Lecture 3: Hash indexing, index selection

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Today's lecture

- Morning session: Hashing
  - Static hashing, hash functions
  - Extendible hashing
  - Linear hashing
  - Newer techniques: Buffering, two-choice hashing
- Afternoon session: Index selection
  - Factors relevant for choice of indexes
  - Rules of thumb; examples and counterexamples
  - Exercises

What data in index?

- At least three possibilities:
  1) Record of key.
  2) Key and pointer to record of key.
  3) Key and list of pointers to the records containing the key (for non-unique keys).

- For simplicity, we consider the case where there is the same number of keys (B) in every disk block.
  - Case 1 with fixed length records.
  - Case 2 with fixed length keys.

Static external hashing

- Hash table:
  - Array of **N** disk blocks. (Notation from RG.)
  - Can access block **i** in 1 I/O, for any **i**.
- Hash function **h**:
  - Maps keys to {0,...,**N**-1}.
  - Should be efficient to evaluate (0 I/Os).
  - Idea: **x** is stored in block **h(x)**.
- Problem:
  - Dealing with overflows.
  - Standard solution: Overflow chains.

Problem session

- Consider the following claim from RG:

If we have N buckets, numbered 0 through $N-1$, a hash function h of the form $h(value) = (a*value+b)$ works well in practice. (The bucket identified is $h(value)$ *mod* N.)

- Donald Ummy uses this hash function in an application, and finds out that it performs terribly, no matter how the constants a and b are chosen.
- What might have gone wrong?

Randomized hash functions

Another approach (not mentioned in RG):

- Choose **h** at random from some set of functions.
- This can make the hashing scheme behave well regardless of the key set.
- E.g., "universal hashing" makes chained hashing perform well (in theory and practice).
- Details out of scope for this course...

Analysis, static hashing

- Notation:
  - N keys inserted,
  - Each block in the hash table can hold B keys.
- Suppose that we insert αN keys in the hash table (”it is a fraction α full”, “load factor α”).
- Assume **h** is truly random.
- Expected number of overflow blocks: $(1-\alpha)^{-2} \cdot 2^{-\Omega(B)}\, N$ (proof omitted!)
- Good to have many keys in each bucket (an advantage of secondary indexes).

Sometimes, life is easy

- If B is sufficiently large compared to N, all overflow blocks can be kept in internal memory.
- Lookup in 1 I/O.
- Update in 2 I/Os.

Too many overflow chains?

Can have too many overflow chains if:

- The hash function does not distribute the set of keys well (”skew”).
  - Solution 1: Choose a new hash function.
  - Solution 2?: Overflow in main memory.
- The number of keys in the dictionary exceeds the capacity of the hash table.
  - Solution: Rehash to a larger hash table.
  - Better solution: ?

