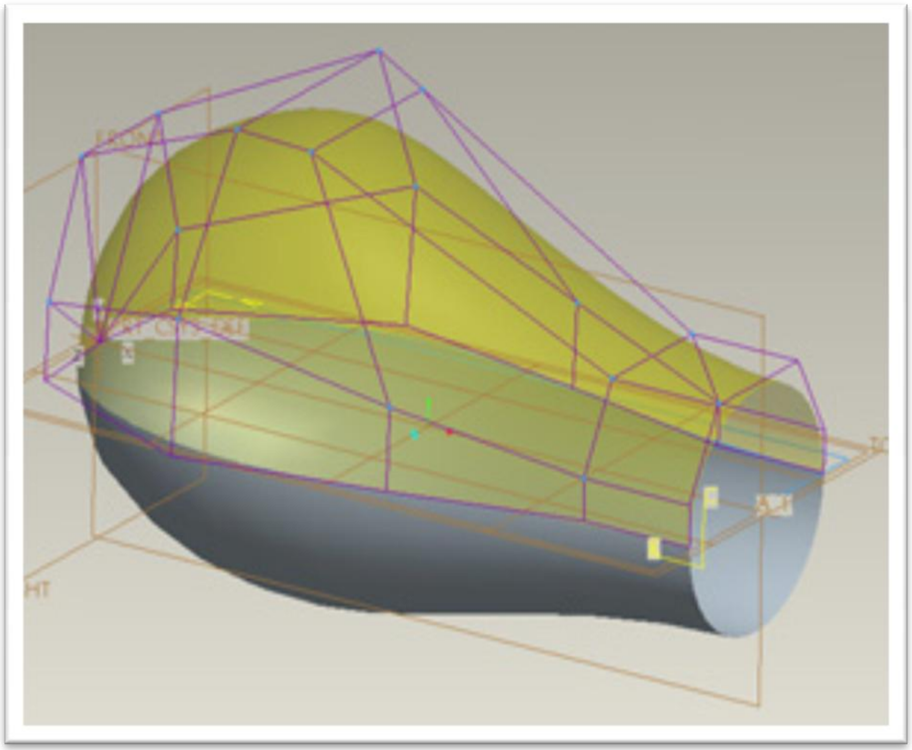
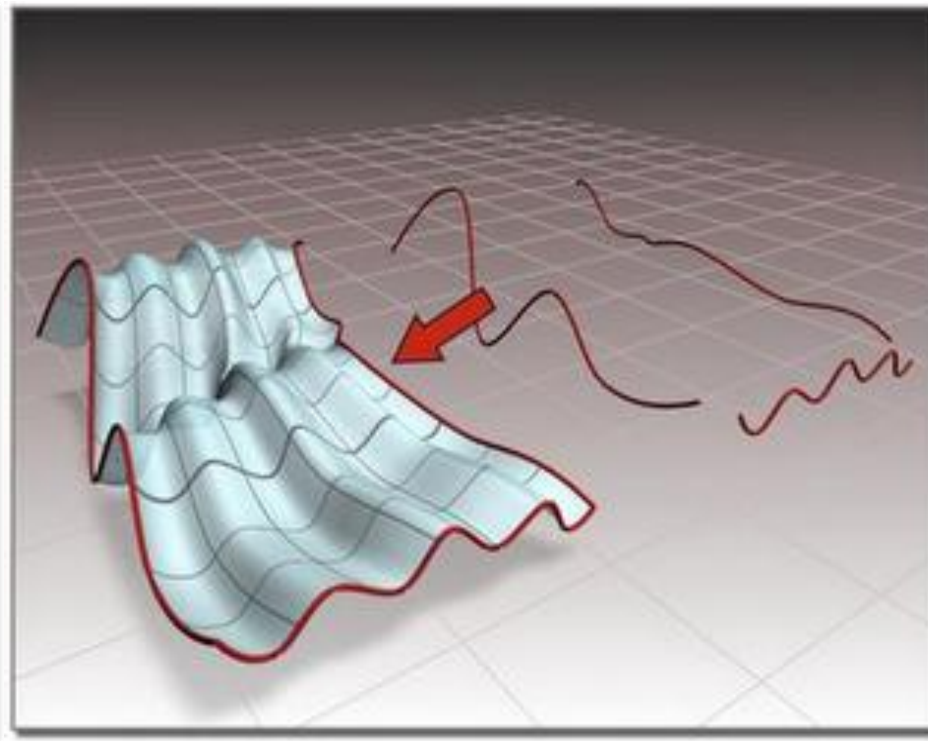
CAD/CAM

MSc. Course / Production Engineering





Lecture [3] : Surface Modeling

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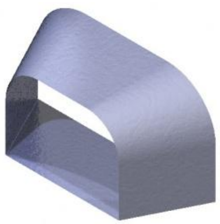
Surface model (Late 1980's):

Surface modeling is more sophisticated than wireframe modeling in that it defines not only the edges of a 3D object, but also its surfaces.

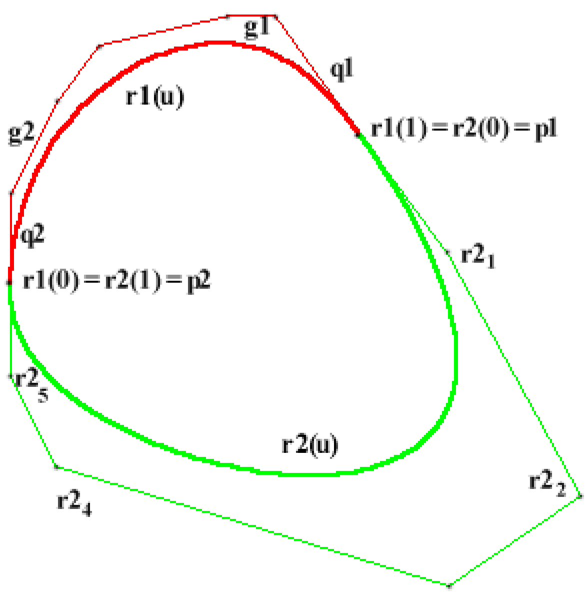
Surface modeling gives designers a great amount of ***control*** and ***flexibility***.

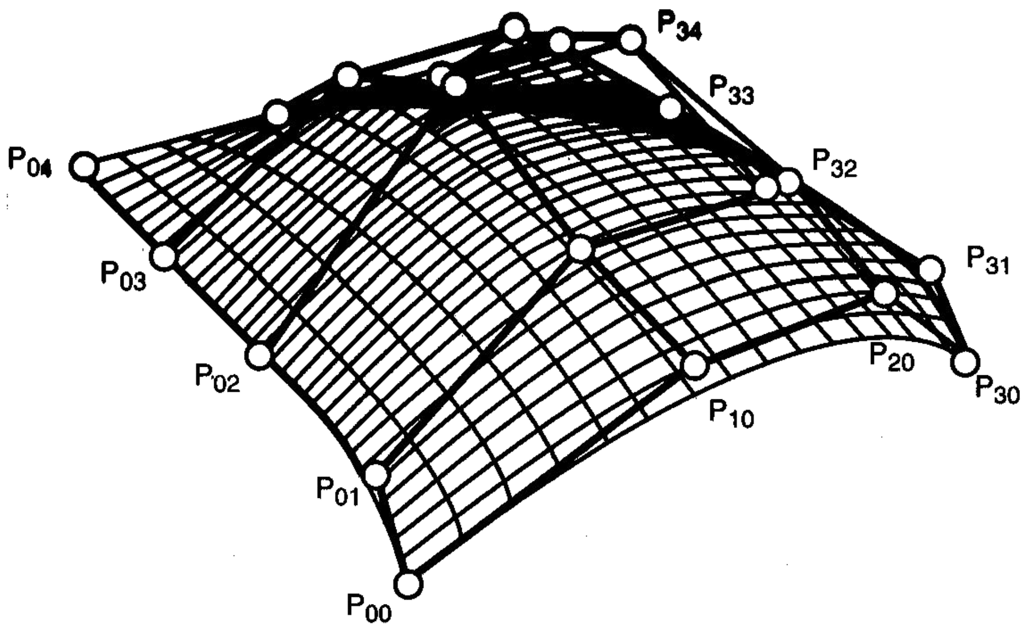


Surface modeling entities

Analytic surfaces (plane surfaces, ruled surfaces, surface of revolution, tabulated surfaces)

Synthesis surfaces (parametric cubic surfaces, Bezier surfaces, B-spline surfaces, ….)



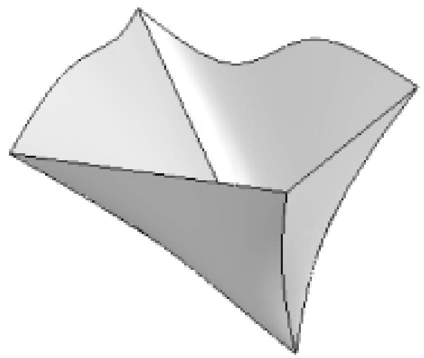


- Surface modeling is a widely used modeling technique in which objects are defined by their *bounding faces*.
- Surface modeling systems contain definitions of *surfaces*, *edges*, and *vertices*
- Complex objects such as car or airplane body can not be achieved utilizing wireframe modeling.

