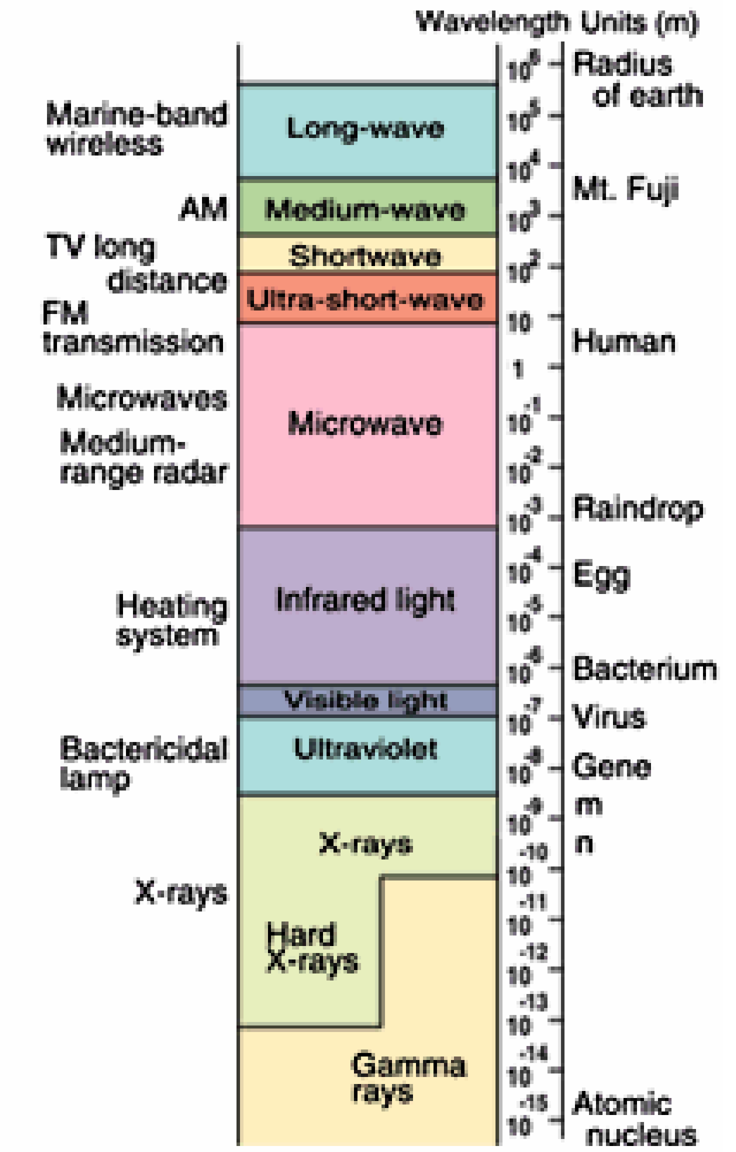
Lecture 26. Blackbody Radiation (Ch. 7)

Two types of bosons:

(a) Composite particles which contain an even number of fermions. These number of these particles is conserved if the energy does not exceed the dissociation energy (~ MeV in the case of the nucleus).

(b) particles associated with a field, of which the most important example is the photon. These particles are not conserved: if the total energy of the field changes, particles appear and disappear. We'll see that the chemical potential of such particles is zero in equilibrium, regardless of density.



Radiation in Equilibrium with Matter

Typically, radiation emitted by a hot body, or from a laser is not in equilibrium: energy is flowing outwards and must be replenished from some source. The first step towards understanding of radiation being in equilibrium with matter was made by Kirchhoff, who considered a ***cavity filled with radiation***, the walls can be regarded as a heat bath for radiation.

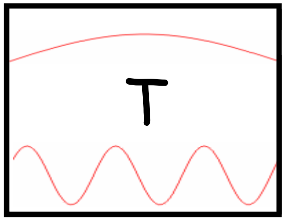
The walls emit and absorb e.-m. waves. In equilibrium, the walls and radiation must have the same temperature T. The energy of radiation is spread over a range of frequencies, and we define $u_S(\nu,T)\,d\nu$ as the energy density (per unit volume) of the radiation with frequencies between ν and $\nu+d\nu$. $u_S(\nu,T)$ is the ***spectral energy density***. The internal energy of the photon gas:

$$u(T)=\int_0^\infty u_S(\nu,T)\,d\nu$$

In equilibrium, $u_S(\nu,T)$ is the same everywhere in the cavity, and is a function of frequency and temperature only. If the cavity volume increases at T=const, the internal energy $U = u(T)\,V$ also increases. The essential difference between the photon gas and the ideal gas of molecules: for an ideal gas, an isothermal expansion would conserve the gas energy, whereas for the photon gas, it is the ***energy density*** which is unchanged, the number of photons is not conserved, but proportional to volume in an isothermal change.

A real surface absorbs only a fraction of the radiation falling on it. The absorptivity α is a function of ν and T; a surface for which $\alpha(\nu)$ =1 for all frequencies is called a ***black body.***

Photons



The electromagnetic field has an infinite number of modes (standing waves) in the cavity. The black-body radiation field is a superposition of plane waves of different frequencies. The characteristic feature of the radiation is that ***a mode may be excited only in units of the quantum of energy h*** ν (similar to a harmonic oscillators) :

$$\varepsilon_i = (n_i + 1/2)h\nu$$

This fact leads to the concept of ***photons as quanta of the electromagnetic field***. The state of the el.-mag. field is specified by the number ***n*** for each of the modes, or, in other words, by enumerating the number of photons with each frequency.

According to the quantum theory of radiation, photons are ***massless bosons of spin*** **1** (in units ***ħ***). They move with the speed of light :

$$E_{ph} = h\nu$$

$$E_{ph} = cp_{ph}$$

$$p_{ph} = \frac{E_{ph}}{c} = h\frac{\nu}{c}$$

The linearity of Maxwell equations implies that ***the photons do not interact with each other***. (Non-linear optical phenomena are observed when a large-intensity radiation interacts with matter).

The mechanism of establishing equilibrium in a photon gas is ***absorption and emission*** of photons by matter. Presence of a small amount of matter is essential for establishing equilibrium in the photon gas. We'll treat a system of photons as ***an ideal photon gas***, and, in particular, we'll apply the BE statistics to this system.

Chemical Potential of Photons = 0

The mechanism of establishing equilibrium in a photon gas is ***absorption and emission*** of photons by matter. The textbook suggests that ***N*** can be found from the equilibrium condition:

$$\left(\frac{\partial F}{\partial N}\right)_{T,V} = 0$$

On the other hand, $\left(\frac{\partial F}{\partial N}\right)_{T,V} = \mu_{ph}$ Thus, in equilibrium, the ***chemical potential*** for a photon gas is ***zero:***

$$\boxed{\mu_{ph} = 0}$$

However, we cannot use the usual expression for the chemical potential, because one cannot increase ***N*** (i.e., add photons to the system) at constant volume and at the same time keep the temperature constant: $\left(\frac{\partial F}{\partial N}\right)_{T,V}$ - does not exist for the photon gas

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Instead, we can use $G = N\mu$ $\quad G = F + PV \quad P = -\left(\frac{\partial F}{\partial V}\right)_T = -\frac{F(T,V)}{V}$

- by increasing the volume at ***T***=const, we proportionally scale ***F***

Thus, $G = F - \frac{F}{V}V = 0$ - the Gibbs free energy of an equilibrium photon gas is 0 !

$$\boxed{\mu_{ph} = \frac{G}{N} = 0}$$

For **μ = 0**, the BE distribution reduces to the ***Planck's distribution***:

$$\boxed{\overline{n}_{ph} = f_{ph}(\varepsilon,T) = \frac{1}{\exp\left(\frac{\varepsilon}{k_B T}\right) - 1} = \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}}$$

Planck's distribution provides the average number of photons in a single mode of frequency $\nu = \varepsilon/h$.

