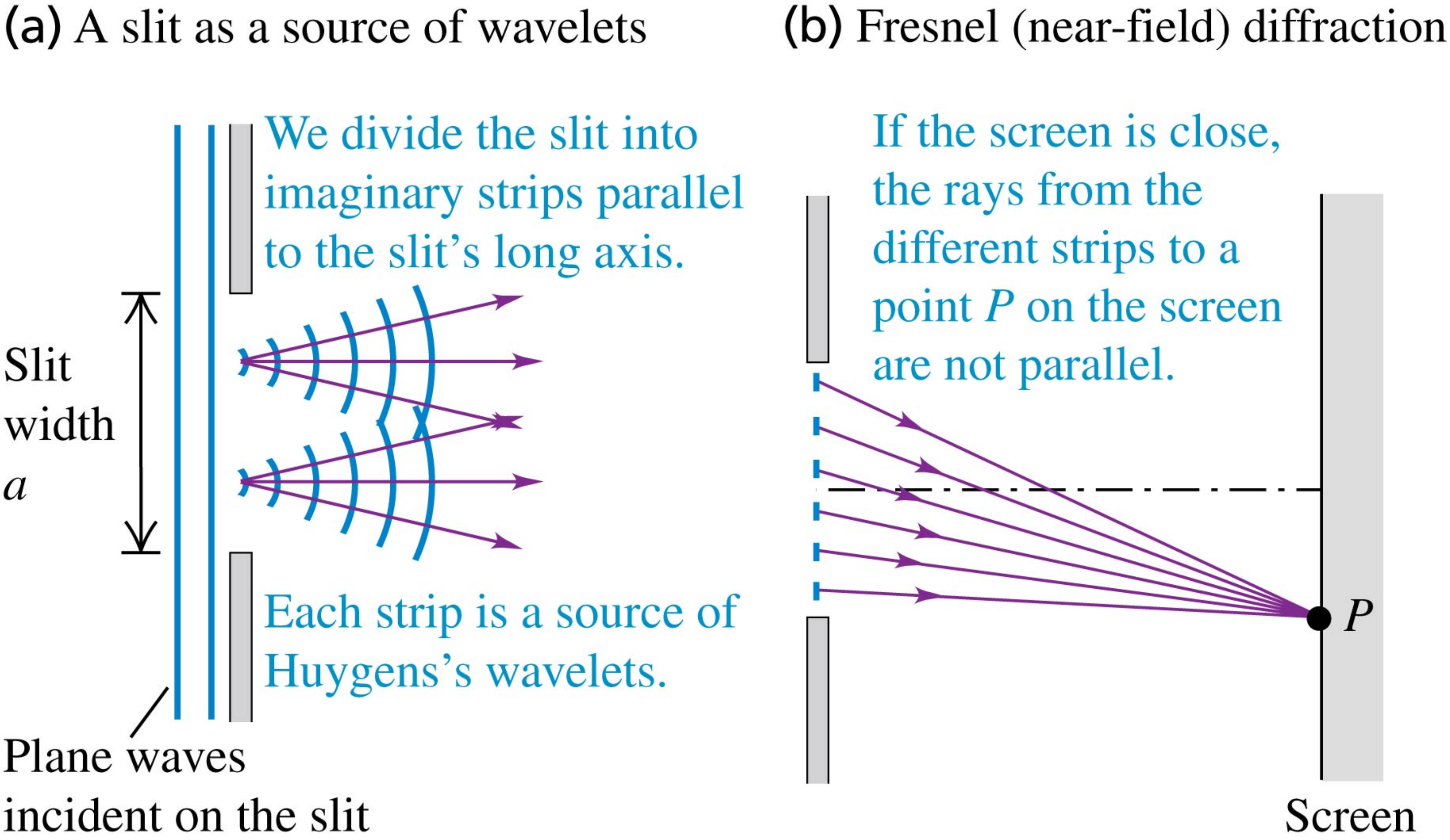
Diffraction from a single slit

WHAT REALLY HAPPENS:

In reality, we see a diffraction pattern—a set of bright and dark fringes.



