Lecture 24 – Prestressed Concrete

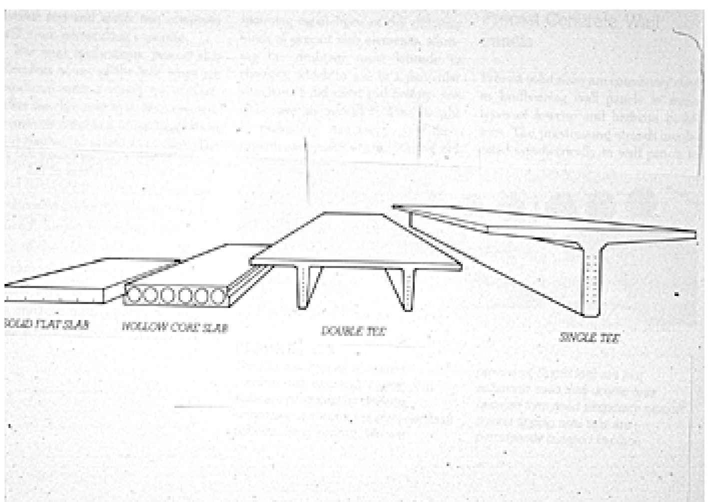
Prestressed concrete refers to concrete that has **applied stresses** induced into the member. Typically, wires or "tendons" are stretched and then blocked at the ends creating compressive stresses throughout the member's entire cross-section. Most Prestressed concrete is precast in a plant.

Advantages of Prestressed concrete vs. non-Prestressed concrete:

- More efficient members (i.e., smaller members to carry same loads)
- Much less cracking since member is almost entirely in compression
- Precast members have very good quality control
- Precast members offer rapid field erection

Disadvantages of Prestressed concrete vs. non-Prestressed concrete:

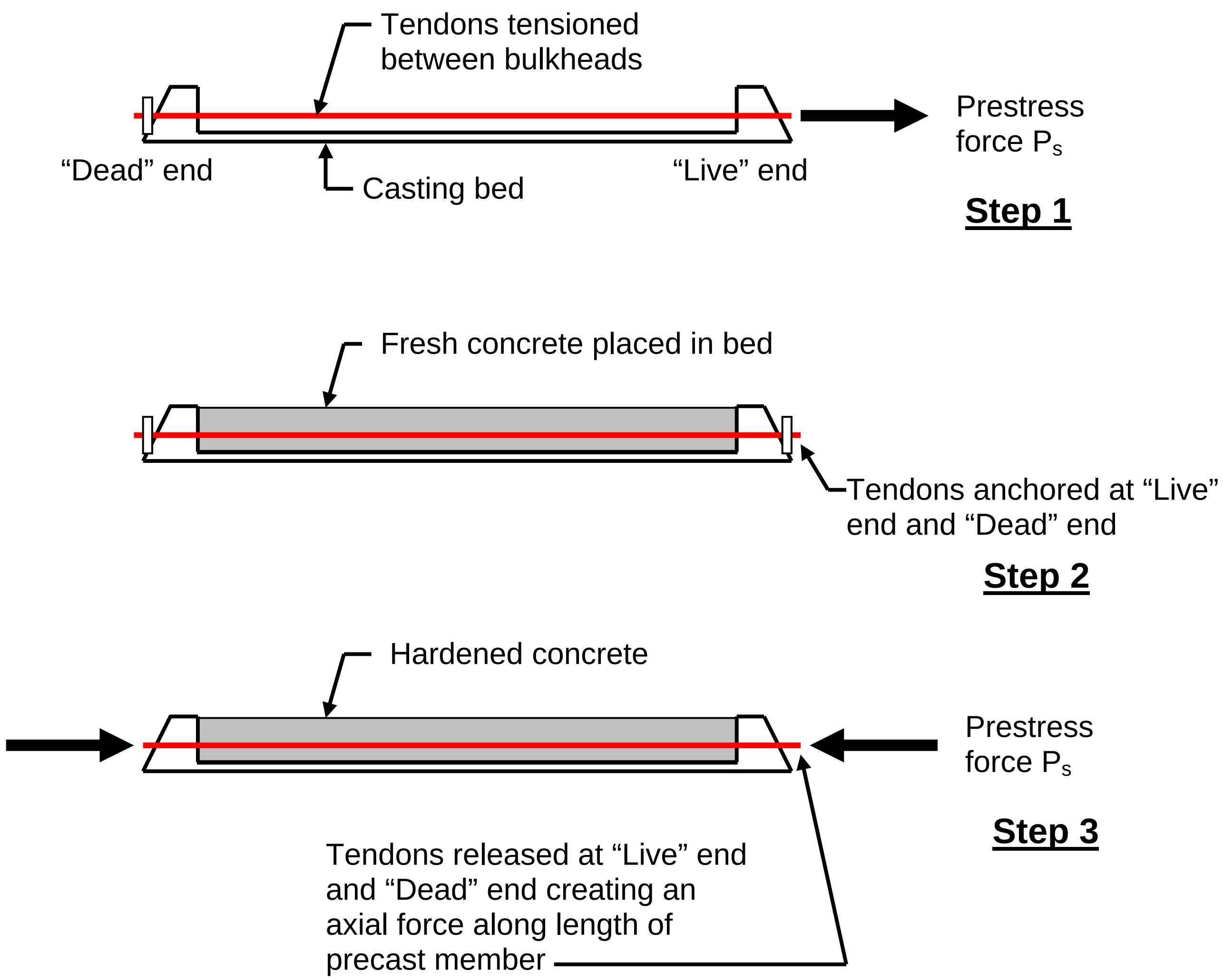
- More expensive in materials, fabrication, delivery
- Heavy precast members require large cranes
- Somewhat limited design flexibility
- Small margin for error
- More complicated design