Doubling the hash table

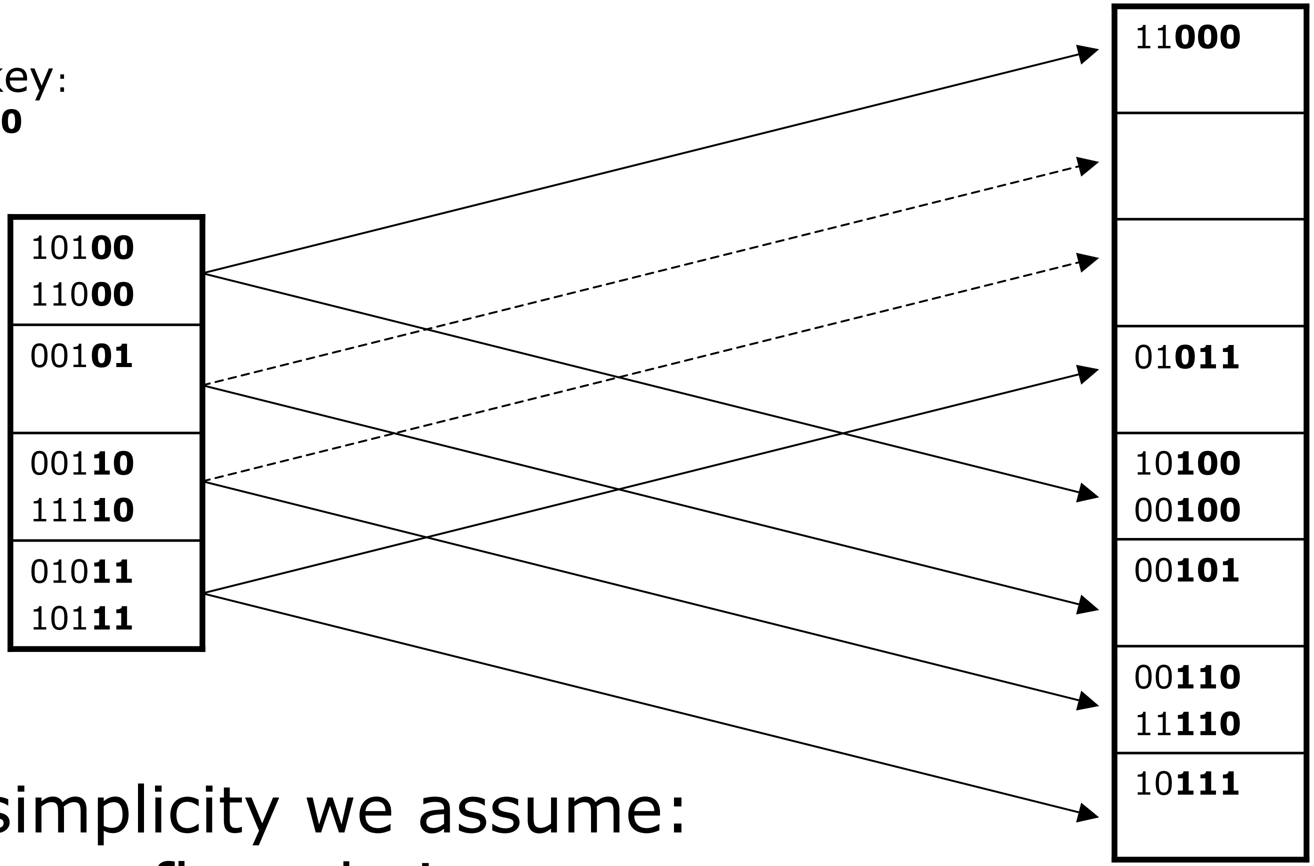
- For simplicity, assume **N** is a power of 2. Suppose **h** is a hash function that has values of ”many” (e.g. 64) bits.
- We can map a key **x** to {0**,...,N**-1} by taking the log **N** least significant bits of **h(x)**.
- Suppose that the hash table has become too small:
  – Want to double the size of the hash table.
  – Just consider one more bit of **h(x)**.

Doubling the hash table, cont.

- Suppose **h(x)**=0111001 (in binary) and the hash table has size 16.
- Then **x** is stored in block number 1001 (binary).
- After doubling to size 32, **x** should be stored in block 11001.
- Generally, all keys in block 1001 should be moved to block 01001 or 11001.
- **Conclusion**: Can rehash by scanning the table and split each block into two blocks.

Doubling, example

New key:
00100



For simplicity we assume:

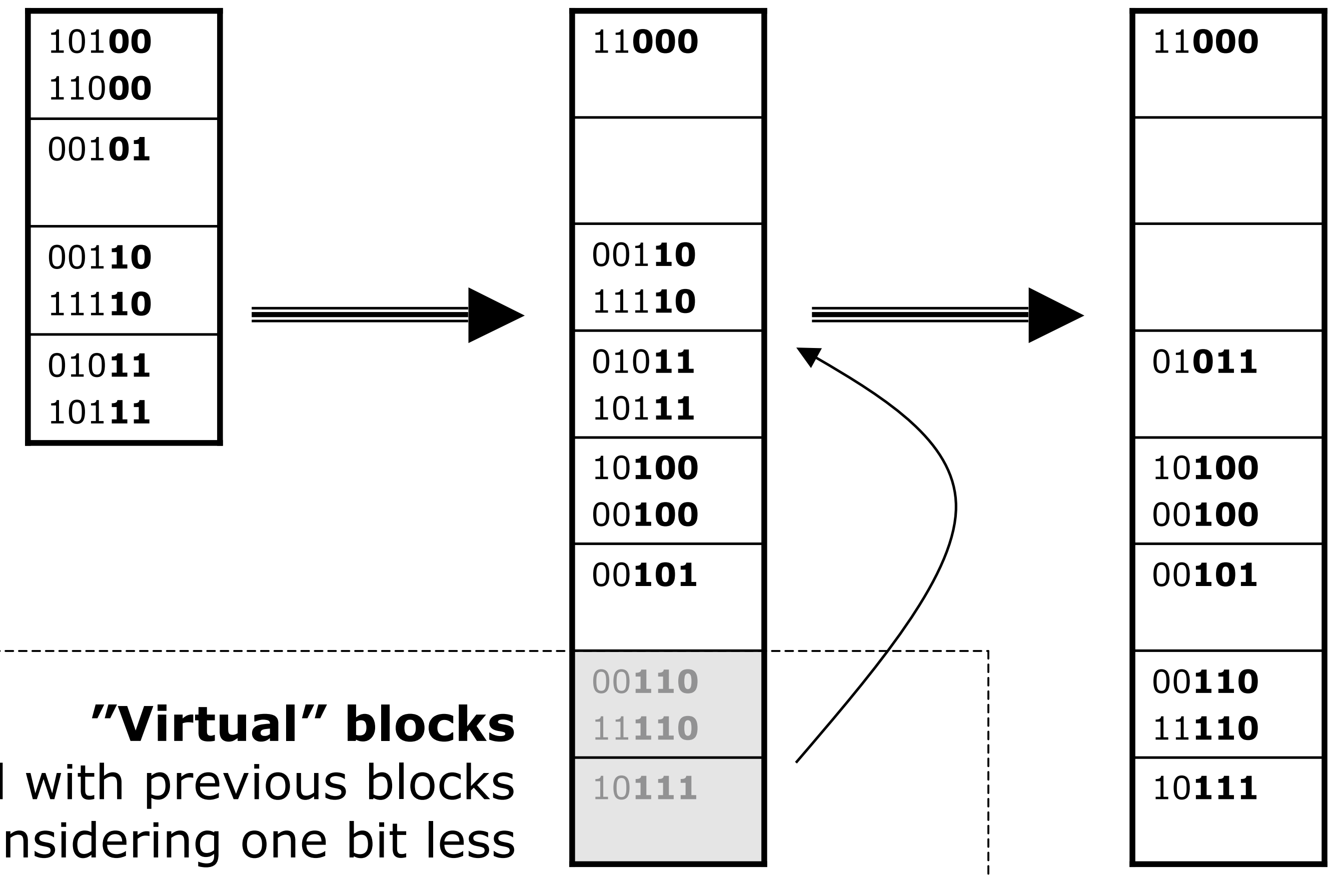
- No overflow chains
- h(x)=x

Problem session

- Find some possible disadvantages of the doubling strategy. Consider:
  - Space usage vs overflows
  - System response time

- **Next**: Alternatives that address some of the disadvantages of doubling.

Linear hashing



"Virtual" blocks

- Merged with previous blocks by considering one bit less
- Turned into physical blocks as the hash table grows