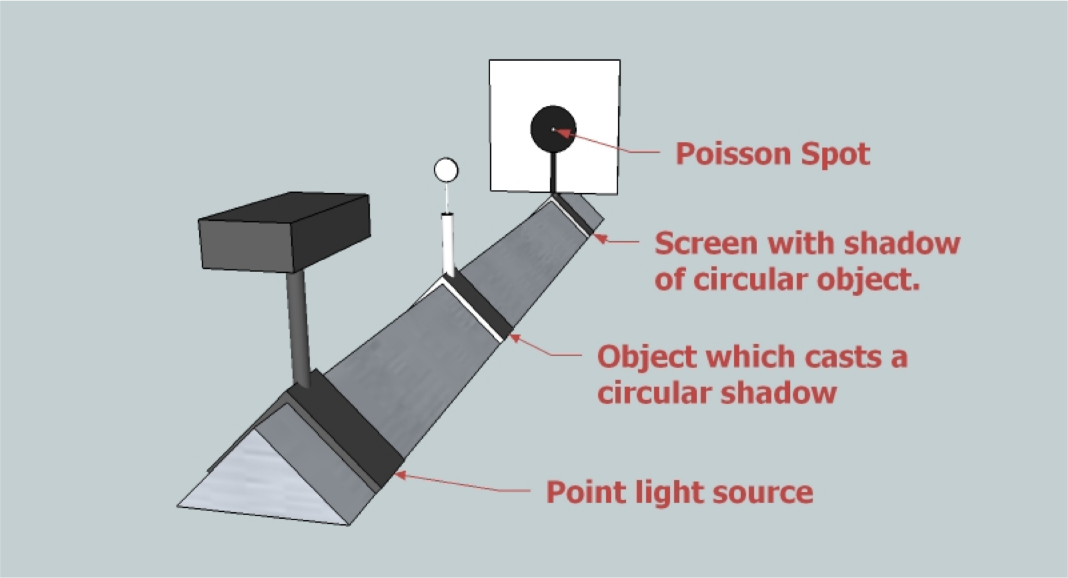
Fresnel diffraction by a single slit



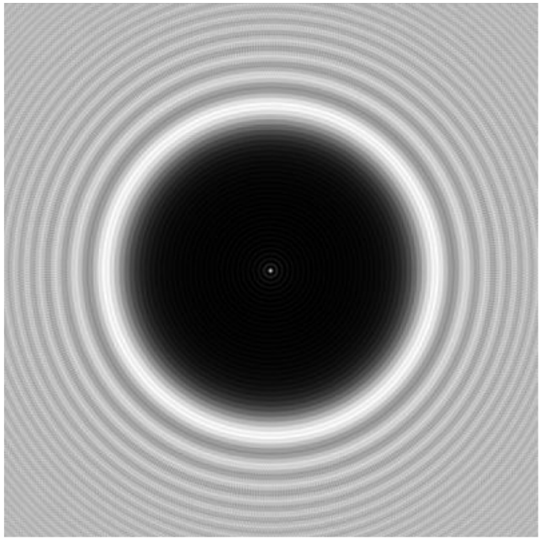
Frensel/Poisson/Arago spot

The weird phenomenon that convinced French academy that light is a wave.

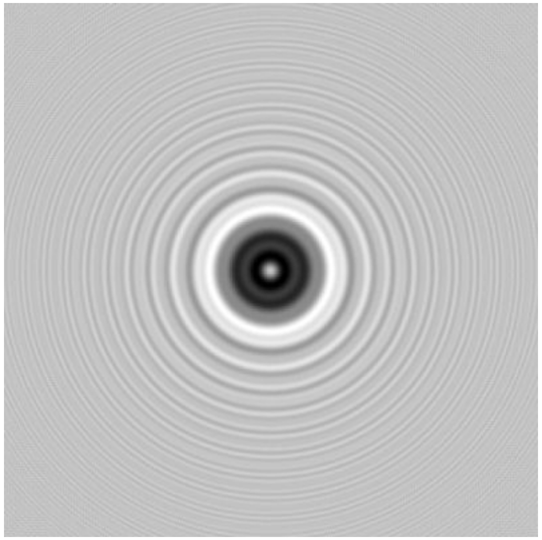
D – diameter of disk
L– distance to screen