Typical Precast Prestressed concrete members

Pre-Tensioned Prestressed Concrete:

Pre-tensioned concrete is almost always done in a precast plant. A pre-tensioned Prestressed concrete member is cast in a preformed casting bed. The BONDED wires (tendons) are tensioned prior to the concrete hardening. After the concrete hardens to approximately 75% of the specified compressive strength f'_c, the tendons are released and axial compressive load is then transmitted to the cross-section of the member.



Analysis of Rectangular Prestressed Members:

The analysis of a member is typically done for various stages of loading under **SERVICE LOADS**. Stresses "f" are obtained as follows:

$$f = \frac{P_s}{A_g} \pm \frac{P_s e y}{I_g}$$

where: P_s = prestress force

A_g = gross cross-sectional area of member

e = eccentric distance between prestressing tendons and member centroid

y = distance from centroid to extreme edge of member

I_g = gross moment of inertia of member about N.A.

$$\mathbf{M_u = 0.9A_{ps}f_{ps}(d_p - \frac{a}{2})}$$

where: M_u = usable moment capacity of prestressed beam

A_{ps} = area of prestressed tendons

$$f_{ps} = f_{pu}\left(1 - \left[\frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right]\right)$$

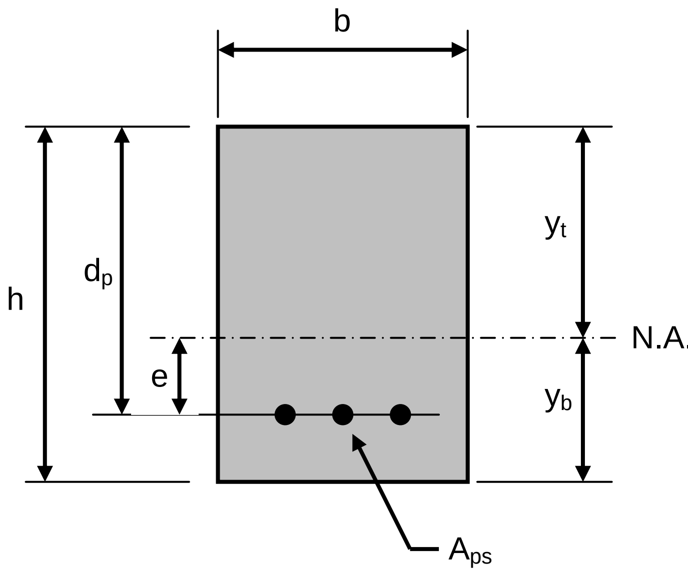
f_{pu} = ultimate tensile strength of prestressing tendon