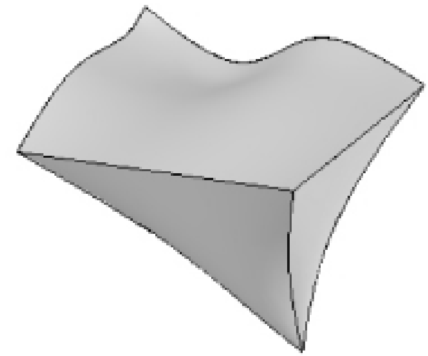
Surface modeling are used in

- calculating mass properties
- checking for interference between mating parts
- generating cross-section views
- generating finite elements meshes
- generating NC tool paths for continuous path machining

- All points on surface are defined
  - useful for machining, visualization, etc.
- Surfaces have no thickness, objects have no volume or solid properties
- Surfaces may be open



Open Surface



Closed Surface

Surface Model: An area bounded by an identifiable **perimeter**.
In Computer Graphics, is an area within which every position is defined by mathematical method.

Surface may be:

- Planar
- Cylindrical/conic
- Sculptured or freeform in shape

Sculptured or Free Form Surfaces

→ mathematical description of "hand drawn" surfaces, analogy to free form curves

→ possibility of using as mathematical approximate description of algebraic and geometric surfaces (e.g. as interpolation surface through given set of points or curves; more points means more accurate description, but more processor load)

→ free form surfaces are given by a rectangular array of control points and a mathematical model defining the way the surface is created

→ by this model the free form surface are divided as:

- **interpolation** free form surfaces – the surface is passing through the control points
- **approximation** free form surfaces – the curve doesn't have to pass through the control points; commonly used are (non) rational bézier surfaces and (non) rational (non) uniform b-spline surfaces

→ each control point of a curve in one direction is replaced by another bézier curve, all this curves has to have the same degree (and number of control points), the control points of these curves are control points of the surface

→ the degree of surface can be different in u and v directions

Quadratic Surfaces

- Sphere

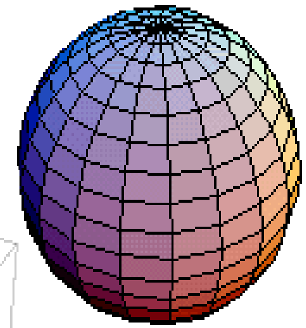
$$x^2 + y^2 + z^2 = r^2$$

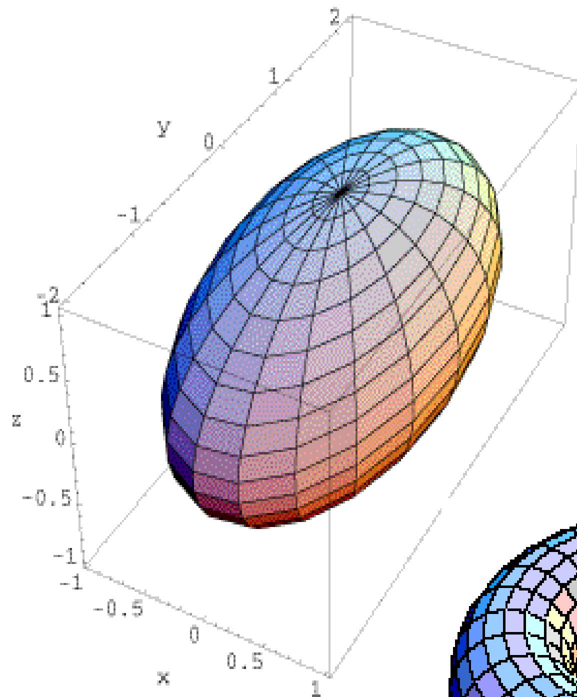
- Ellipsoid

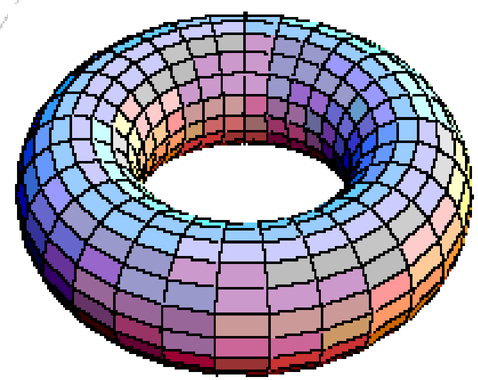
$$\frac{x}{r_x}^2 + \frac{y}{r_y}^2 + \frac{z}{r_z}^2 = 1$$

- Torus

$$r \sqrt{\frac{x}{r_x}^2 + \frac{y}{r_y}^2}^{\,2} + \frac{z}{r_z}^2 = 1$$





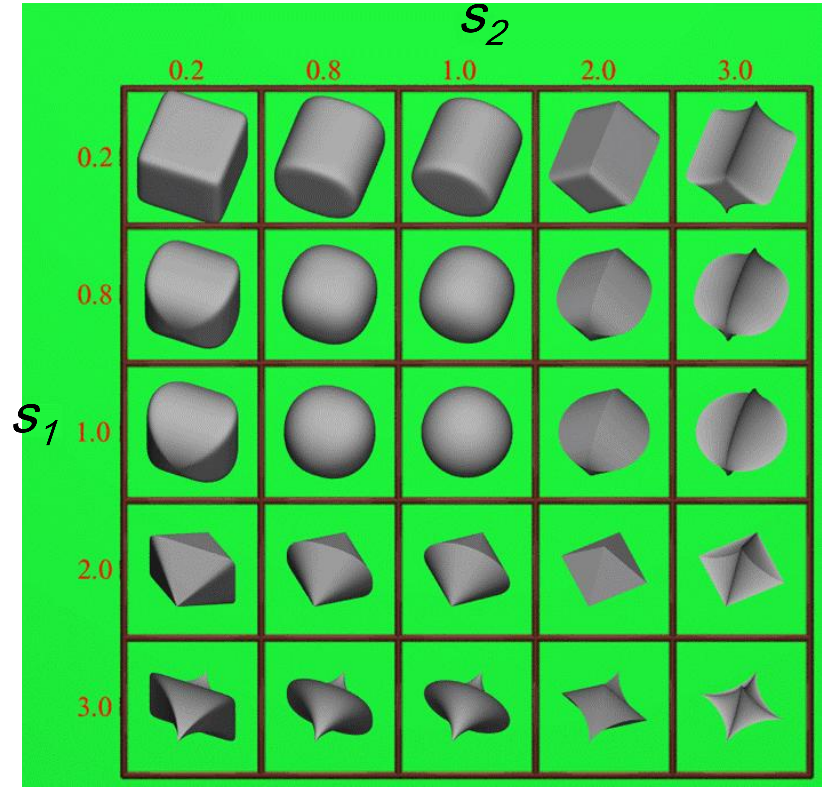


Superellipsoid Surfaces

- Generalization of ellipsoid
- Control parameters s_1 and s_2

$$\left\{ {\frac{x}{r_x}}^{2/s_2} + {\frac{y}{r_y}}^{2/s_2} \right\}^{s_2/s_1} + {\frac{z}{r_z}}^{2/s_1} = 1$$

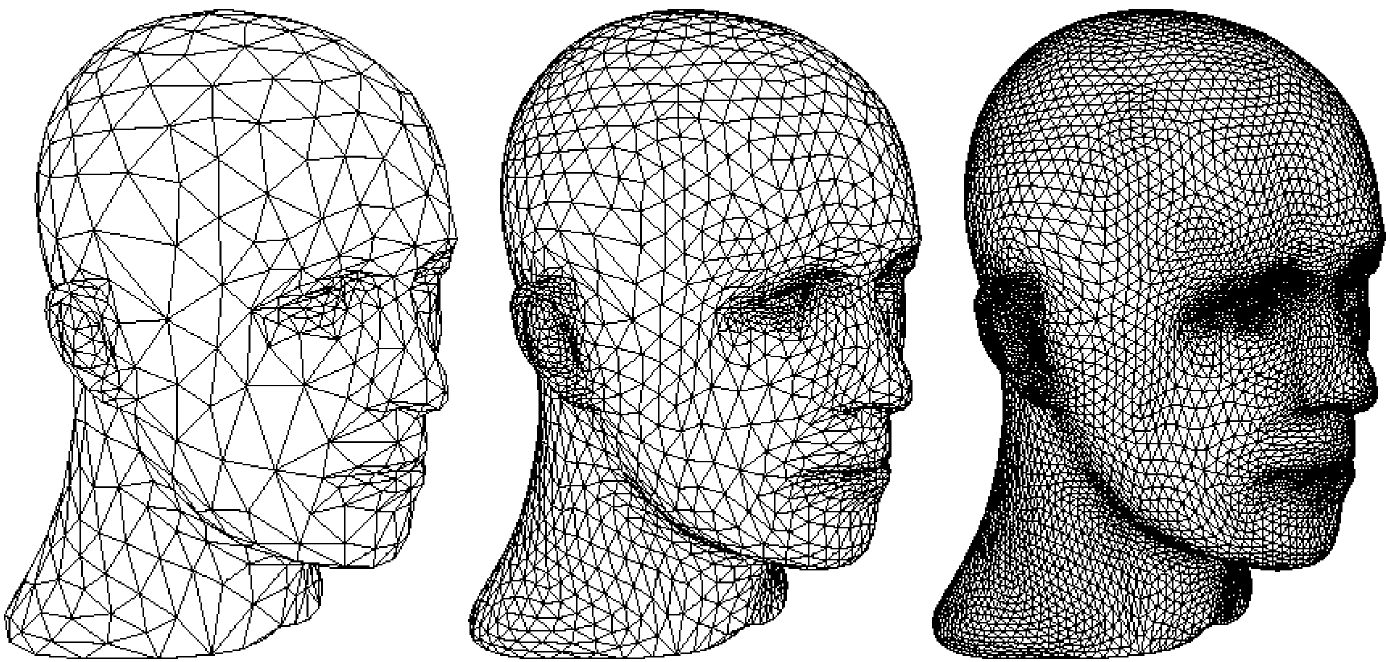
- If $s_1 = s_2 = 1$ then regular ellipsoid
- Has an implicit and parametric form.