D=2mm
L=1m

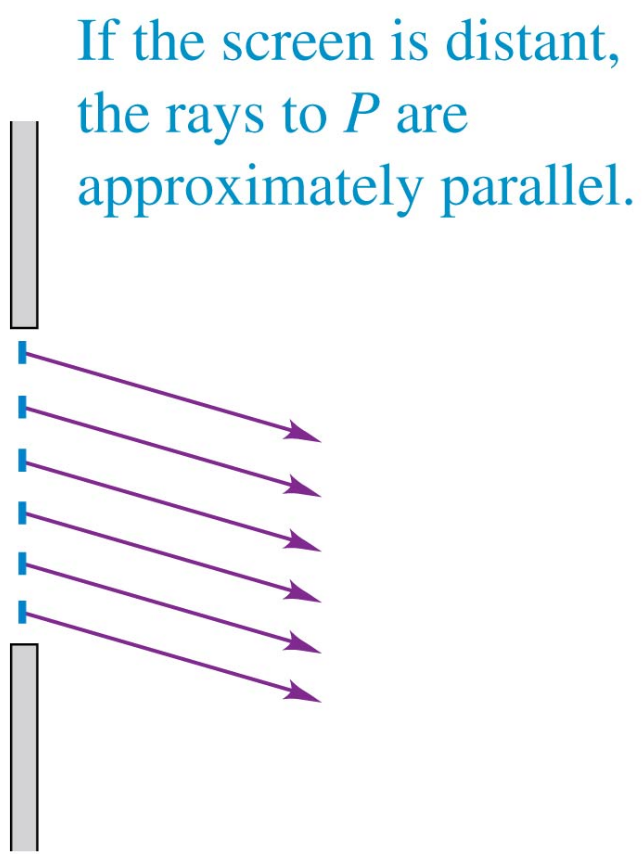


D=1mm
L=1m

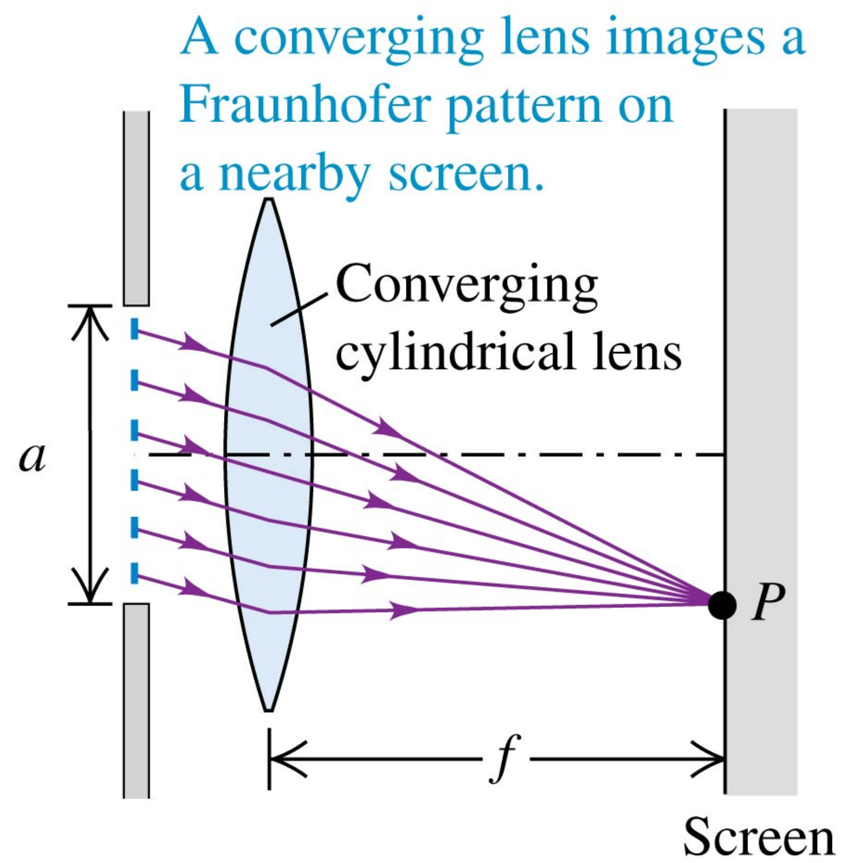


Fraunhofer diffraction by a single slit

(c) Fraunhofer (far-field) diffraction

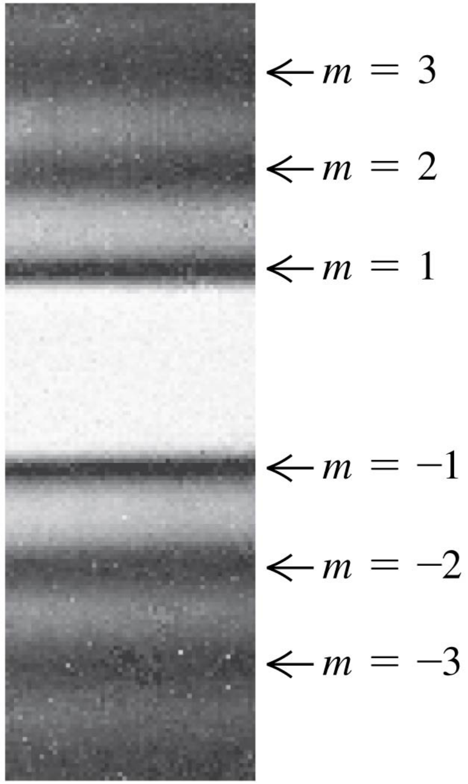


(d) Imaging Fraunhofer diffraction



Locating the dark fringes

- Shown is the Fraunhofer diffraction pattern from a single horizontal slit.
- It is characterized by a central bright fringe centered at $\theta = 0$, surrounded by a series of dark fringes.
- The central bright fringe is twice as wide as the other bright fringes.
- Opposite to the 2-slit interference:



Dark fringes, single-slit diffraction:

Angle of line from center of slit to *m*th dark fringe on screen

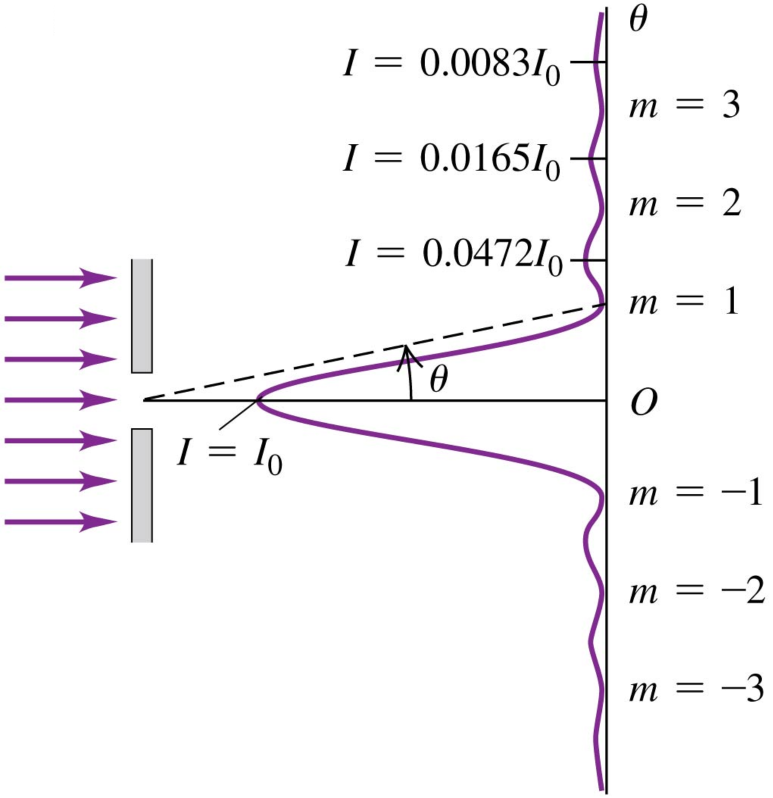
$$\sin\theta = \frac{m\lambda}{a} \quad (m = \pm 1, \pm 2, \pm 3, \ldots)$$