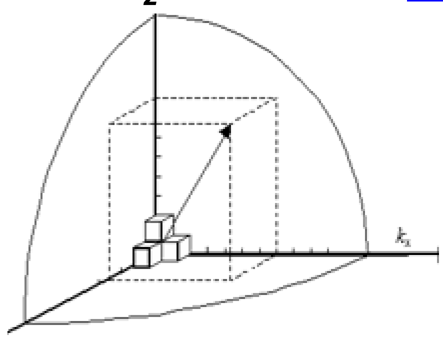
The average energy in the mode:

$$\bar{\varepsilon} = \bar{n}\,h\nu = \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

In the classical (***hν << k_BT***) limit: $\bar{\varepsilon} = k_B T$

In order to calculate the average number of photons per small energy interval ***dε***, the average energy of photons per small energy interval ***dε***, etc., as well as the total average number of photons in a photon gas and its total energy, we need to know the ***density of states for photons*** as a function of photon energy.

Density of States for Photons



$$N(k) = \frac{1}{8}\frac{(4/3)\pi k^3}{\frac{\pi}{L_x}\times\frac{\pi}{L_y}\times\frac{\pi}{L_z}} = \frac{k^3(volume)}{6\pi^2} \qquad G(k) = \frac{k^3}{6\pi^2}$$

$$g(\varepsilon) = \frac{dG(\varepsilon)}{d\varepsilon} \qquad \boxed{\varepsilon = cp = c\hbar k} \qquad G(\varepsilon) = \frac{\varepsilon^3}{6\pi^2(c\hbar)^3} \qquad g_{ph}^{3D}(\varepsilon) = \frac{\varepsilon^2}{2\pi^2(c\hbar)^3}$$

extra factor of 2 due to two polarizations:

$$g_{ph}^{3D}(\nu) = g_{ph}^{3D}(\varepsilon)\frac{d\varepsilon}{d\nu} = h\frac{(h\nu)^2}{\pi^2(c\hbar)^3} = \frac{8\pi\nu^2}{c^3}$$

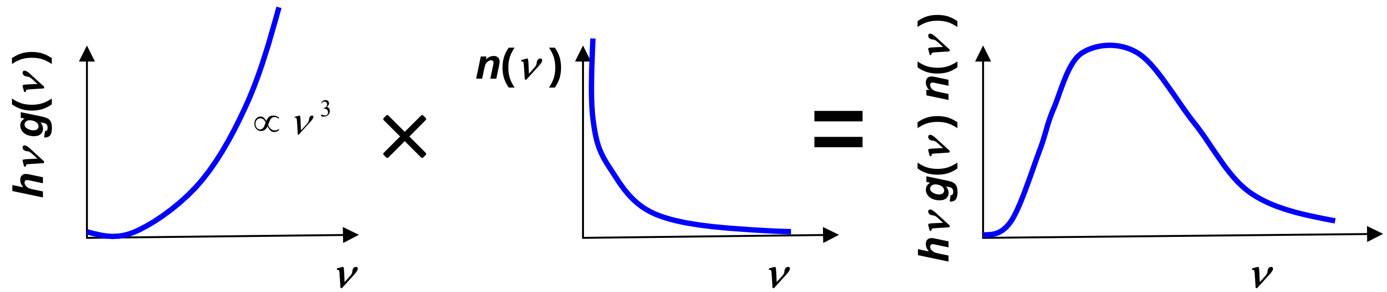
$$\boxed{g_{ph}^{3D}(\nu) = \frac{8\pi\nu^2}{c^3}}$$

Spectrum of Blackbody Radiation

The average energy of photons with frequency between ν and ν+dν (per unit volume):

$$\underbrace{h\nu}_{\text{photon energy}}\ \underbrace{g(\nu)\,\overline{n}(\nu)}_{\text{average number of photons}}\,d\nu = u_S(\nu,T)\,d\nu$$

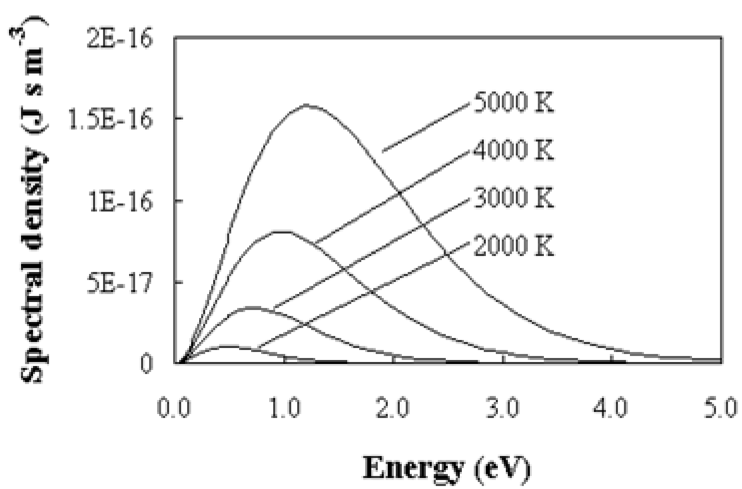
$$g_{ph}^{3D}(\nu) = \frac{8\pi\nu^2}{c^3}$$



$$u_s(\nu,T) = h\nu\, g(\nu) f(\nu) = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp(\beta h\nu)-1}$$

- ***the spectral density of the black-body radiation*** (***the Plank's radiation law***)

u as a function of the energy: $u(\varepsilon,T)d\varepsilon = u(\nu,T)d\nu \quad u(\nu,T) = u(\varepsilon,T)\frac{d\varepsilon}{d\nu} = u(h\nu,T)\times h$



$$u(\varepsilon,T) = \frac{8\pi}{(hc)^3}\frac{\varepsilon^3}{\exp\left(\dfrac{\varepsilon}{k_B T}\right)-1}$$

u(ε,T) - the energy density per unit photon energy for a photon gas in equilibrium with a blackbody at temperature ***T***.

Classical Limit (small *f*, large λ), Rayleigh-Jeans Law

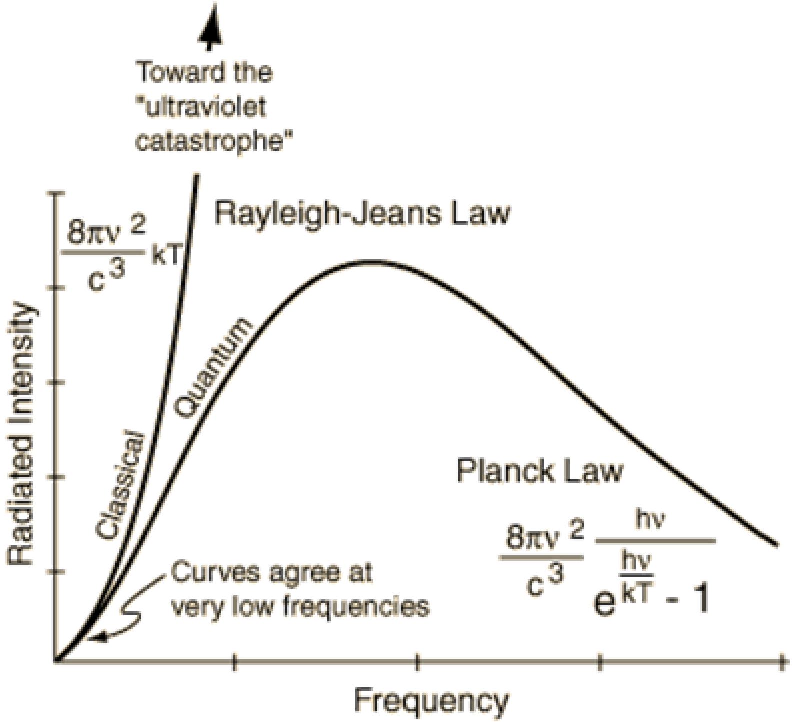
At low frequencies or high temperatures: $\beta h\nu << 1 \quad \exp(\beta h\nu) - 1 \cong \beta h\nu$

$$u_s(\nu,T) = \frac{8\pi h}{c^3}\frac{\nu^3}{\exp(\beta h\nu)-1} \cong \frac{8\pi\nu^2}{c^3}k_B T$$

Rayleigh-Jeans Law

- purely classical result (no ***h***), can be obtained directly from equipartition

This equation predicts the so-called ***ultraviolet catastrophe*** – an infinite amount of energy being radiated at high frequencies or short wavelengths.



