Lecture 46
Bode Plots of Transfer Functions:II

A. Low Q Approximation for Two Poles

$$w_o$$

$$|\leftarrow\text{-----------}|\text{-----------}\rightarrow|$$

$$w_L = Q^{-1} w_o \qquad 2\pi f_o \qquad w_h = Q w_o$$

$$w_L \sim \frac{1}{RC} \qquad\qquad w_h \approx \frac{R}{L}$$

$$w_L \neq f(L) \qquad\qquad w_h \neq f(c)$$

B. Construction from T(s) Asymptotes the Actual T(s) Analytical Forms

1. Erickson'sProblems 8.2 and 8.3
2. Problem 8.5

C. Algebra on a Graph Graphically Combining Asymptotes to Find Z(total)

1. Series impedance's
2. Parallel Impedance's
3. Series / Parallel
4. Voltage Dividers: Division of Asymptotes

D. Measuring T(s) and Impedance

1. **Narrow Band Tracking Voltmeter**
2. **Methods for Measuring T(s)**
   a. **Overview** and Simple X_C Measurement
   b. **Choice of proper injection point for T(s) Measurements is artful**
   c. **Measure Z_{out} of a System**
   d. **Grounding Issues In Measurement**

Lecture 46
Bode Plots of Transfer Functions:II

A. Low Q Approximation for Two Pole T(s)

Often our two pole transfer functions have widely separated poles in frequency space allowing some nice approximate solutions to G(s).

Given a second-order denominator polynomial, of the form

$$G(s) = \frac{1}{1 + a_1 s + a_2 s^2} \qquad \text{or} \qquad G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

When the roots are real, i.e., when $Q < 0.5$, then we can factor the denominator, and construct the Bode diagram using the asymptotes for real poles. We would then use the following normalized form:

$$G(s) = \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

This is a particularly desirable approach when $Q << 0.5$, i.e., when the corner frequencies ω_1 and ω_2 are well separated.

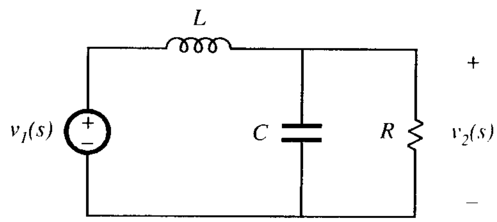
The T(s) function,

$$\frac{V_2(s)}{V_1(s)} = \frac{1}{1 + \frac{S}{Qw_o} + \left(\frac{S}{w_o}\right)^2}$$

, needs to be solved for the pole locations. From the pole locations we can sketch the Bode plot by inspection when the poles are far apart. When the poles are close a resonant response can be constructed from the asymptotic curves by employing a method called “Q peaking”.

R-L-C network example:

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$



Use quadratic formula to factor denominator. Corner frequencies are:

$$\omega_1, \omega_2 = \frac{L/R \pm \sqrt{\left(L/R\right)^2 - 4\,LC}}{2\,LC}$$

We can solve for the two roots.

$$\omega_1, \omega_2 = \frac{L/R \pm \sqrt{\left(L/R\right)^2 - 4\,LC}}{2\,LC}$$

This complicated expression yields little insight into how the corner frequencies ω_1 and ω_2 depend on R, L, and C.

When the corner frequencies are well separated in value, it can be shown that they are given by the much simpler (approximate) expressions

$$\omega_1 \approx \frac{R}{L}, \qquad \omega_2 \approx \frac{1}{RC}$$

ω_1 is then independent of C, and ω_2 is independent of L.

These simpler expressions can be derived via the Low-Q Approximation.

The roots can be put into the Q form as follows in order to better see how the poles change as Q of the circuit changes.

Given

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

Use quadratic formula to express corner frequencies ω_1 and ω_2 in terms of Q and ω_0 as:

$$\omega_1 = \frac{\omega_0}{Q}\,\frac{1 - \sqrt{1 - 4Q^2}}{2} \qquad\qquad \omega_2 = \frac{\omega_0}{Q}\,\frac{1 + \sqrt{1 - 4Q^2}}{2}$$

Can you see what occurs for low Q?

The high frequency pole of the two pole pair is.

$$\omega_2 = \frac{\omega_0}{Q}\,\frac{1+\sqrt{1-4Q^2}}{2}$$

can be written in the form

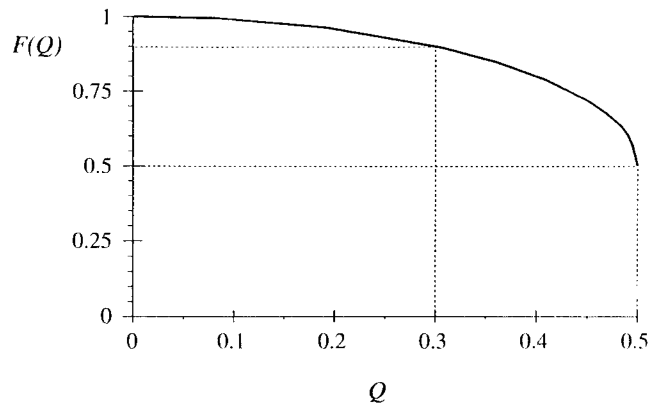
$$\omega_2 = \frac{\omega_0}{Q}\,F(Q)$$

where

$$F(Q) = \frac{1}{2}\left(1+\sqrt{1-4Q^2}\right)$$

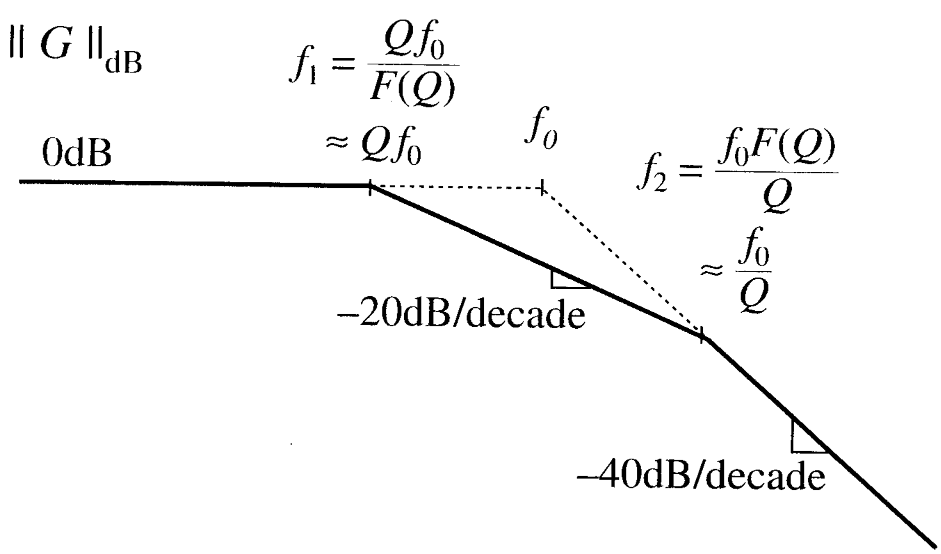
For small Q, $F(Q)$ tends to 1.
We then obtain

$$\omega_2 \approx \frac{\omega_0}{Q} \quad \text{for } Q \ll \frac{1}{2}$$



For Q < 0.3, the approximation $F(Q)$ = 1 is within 10% of the exact value.