Slit width (a); Wavelength (λ)

Intensity maxima in a single-slit pattern

- Shown is the intensity versus angle in a single-slit diffraction pattern.
- Most of the wave power goes into the central intensity peak (between the $m = 1$ and $m = -1$ intensity minima).

(For illustration only, won't appear on the Final)



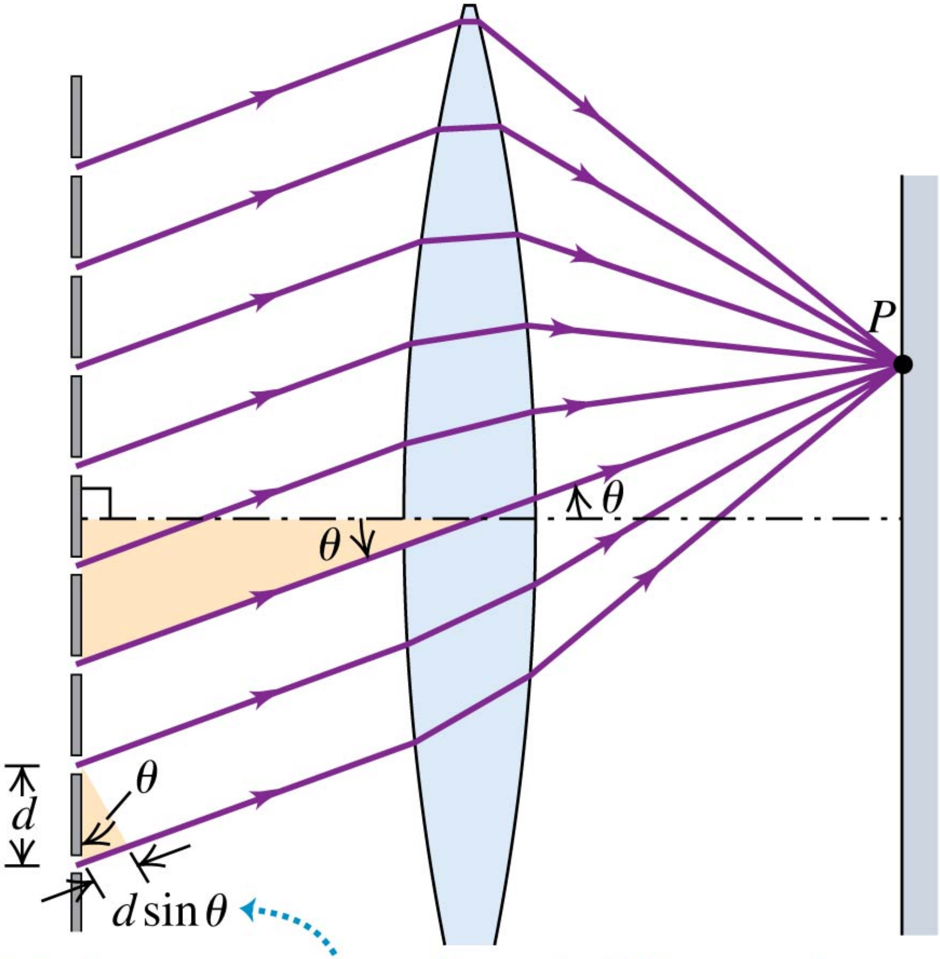
Angle of line from center of slit to position on screen

Intensity in single-slit diffraction

$$I = I_0 \left\{ \frac{\sin[\pi a (\sin\theta)/\lambda]}{\pi a (\sin\theta)/\lambda} \right\}^2$$