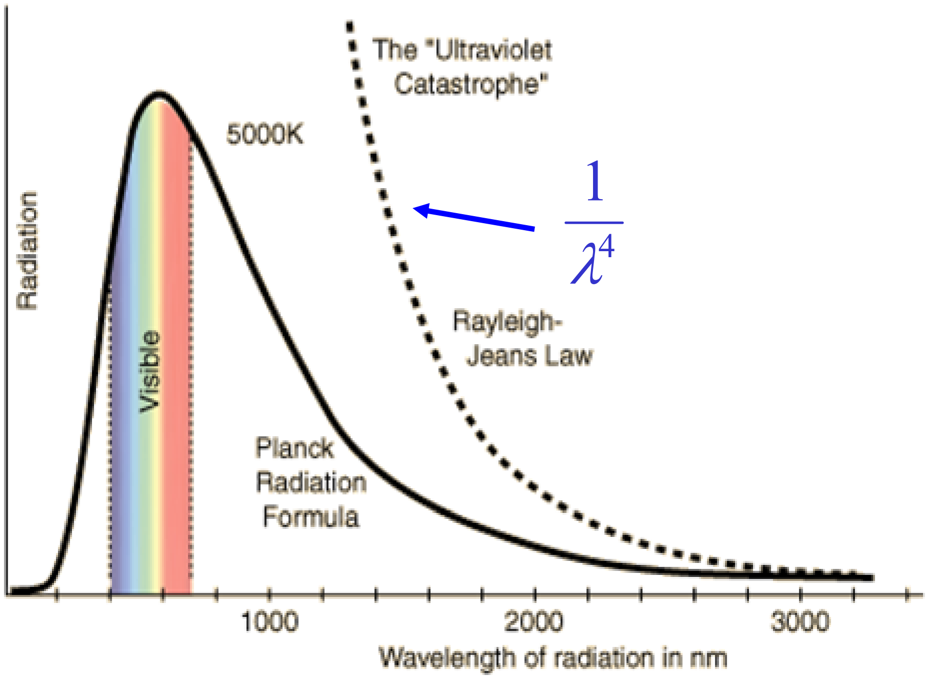
Rayleigh-Jeans Law (cont.)

u as a function of the wavelength:

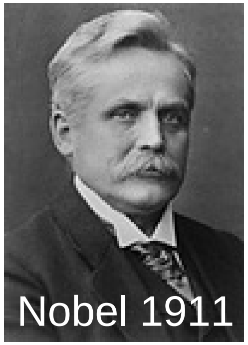
$$u(\lambda,T)d\lambda = -u(\varepsilon,T)d\varepsilon \quad \left[\frac{d\varepsilon}{d\lambda} = -\frac{hc}{\lambda^2}\right] \quad u(\lambda,T) = \frac{8\pi}{(hc)^3}\frac{\left(h\frac{c}{\lambda}\right)^3}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}\left(\frac{hc}{\lambda^2}\right) = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}$$

In the classical limit of large λ:

$$u(\lambda,T)\Big|_{\text{large }\lambda} \approx \frac{8\pi k_B T}{\lambda^4}$$



High ν limit, Wien's Displacement Law



At high frequencies/low temperatures: $\beta h\nu >> 1 \quad \exp(\beta h\nu)-1 \cong \exp(\beta h\nu)$

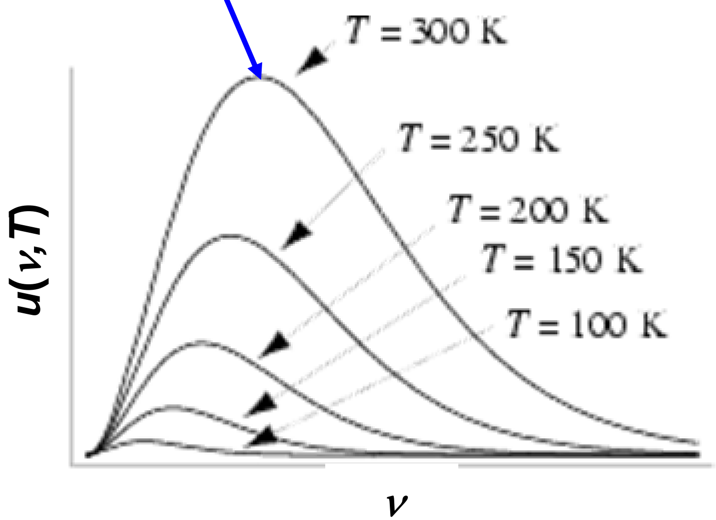
$$u_s(\nu,T)=\frac{8\pi h}{c^3}\nu^3\exp(-\beta h\nu)$$

The maximum of ***u*(*ν*)** shifts toward higher frequencies with increasing temperature. The position of maximum:

$$\frac{du}{d\nu}=const\times\frac{d}{d\left(\frac{h\nu}{k_BT}\right)}\left[\frac{\left(\frac{h\nu}{k_BT}\right)^3}{\exp\left(\frac{h\nu}{k_BT}\right)-1}\right]=const\times\left[\frac{3x^2}{e^x-1}-\frac{x^3e^x}{(e^x-1)^2}\right]=0$$

$$\nu_{\max}\approx 2.8\frac{k_BT}{h}$$

$$(3-x)e^x=3 \quad\rightarrow\quad x\approx 2.8$$



$$\frac{h\nu_{\max}}{k_BT}\approx 2.8$$

Wien's displacement law

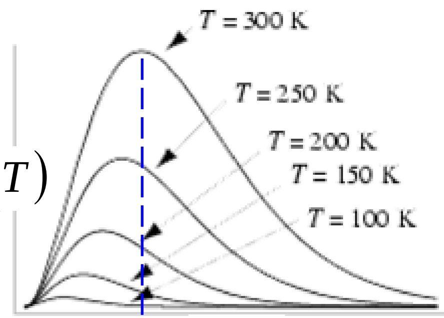
- discovered experimentally by Wilhelm Wien

- the "most likely" frequency of a photon in a blackbody radiation with temperature ***T***

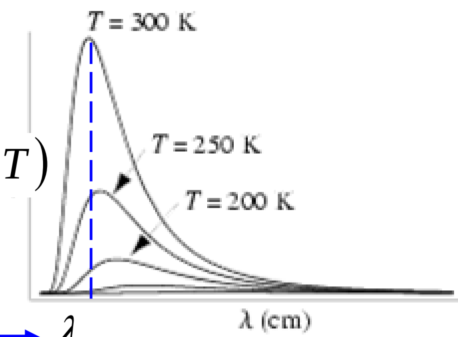
Numerous applications (e.g., non-contact radiation thermometry)