The low frequency pole can be shown to be Qf_0. Both poles appear in the low Q approximation Bode plot as follows:



Note above, by knowing that Q is low and the value of f_0 we can rapidly draw the Bode plot of T(s). When the exact value of Q is specified so are the two pole locations. F_0 is easily known from L and C values.

As Q reduces below 1/2 the <u>roots separate</u> from their original position,both located at w_o , to widely separate and ultimately isolated.
For Q $\leq$ ½ we find:

$$w_{low} = \frac{Q\,w_o}{F(Q)}$$

$$w_{high} = \frac{w_o}{Q}$$

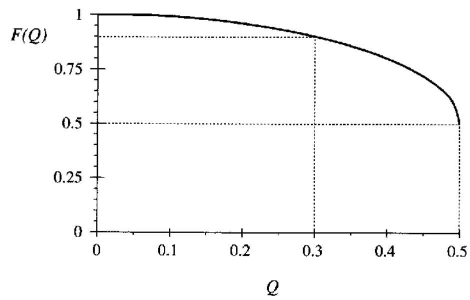


Fig. 8.24 $F(Q)$ vs. Q, as given by Eq. (8.72). The approximation $F(Q) \approx 1$ is within 10% of the exact value for $Q < 0.3$.

|←---------------|---------------→|

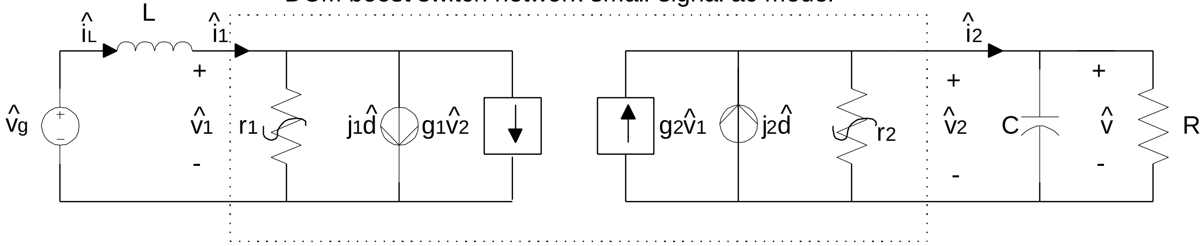
$w_L = Qw_o$ $\qquad\qquad$ w_o $\qquad\qquad$ $w_H = \frac{w_o}{Q} = \frac{R}{L}$

$\approx 1/RC$ $\qquad\qquad$ $w_H \neq f(c)$

$w_L \neq f(L)$	Notice as $L \rightarrow 0$
For very large C	this pole heads out
$w_L \rightarrow 0$	to high w

Future use of low Q approximation in analyzing two-pole transfer functions will occur. Specifically, the DCM mode of operation of the boost circuit in Chapter 10 Figure 10.17b, is reproduced below on page 6. Looking ahead we will find an equivalent circuit as shown below whose transfer function ,V_O/ d we need to solve. The very low Q approximation we have been playing with will be employed. That is we will end up the discussion with a live example. It may be hard to see because the transfer function is from the input duty cycle to the output voltage. We are not yet familiar with this concept.

DCM boost switch network small-signal ac model



Find $G_{vd}(s) = \dfrac{V_o(s)}{d(s)}\Big|_{V_g=0}$ $\Rightarrow$ L shorts the input

We will find the output voltage to duty cycle transfer function:

$$G_{vd}(s) = \frac{G_{vdo}(s - w_{z_1})}{1 + a_1 s + a_2 s^2}$$ with two poles and a RHP zero

For now forget about the right-half plane zero. In standard form the denominator will be: $1 + \dfrac{s}{Qw_o} + \left(\dfrac{s}{w_o}\right)^2$, $w_o = \dfrac{1}{\sqrt{LC}}$

We can rapidly sketch out the pole locations using the low Q

For the previous example:

$$G(s) = \frac{v_2(s)}{v_1(s)} = \frac{1}{1 + s\frac{L}{R} + s^2 LC}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}}$$

Use of the Low-Q Approximation leads to

$$\omega_1 \approx Q\,\omega_0 = R\sqrt{\frac{C}{L}}\,\frac{1}{\sqrt{LC}} = \frac{R}{L}$$

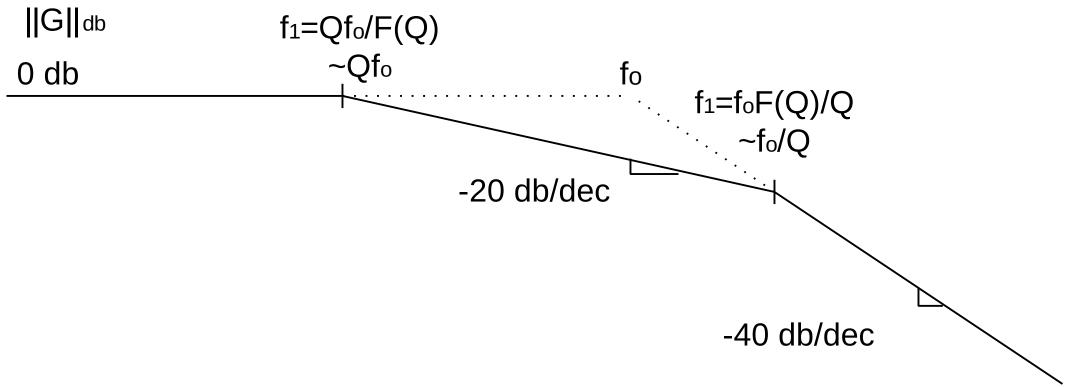
$$\omega_2 \approx \frac{\omega_0}{Q} = \frac{1}{\sqrt{LC}}\,\frac{1}{R\sqrt{\frac{C}{L}}} = \frac{1}{RC}$$

approximation method.

$w_{high} = \dfrac{R}{L} \rightarrow \infty$ or above the f_{sw} as L $\downarrow$

$w_{low} = \dfrac{1}{RC} \rightarrow 0$ for very high C

This allows us to see circuit conditions for which we go from two poles to a single pole. $\Rightarrow G_{vd}(s)$ has a <u>single pole</u> when L values are chosen very low, see Erickson Table 10.3 for other conditions.

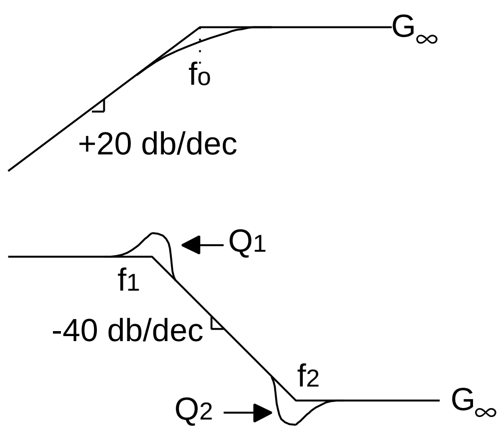


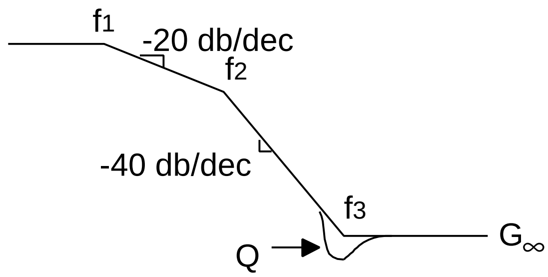
From T(s) plots we can estimate system stability, via the Nyquist criteria. **Explain the Nyquist criter**ia.