Intensity at $\theta = 0$ Slit width Wavelength

Several slits

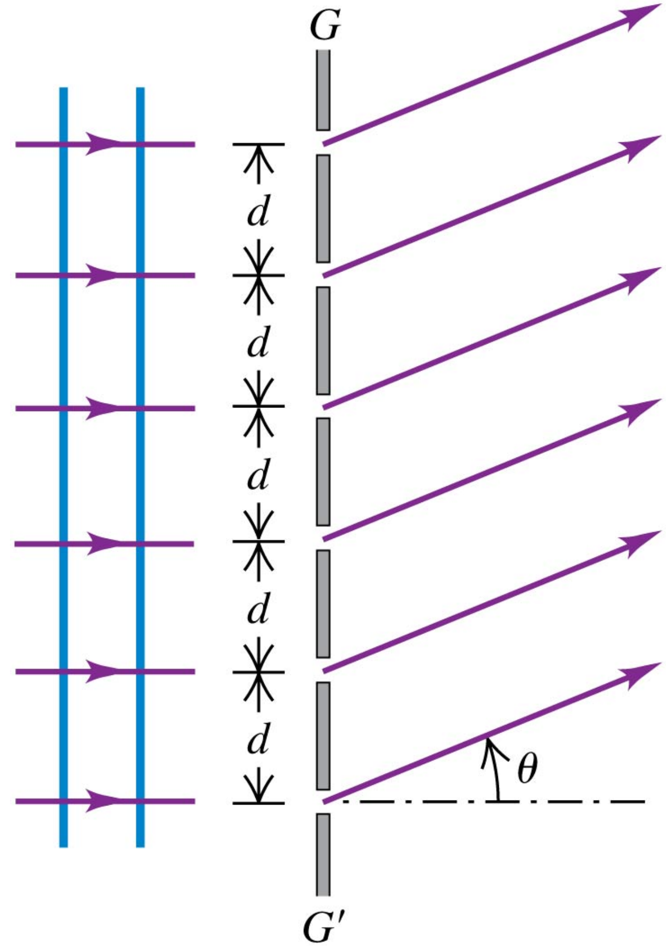


Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d \sin\theta = m\lambda$.

- Shown is an array of eight narrow slits, with distance *d* between adjacent slits.
- Constructive interference occurs for rays at angle θ to the normal that arrive at point *P* with a path difference between adjacent slits equal to an integer number of wavelengths.

The diffraction grating

- An array of a large number of parallel slits is called a **diffraction grating**.
- In the figure, GG' is a cross section of a transmission grating.
- The slits are perpendicular to the plane of the page.
- The diagram shows only six slits; an actual grating may contain several thousand.



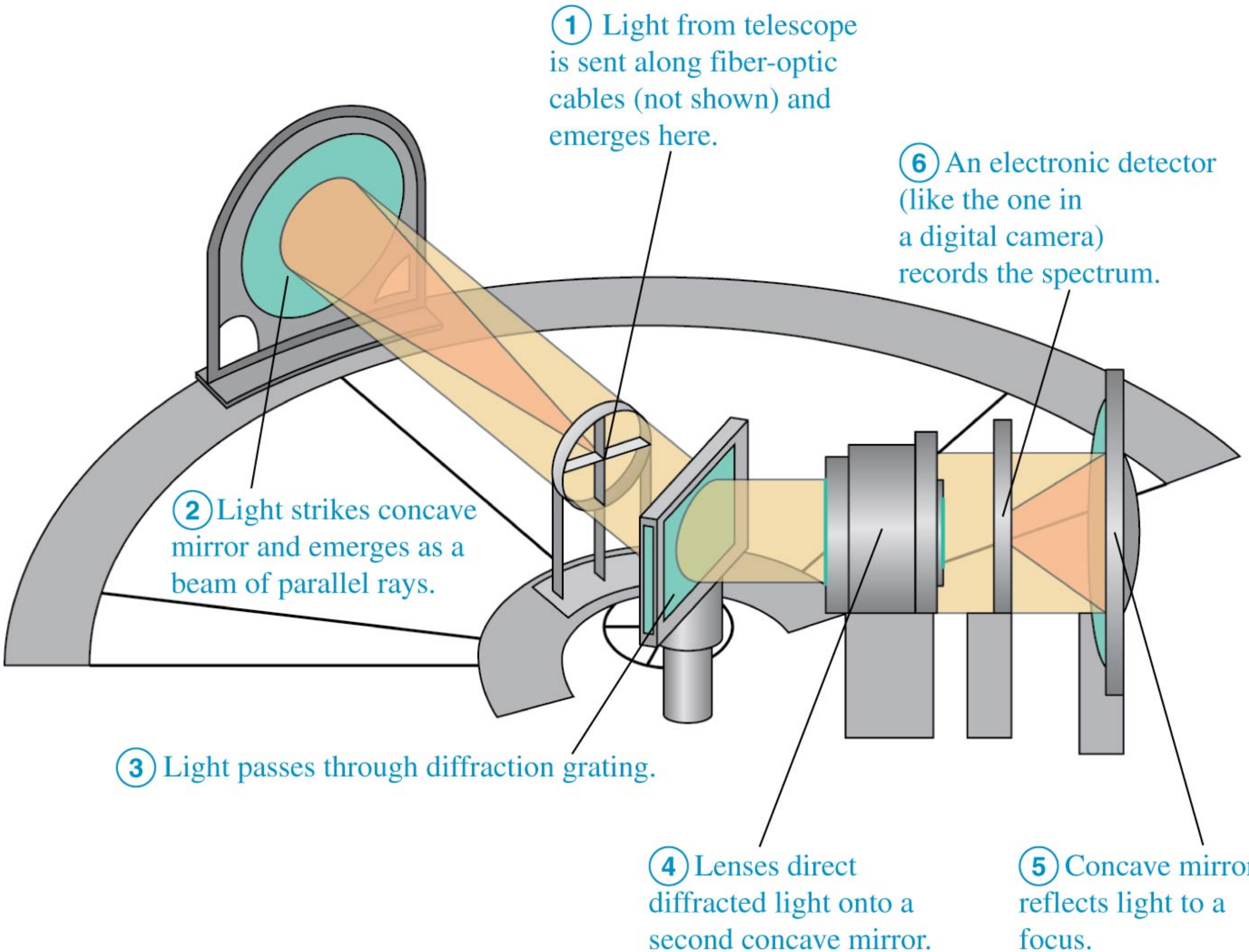
Intensity maxima, multiple slits:

Distance between slits; Wavelength

$$d \sin\theta = m\lambda \quad (m = 0, \pm 1, \pm 2, \dots)$$

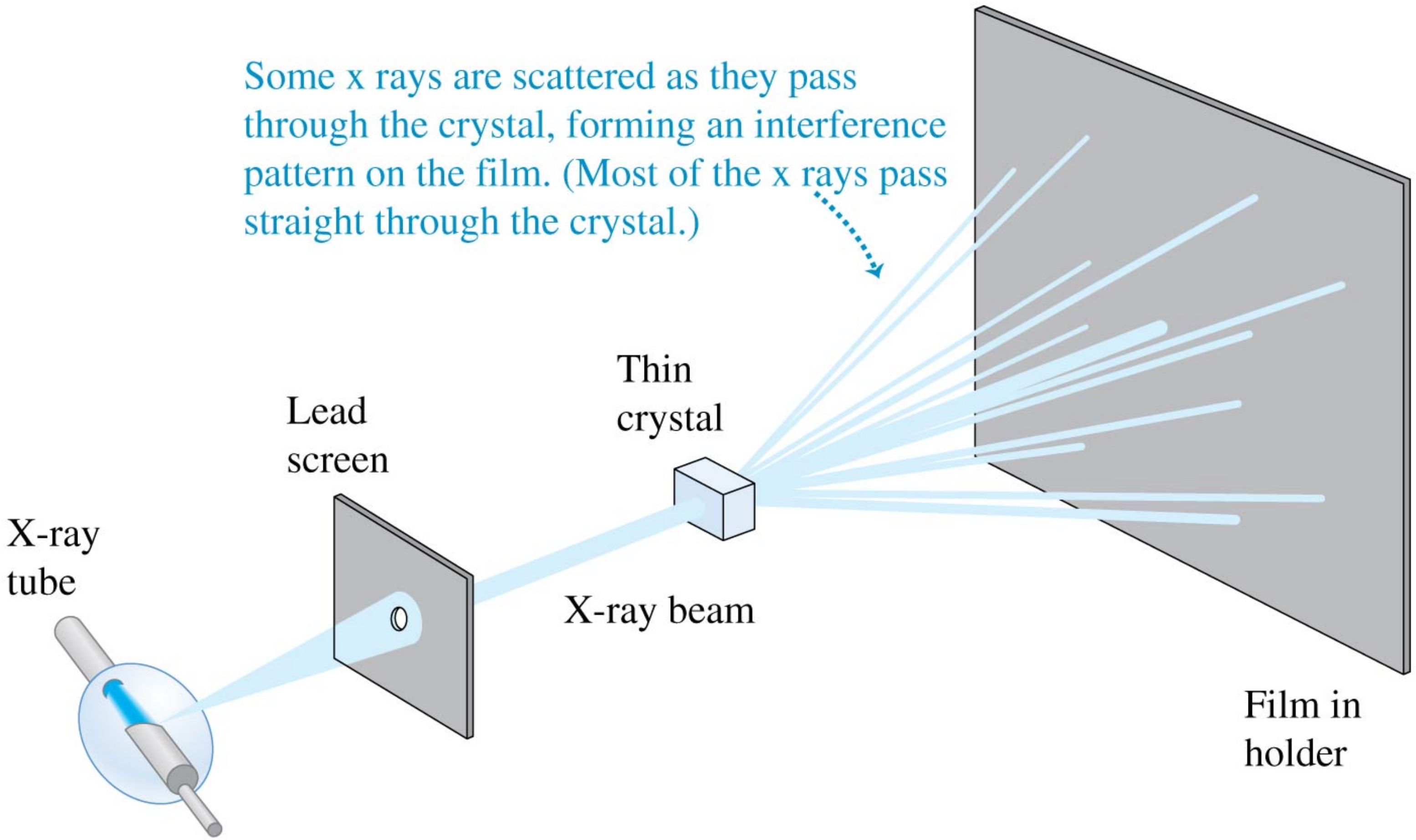
Angle of line from center of slit array to *m*th bright region on screen