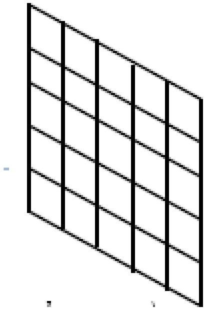
Subdivision Surfaces

Coarse Mesh & Subdivision Rule

- Define smooth surface as limit of sequence of algorithmic refinements
  - Modify topology & interpolate neighboring vertices
  - Used in graphics, animation and digital arts applications

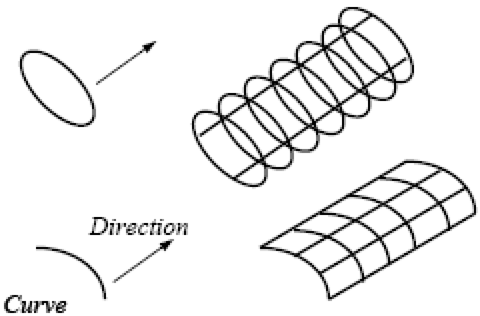


Surfaces Entities

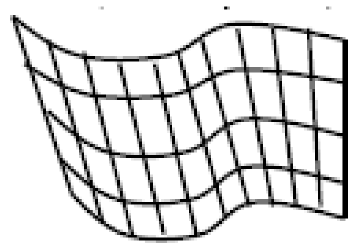


Plane surface

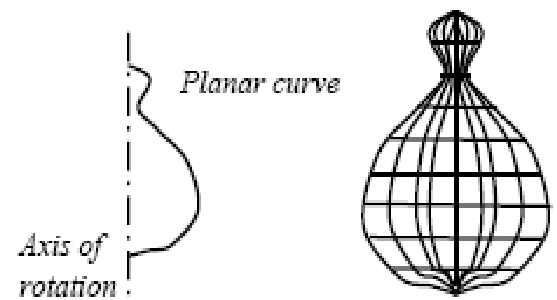
1- Analytical surface entities



Tabulated cylinder



Ruled (lofted) surface

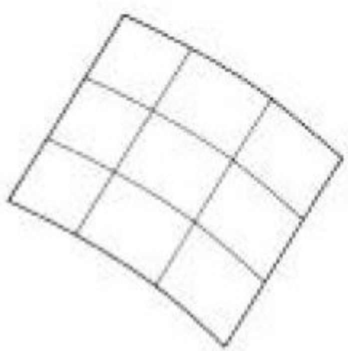


Surface of revolution

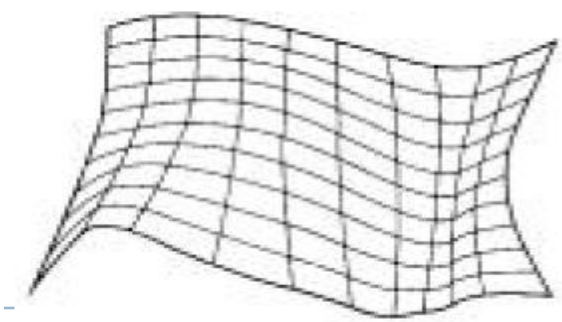
2- Synthesis surface entities

- Bezier surface

- B-spline surface



Bezier surface



B-Spline surface

Analytic Surface Representations:

Like a general analytic curve, general analytic surface can also be defined by either an implicit or an explicit equation.

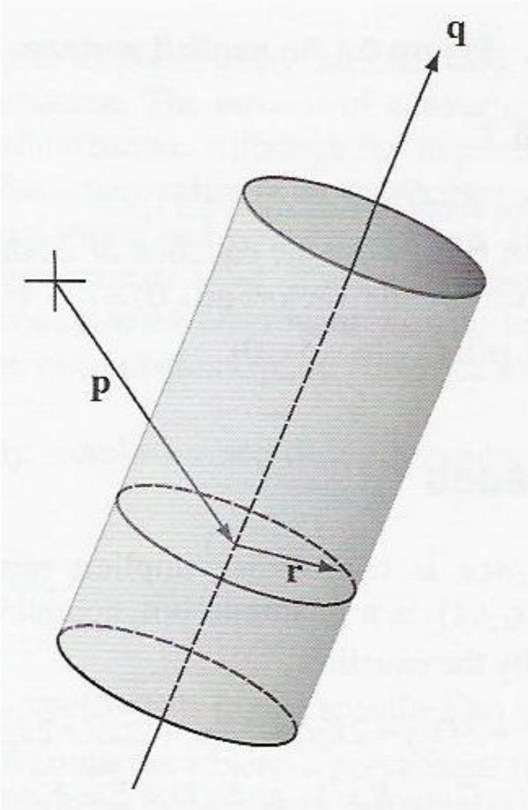
Implicit Equation

$F(x, y, z) = 0$

Its geometric meaning is that the locus of the points that satisfy the above constraint equation defines the surface.

Example:
Right circular cylinder

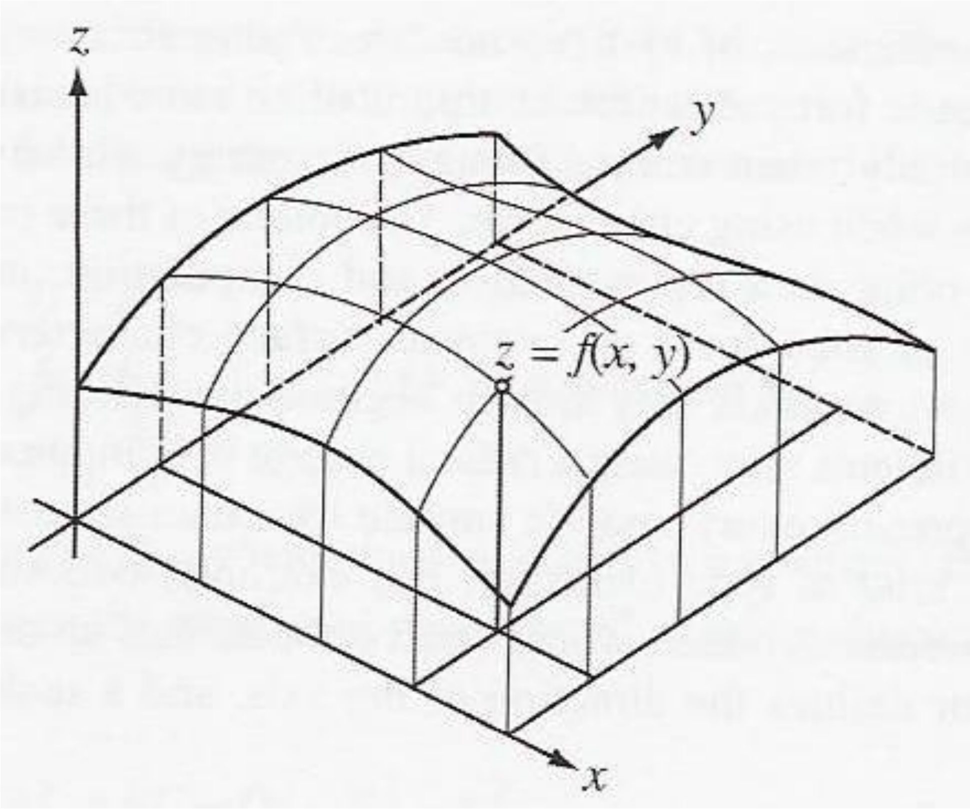
- One vector gives a point on its axis
- One vector defines axis direction
- Scalar gives radius

