Lecture 3: advection – diffusion, mixing in micro-channels

Introduction

For obvious reasons, mixing is crucial in microfluidics. Indeed, many applications deal with chemical reactions where components should mix as well as possible to ensure a controlled synthesis and a good yield. Diagnostic applications also require that the tested fluid and the reagent are well mixed. For example, identification of proteins which is performed once the proteins have been fragmented by enzymes requires the good mixing of the proteins with the enzyme. Indeed mixing turned out to be one of the most challenging processes to achieve within the field of microfluidics and has triggered a lot of experimental and theoretical work. Several reviews about mixing in microfluidcs are available presenting the last technological developments for both passive and active mixers.

In this lecture, the principles of mixing are presented underlining the differences between mixing in daily life and mixing at small Reynolds number. To do so, the notion of diffusion is introduced and the theoretical approach is completed and illustrated by state-of-the-art examples of micro-mixers – both passive and active.

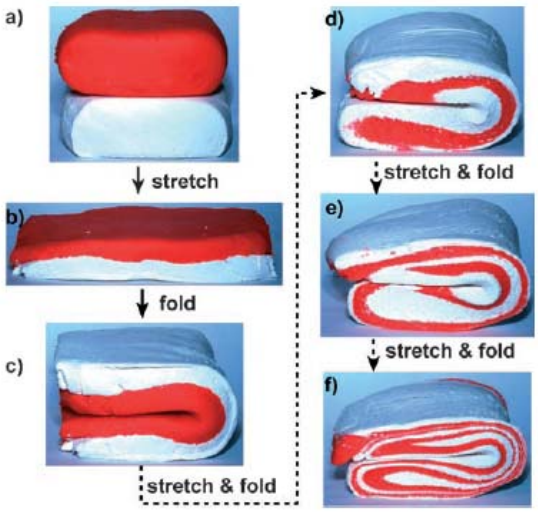
1) Daily life mixers

To explain how mixing is typically achieved in daily life, it is useful to estimate typical Reynolds number.

$\boldsymbol{Re} = \frac{\rho DU}{\eta}$ with $\rho \sim 10^3$ kg.m^3, D ~ 1 m, U ~ 10^{-1} m/s, $\eta \sim 10^{-3}$ Pa.s → $\boldsymbol{Re} \sim \mathbf{10^5}$ turbulent flows.

In daily life, turbulences and/or hydrodynamics instabilities allow streamlines to fold and diffusion ensures the mixing at lower length scales.

At low Reynolds number, there are no turbulences, no hydrodynamic instabilities and mixing is ***a priori*** purely diffusive.



Ismagilov et al (2004)

2) Advection-Diffusion

Advection refers to the transport mechanism of a substance (or conserved property i.e. mass) by a fluid due to the fluid's bulk motion (i.e. to the flow).

Diffusion refers to another transport mechanism which occurs without any motion of the fluid´s bulk. It is a time dependant process which finds its physical origin in the random motion of individual particles.

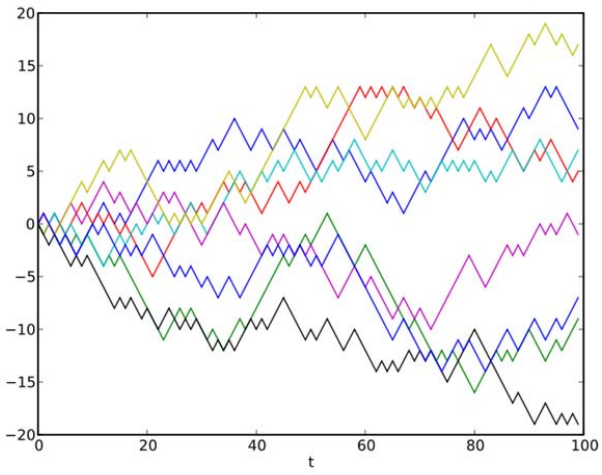
2)1) Microscopic origin of diffusion processes

Brownian motion (discovered by Brown 1827)

It is a very general description where diffusing objects are moving in a free environment where the only relevant events are molecular collisions (molecule/molecule and molecule/diffuser).

The description of 1 D Brownian motion is often done via the analogy with a walker on a line taking a succession of steps of length of +/-*l* in a random manner.