Linear hashing - performance

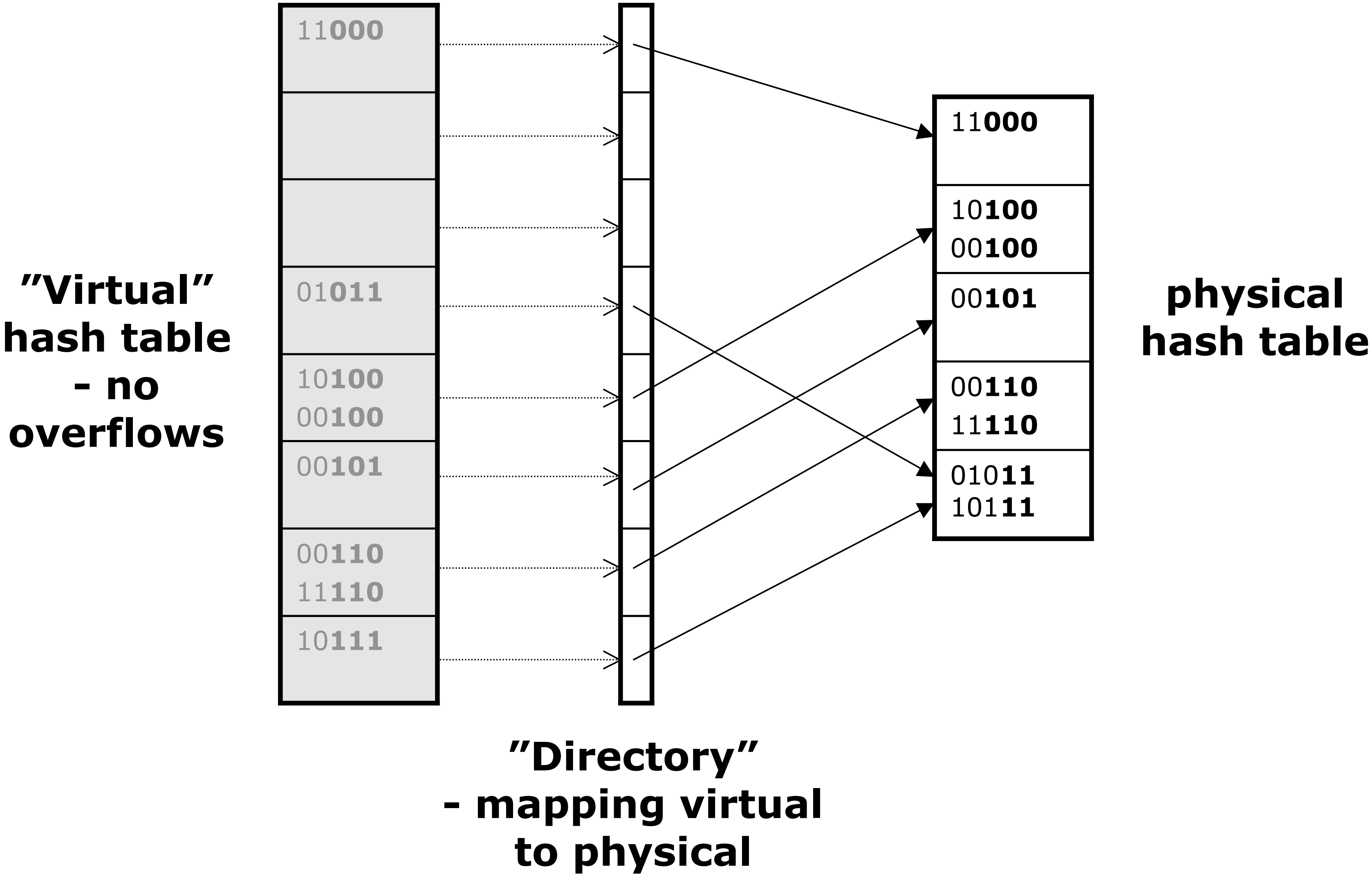
The good:

- Resizes hash table one block at a time: Split a block or merge two blocks.
- Cost of resize: 3 I/Os. Cheap!

The bad:

- Increasing size of hash table may not eliminate any overflow chain.
- Uneven distribution of hash values; works best for relatively low load factors, 50-80%. (But variants of linear hashing improve this.)
- No worst-case guarantee on query time.

Extendible hashing



Extendible hashing invariants

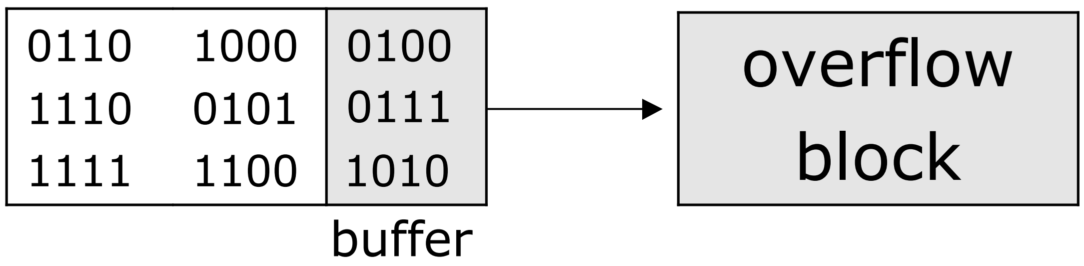
- Virtual hash table has no overflows - may need to increase in size.
- Physical hash table has no overflows.
- Virtual hash table is as small as possible - may need to shrink.
- **”Compression”**: For any bit string **s**, if we consider the virtual hash table blocks whose index ends with **s** then either:
  - These blocks contain more than B keys, or
  - The corresponding entries in the directory all point to the same block. (In other words, these blocks are merged.)

Extendible hashing performance

- At most 2 I/Os for every lookup.
- Only 1 I/O if directory fits in internal memory.
- Space utilization in physical hash table is 69% (expected).
- Size of directory is roughly $4N\sqrt[B]{N}$ (expected) - this is much smaller than the hash table if B is moderately large.

Buffering

- Same trick as in buffered B-trees: Don't do updates right away, but put them in a buffer.



- Advantage: Several keys moved to the overflow block at once.
- Disadvantage: Buffer takes space.