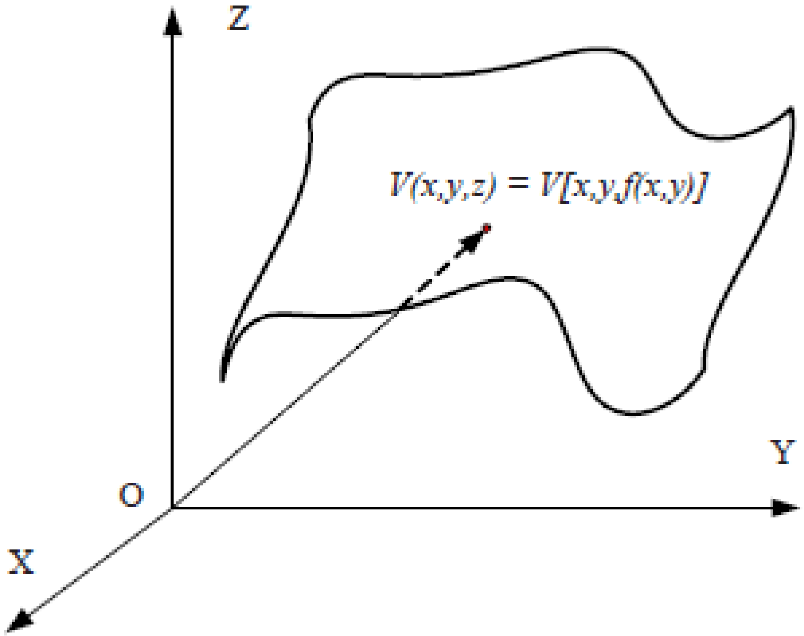


Explicit Equation

$$V = [x, y, z]^T = [x, y, f(x, y)]^T$$

where *V is the position vector of a variable point on the surface. In this equation, the variable* point coordinates *x, y, z are directly defined. The z coordinates of the position vector of* the variable points are defined by *x and y through function f(x, y), as shown in Figure*





Parametric Equation

The above equations illustrate that the points on a surface have two degrees of freedom that are directly controlled by the *x and y coordinates*. *There are no extra parameters in* these equations. Therefore, this type of surface representation is called nonparametric representation.

The fact that the surface can be controlled by *x and y coordinates, also means* that two parameters (e.g. *s and t) can always be found as the controlling parameters as the x and y coordinates do. Understandably, the equations that utilize this type of parameter* are called **parametric equations** and can be expressed as follows,

$$V(s,t) = [x, y, z]^T = [X(s,t), Y(s,t), Z(s,t)]^T, \quad s_{min} < s < s_{max},\ t_{min} < t < t_{max}$$

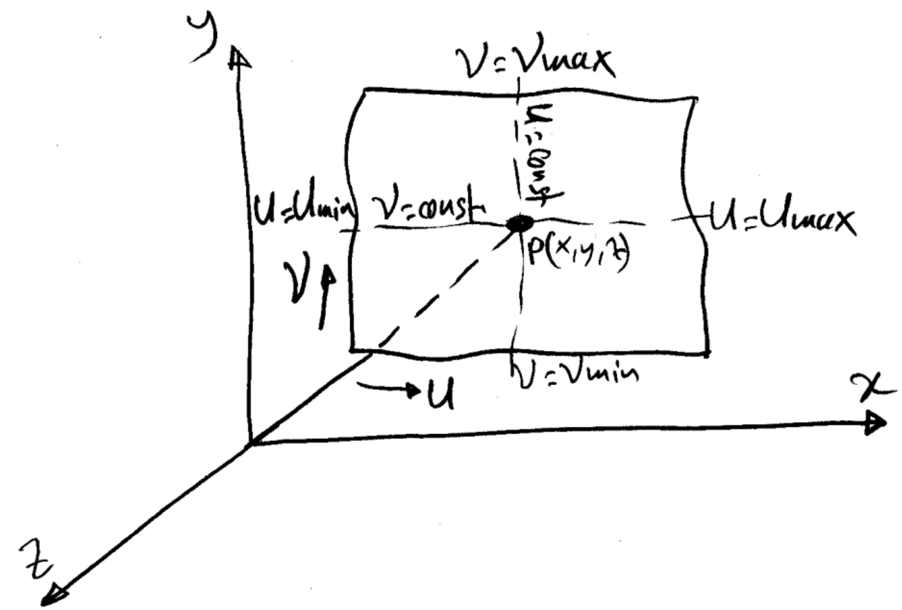
where *X, Y, and Z are the functions of the two parameters, s and t.*

$$P(u,v) = [x \quad y \quad z]^T$$

$$P(u,v) = [x(u,v) \quad y(u,v) \quad z(u,v)]$$

$$u_{\min} \le u \le u_{\max}$$

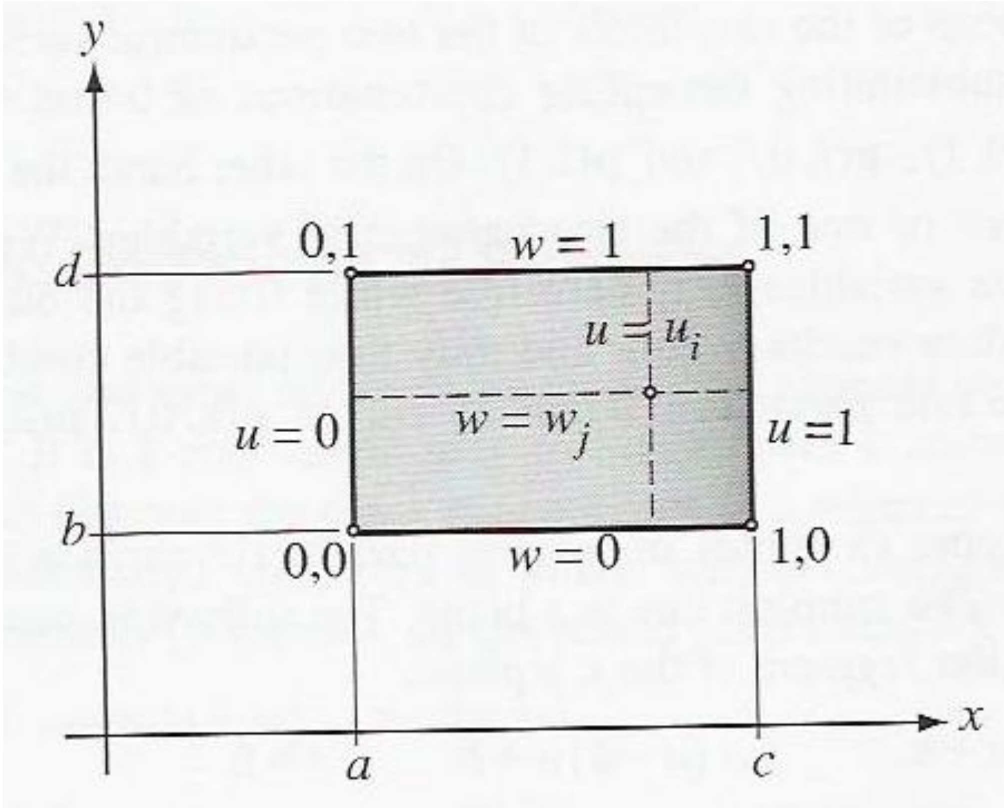
$$v_{\min} \le v \le v_{\max}$$



- **Sample patch**: rectangular segment of x, y plane

$$x = (c - a)u + a$$
$$y = (d - b)w + b$$
$$z = 0$$

- Here:
  - Curves of constant w are horizontal lines.
  - Curves of constant u are vertical lines.



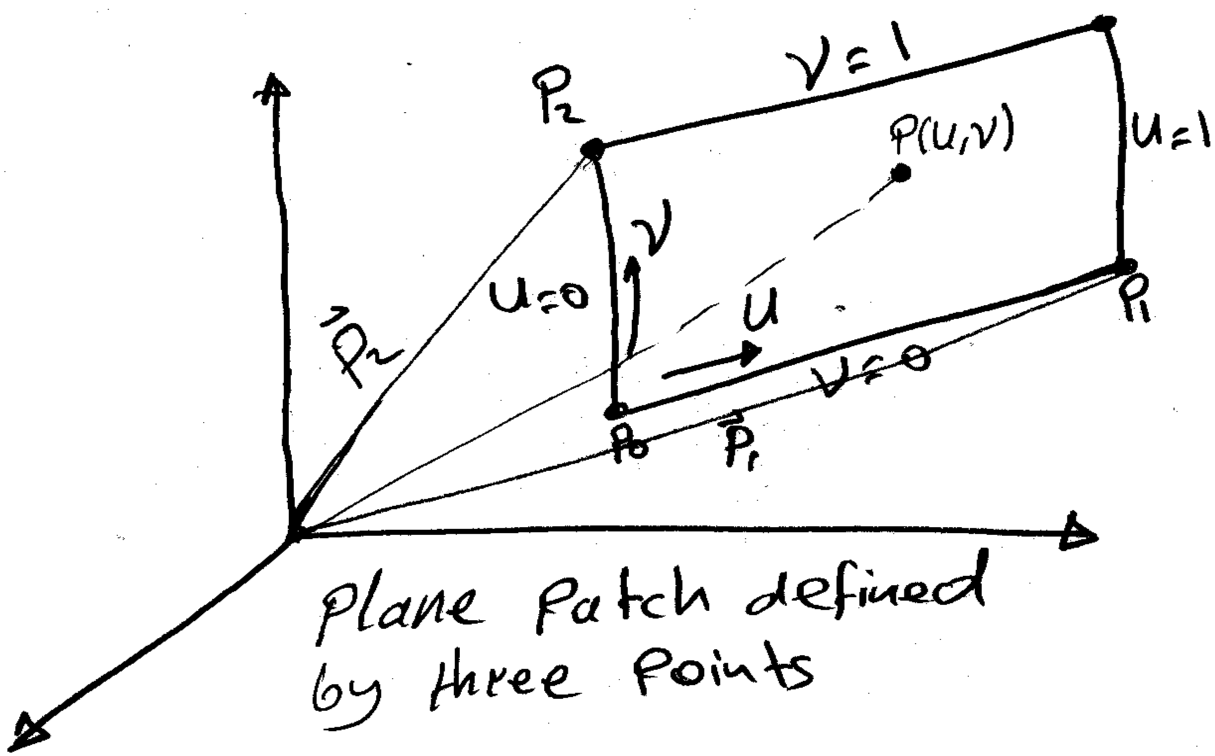
Parametric and x,y coordinates of a plane