After N steps, the Brownian walker is found at the position $\boldsymbol{X(N) = \langle \sum_1^N l_i \rangle \rightarrow 0}$. The squared value of the walker displacement is thus: $\boldsymbol{X^2(N) = \langle \sum_1^N {l_i}^2 \rangle = N \langle l^2 \rangle}$ with $\boldsymbol{N \propto t}$ this yields: $\boldsymbol{X^2(N) \sim Dt}$ where D is the diffusion coefficient.



Trajectories of a 1D Brownian walker as a function of time

A Brownian walker makes larger and larger excursions as time goes → $\mathbf{L \sim \sqrt{Dt}}$ **;** but remains on average at its initial position.

Einstein´s relation

- Diffusion equation for brownian particles → similar to random walk but for continuous media →$\boldsymbol{\overline{x^2} = 2Dt}$
- Expression of diffusion coefficients from measurable quantities (typically → Stokes Einstein relation with $\boldsymbol{D = \frac{kT}{6\pi r \eta}}$)

2)2) Fick law

Very general (very large validity domain):

$$\overrightarrow{J_{dif}} = -D\,\overrightarrow{grad}\,C$$

Where $\vec{J}_{dif}$ is the diffusive flux (of substance or conserved property) and C the concentration (of substance or conserved property).

i) For gas $D = \frac{1}{3}\lambda u$ with λ mean free path, *u* velocity of thermal agitation (theory of gas kinetics)

ii) For liquid (Stokes Einstein formula) $D = \frac{kT}{6\pi r\eta}$ with r: the typical radius of the diffusing particle and η the viscosity of the fluid.

Orders of magnitude

Particles	**Ion**	**Virus**	**Cells**	**Gas**
Size	0.1 nm	100 nm	10 µm	0.1 nm
D	10^3 µm^2/s	1 µm^2/s	0.01 µm^2/s	10^8 µm^2/s

2)3) Advection diffusion equation

Derived from continuity equation where s : source term (s=0 for mass normally)

$$\frac{\partial C}{\partial t} + div\,\vec{J}_{tot} = s$$

Considering that the total flux is the sum of the advective and the diffusive fluxes: $\vec{J}_{tot} = \vec{J}_{adv} + \vec{J}_{dif}$

and using the definition of the advective flux: $\vec{J}_{adv} = C\vec{v}$, and Fick law for the diffusive flux, it yields:

$$\frac{DC}{Dt} = \frac{\partial C}{\partial t} + (\vec{v}.\overrightarrow{grad})C = D\,\Delta C + s$$ Advection-diffusion equation

With no source term → $\frac{\partial C}{\partial t} + (\vec{v}.\overrightarrow{grad})C = D\,\Delta C$

2)4) Peclet number

Making the advection diffusion equation dimensionless, we can define the Peclet number – a dimensionless number which compares the relative importance of advection and diffusion for transport

phenomena. This leads to: $\boldsymbol{Pe} = \frac{\boldsymbol{Ua}}{\boldsymbol{D}}$ with U typical velocity in the flow, a typical dimension of the objects of interests (particles, molecules,…) and D their diffusion coefficient.

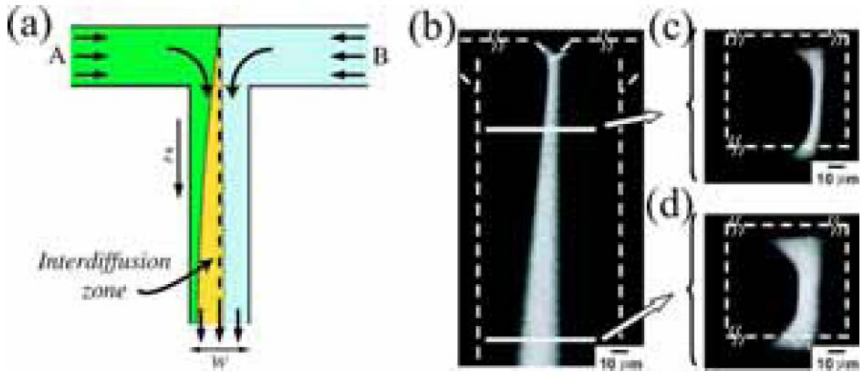
$$\boldsymbol{Pe} = \frac{\boldsymbol{typical\ time\ scale\ of\ diffusion}}{\boldsymbol{typical\ time\ scale\ of\ advection}}$$