B. Construction from T(s) Asymptotes the Actual T(s) Analytical Forms

1. Given the three T(s) plots below

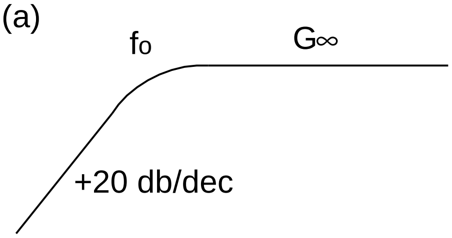
Problems 8.2 / 8.3 of Erickson. Find analytic expressions for the low f asymptote in terms of G_∞ and the break frequencies.

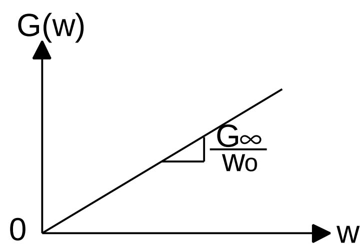




Solutions to Erickson Problems 8.2 and 8.3 follow. Given the Bode plots on the left find analytical expressions for low f asymptotes. Also get G(s) in factored pole zero form. First find $T(s)_{s \to 0}$ then $T(s) \to$ low frequency asymptote.

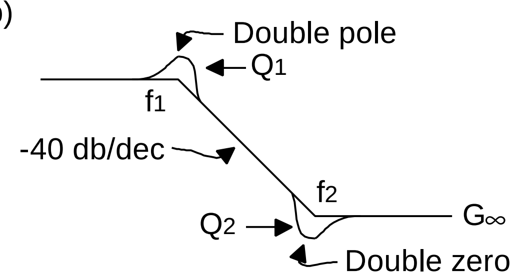
(a)

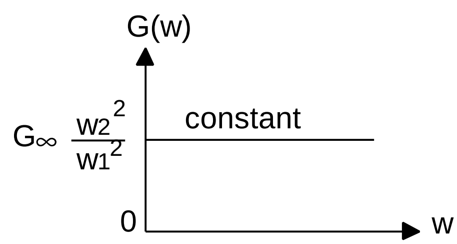




$$G(s) = \frac{G_\infty s}{s + w_0} \;\rightarrow\; \frac{G_\infty}{w_0} s \quad \text{as } s \rightarrow 0$$

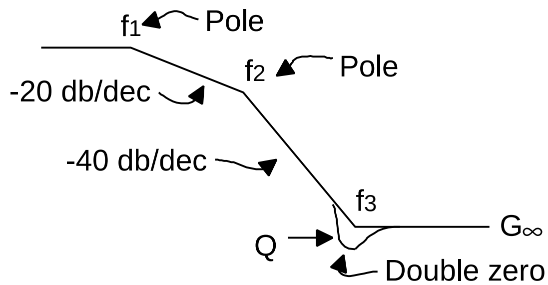
(b)

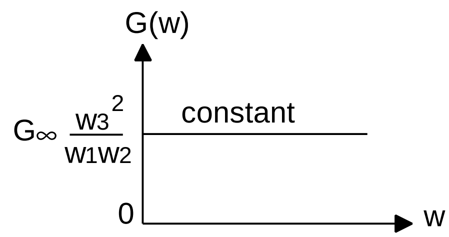




$$G(s) = G_\infty \frac{1 + \frac{w_2}{Q_2 s} + \left(\frac{w_2}{s}\right)^2}{1 + \frac{w_1}{Q_1 s} + \left(\frac{w_1}{s}\right)^2} \text{as } s \rightarrow 0 \;\rightarrow\; G_\infty \frac{w_2^{\,2}}{s^2} \cdot \frac{s^2}{w_1^{\,2}} = \frac{G_\infty w_2^{\,2}}{w_1^{\,2}} \quad s \rightarrow 0$$

(c)



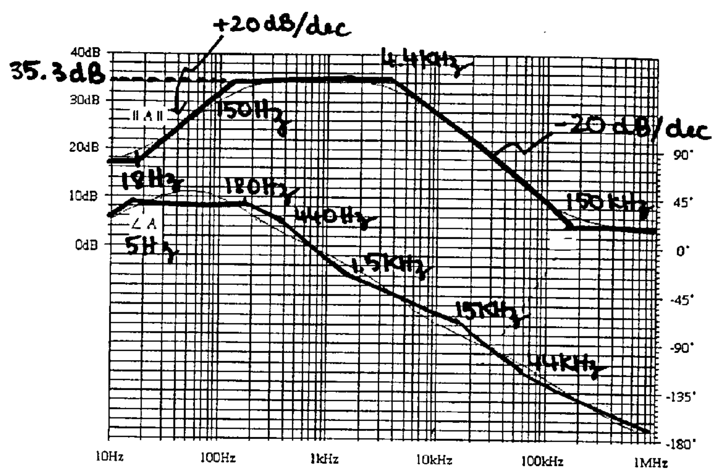


$$G(s) = G_\infty \frac{1 + \frac{w_3}{Q_3 s} + \left(\frac{w_3}{s}\right)^2}{\left(1 + \frac{w_1}{s}\right) + \left(1 + \frac{w_2}{s}\right)} \rightarrow G_\infty \frac{w_3^2}{s^2} \cdot \frac{s}{w_1} \cdot \frac{s}{w_2} = \frac{G_\infty w_3^2}{w_1\, w_2} \quad s \rightarrow 0$$

2. Erickson Problem 8.5

Given experimental data of Figure 8.56 find the <u>proper</u> asymptotes.

After some noodling,
Zero $\rightarrow f_1 = 180\text{Hz}$
pole $\rightarrow f_2 = 150\text{Hz}$
pole $\rightarrow f_3 = 4.4\text{KHz}$
RHP zero $\rightarrow f_4 =$ 150KHz
(since $|A| \uparrow$ and $\angle A \downarrow$

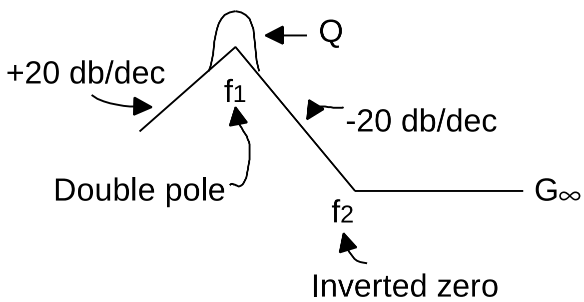


So F $A(s) = \frac{K(s + w_1)(s - w_4)}{(s + w_2)(s + w_3)}$ $\leftarrow$ Right Half Plane zero pushes

gain up vs. f, Phase down

From graph, K = 35.3 dB $\approx$ 58.3

3. Given T(s) Plot, Find the analytical form in the standard format with poles and zeros. This is Erickson problem 8.1 for plot(c) found on page 315 of the text.