1- Plane Surface

The parametric equation of a plane defined by three points, P_0, P_1, and P_2

$$P(u,v) = P_0 + u(P_1 \quad P_0) + v(P_2 \quad P_0)$$

$$0 \le u \le 1 \qquad 0 \le v \le 1$$



Plane Patch defined by three Points

2- Ruled Surface

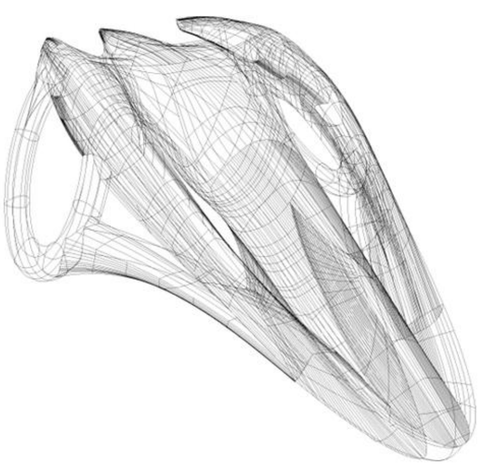
A ruled surface is generated by joining corresponding points on two space curves (rails) G(u) and Q(u) by straight lines

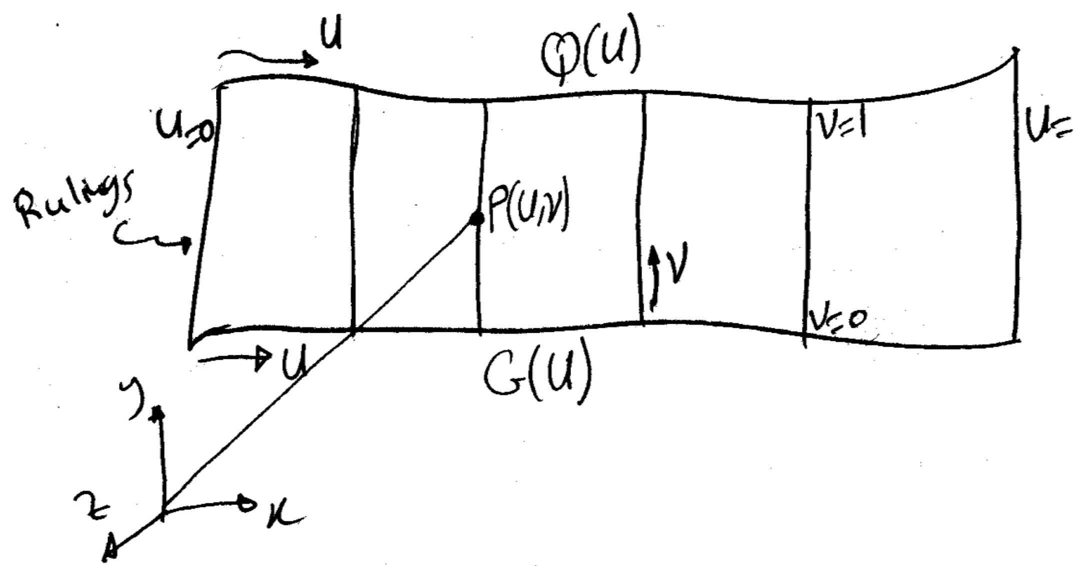
- The parametric equation of a ruled surface defined by two rails is given as

$$P(u,v) = (1 \quad v)G(u) + vQ(u)$$

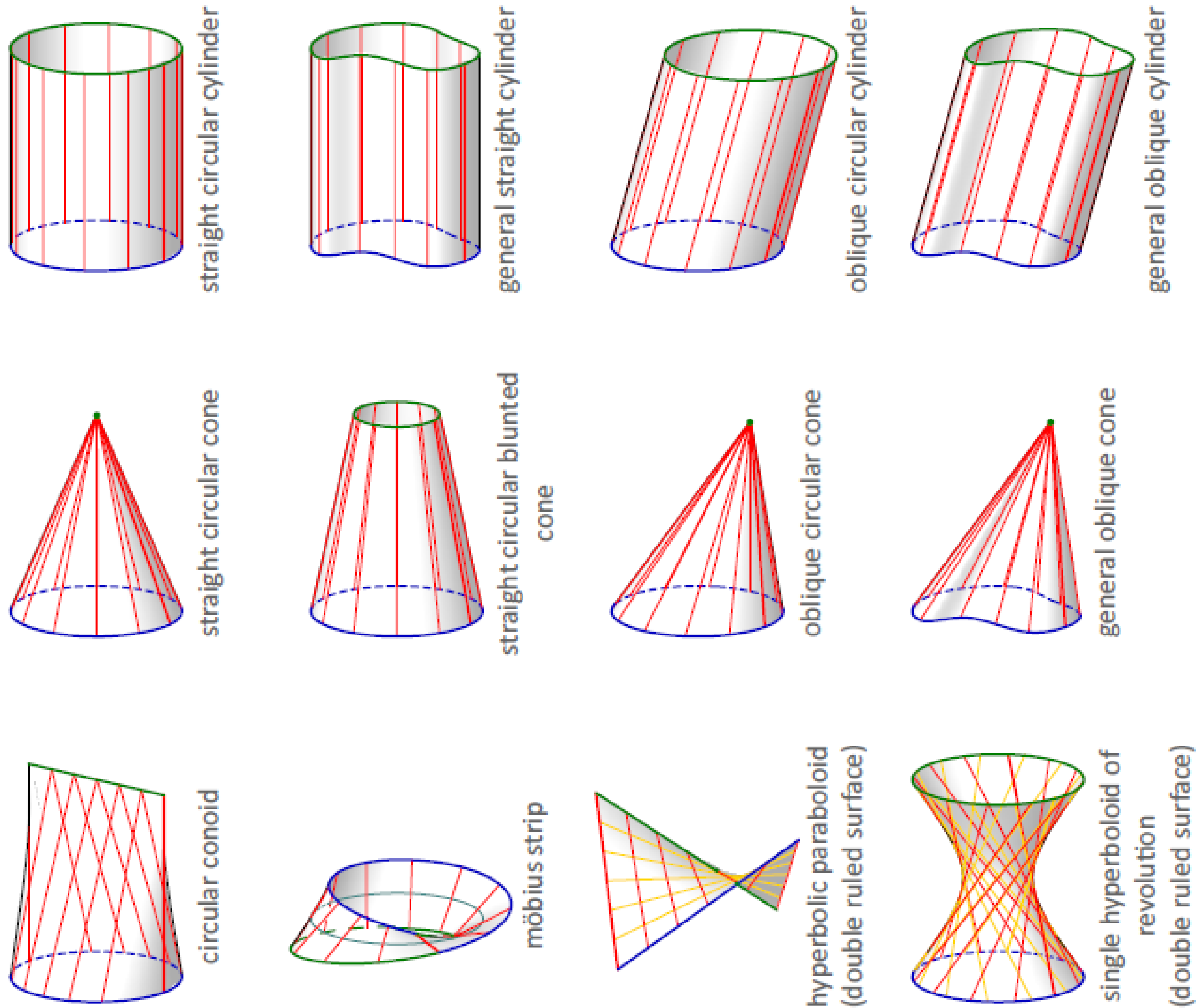
$$0 \leq u \leq 1 \quad 0 \leq v \leq 1$$

Holding the u value constant in the above equation produces the rulings in the v direction of the surface, while **holding the v value constant** yields curves in the u direction.





- → sweeping movement of a **line** along **two** (in general 3D) **curves**, some has additional conditions (like parallel plane for conoids, or third line path for conusoids).
- → **single** (lines in one direction) ruled surfaces
- → **double** (lines in two directions) ruled surfaces; the only double ruled surfaces are:
  - hyberbolic paraboloid
  - single hyperboloid of revolution
  - a plane
- → **developable surfaces** are:
  - cylindrical
  - conical
  - swept surfaces, where the profile is a tangent to path
- → **skew surface** is the opposite of developable


straight circular cylinder
general straight cylinder
oblique circular cylinder
general oblique cylinder
straight circular cone
straight circular blunted cone
oblique circular cone
general oblique cone
circular conoid
möbius strip
hyperbolic paraboloid (double ruled surface)
single hyperboloid of revolution (double ruled surface)

3- Tabulated Cylinder

A tabulated cylinder has been defined as a surface that results from translating a space planar curve along a given direction.