2)5) Mixing thickness

$$\boldsymbol{\delta \sim \sqrt{D\,t}}$$

3) Applications of the diffusive regime

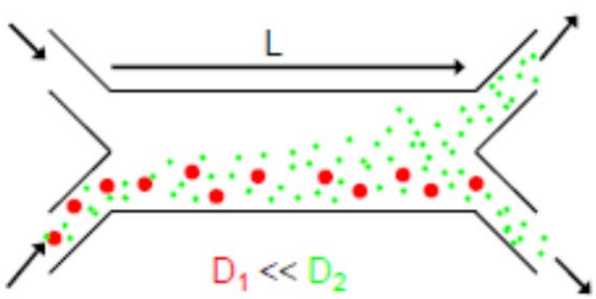
3)1) T sensor for measuring the diffusion coefficient



Ismagilov et al (2000)

3)2) H sensor for filtration

Filtration:separation of large and small particles



example:

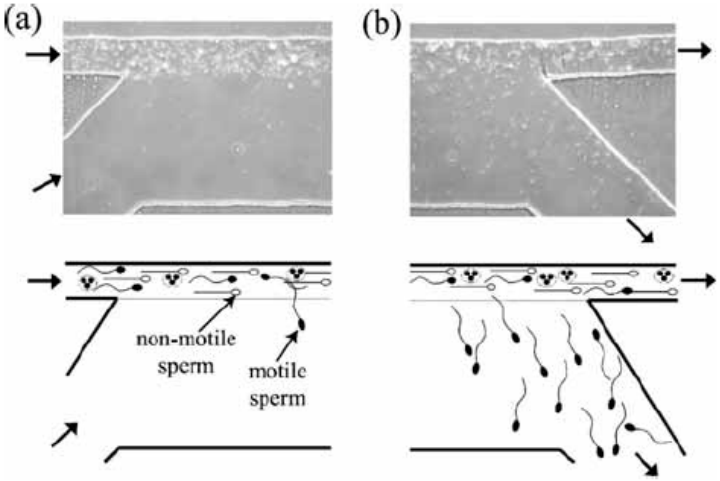
small mocules ($D_1 = 10^{-9}$ m²/s)
dust (colloids > 100 nm, $D_2 < 10^{-12}$ m²/s)

v = 1 mm/s, L = 1 cm } residence time = 10 s

$w_1 = (D_1 t) = 100$ µm
$w_2 = (D_2 t) < 3$ µm

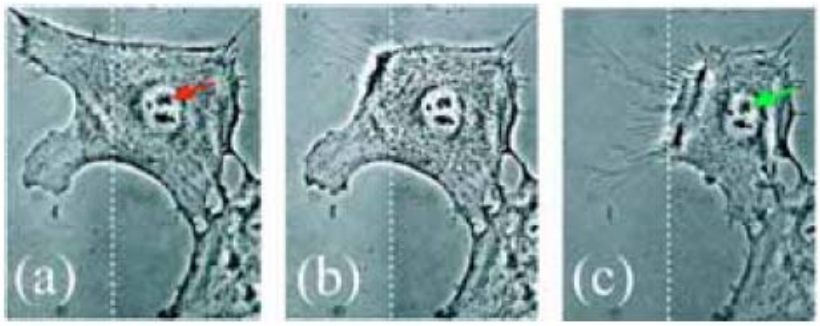
Yager et al. adapted from Salmon (2010)

Measurements of the spermatozoa mobility



Cho (2000)

3)3) Local probing of cells



Takayama (2003)

4) Mixing in microchannels

4)1) Taylor dispersion and annular mixer

Active mixing strategy.

Molecular diffusion can be enhanced making use of a non flat velocity profile.

The classical diffusion coefficient is thus replaced by an effective diffusion coefficient