γ_p = factor based on the type of prestressing steel
= 0.40 for ordinary wire strand
= 0.28 for low-relaxation wire strand

$\beta 1$ = 0.85 for concrete f'_c = 4000 PSI
= 0.80 for concrete f'_c = 5000 PSI

$$\rho_p = \frac{A_{ps}}{bd_p}$$

a = $\frac{A_{ps} f_{ps}}{0.85 f'_c b}$



Rectangular Prestressed Beam

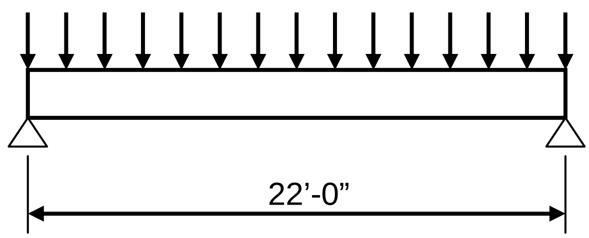
Example

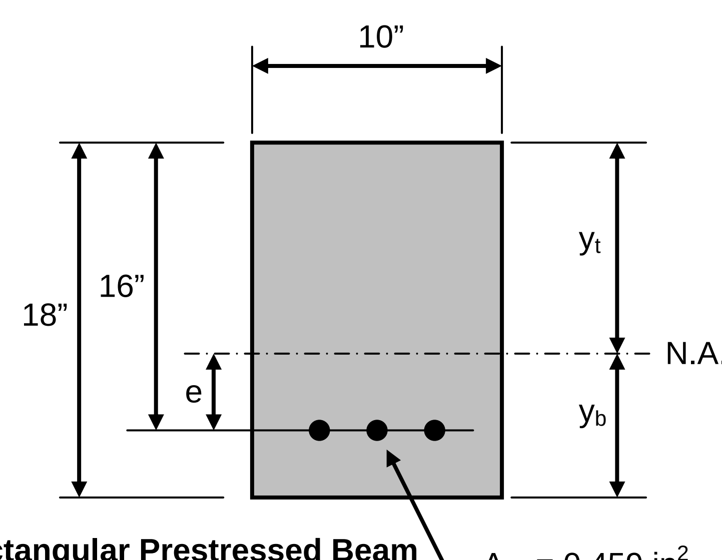
GIVEN: The rectangular prestressed concrete beam as shown below. Use the following:

- Concrete f'_c = 5000 PSI
- Concrete strength = 75%(f'_c) at time of prestressing
- A_{ps} = 3 – ½" dia. 7-wire strands @ 0.153 in^2 per strand = 0.459 in^2
- f_{pu} = 270 KSI (using an ordinary 7-wire strand)
- Initial prestress force, P_s = 70%(f_{pu})(A_{ps})
- Service dead load, (NOT including beam weight) = 400 PLF
- Service beam weight = 188 PLF
- Service live load = 1500 PLF

REQUIRED:

1) Determine the location of the neutral axis and prestress eccentricity "e".
2) Determine the moment of inertia about the neutral axis, I_g.
3) Determine the stresses during prestressing.
4) Determine the stresses during initial applied service beam weight.
5) Determine the stresses due to service applied dead load + live load.
6) Determine the final stresses due to all service loads and prestressing.
7) Determine the maximum **actual** factored moment on the beam M_{max}.
8) Determine the factored **usable moment capacity** M_u of the beam.

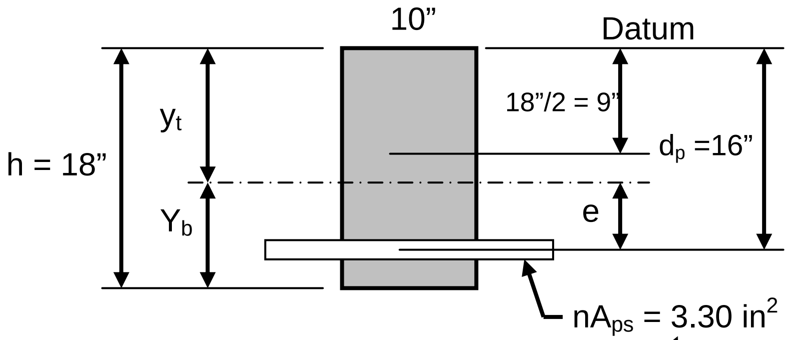




Rectangular Prestressed Beam

Step 1 – Determine the location of the neutral axis and prestress eccentricity "e":