Diagram of a grating spectrograph



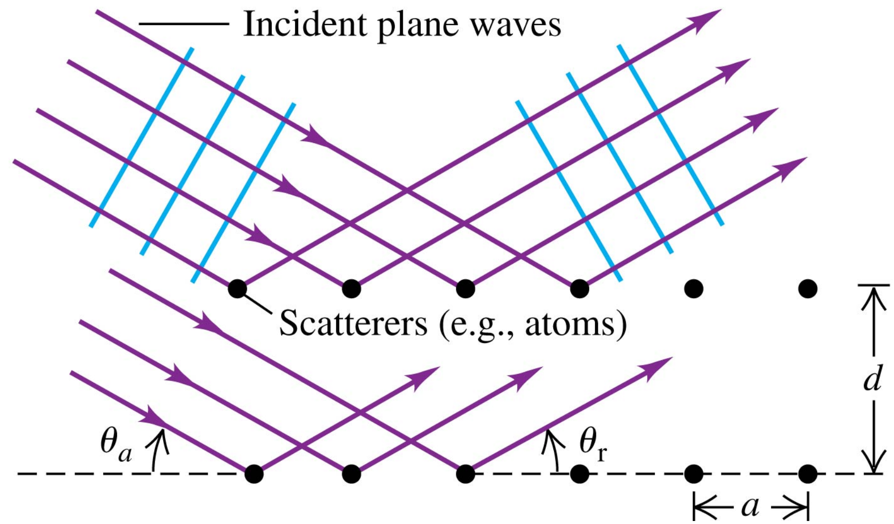
X-ray diffraction

- When x rays pass through a crystal, the crystal behaves like a diffraction grating, causing **x-ray diffraction**.



A simple model of x-ray diffraction

- To better understand x-ray diffraction, we consider a two-dimensional scattering situation.
- The path length from source to observer is the same for all the scatterers in a single row if $\theta_a = \theta_r = \theta$.



Bragg condition for constructive interference from an array:

Distance between adjacent rows in array

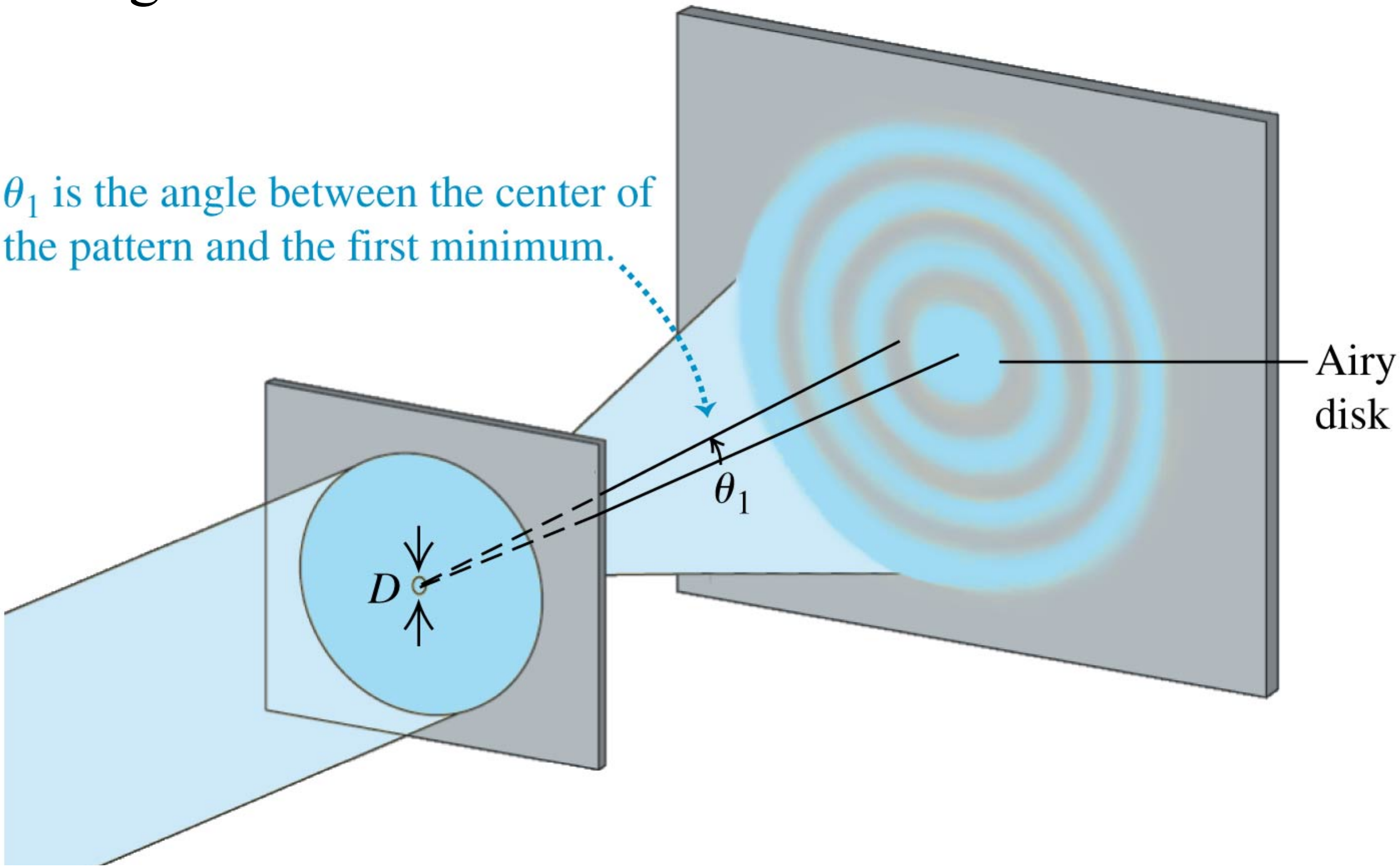
Wavelength

$$2d \sin\theta = m\lambda \quad (m = 1, 2, 3, \ldots)$$

Angle of line from surface of array to *m*th bright region on screen

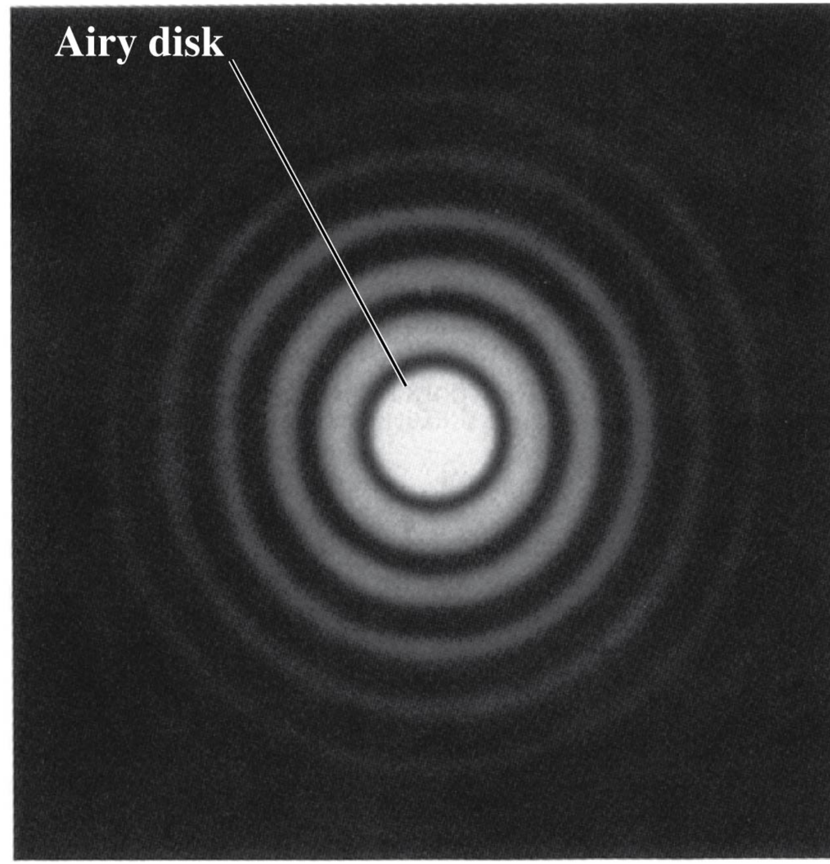
Circular apertures

- The diffraction pattern formed by a circular aperture consists of a central bright spot surrounded by a series of bright and dark rings.



Diffraction by a circular aperture

- The central bright spot in the diffraction pattern of a circular aperture is called the Airy disk.
- We can describe the radius of the Airy disk by the angular radius θ_1 of the first dark ring:



Diffraction by a circular aperture:

Angular radius of first dark ring = angular radius of Airy disk

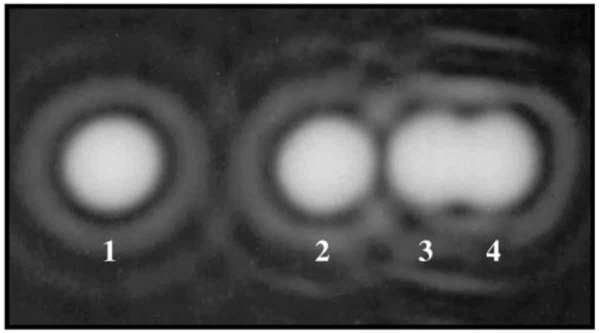
$$\sin\theta_1 = 1.22\frac{\lambda}{D}$$

λ: Wavelength; D: Aperture diameter

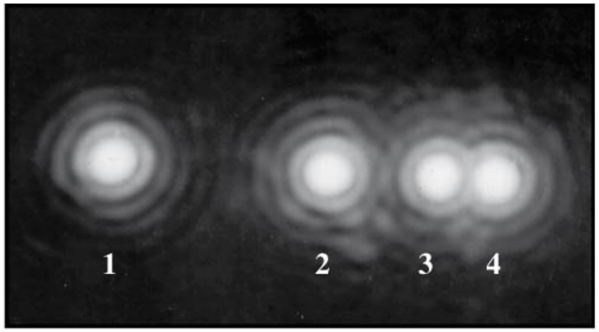
Diffraction and image formation

- Diffraction limits the resolution of optical equipment, such as telescopes.
- The larger the aperture, the better the resolution.
- A widely used criterion for resolution of two point objects, is called **Rayleigh's criterion**:
  - Two objects are just barely resolved (that is, distinguishable) if the center of one diffraction pattern coincides with the first minimum of the other.

(a) Small aperture



(b) Medium aperture



(c) Large aperture