$\nu_{max} \Leftrightarrow \lambda_{max}$

$u(\nu,T)$



$u(\lambda,T)$

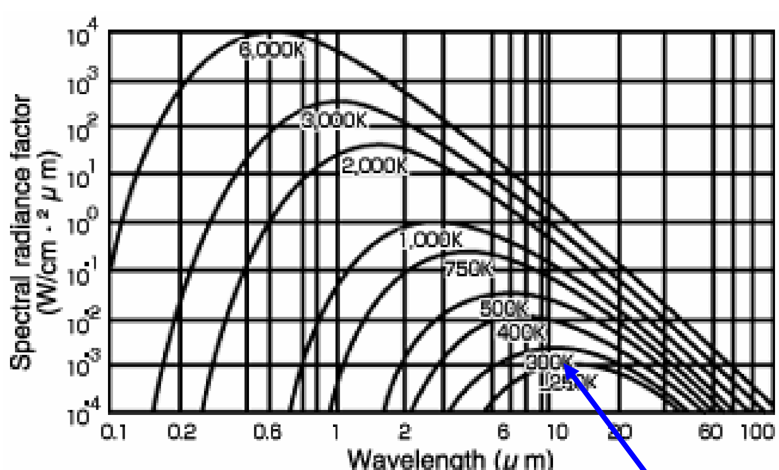


$\nu_{\max} \rightarrow \lambda_{\max}$

$$\frac{h\nu_{\max}}{k_B T} \approx 2.8$$ - does this mean that $$\frac{hc}{k_B T\,\lambda_{\max}} \approx 2.8$$? **No!**

$$u(\lambda,T)d\lambda = -u(\varepsilon,T)d\varepsilon \quad \left[\frac{d\varepsilon}{d\lambda} = -\frac{hc}{\lambda^2}\right] \quad u(\lambda,T) = \frac{8\pi}{(hc)^3}\frac{\left(h\frac{c}{\lambda}\right)^3}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}\left(\frac{hc}{\lambda^2}\right) = \frac{8\pi\, hc}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right)-1}$$

$$\frac{du}{df} = const \times \frac{d}{dx}\left[\frac{1}{x^5\{\exp(1/x)-1\}}\right] = const \times \left[-\frac{5}{x^6\{\exp(1/x)-1\}} - \frac{\left(-x^{-2}\right)\exp(1/x)}{x^5\{\exp(1/x)-1\}^2}\right] = 0$$

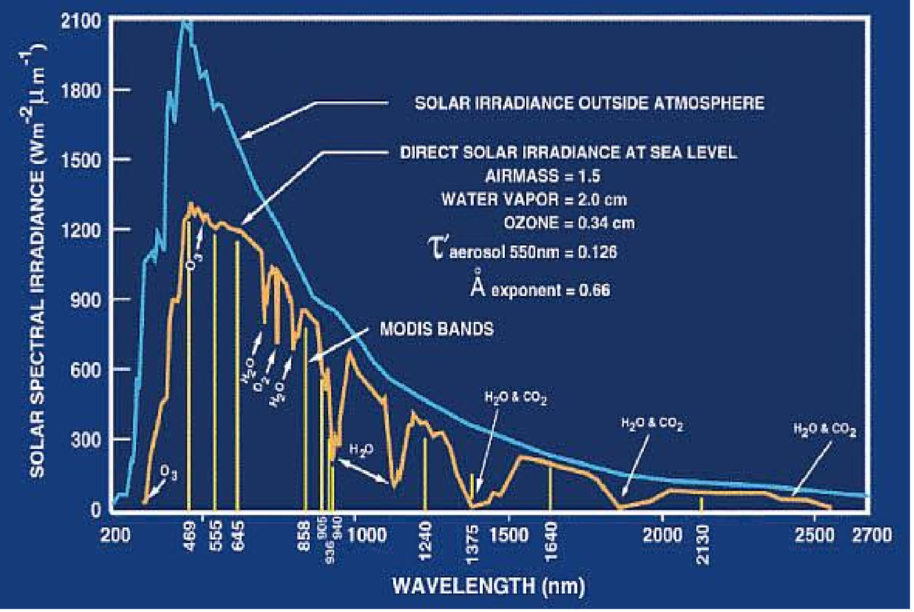


$$5x\{\exp(1/x)-1\} = \exp(1/x) \rightarrow \lambda_{\max} \approx \frac{hc}{5k_B T}$$

T **= 300 K** $\rightarrow$ $\lambda_{max} \approx$ **10 μm**

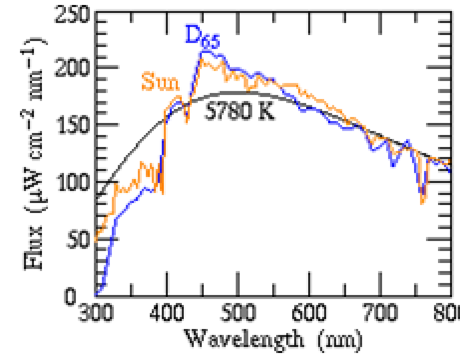
“night vision” devices

Solar Radiation



The surface temperature of the Sun - 5,800K.

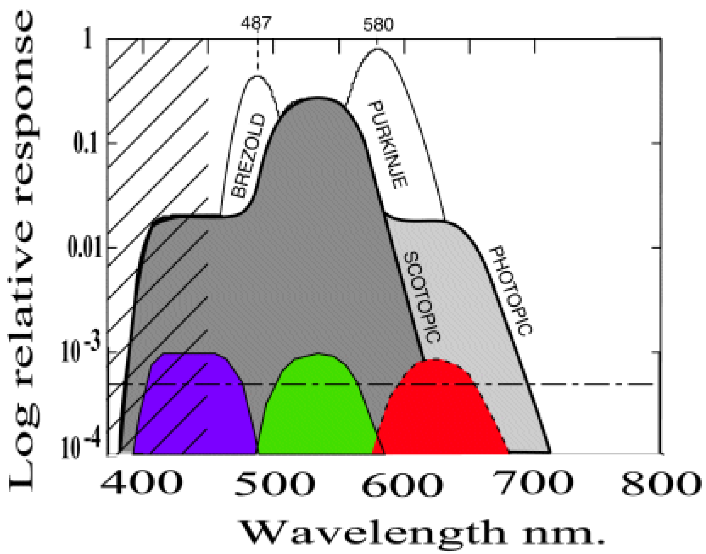
$$\lambda_{\max} = \frac{hc}{5\,k_B T} \approx 0.5\,\mu\text{m}$$



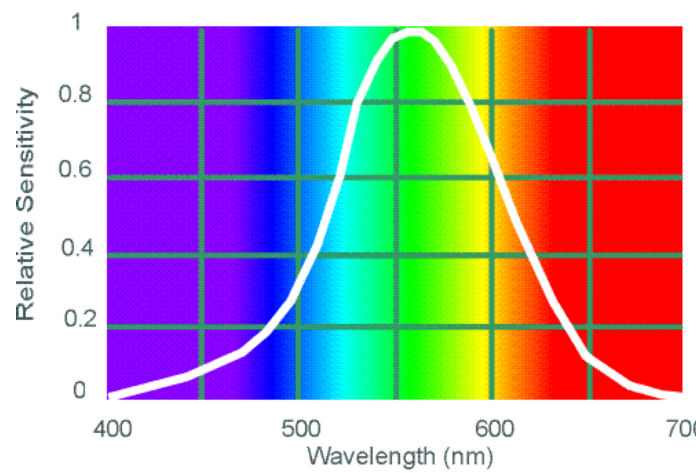
As a ***function of energy***, the spectrum of sunlight peaks at a photon energy of

$$u_{\max} = h\nu_{\max} = 2.8 k_B T \approx 1.4\,eV$$

- close to the energy gap in Si, ~1.1 eV, which has been so far the best material for solar cells



Spectral sensitivity of human eye:



Stefan-Boltzmann Law of Radiation

The total number of photons per unit volume:

$$\bar{n} \equiv \frac{\bar{N}}{V} = \int_0^\infty \bar{n}(\varepsilon) g(\varepsilon) d\varepsilon = \frac{8\pi}{c^3} \int_0^\infty \frac{\nu^2}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} d\nu = \frac{8\pi}{c^3} \left(\frac{k_B T}{h}\right)^3 \int_0^\infty \frac{x^2 dx}{e^x - 1} = 8\pi \left(\frac{k_B}{hc}\right)^3 T^3 \times 2.4$$

- increases as T^3

The total energy of photons per unit volume :
(the energy density of a photon gas)

$$u(T) \equiv \frac{U}{V} = \int_0^\infty \frac{\varepsilon \times g(\varepsilon)}{\exp(\beta\varepsilon) - 1} d\varepsilon = \frac{8\pi^5 (k_B T)^4}{15 (hc)^3}$$

$$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2}$$

the Stefan-Boltzmann constant

$$u(T) = \frac{4\sigma}{c} T^4$$

the Stefan-Boltzmann Law

The average energy per photon:

$$\bar{\varepsilon} = \frac{u(T)}{\bar{N}} = \frac{8\pi^5 (k_B T)^4 (hc)^3}{15 (hc)^3 8\pi (k_B T)^3 \times 2.4} = \frac{\pi^4}{15 \times 2.4} k_B T \approx 2.7\, k_B T$$

(just slightly less than the “most” probable energy)

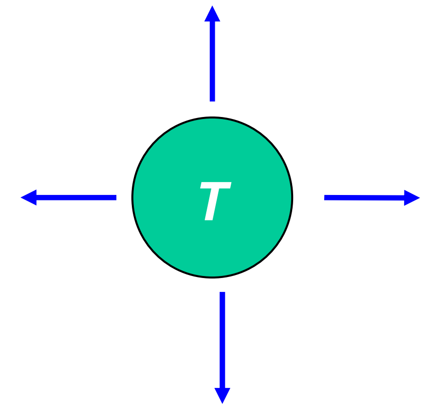
<u>Some numbers:</u> The value of the Stefan-Boltzmann constant: $\sigma = 5.76 \times 10^{-8}\, W / \left(K^4 m^2\right)$

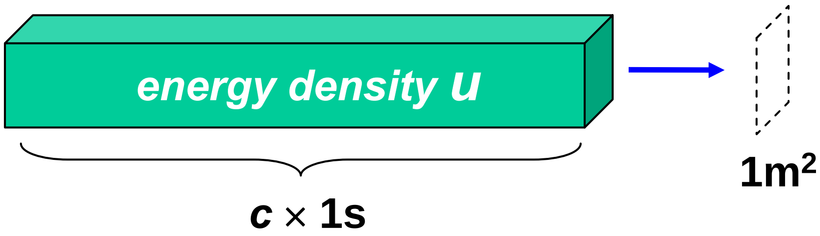
Consider a black body at 310K. The power emitted by the body: $\sigma T^4 \approx 500\, W / m^2$

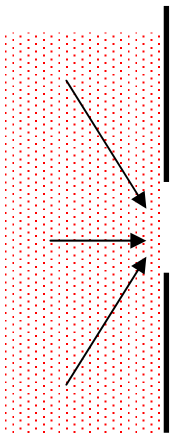
While the emissivity of skin is considerably less than 1, it still emits a considerable power in the infrared range. For example, this radiation is easily detectable by modern techniques (night vision).

Power Emitted by a Black Body

For the "uni-directional" motion, the flux of energy per unit area $= c \times u$







Integration over all angles provides a factor of ¼:

$$\text{power emitted by unit area} = \frac{1}{4} c \times u$$

(the hole size must be >> the wavelength)

Thus, the power emitted by a unit-area surface at temperature ***T*** in all directions:

$$\text{power} = \frac{c}{4} u(T) = \frac{c}{4} \times \frac{4\sigma}{c} T^4 = \sigma T^4$$

The total power emitted by a black-body sphere of radius *R*:

$$= 4\pi R^2 \sigma T^4$$

Sun's Mass Loss

The spectrum of the Sun radiation is close to the black body spectrum with the maximum at a wavelength λ = 0.5 μm. Find the mass loss for the Sun in one second. How long it takes for the Sun to loose 1% of its mass due to radiation? Radius of the Sun: $7 \cdot 10^8$ m, mass - $2 \cdot 10^{30}$ kg.

λ_{max} = 0.5 μm $\rightarrow$ $\lambda_{max} = \frac{hc}{5 k_B T}$ $\rightarrow$ $T = \frac{hc}{5 k_B \lambda_{max}}$ $\left(= \frac{6.6 \cdot 10^{-34} \times 3 \cdot 10^8}{5 \times 1.38 \cdot 10^{-23} \times 0.5 \cdot 10^{-6}} K = 5{,}740 K \right)$

$$P\left(\text{power emitted by a sphere}\right) = 4\pi R^2 \sigma T^4 \qquad \sigma = \frac{2\pi^5 k_B^{\ 4}}{15 h^3 c^2} \approx 5.7 \cdot 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2 \mathrm{K}^4}$$

This result is consistent with the flux of the solar radiation energy received by the Earth (1370 W/m^2) being multiplied by the area of a sphere with radius $1.5 \cdot 10^{11}$ m (Sun-Earth distance).

$$P = 4\pi \left(R_{Sun}\right)^2 \sigma \left(\frac{hc}{2.8 k_B \lambda_{max}} \right)^4 = 4\pi \left(7 \cdot 10^8 \text{ m}\right)^2 \times 5.7 \cdot 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2 \mathrm{K}^4} \times \left(5{,}740\mathrm{K}\right)^4 = 3.8 \cdot 10^{26} \,\mathrm{W}$$

the mass loss per one second $\frac{dm}{dt} = \frac{P}{c^2} = \frac{3.8 \cdot 10^{26} \,\mathrm{W}}{\left(3 \cdot 10^8 \text{ m}\right)^2} = 4.2 \cdot 10^9 \text{ kg/s}$

1% of Sun's mass will be lost in $\Delta t = \frac{0.01 M}{dm/dt} = \frac{2 \cdot 10^{28} \text{ kg}}{4.2 \cdot 10^9 \text{ kg/s}} = 4.7 \cdot 10^{18} \text{ s} = 1.5 \cdot 10^{11} \text{ yr}$

Radiative Energy Transfer

Liquid nitrogen and helium are stored in a vacuum or Dewar flask, a container surrounded by a thin evacuated jacket. While the thermal conductivity of gas at very low pressure is small, energy can still be transferred by radiation. Both surfaces, cold and warm, radiate at a rate:

$$J_{rad} = (1-r)\sigma T_i^4 \quad W/m^2$$

i*=*a for the outer (hot) wall, ***i*=*b*** for the inner (cold) wall,
r – the coefficient of reflection, (1-***r***) – the coefficient of emission

Let the total ingoing flux be ***J***,
and the total outgoing flux be ***J'***:

$$J = (1-r)\sigma T_a^4 + rJ' \qquad J' = (1-r)\sigma T_b^4 + rJ$$

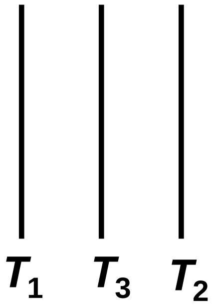
The net ingoing flux:

$$J - J' = \frac{1-r}{1+r}\sigma\left(T_a^4 - T_b^4\right)$$

If ***r***=0.98 (walls are covered with silver mirror), the net flux is reduced to 1% of the value it would have if the surfaces were black bodies (***r***=0).

Superinsulation

Two parallel black planes are at the temperatures T_1 and T_2 respectively. The energy flux between these planes in vacuum is due to the blackbody radiation. A third black plane is inserted between the other two and is allowed to come to an equilibrium temperature T_3. Find T_3, and show that the energy flux between planes 1 and 2 is cut in half because of the presence of the third plane.