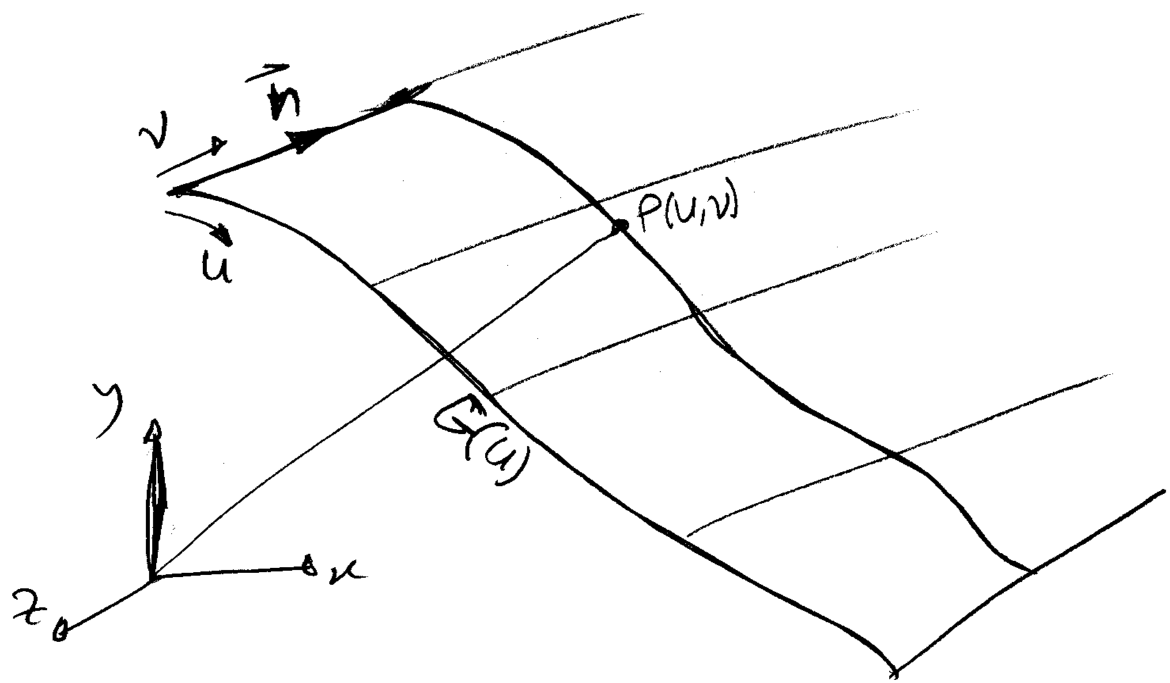
- The parametric equation of a tabulated cylinder is given as

$$P(u,v) = G(u) + v\vec{n} \qquad 0 \le u \le u_{max} \qquad 0 \le v \le v_{max}$$

Where:
G(u) can be any wireframe entities to form the cylinder

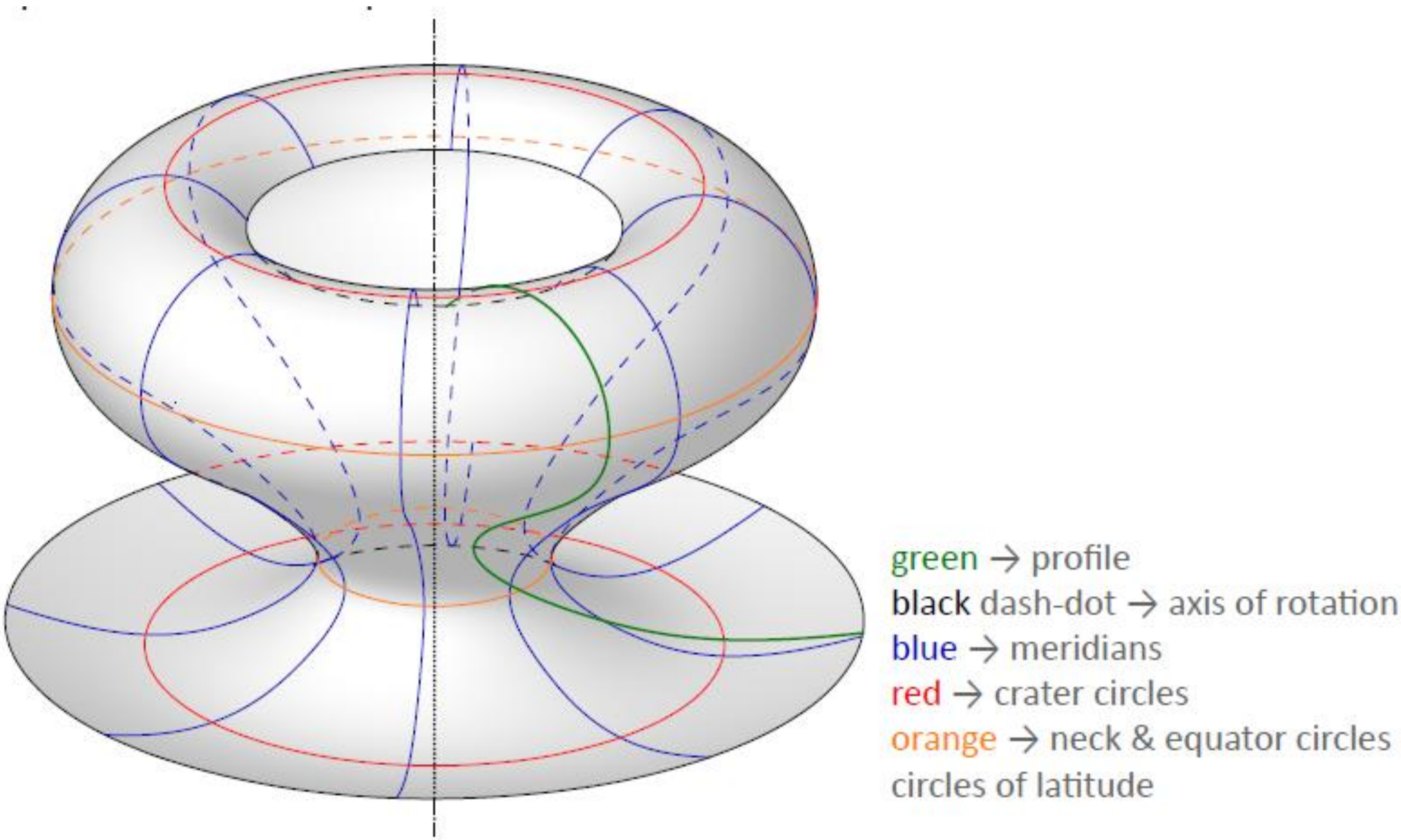
v is the cylinder length

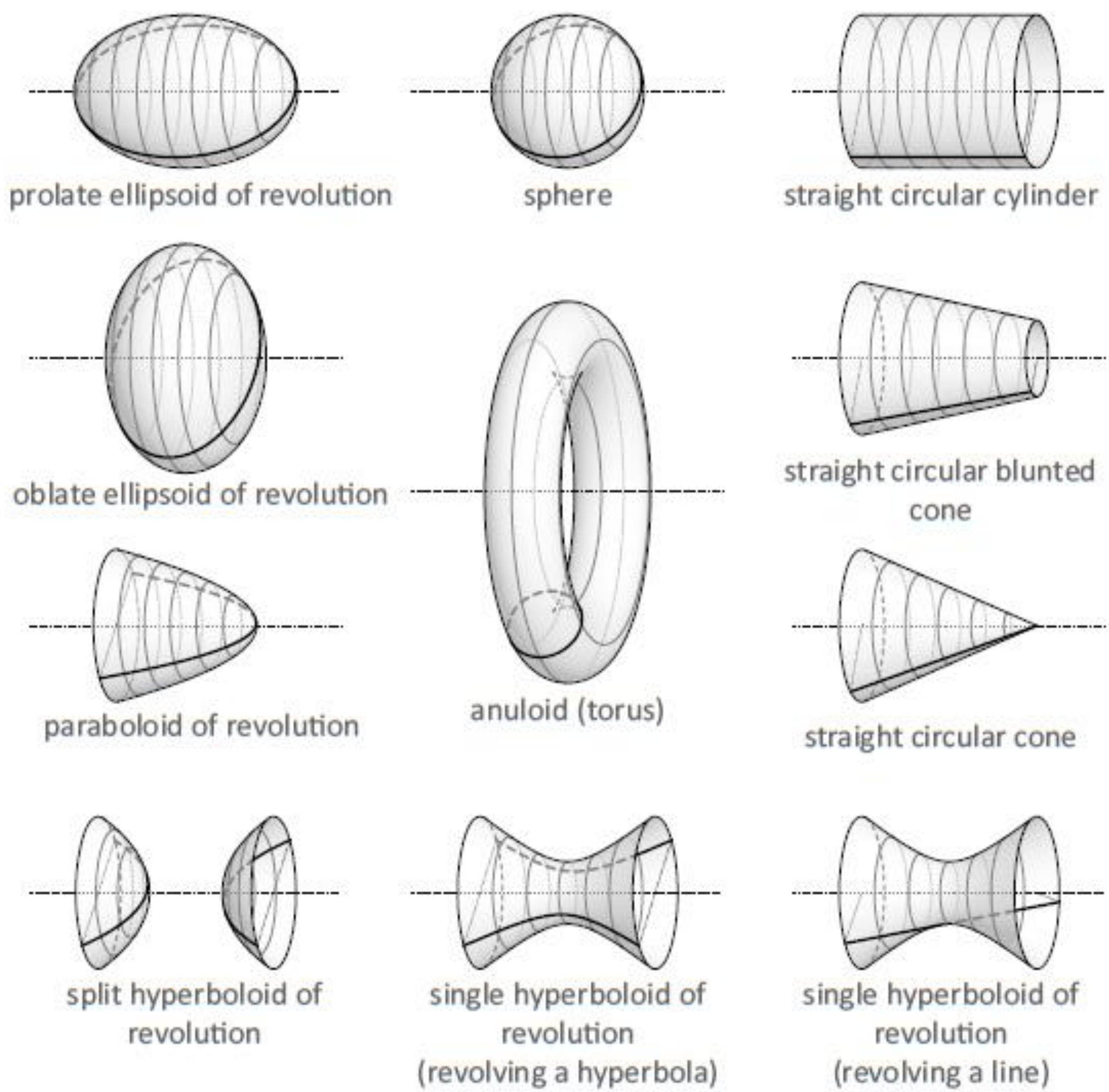
n is the cylinder axis (defined by two points)