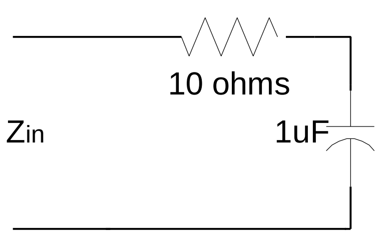


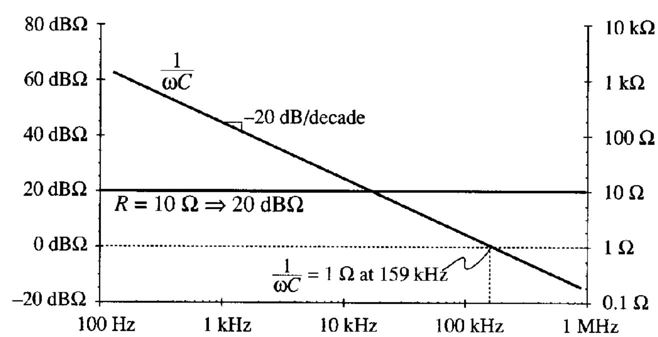
$$G(s) = G_{\infty} \frac{\left(1 + \frac{w_2}{s}\right)}{1 + \frac{s}{Q_1 w_1} + \left(\frac{s}{w_1}\right)^2}$$

C. Algebra on a Graph Graphically Adding Asymptotes to calculate Z(total)

1. Combining Series R-C Elements

Plot each Z(f) part separately first





8.32 Impedance magnitudes of the individual elements in the network of Fig. 8.31.

$$f_{crossover} = \frac{1}{2pRC}$$

"Algebra on a graph"
For Z(total) in series always "Take the bigger of the two"

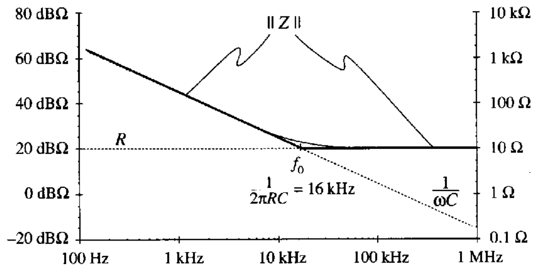
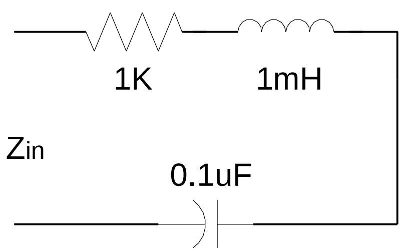


Fig. 8.33 Construction of the composite asymptotes of ‖Z‖. The asymptotes of the series combination can b approximated by simply selecting the larger of the individual resistor and capacitor asymptotes.

2. Series R - L - C

Plot each, X(f) ,individually. Then for a series connection always take the largest value for Z(total)=Z(f).



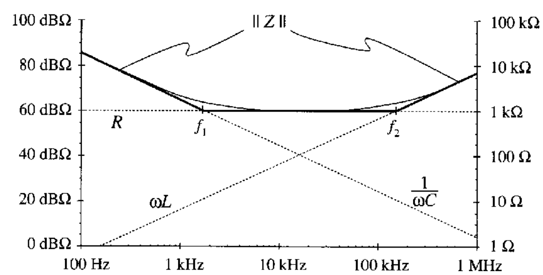


Fig. 8.35 Graphical construction of ‖Z‖ of the series R–L–C network of Fig. 8.34, for the element values specified by Eq. (8.139).

Plotted on the left is the case when the resistance, R, is larger than the value of R_0 = wL= 1/wC. What if R lies <u>below</u> , R_0, the crossing of wL and 1/wc? Then we take a "nose dive" from the reactive Z asymptotes to reach R and observe Q peaking in the impedance plot at ω_0. Clearly Z_{in} at resonance is R because @ f_o + jwL cancels -j/wC. Both are equal in magnitude.

$$R_o = \left|\frac{1}{wC}\right| = |wL|$$, which occurs only at ω_0

The relative value or R compared to R_0 is then the key to determining whether or not we see Q peaking in the Z(f) plots. The case of Q peaking for the series R-L-C is seen on page 12, while the non-peaking plots are shown above.

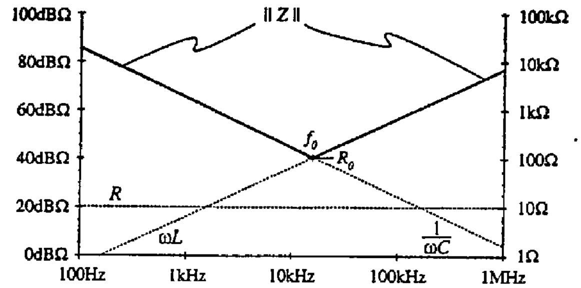


Fig. 8.36. Graphical construction of impedance asymptotes for the series *R-L-C* network example, with *R* decreased to 10Ω.

For Z(total) take the largest except just at resonance when R is less than $R_o = wL = \dfrac{1}{wC}$

Note: $|Q| = |R_o / R|$ and is plotted symmetrically.

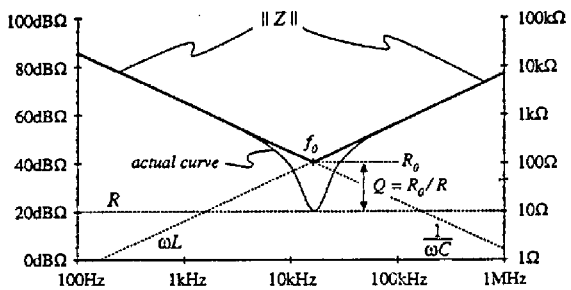


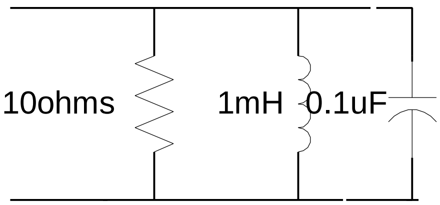
Fig. 8.37. Actual impedance magnitude (solid line) for the series *R-L-C* example. The inductor and capacitor impedances cancel out at $f = f_0$, and hence $Z(j\omega_0) = R$.

What if $R > R_o$? Return to top of page 11 and you are done.

If we combine series X(f) plots using the largest value for the total impedance, what do you think would be the rule for combining parallel X(f) plots?

3. Combining Parallel Impedance's

Plot each X(f) separately and then combine for Z(total)



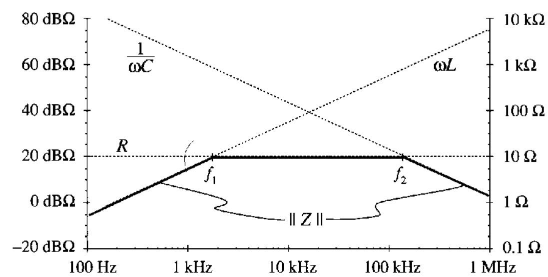


Fig. 8.39 Construction of the composite asymptotes of $\|Z\|$, for the parallel R–L–C example. The asymptotes of the parallel combination can be approximated by simply selecting the smallest of the individual resistor, inductor, and capacitor asymptotes.