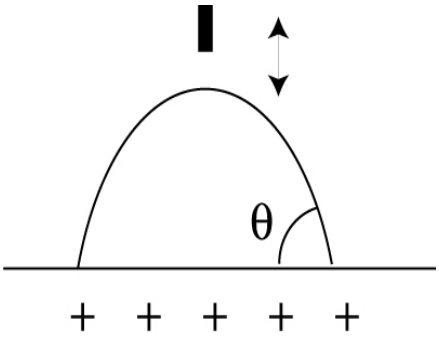
$$D_{eff} = D\left(1 + a\,Pe^2\right)$$

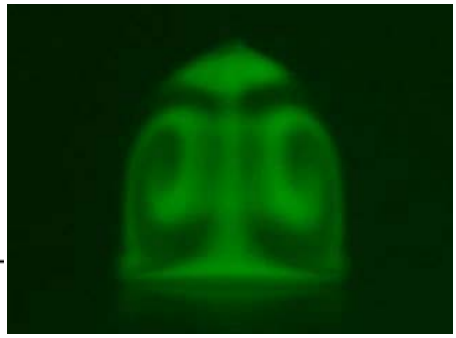
a is function of the canal geometry. For a cylinder a=1/48. One can make use of the Taylor dispersion to enhance mixing → annular micro mixer (see exercise 3)

4)2) Chaotic mixing

The principle of chaotic mixing is to stretch and fold streamlines in order to increase the surface of exchange between reactive and in this way facilitate the diffusion and homogenization. This can be done using an external actuator or taking advantage of the canal geometry.

i) External actuator: modification of the contact angle of a droplet using electrical current.



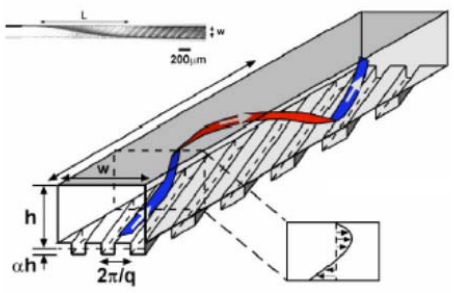


Mugele (2002)

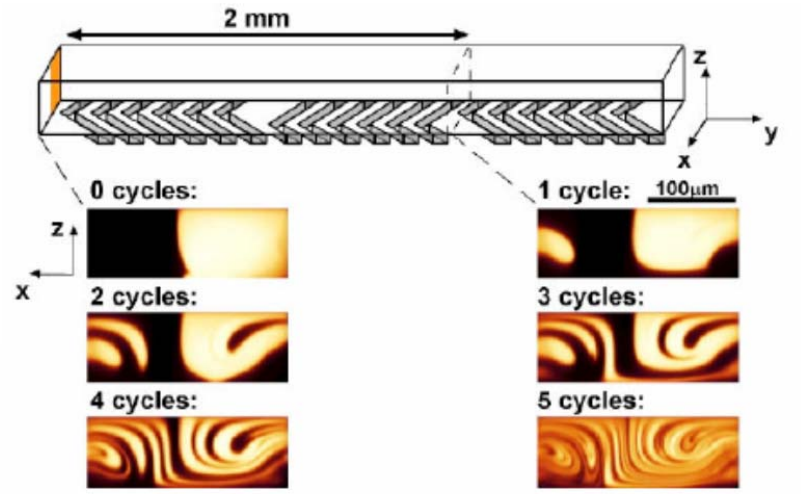
ii) Passive mixer using Geometry

Grooves can be made at the canal wall to force the liquid flowing alternatively from left to right creating a secondary flow which is chaotic.

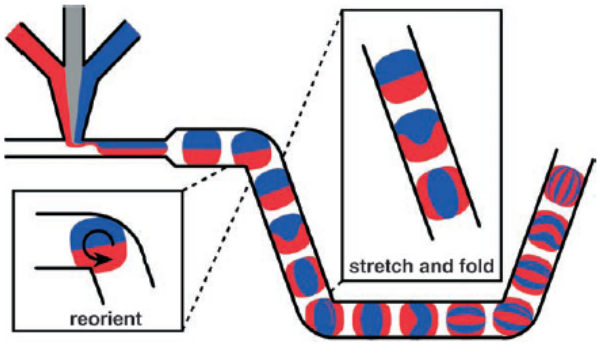
Typical actuator based on canal geometry



Stroock (2002)



Another typical mixer based on canal geometry is a sinusoidal canal as depicted below. This principle works for droplets dispersed in a continuous phase and is of great interest when each droplet is used as an independent micro reactor.