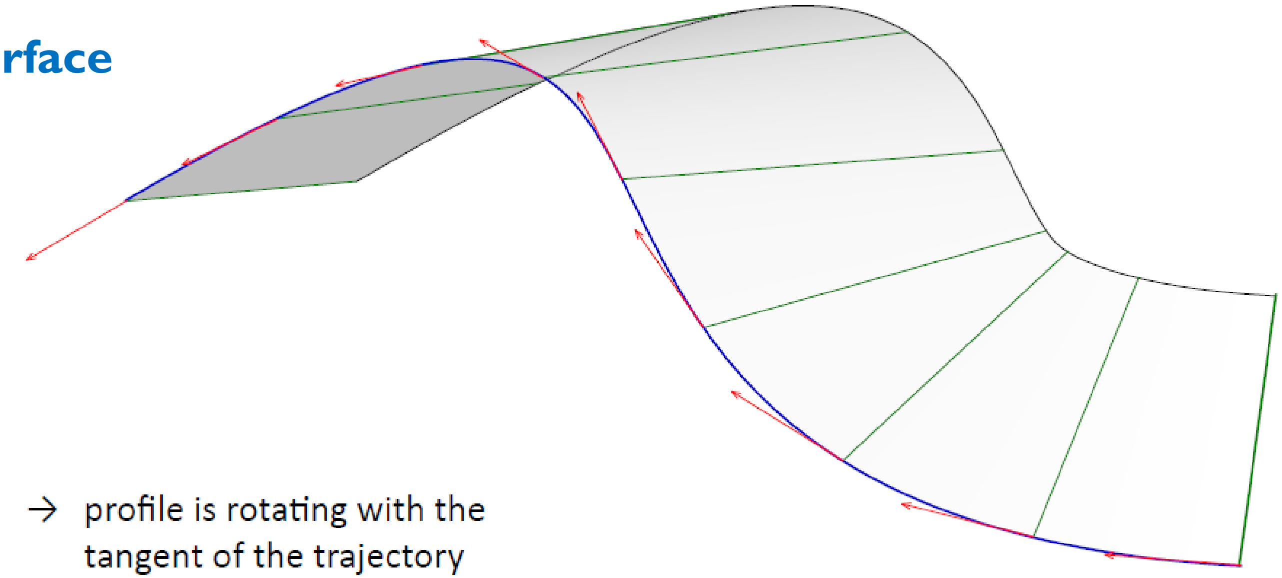
4- Surface of Revolution

Surface of revolution is generated by rotating a planar curve in space about an axis at a certain angle.



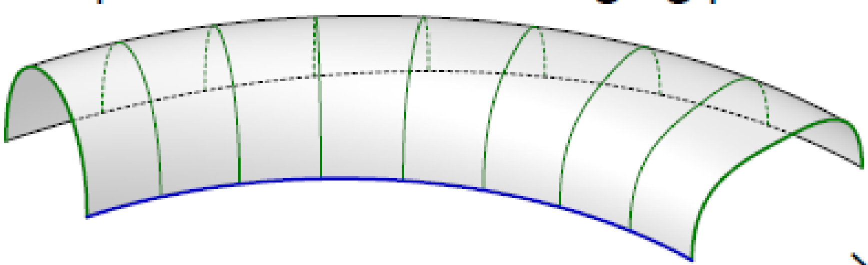

prolate ellipsoid of revolution
sphere
straight circular cylinder
oblate ellipsoid of revolution
anuloid (torus)
straight circular blunted cone
paraboloid of revolution
straight circular cone
split hyperboloid of revolution
single hyperboloid of revolution (revolving a hyperbola)
single hyperboloid of revolution (revolving a line)

Swept Surface

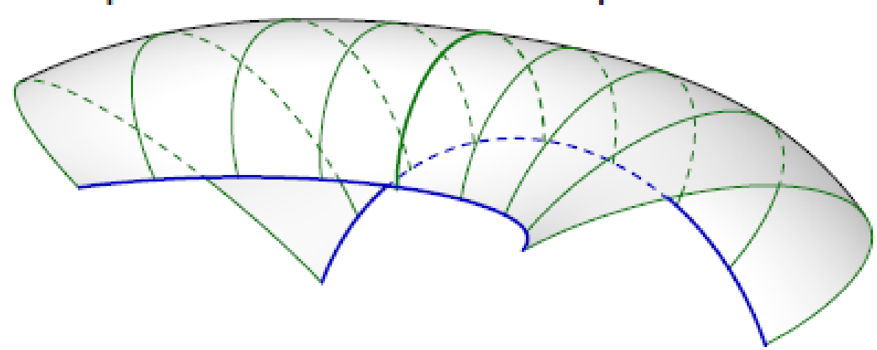


→ profile is rotating with the tangent of the trajectory

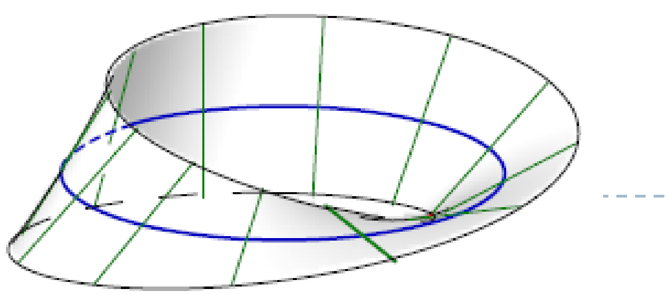
→ swept surfaces with changing profile



→ swept surfaces with two profiles

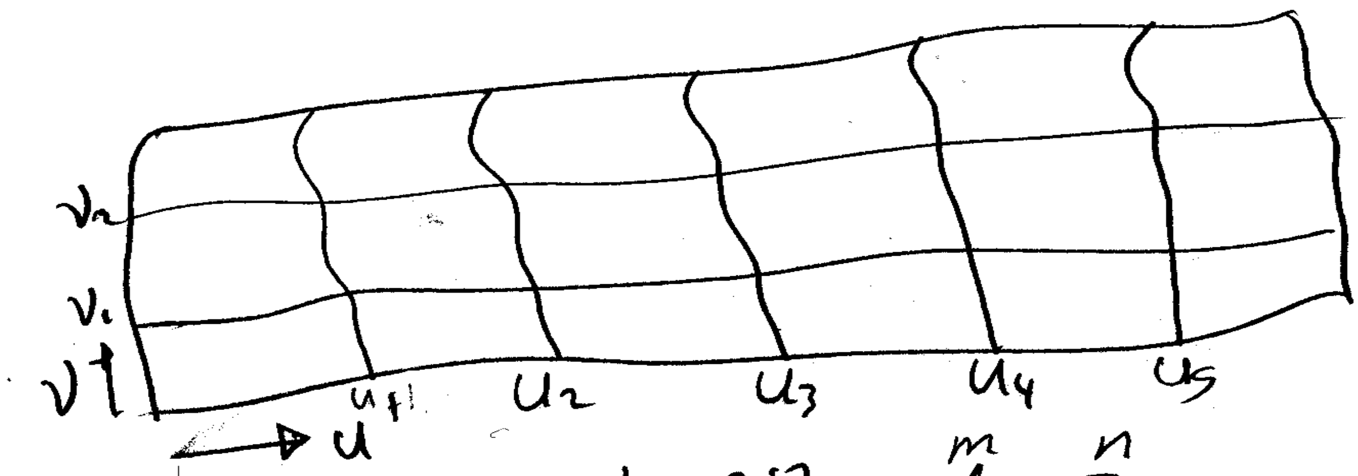


→ twisted surfaces



Mesh Generation

- Whenever the user requests the display of the surface with a **mesh size m x n**
  - ➢ The u range is divided equally into (m-1) divisions and m values of u are obtained.
  - ➢ The v range is divided equally into (n-1) divisions and n values of v are obtained.