Using a datum as measured from the top of the beam:



$$n = \frac{E_{steel}}{E_{conc}}$$

$$= \frac{29{,}000{,}000 PSI}{57{,}000\sqrt{f'_c = 5000 PSI}}$$

$$= 7.2$$

nA_{ps} = 7.2(0.459 in^2)
= 3.30 in^2

$$y_t = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{(10''x18'')9''+(3.30in^2)16''}{(10''x18'')+(3.30in^2)}$$

y_t = 9.13"

y_b = 18" – 9.13"

y_b = 8.87"

e = d_p – y_t
= 16" – 9.13"

e = 6.87"

Step 2 – Determine the moment of inertia about the neutral axis, I_g:

$$I_g = \frac{bh^3}{12} + bh\left(y_t - \frac{h}{2}\right)^2 + nA_{ps}(e)^2$$

$$= \frac{(10")(18")^3}{12} + (10")(18")\left(9.13" - \frac{18"}{2}\right)^2 + (3.30in^2)(6.87")^2$$

$= 4860\ in^4 + 3.0\ in^4 + 155.7\ in^4$

I_g = 5018.7 in^4

Step 3 – Determine the stresses during prestressing:

$$f = -\frac{P_s}{A_g} \pm \frac{P_s ey}{I_g}$$

where: P_s = prestress force
= 70%(f_{pu})(A_{ps})
= 0.70(270 KSI)(0.459 in^2)
= 86.8 KIPS

y = y_t for tensile stresses at **top** of beam
= y_b for compressive stresses at **bottom** of beam

a) **Check stresses at TOP of beam:**

f_{top} = stress at top of beam

$$= -\frac{P_s}{A_g} + \frac{P_s e y_t}{I_g}$$

$$= -\frac{86.8KIPS}{(10" x 18")} + \frac{(86.8KIPS)(6.87")(9.13")}{5018.7in^4}$$

= -0.48 KSI + 1.08 KSI

f_{top} = 0.60 KSI Tension

b) <u>Check stresses at BOTTOM of beam</u>:

f_{bottom} = stress at bottom of beam

$$= -\frac{P_s}{A_g} - \frac{P_s e y_b}{I_g}$$

$$= -\frac{86.8KIPS}{(10''x18'')} - \frac{(86.8KIPS)(6.87'')(8.87'')}{5018.7in^4}$$

= -0.48 KSI - 1.05 KSI

f_{bottom} = -1.53 KSI Compression

<u>Step 4 – Determine the stresses during initial applied service beam weight</u>:

$$f = \pm\frac{M_{beam}(y)}{I_g}$$

where: M_{beam} = maximum unfactored moment due to beam wt.

$$= \frac{w_{beam}(L)^2}{8}$$

$$= \frac{(188PLF)(22'-0'')^2}{8}$$

= 11,374 Lb-Ft
= 11.4 KIP-FT

$y = y_t$ for compression in **top**
$= y_b$ for tension in **bottom**

a) <u>Check stresses at TOP</u>:

$$f_{top} = -\frac{M_{beam}(y_t)}{I_g}$$

$$= -\frac{(11.4KIP-FT(12''/ft))(9.13'')}{5018.7in^4}$$

f_{top} = -0.25 KSI Compression

b) Check stresses at BOTTOM:

$$F_{bottom} = +\frac{M_{beam}(y_b)}{I_g}$$

$$= +\frac{(11.4KIP-FT(12''/ft))(8.87'')}{5018.7in^4}$$

f_{bottom} = 0.24 KSI Tension

Step 5 – Determine the stresses due to service applied dead load + live load:

$$f = \pm\frac{M_{DL+LL}(y)}{I_g}$$

where: M_{DL+LL} = maximum unfactored moment due to DL+LL

$$= \frac{w_{DL+LL}(L)^2}{8}$$

$$= \frac{(400PLF+1500PLF)(22'-0'')^2}{8}$$

= 114,950 Lb-Ft
= 115.0 KIP-FT

y = y_t for compression in **top**
= y_b for tension in **bottom**

a) Check stresses at TOP:

$$f_{top} = -\frac{M_{DL+LL}(y_t)}{I_g}$$

$$= -\frac{(115.0KIP-FT(12''/ft))(9.13'')}{5018.7in^4}$$

f_{top} = -2.51 KSI Compression

b) Check stresses at BOTTOM:

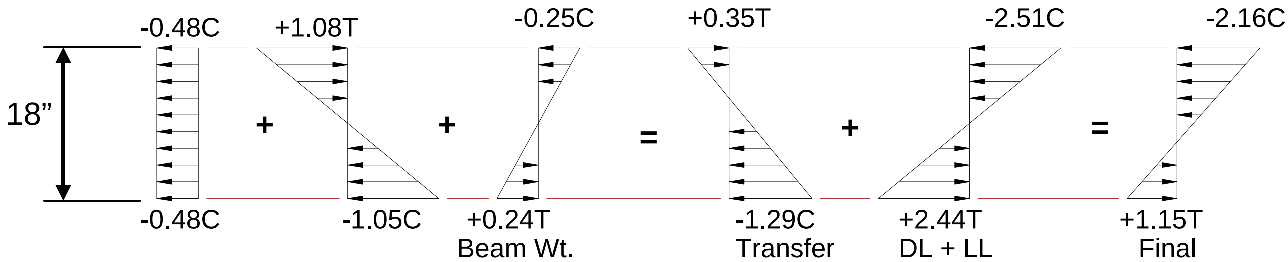
$$f_{bottom} = +\frac{M_{DL+LL}(y_b)}{I_g}$$

$$= +\frac{(115.0KIP - FT(12''/ft))(8.87'')}{5018.7in^4}$$

f_{bottom} = 2.44 KSI Tension

Step 6 – Determine the final stresses due to all service loads and prestressing:

NOTE: All stresses shown have units of KSI



$$\frac{-P_s}{A_g} \qquad \frac{+P_s ey}{I_g}$$

Initial Prestress

ACI dictates the following maximum permissible stresses at service loads:

a) Compression = 0.60f’c

b) Tension = $6\sqrt{f'_c}$

Step 7 – Determine the maximum actual factored moment on the beam M_{max}:

$$M_{max} = \frac{w_u L^2}{8}$$

w_u = 1.2D + 1.6L
= 1.2(400 PLF + 188 PLF) + 1.6(1500 PLF)
= 3106 PLF
= 3.1 KLF

$$M_{max} = \frac{3.1(22'-0)^2}{8}$$

M_{max} = 188 KIP-FT

Step 8 – Determine the factored usable moment capacity M_u of the beam:

$$M_u = 0.9A_{ps}f_{ps}(d_p - \frac{a}{2})$$

where:

$$f_{ps} = f_{pu}\left(1-\left[\frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f'_c}\right]\right)$$

f_{pu} = ultimate tensile strength of prestressing tendon
= 270 KSI

γ_p = factor based on the type of prestressing steel
= 0.40 for ordinary wire strand

$\beta 1$ = 0.80 for concrete f'_c = 5000 PSI

$$\rho_p = \frac{A_{ps}}{bd_p}$$

$$= \frac{0.453in^2}{(10'')(16'')}$$

= 0.00283

$$f_{ps} = 270KSI\left(1-\left[\frac{0.40}{0.80}(0.00283)\frac{270KSI}{5KSI}\right]\right)$$

= 249.4 KSI

$$a = \frac{A_{ps}f_{ps}}{0.85f'_c b}$$

$$= \frac{(0.453in^2)(249.4KSI)}{0.85(5KSI)(10'')}$$

= 2.66"

$$M_u = 0.9A_{ps}f_{ps}(d_p - \frac{a}{2})$$

$$= 0.9(0.453\ in^2)(249.4\ KSI)(16" - \frac{2.66''}{2})$$

= 1492 Kip-In

M_u = 124.3 KIP-FT < M_{max} = 188 KIP-FT → NOT ACCEPTABLE