"Take the smaller of the three" for Z(total) in parallel

What if $R > |1/w_c| = |w_L| = R_o$

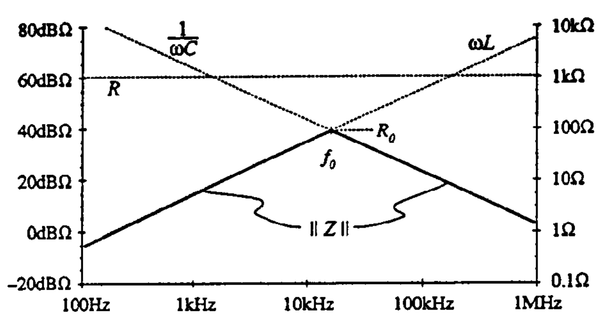


Fig. 8.40. Graphical construction of impedance asymptotes for the parallel *R-L-C* example, with *R* increased to 1kΩ.

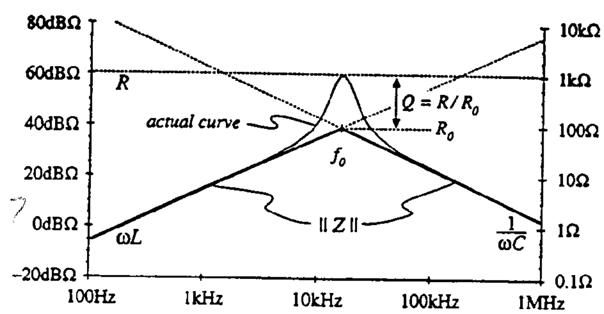
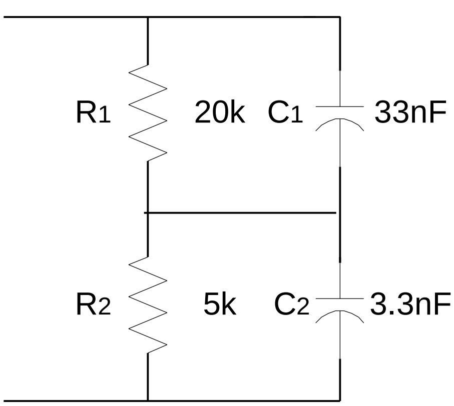
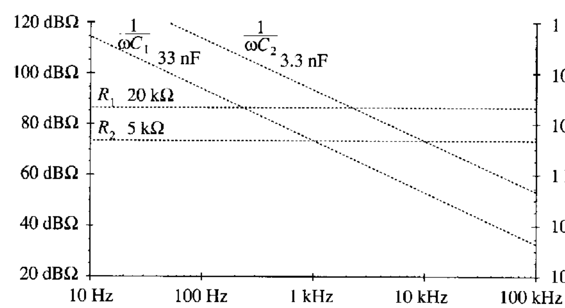


Fig. 8.41. Actual impedance magnitude (solid line) for the parallel *R-L-C* example. The inductor and capacitor impedances cancel out at $f = f_0$, and hence $Z(j\omega_0) = R$.

4. Combining Series / Parallel Combinations

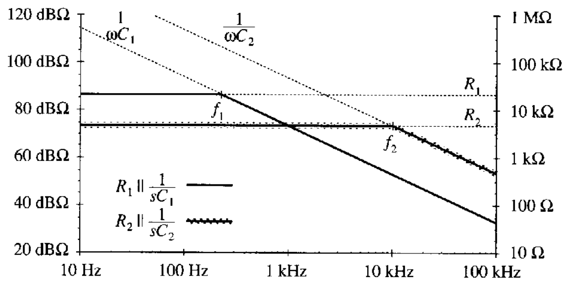




8.43 Impedance magnitudes of the individual elements in the network of Fig. 8.42.

First do $\frac{1}{wC_1}||R_1$ and for Z(total) in parallel “Take the smaller of the two”

Next do $\frac{1}{wC_2}||R_2$ and for Z(total) in parallel “Take the smaller of the two”



8.44 Graphical construction of asymptotes for parallel combinations (solid and shaded lines).

R_1 to $\frac{1}{wC_1}$ is one curve

R_2 to $\frac{1}{wC_2}$ is the 2nd curve

Add these two curves in series

For series combinations Z(total) = the higher of the two

f_1 is one curve f_2 is 2nd curve “Take the higher of the two”

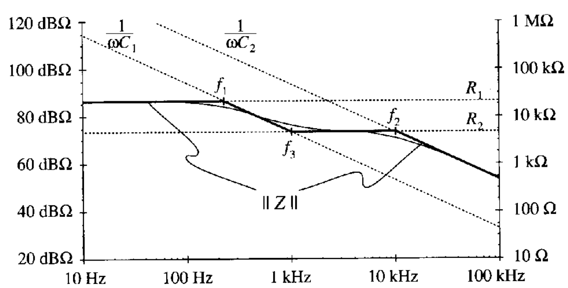


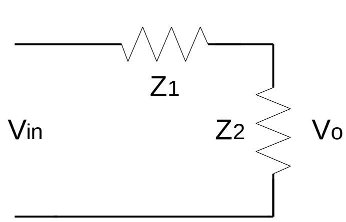
Fig. 8.45 Graphical construction of composite asymptotes for $\|Z\|$.

Note in the middle a new frequency at the crossing

$f_3 = \frac{1}{2p R_2 C_1}$ since its

“R_2"|| $\frac{1}{wC_1}$

5. Voltage Dividers : Division of Asymptotes



$$\frac{V_o(s)}{V_{in}(s)} = \frac{z_2}{z_1 + z_2} + \frac{z_2}{z_{in}}$$

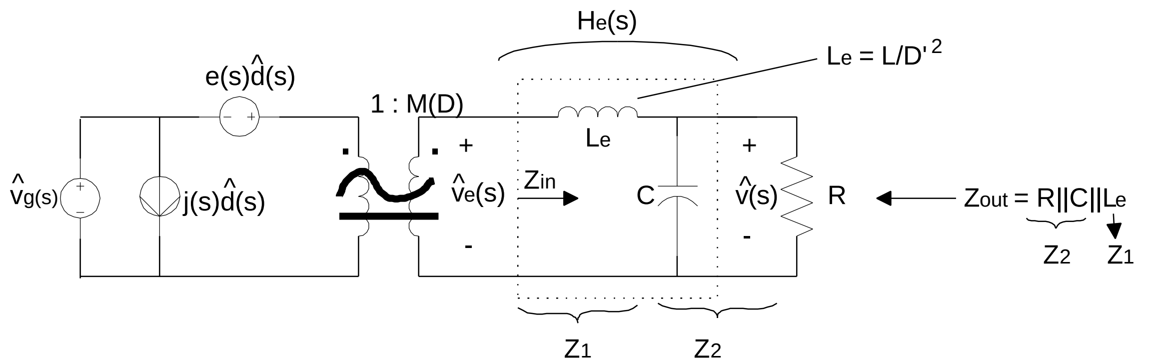
z_{in} the is series combo seen at V_{in}

In some other cases we may already know Z_{out} (looking in from

V_o) so we want to express the transfer function differently:

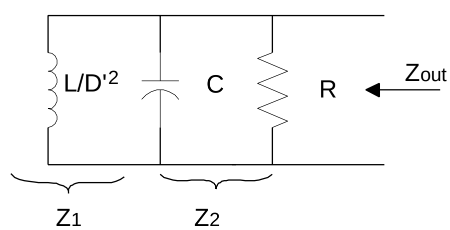
$$\frac{V_o(s)}{V_{in}(s)} = \frac{Z_2 Z_1}{Z_1 + Z_2}\frac{1}{Z_1} = \frac{Z_{out}}{z_1}$$

Sometimes it's easier to do a Series Combo or a Parallel Combo
Consider the two pole circuit model below where $L_{effective} = L/(D')^2$ as occurs in the boost or buck-boost circuit model. That is, the duty cycle choice effects the effective inductance see by the small signal model. For small values of duty cycle, D', to achieve DC operation goals L appears bigger in the dynamic model than its circuit value. By equations alone it is sometimes difficult to see these duty cycle ,"D", design choice effects" on ac models.