Without the third plane, the energy flux per unit area is: $J^0 = \sigma\left(T_1^4 - T_2^4\right)$

The equilibrium temperature of the third plane can be found from the energy balance:

$$\sigma\left(T_1^4 - T_3^4\right) = \sigma\left(T_3^4 - T_2^4\right) \qquad T_1^4 + T_2^4 = 2T_3^4 \qquad T_3 = \left(\frac{T_1^4 + T_2^4}{2}\right)^{1/4}$$

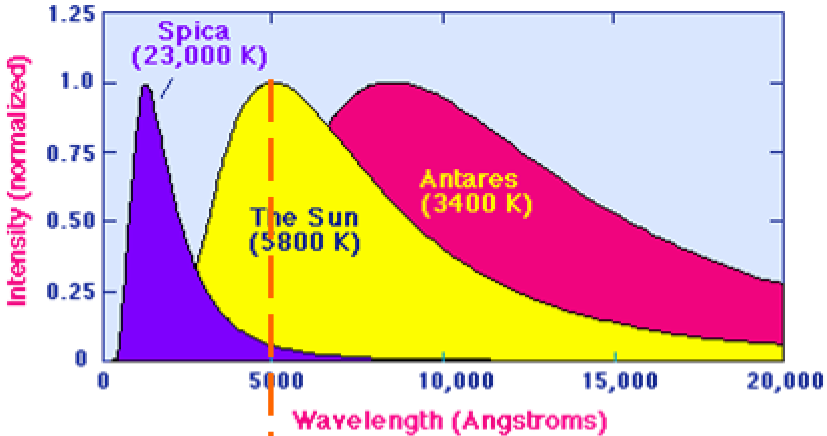
The energy flux between the 1st and 2nd planes in the presence of the third plane:

$$J = \sigma\left(T_1^4 - T_3^4\right) = \sigma\left(T_1^4 - \frac{T_1^4 + T_2^4}{2}\right) = \frac{1}{2}\sigma\left(T_1^4 - T_2^4\right) = \frac{1}{2}J^0$$

- cut in half

Superinsulation: many layers of aluminized Mylar foil loosely wrapped around the helium bath (in a vacuum space between the walls of a LHe cryostat). The energy flux reduction for N heat shields:

$$J_N = \frac{J^0}{N+1}$$



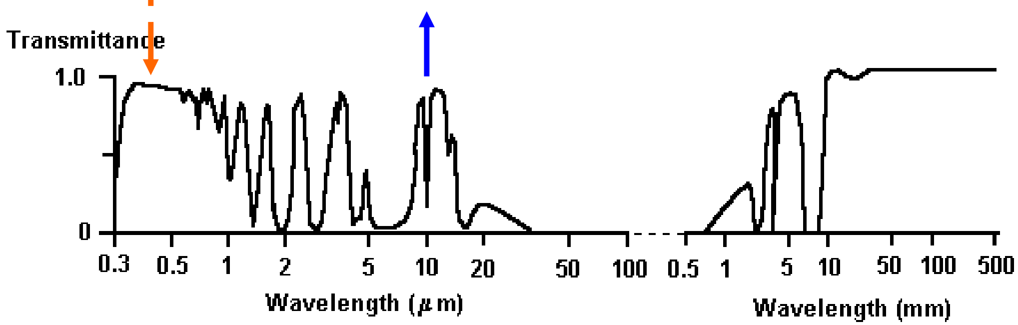
The Greenhouse Effect

Absorption:

$$\text{Power in} = \alpha \left(\pi R_E^{\,2}\right) \sigma \left(T_{Sun}\right)^4 \left(\frac{R_{Sun}}{R_{orbit}}\right)^2$$

the flux of the solar radiation energy received by the Earth ~ 1370 W/m^2

Emission: $\text{Power out} = 4\pi R_E^{\,2} \sigma T_E^{\,4}$



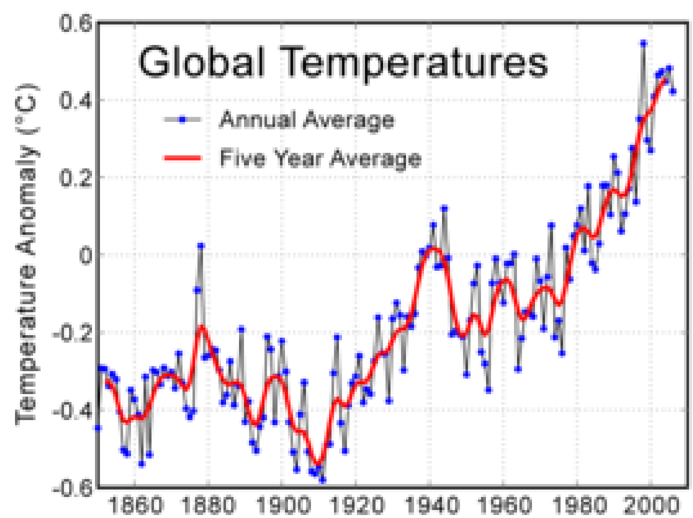
$$T_E = \left[\frac{\alpha}{4}\left(\frac{R_{Sun}}{R_{orbit}}\right)^2\right]^{1/4} T_{Sun}$$

$\boldsymbol{R_{orbit}}$ = 1.5·10^{11} m $\quad$ $\boldsymbol{R_{Sun}}$ = 7·10^{8} m

Transmittance of the Earth atmosphere

α **= 1 – T_{Earth} = 280K**

In reality α **= 0.7 – T_{Earth} = 256K**



To maintain a comfortable temperature on the Earth, we need the Greenhouse Effect !

However, too much of the greenhouse effect leads to ***global warming***:

Thermodynamic Functions of Blackbody Radiation

The heat capacity of a photon gas at constant volume:

$$c_V \equiv \left[\frac{\partial u(T)}{\partial T}\right]_V = \frac{16\sigma}{c} VT^3$$

This equation holds for all ***T*** (it agrees with the Nernst theorem), and we can integrate it to get the entropy of a photon gas:

$$S(T) = \int_0^T \frac{c_V(T')}{T'} dT' = \frac{16\sigma V}{c} \int_0^T T'^2 dT' = \frac{16\sigma}{3c} VT^3$$

Now we can derive all thermodynamic functions of blackbody radiation:

the Helmholtz free energy:

$$F = U - TS = \frac{4\sigma}{c} VT^4 - \frac{16\sigma}{3c} VT^4 = -\frac{4\sigma}{3c} VT^4$$

the Gibbs free energy:

$$G = U - TS + PV = F + PV = 0 \quad (= \mu N)$$

the pressure of a photon gas (radiation pressure)

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = \frac{4\sigma}{3c} T^4 = \frac{1}{3} u$$

$$PV = \frac{1}{3} U$$

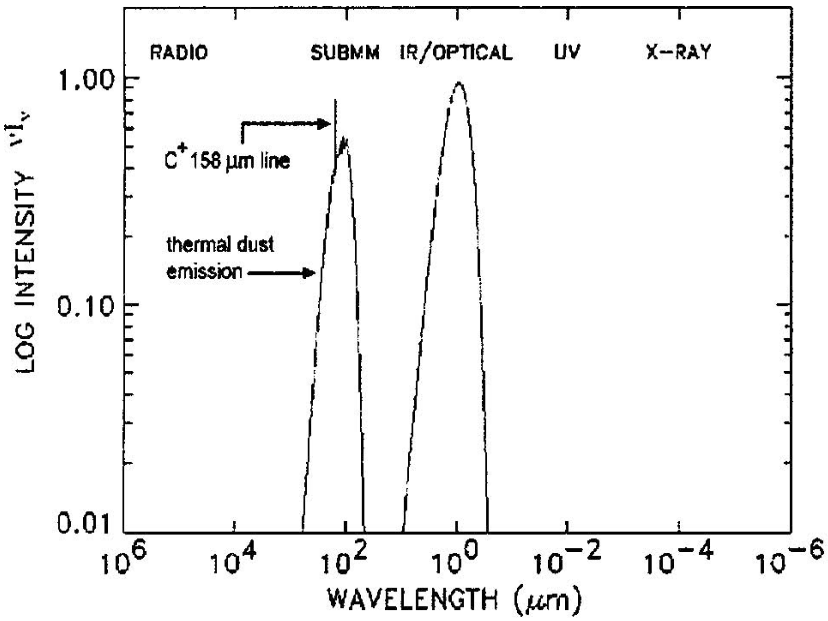
For comparison, for a non-relativistic monatomic gas – ***PV* = (2/3)*U*.** The difference – because the energy-momentum relationship for photons is ultra-relativistic, and the number of photon depends on ***T***.