mesh size $\overset{m}{6} \times \overset{n}{3}$

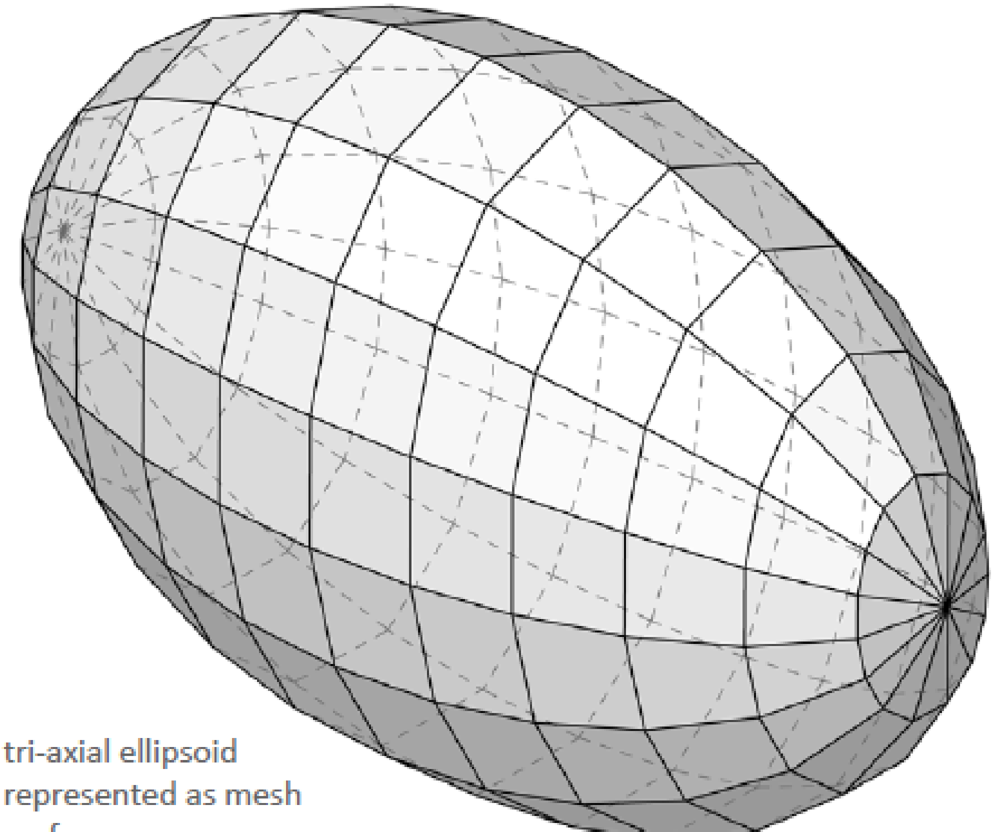
u divisions $= m - 1 = 5$

v divisions $= n - 1 = 2$

→ representation of surfaces by triangular or quadrangular small surfaces, given by mesh of vertices and edges

→ not exact, more accurate with more vertices

→ the best results are with changing size respectively to curvature



tri-axial ellipsoid represented as mesh surface

Synthetic Surface Representations

Hermite Bicubic Surface

As discussed before, synthetic curves are dealt with as curve segments in a single parameter (e.g. *s*) *domain.*
Likewise, synthetic surfaces are defined in **patches**, *each corresponding* to a **rectangular domain** in the *s* - *t space. Hermite Bicubic Surface is one of the common* types of synthetic surfaces used in CAD systems. In mathematic terms, a Hermite Bicubic surface can be described using the following cubic parametric equation,

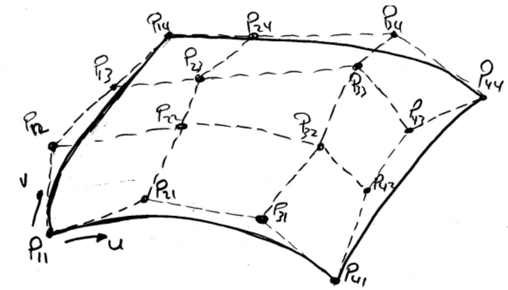
$$r = V(s,t)$$
$$= \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} s^i t^j, \qquad 0 \le s \le 1, \quad 0 \le t \le 1$$

Note that this is a 16-term, *third-power series*. Like Hermite bicubic curves, a Hermite surface also requires the values of the **tangent vectors** at the **corners** of the surface.

Bézier Surface Patches

Mathematically, the only difference between a Hermite surface patch and a Bézier surface patch is that *different basis functions* are used. As with the Bézier curve, the Bernstein basis function is used for the Bézier surface patch.

Generally, the most common use of Bézier surfaces is as **nets of bi-cubic patches**. The geometry of a single bi-cubic patch is thus completely defined by a set of **16 control points**. The cubic Bézier surface can then be expressed as,



$$r = V(s,t)$$
$$= \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} b_i^3(s) b_j^3(t), \qquad 0 \le s \le 1, \quad 0 \le t \le 1$$

where, $b_i^3(s) = \binom{3}{i} s^i (1-s)^{3-i}, \qquad b_j^3(t) = \binom{3}{i} t^j (1-t)^{3-j}$ are Bernstein polynomials.

Bézier patch meshes are superior to meshes of triangles as a representation of smooth surfaces, since they are much more **compact, easier to manipulate**, and have much better **continuity** properties. In addition, other common parametric surfaces such as spheres and cylinders can be well approximated by relatively small numbers of cubic Bézier patches. However, Bézier patch meshes are difficult to render directly. Another problem with Bézier patches is that ***calculating their intersections with lines is difficult***, making them awkward for pure ray tracing or other direct geometric techniques which do not use subdivision or successive approximation techniques. They are also *difficult to combine directly with perspective projection algorithms.*

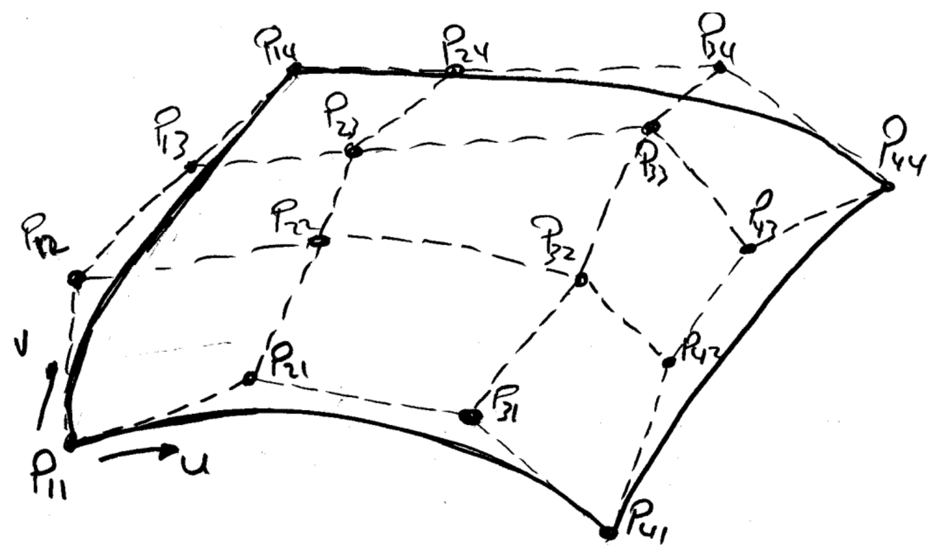
Uniform Cubic B-Spline Surfaces

Using a corresponding basis function, uniform cubic B-Spline surface can be formed and has a net of control points that define the surface, none of which interpolate the patch, as in the case of the B-spline curve. Likewise, an advantage of B-spline surface is that it *supports local control of the surface.*

Cubic B-splines with uniform *knot-vector* is the most commonly used form of B-spline. The blending function can easily be precalculated, and is equal for each segment in this case. Put in matrix-form, it is:

$$\mathbf{S}_i(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_{i-1} \\ \mathbf{p}_i \\ \mathbf{p}_{i+1} \\ \mathbf{p}_{i+2} \end{bmatrix}$$

for $t \in [0, 1]$.



Surface Manipulation

Various surface manipulation techniques are employed in CAD systems. The simplest and most widely used method is to display a surface by a mesh of curves. This is usually called *a mesh in the CAD software. By holding one parameter constant at a time, a mesh* of curves can be generated to represent the surface.

Shading of a surface is an effective way of rendering a design model and is available in many CAD systems.

Segmentation and **trimming** is a way of representing part of a surface with localised interests. Some surfaces can present computational difficulties when split and partitioned.

Similar to segmentation and trimming, **intersection** is another useful function where curves can be defined as a result of intersection.

Sometimes, **projection** is required by projecting an entity onto a plane or surface. When a curve or surface is projected, the point projections are performed repeatedly. This function is often used in determining shadows of entities.

As with the curve transformation, one can translate, rotate, mirror and scale a surface in most CAD systems.

To **transform** a surface, the control points of the surface are evaluated and then transformed to new positions and/or orientations. The new surface is then created according to the newly transformed control points.