Algebra on the graph however provides a visualization aid.

From the secondary of the transformer to the output.



$$\frac{V_o(s)}{V_e(s)} = \frac{z_2}{z_1 + z_2} = \frac{z_2}{z_{in}} = \frac{z_{out}}{z_1}$$

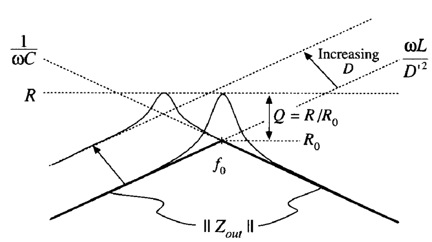
Now lets plot versus frequency

$\frac{Z_{out}}{z_1}$ via algebra on a graph

$\Rightarrow$

$\uparrow$

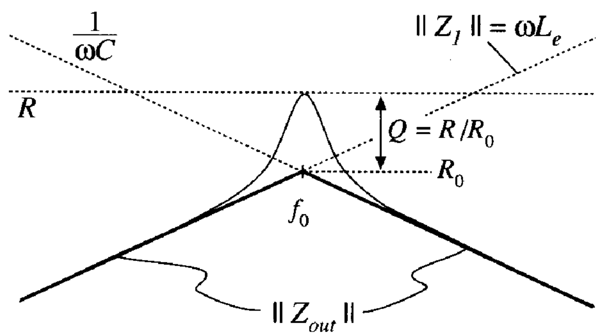
$\frac{wL}{(D')^2}$



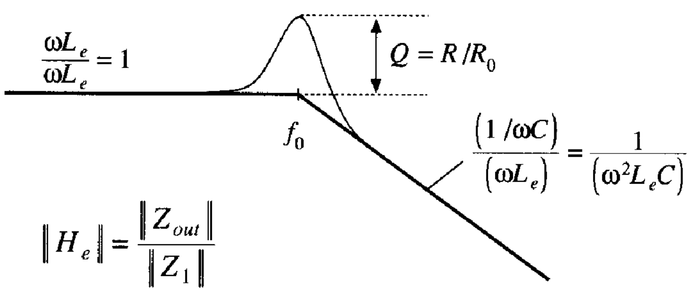
Z_{out} is the smallest of three asymptotes because it represents all three elements in parallel. Z_1 is just an increasing positive slope but it is offset by the duty cycle,D', choice. For D ↑ D' ↓ just shift the wL/ $(D')^2$ asymptote as shown upwards toward the left.

$\frac{Z_{out}}{Z_1}$ ratio for $w < w_o$ is clearly unity but for $w > w_o$ what is this ratio?

$$\frac{z_{out}(w < w_o)}{z_1(w > w_o)} = \frac{\frac{1}{wC}}{wL_c} = \frac{1}{w^2 L_c C}$$ } Sharp 40 db/decade slope



Z_{out} and Z_1 plotted together versus f



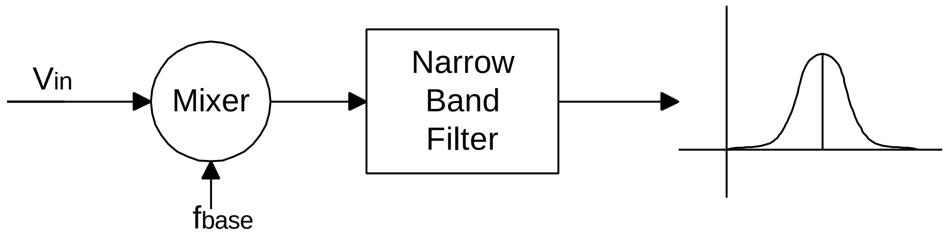
Z_{out}/Z_1 plotted versus f

Section C, I hope, illustrates the value of algebra on a graph to **better visualize changes in transfer functions**:

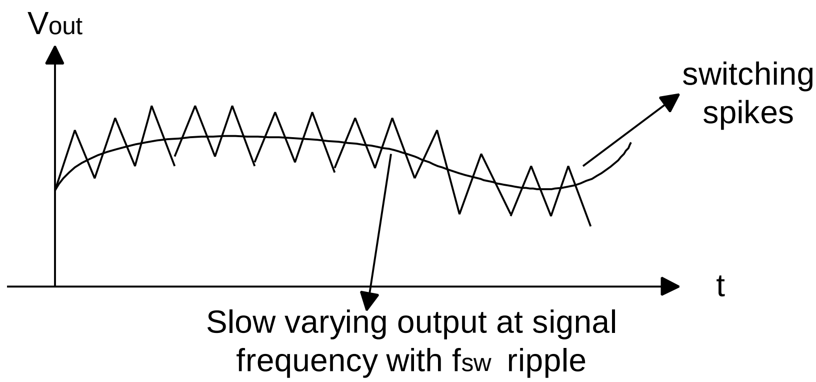
* Effects of varying quiescent point via duty cycle, D ,which in turn changes the dynamic model.
* Effect of changing element values like C, L, or R on the transfer functions

D. Measuring Transfer Functions and Impedance's

1. Narrow Band Tracking Voltmeter



Useful tool for PWM Converters where signals are typically complex with various frequency components present.



We will need to make Z(s) and T(s) measurements versus f.

Frequency Response Measurements

Why Measure?

- Models often overlook important effects
- Models are often missing for important technologies
- Component parasitics frequently dominate response
- Measurement and model almost always disagree first time

Why Teach?

- Most power electronics students are poorly prepared for real circuit design
- Many products are inadequately designed for control
- University models are often rendered unusable since they are not confirmed by lab data, and are often contradicted.
- Conferences, industry do not fill this role.
- If time is short, measurement is far more critical than modeling

Frequency Response Measurements

Equipment needed
Control measurements
Signal Injection
Grounding
Technique
Impedance measurements
Examples

The required test equipment for T(s) and Z(s) measurements includes:

Essential Laboratory Equipment

Network Analyzer

Outputs test signal, measures ratio (gain and phase) of 2 test signals at the same frequency as test signal.