Two-choice hashing

- **Idea**:
  - Use two hash functions, h_1 and h_2.
  - x is stored in either block $h_1(x)$ or $h_2(x)$, use two I/Os for lookup.
  - When inserting x, choose the least loaded block among h_1 and h_2.
- Can be shown that overflow probabilities are much smaller than with one function, especially when B is small.
- If two disks are available, the 2 I/Os can be done in parallel.

Today's lecture, part 2

- Index selection
  - Factors relevant for choice of indexes
  - Rules of thumb; examples and counterexamples
- Exercises

Workload

- The workload (mix of operations to be carried out by the DBMS) has a large influence on what indexes should be created in a database.
- Other factors are:
  - the data in relations, and
  - the query plans produced by the DBMS.

Rules of thumb

- Rules of thumb can be used to guide thinking, and as a checklist.
- Are often valid in most cases, but there are always important exceptions.
- Quote from SB:

  The point of the example is that the tuner must understand the reason for the rule

- You don’t yet have the entire picture (query optimization, concurrency), but we can start reasoning about rules anyway.

Rule of thumb 1:
Index the most selective attribute

- Argument: Using an index on a selective attribute will help reducing the amount of data to consider.
- Example:

```
SELECT count(*) FROM R
WHERE a>'UXS' AND b BETWEEN 100 AND 200
```

- Counterexamples:
  - Full table scan may be faster than an index
  - It may not be possible/best to apply an index.

Rule of thumb 2:
Cluster the most important index of a relation

- Argument:
  - Range and multipoint queries are faster.
  - Usually sparse, uses less space.
- Counterexamples:
  - May be slower on queries ”covered” by a dense index.
  - If there are many updates, the cost of maintaining the clustering may be high.
  - Clustering does not help for point queries.
  - Can cluster according to several attributes by duplicating the relation!

Rule of thumb 3:
Prefer a hash index over a B-tree if point queries are more important than range queries

- Argument:
  - Hash index uses fewer I/Os per operation than a B-tree.
  - Joins, especially, can create many point queries.
- Counterexamples:
  - If a real-time guarantee is needed, hashing can be a bad choice.
  - Might be best to have **both** a B-tree and a hash index.

Hashing and range queries

RG page 371:

Hash-based indexing techniques cannot support range searches, unfortunately.

- **But**: they can be *used* to answer range queries in **O(1+Z/B)** I/Os, where Z is the number of results. (Alstrup, Brodal, Rauhe, 2001; Mortensen, Pagh, Patrascu 2005)
- Theoretical result on external memory (why?) - and out of scope for DBT.

Problem session

- Comparison of B-trees and extendible hashing.
  - Case 1: Directory fits internal memory.
  - Case 2: Directory on external memory.
  - Case A: $B=4$, $N=2^{20}$.
  - Case B: $B=2^8$, $N=2^{20}$.
- Consider cases 1A, 1B, 2A, 2B.

Rule of thumb 4:

Balance the increased cost of updating with the decreased cost of searching

- Argument: The savings provided by an index should be bigger than the cost.
- Counterexample:
  - If updates come when the system has excess capacity, we might be willing to work harder to have indexes at the peaks.
- If buffered B-trees are used, the cost per update of maintaining an index may be rather low. Especially if binary (!) trees are used.

Rule of thumb 5:
A non-clustering index helps when the number of rows to retrieve is smaller than the number of blocks in the relation.

- Argument: In this case it surely reduces I/O cost.
- Counterexample:
  - Even for a non-clustered index, the rows to retrieve can sometimes be found in a small fraction of the blocks (e.g. salary, clustered on date of employment).

Rule of thumb 6:
Avoid indexing of small tables.

- Argument: Small tables can be kept in internal memory, or read entirely in 1 or 2 I/Os.
- Counterexample:
  - If the index is in main memory, it might still give a speedup.

Conclusion

- Indexing is a complicated business!
- Understanding the various index types and their performance characteristics, as well as the characteristics of the database at hand and its workload allows informed indexing decisions.
- Rules of thumb can be used to guide thinking.
- More complications to come!

Tip: Clustered indexing in Oracle

- Default in Oracle is to store tuples in a heap (think insertion order).
- Is clustered according to the **primary key**, if ”`ORGANIZATION INDEX`” is added after the schema when creating the relation.
- To cluster according to a non-unique attribute A, declare a composite primary key (A,P), where P is a unique key.

Exercises

Hand-outs:

- ADBT exam, June 2005, problem 1.
- ADBT exam, June 2006, problem 1.