In terms of the average density of phonons:

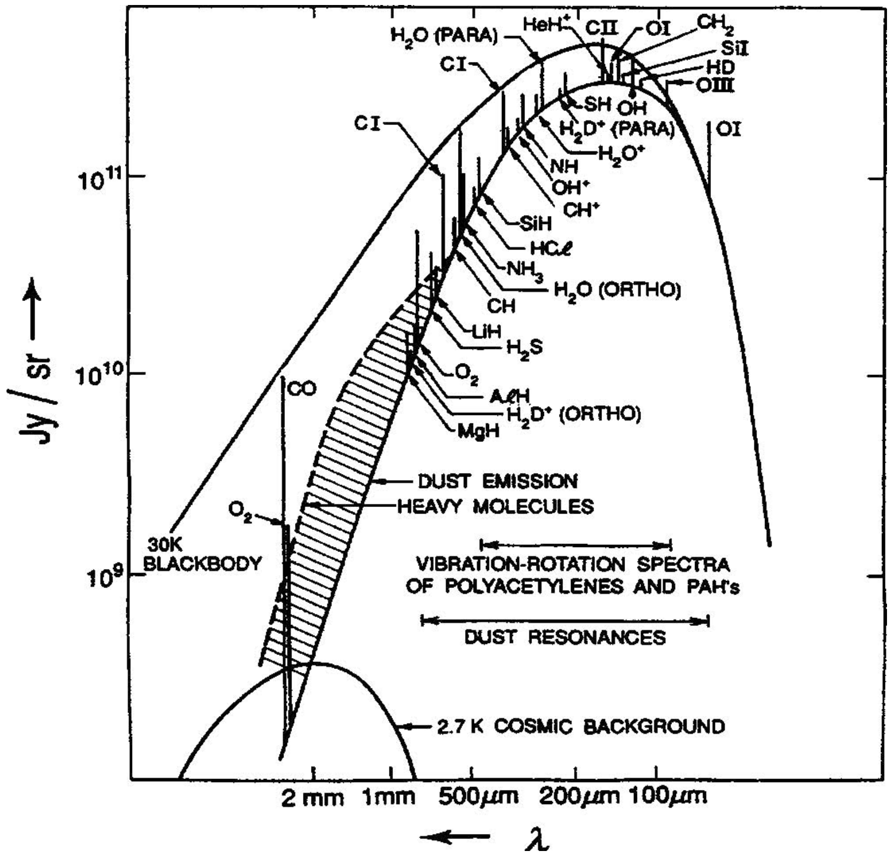
$$PV = \frac{1}{3} U = \frac{1}{3} V \times \bar{n} \times \bar{\varepsilon} = \frac{1}{3} V \times \bar{n} \times 2.7 k_B T = 0.9 \bar{N} k_B T$$

Radiation in the Universe

Approximately 98% of all the photons emitted since the Big Bang are observed now in the submillimeter/THz range.

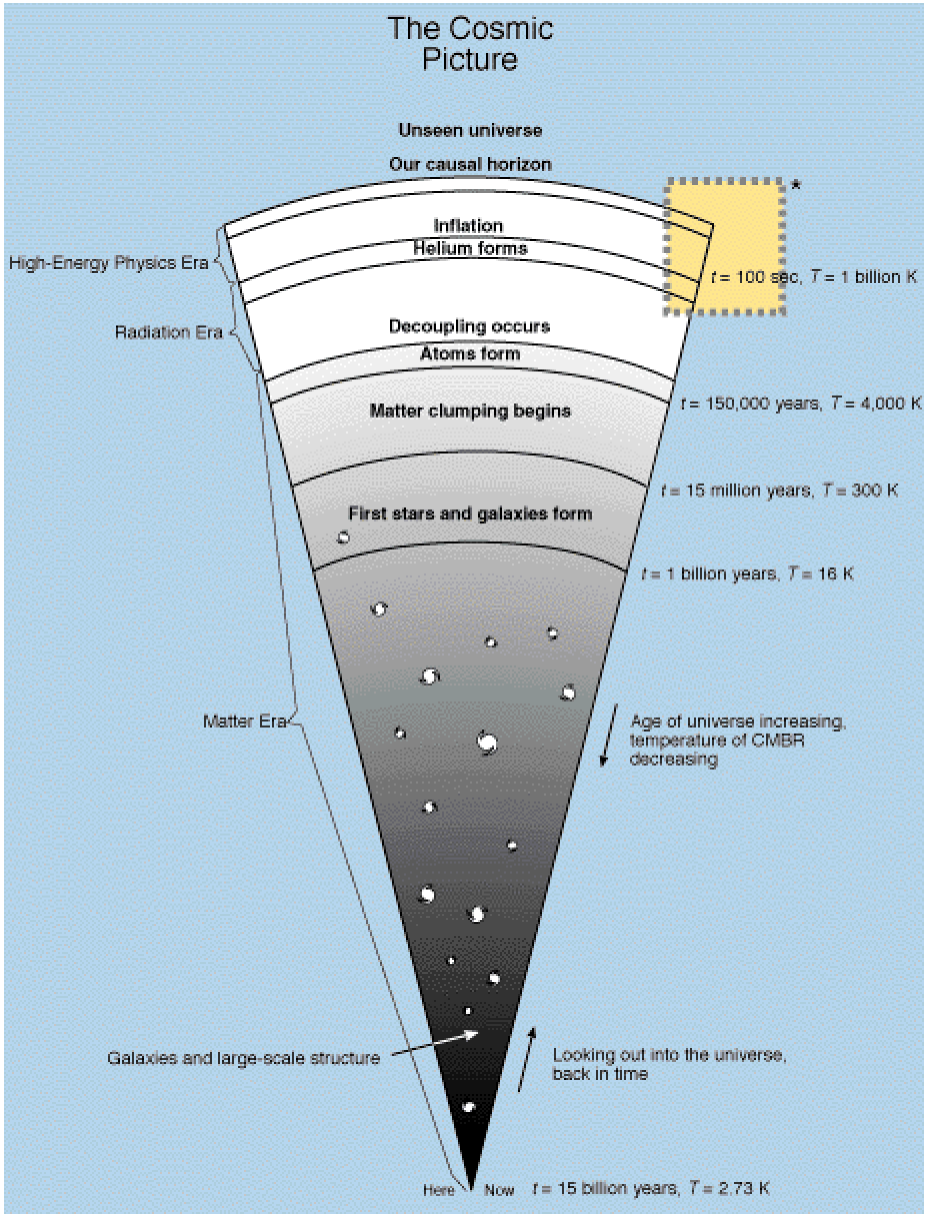


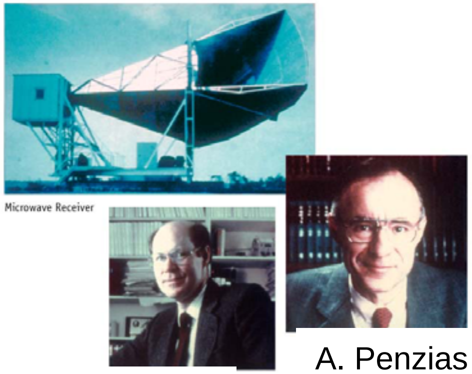
In the spectrum of the Milky Way galaxy, at least one-half of the luminous power is emitted at sub-mm wavelengths



The dependence of the radiated energy versus wavelength illustrates the main sources of the THz radiation: the interstellar dust, emission from light and heavy molecules, and the 2.7-K cosmic background radiation.