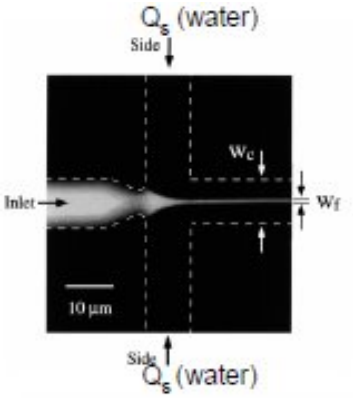


Ismagilov (2004)

4)3) Reducing size

A passive mixing strategy → size reduction in order to make the diffusion effective enough even at low Reynolds number

i) Flow focusing



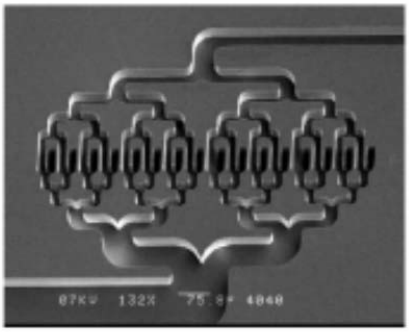
w_f = 100 nm – 1 µm

$t_d = w_f^2/D$ = 10 µs – 1 ms ⇒ fast mixing

⇒ but hydrodynamic dispersion due to the top and bottom walls
⇒ small heights are needed

Knight *et al.* Phys. Rev. Lett. 1998

i) -Multi-laminar flow



Manz *et al.* 2004

$L_o = U/t = UD/a^2 = Pe/a$

$L_n = U/t_n = UD/(a/N)^2 = Pe/a\ 1/N^2$

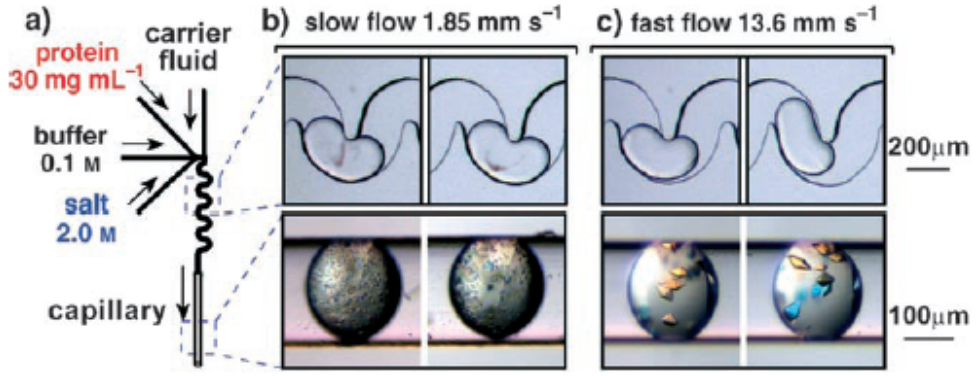
L is divided by N^2

4)4) Controlling mixing and protein crystallization

It is very difficult to determine the function of a protein without knowing its structure (even if the sequence of amino acids is well known).

On line X-ray monitoring

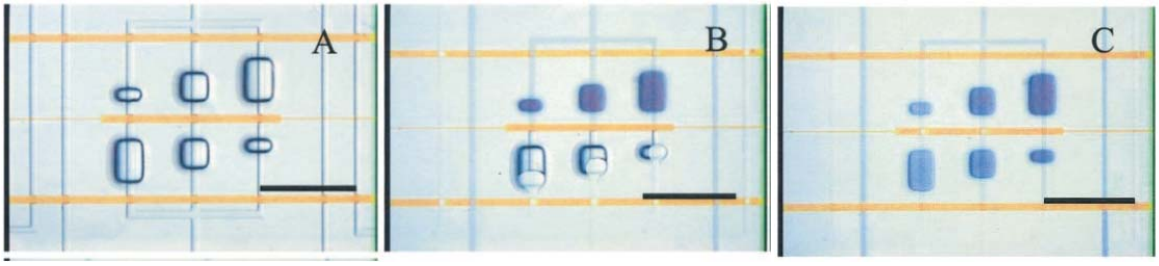
One very promising way to access this information is to perform X ray crystallography in microdrops produced in microfluidics chips. In this way, the volume of required material and especially of required protein is reduced which leads to reduced costs. Additionally it allows screening the space parameter to optimize the growth of single crystals of the desired quality.



Chen (2005)

Free interface diffusion and kinetic path

Hansen 2002



To screen various kinetic path of protein crystallisation and find optimal conditions. This is not possible with microbatch (only 1 composition can be studied) or with hanging drop (where only a concentration process of all compounds can be achieved via solvent evaporation).