0.1 Hz to 10 MHz operation
Narrowband receiver selectivity to 1 Hz
Good output signal (1 V or more)
Automatic sweep (save time)
Automatic averaging (better noise rejection)

HP4194A ($30k)
HP4195 ($35k +)
Used HP ($15k +)

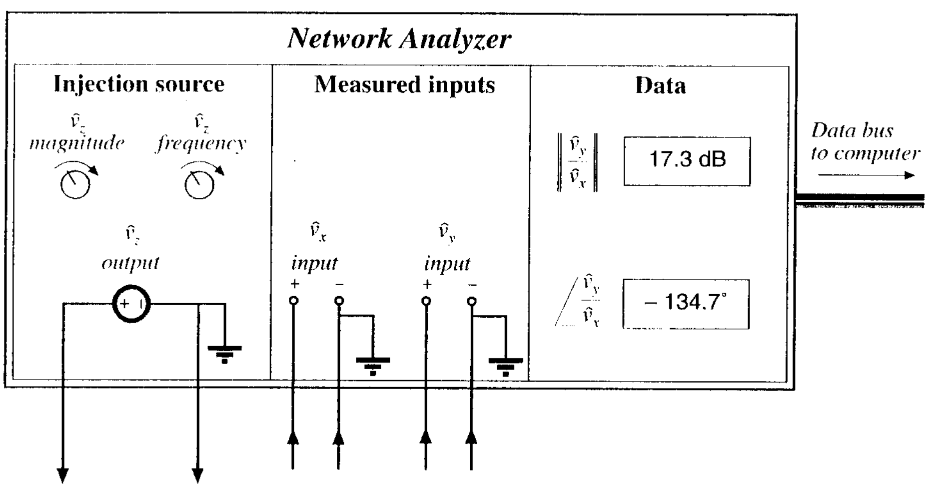
AP Instruments ($7k)

Signal Generator and scope ($500) (manual, low noise only)

Accessories

Injection devices (isolated)
Reference impedances
Inpedance accessories

The front panel of a network analyzer looks like:



The Z(s) or T(s) data is in db and angles. We also have a probe signal $V_Z(f)$ available.

2. Measuring T(s) Overview and Simple X_C Measurement

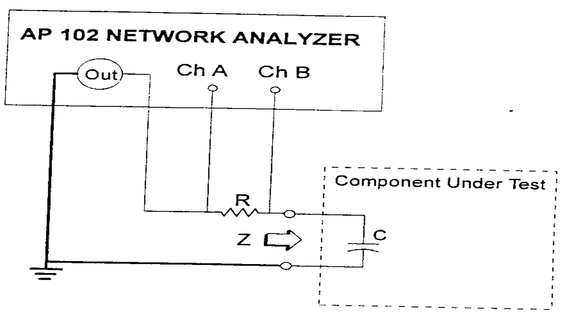
- Injection source produces sinusoid $\hat{v}_z$ of controllable amplitude and frequency
- Signal inputs $\hat{v}_x$ and $\hat{v}_y$ perform function of narrowband tracking voltmeter:

  Component of input at injection source frequency is measured

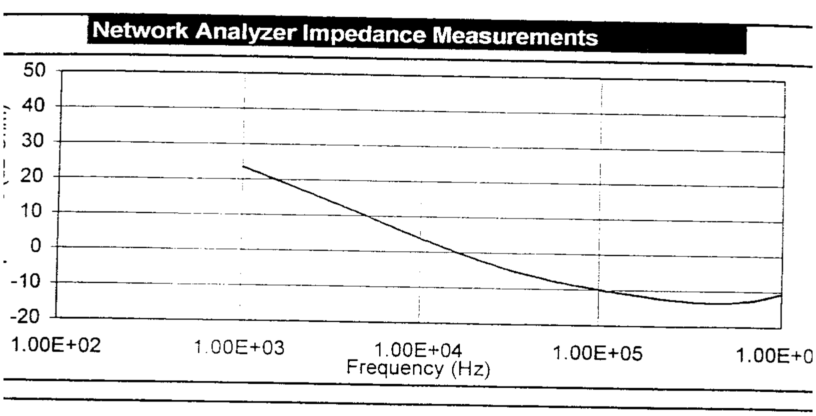
  Narrowband function is essential: switching harmonics and other noise components are removed
- Network analyzer measures

$$\left\| \frac{\hat{v}_y}{\hat{v}_x} \right\| \quad \text{and} \quad \angle \frac{\hat{v}_y}{\hat{v}_x}$$

Below we measure X(f) for a Capacitor.



For a tantalum capacitor we find $X_C(f)$:

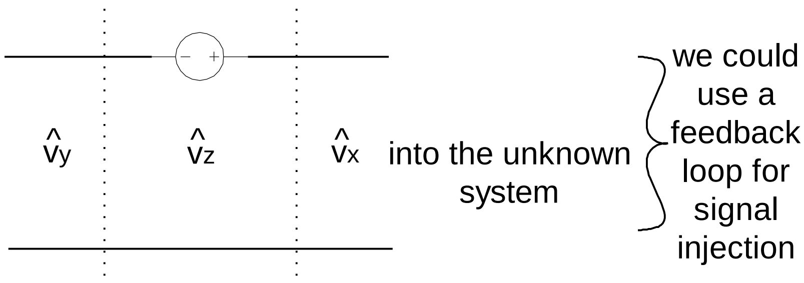


Tantalum Capacitor			
Measured Frequency	Value F	Real Ohm	Im Ohm
100000 Hz	9.50E-06	2.77E-01	1.75E-01

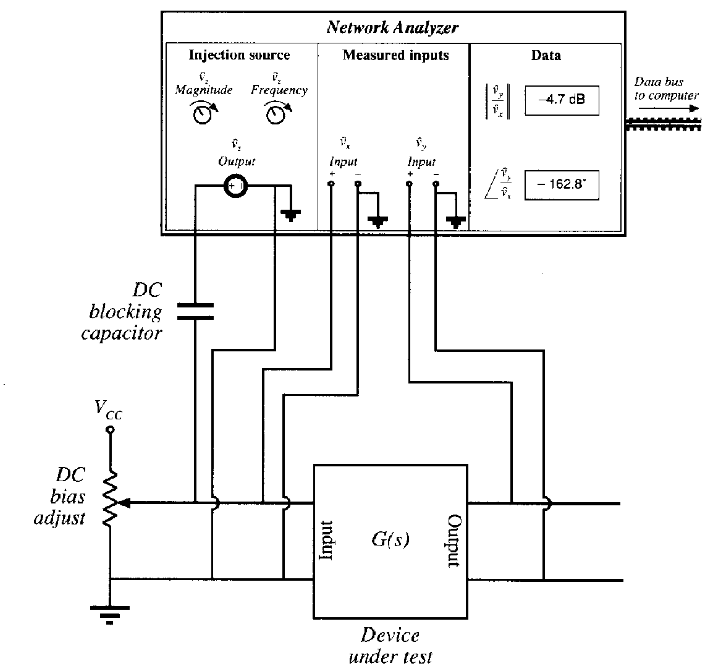
C values and ESR values are both found.

a. Choice of proper injection point for T(s) Measurements is artful

We use the signal frequency to <u>inject an input</u> into a system to measure the transfer function, T(s). To avoid undesired loading when we break a feedback loop, $V_z(f)$ is injected and V_x/V_y is measured versus f(applied) as shown below schematically.



V_y is the return signal from the unknown system. The above measurement scheme is for a feedback loop analysis where we need to inject a signal to measure the loop gain.