Cosmic Microwave Background





R. Wilson

A. Penzias

Nobel 1978

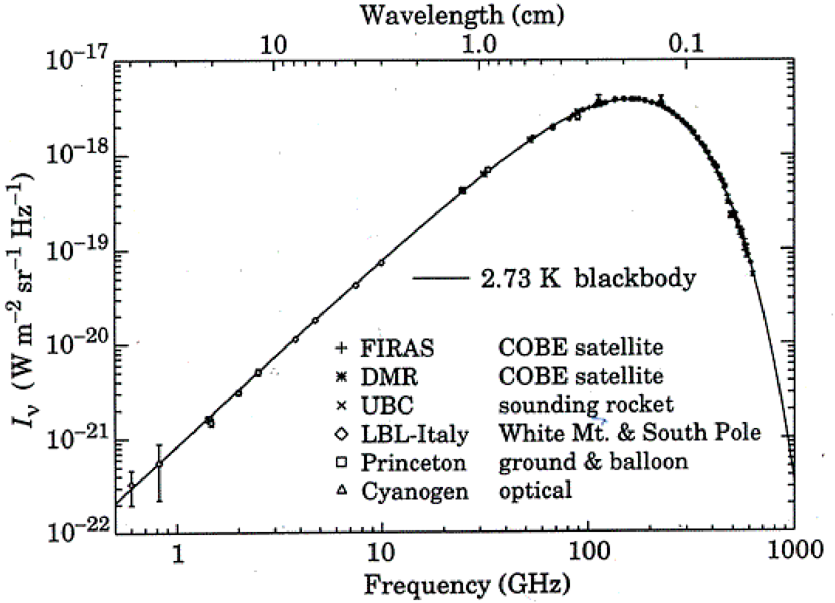
In the standard Big Bang model, the radiation is decoupled from the matter in the Universe about 300,000 years after the Big Bang, when the temperature dropped to the point where neutral atoms form (***T***~3000K). At this moment, the Universe became transparent for the "primordial" photons. The further expansion of the Universe can be considered as quasistatic adiabatic (***isentropic***) for the radiation:

$$S(T) = \frac{16\sigma}{3c} V T^3 = const$$

Since $\boldsymbol{V} \propto \boldsymbol{R}^3$, the isentropic expansion leads to :

$$T \propto R^{-1}$$

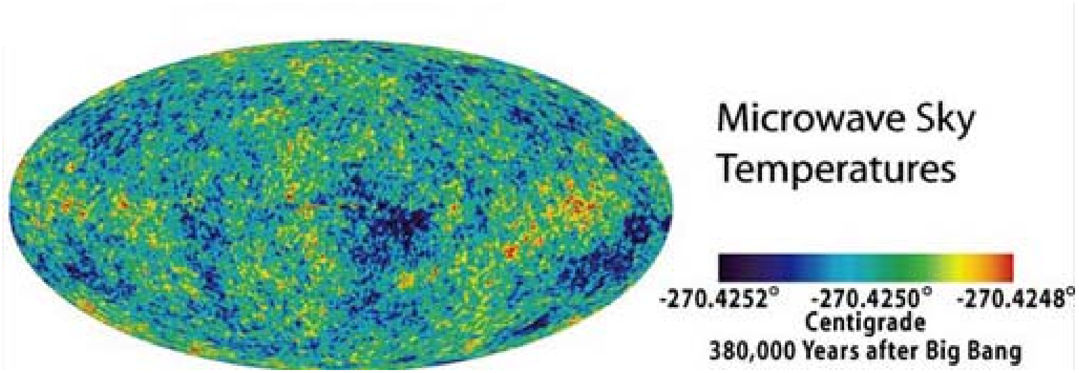
CMBR (cont.)



"… for their discovery of the blackbody form and anisotropy of the CMBR".

Mather, Smoot, Nobel 2006

At present, the temperature of the Planck's distribution for the CMBR photons is 2.735 K. The radiation is coming from all directions and is quite distinct from the radiation from stars and galaxies.



Alternatively, the later evolution of the radiation temperature may be considered as a result of the red (Doppler) shift (z). Since the CMBR photons were radiated at ***T***~3000K, the red shift z~1000.

Problem 2006 (blackbody radiation)

The cosmic microwave background radiation (CMBR) has a temperature of approximately 2.7 K.

(a) (5) What wavelength λ_{max} (in m) corresponds to the maximum spectral density $\boldsymbol{u}(\lambda, \boldsymbol{T})$ of the cosmic background radiation?

(b) (5) What frequency ν_{max} (in Hz) corresponds to the maximum spectral density $\boldsymbol{u}(\nu, \boldsymbol{T})$ of the cosmic background radiation?

(c) (5) Do the maxima $\boldsymbol{u}(\lambda, \boldsymbol{T})$ and $\boldsymbol{u}(\nu, \boldsymbol{T})$ correspond to the same photon energy? If not, why?

(a) $$\lambda_{\max} \approx \frac{hc}{5k_B T} = \frac{6.6\cdot 10^{-34}\times 3\cdot 10^{8}}{5\times 1.38\cdot 10^{-23}\times 2.7} = 1.1\cdot 10^{-3}\,m = 1.1mm \qquad \frac{hc}{\lambda_{\max}} = 1.1\,meV$$

(b) $$\nu_{\max} \approx 2.8\frac{k_B T}{h} = \frac{2.8\times 1.38\cdot 10^{-23}\times 2.7}{6.6\cdot 10^{-34}} = 1.58\cdot 10^{11}\,Hz \qquad h\nu_{\max} = 0.65\,meV$$

(c) the maxima $\boldsymbol{u}(\lambda, \boldsymbol{T})$ and $\boldsymbol{u}(\nu, \boldsymbol{T})$ do not correspond to the same photon energy. The reason of that is

$$u(\lambda, T)d\lambda = -u(\nu, T)d\nu \quad \left[\frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}\right] \quad u(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

Problem 2006 (blackbody radiation)

(d) (15) What is approximately the number of CMBR photons hitting the earth per second per square meter [*i.e.* photons/(s·m^2)]?

(d)

$$J = \sigma T_{CMBR}{}^4 = 5.7 \cdot 10^{-8} \left(W / K^4 \cdot m^2\right) \times (2.7)^4 K^4 = 3 \cdot 10^{-6} W / m^2$$

$$\bar{\varepsilon} = \frac{u(T)}{\bar{N}} = \frac{8\pi^5 (k_B T)^4 (hc)^3}{15 (hc)^3 8\pi (k_B T)^3 \times 2.4} = \frac{\pi^4}{15 \times 2.4} k_B T \approx 2.7\, k_B T$$

$$N\left(\frac{photons}{s \cdot m^2}\right) = \frac{J\left(\frac{W}{m^2}\right)}{\bar{\varepsilon}(J)} \approx \frac{3 \cdot 10^{-6}}{2.7 \times 1.38 \cdot 10^{-23} \times 2.7} \approx 3 \cdot 10^{16} \frac{photons}{s \cdot m^2}$$