⇒ Plots of V_o/V_{in} amplitude versus f and $\angle V_o/V_{in}$ phase versus f can both be measured

The DC bias shown should simulate actual operating conditions before we broke the loop for signal injection, so it will have no effect on the measurement $V_o(s)/V_{in}(s)$

c. Measure Z_{out}

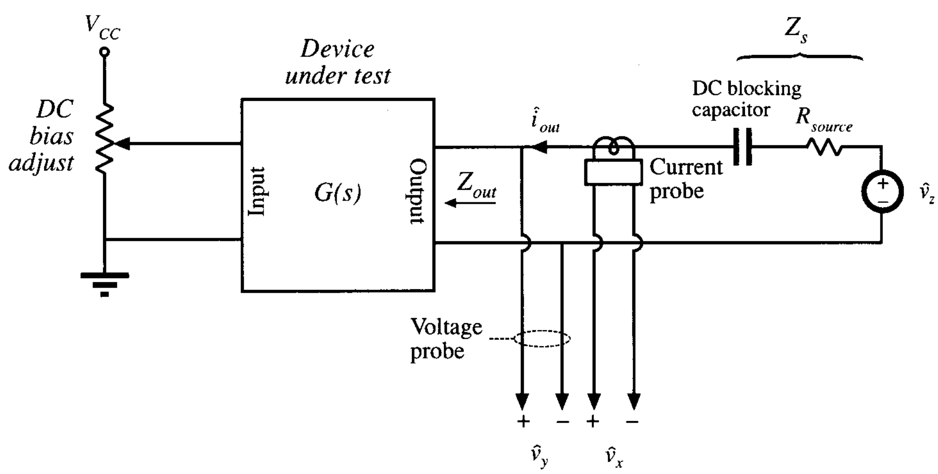
- Treat output impedance as transfer function from output current to output voltage:

$$Z(s) = \frac{\hat{v}(s)}{\hat{i}(s)} \qquad\qquad Z_{out}(s) = \frac{\hat{v}_y(s)}{\hat{i}_{out}(s)}\Bigg|_{\substack{amplifier\\ ac\ input}=0}$$

- Potentiometer at device input port establishes correct quiescent operating point
- Current probe produces voltage proportional to current; this voltage is connected to network analyzer channel $\hat{v}_x$
- Network analyzer result must be multiplied by appropriate factor, to account for scale factors of current and voltage probes

$$Z_o(s) = \frac{V_o(s)}{i_o(s)}$$

V_x from i_o , V_y from V_o $\quad Z_o(s) = \dfrac{V_o(s)}{i_o(s)} = \dfrac{V_y(s)}{V_x(s)}$



The test set-up requires us to simulate the input impedance that normally drives the circuit to accurately measure Z_{OUT}.

The DC bias simulates V_o of prior stage into input. Injection of V_z via Z_s, composed of R_s & 1/wC, creates a current into Z_o. AC input to the device must be zero.

Value of $Z_S = (R + \frac{1}{jwC})$ and amplitude of V_z do not effect Z_{out} measurement

d. Grounding Issues

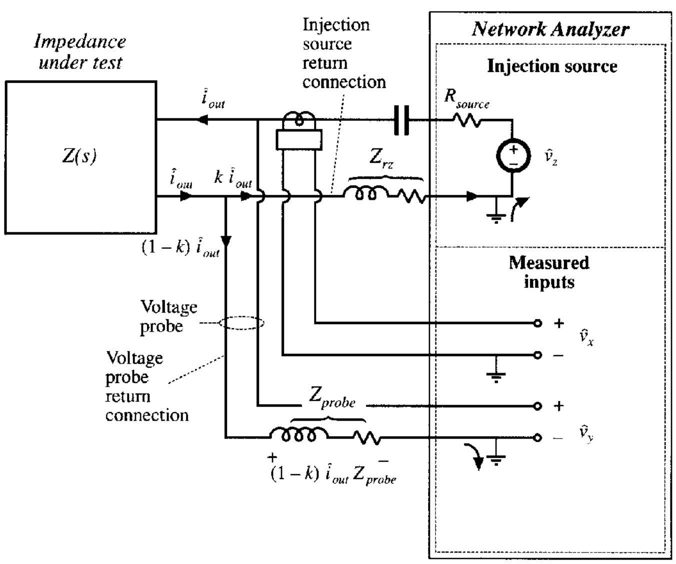
1. Statement of the problem

i_{out} above the injected into Z_o at the top terminal in the figure above comes back towards - V_z along the bottom return path. The current return can choose either the -V_y path or the -V_z path when it exits the G(s) at the bottom terminal.

i_{out} splits < ki_o to - V_z ground

< $(1 - k)i_o$ to - V_y ground

But the sum of both paths is I_o.



K depends on the relative impedance's of the two paths

Z_{probe} into - V_y gnd

Z_{rz} into - V_z gnd

Z is actual impedance of the sample

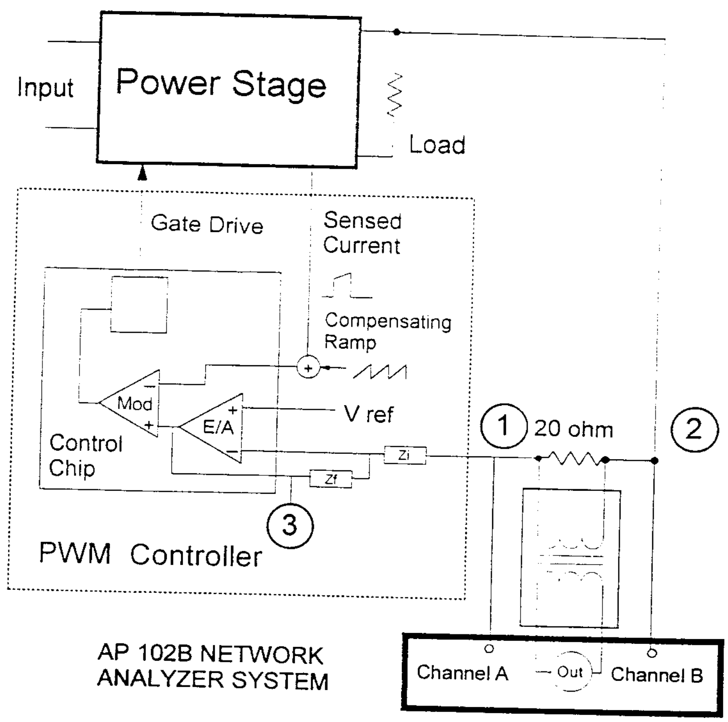
Z_o measured is: $Z_o = Z + Z_{probe}(\text{into} - V_y) || Z_{rc}$. This situation

sets a lower limit on the smallest Z you can accurately measure.
$Z >> Z_{probe}||Z_{rc}$
Easy solution for low Z: Buffer V_z with a transformer

Finally, For HW#3 Due in 1 week:

1. Answer any Questions asked throughout lectures 45-46.
2. Chapter 8 of Erickson do Problems 6 and 10.
3. Explain the measurement below in as much detail as possible.

Measuring a Power Supply Loop Gain



Put Channel B at 2, and Channel A at 1 to measure loop gain

Put Channel B at 2, and Channel A at 3 to measure power stage control-to-output gain

Put Channel B at 3, and Channel A at 1 to measure compensator gain